ISET Journal of Earthquake Technology, Paper No. 603, Vol. 62, No. 4, December 2025, pp. 193-210

PROBABILISTIC STABILITY ANALYSIS OF REINFORCED SOIL RETAINING STRUCTURES UNDER STATIC AND SEISMIC CONDITIONS

A. Pain*
Scientist F, Geotechnical Engineering and Geo-Hazards Group
CSIR-Central Building Research Institute, Roorkee (India) - 247667, Email id: apain.cbri@csir.res.in

E. Agarwal
Postdoctoral Fellow, Department of Civil and Environmental Engineering
University of Alberta (Canada) - T6G 1H9, Email id: ekansh1294@gmail.com

ABSTRACT

This keynote explores the growing need for probabilistic methods in assessing the stability of geosynthetic-reinforced soil retaining structures, under seismic and uncertain field conditions. While traditional deterministic approaches remain widely used, they often overlook the natural variability in soil properties, environmental effects like partial saturation, and complex seismic forces. Probabilistic analysis offers a more realistic way to account for these uncertainties and make safer, better-informed design decisions. The keynote brings together insights from recent research, including surrogate-based modeling techniques like the stochastic response surface method and fourth-moment normal transformation, which help balance accuracy and efficiency in high-dimensional or nonlinear problems. Case studies, such as the Northridge earthquake, demonstrate the practical use of these tools in real-world situations. The keynote also reflects on challenges in choosing parameters like the coefficient of variation, the role of damping and frequency in seismic stability, and the practical limitations of software and field data. Looking ahead, it suggests pathways for integrating probabilistic tools into everyday engineering practice such as combining machine learning with physics-based models, developing partial safety factors from rigorous simulations, and creating open-source toolkits for field use. Making these tools accessible and practical is key to improving the reliability and safety of reinforced soil structures amid real-world uncertainties.

KEYWORDS: Geosynthetic-Reinforced Soil Retaining Structures, Uncertainty Quantification, Reliability-Based Geotechnical Design, Seismic Slope Stability Analysis, Surrogate Modeling in Geomechanics

INTRODUCTION

The design and stability assessment of geosynthetic-reinforced soil retaining structures (GRSRS) remains a complex challenge in geotechnical engineering due to the inherent variability and uncertainty associated with soil properties, reinforcement materials, and loading conditions, particularly under seismic events. These uncertainties are not only a result of spatial heterogeneity and limited subsurface investigations but also arise from simplifications in modeling, assumptions in analytical methods, and potential construction errors.

Traditional design approaches predominantly rely on deterministic methods, where soil parameters such as cohesion (c), internal friction angle (ϕ), and unit weight (γ) are treated as fixed values, typically the mean or a conservative estimate based on limited site investigation data. Likewise, reinforcement properties such as tensile strength and stiffness are considered uniform, despite variations that may arise from manufacturing inconsistencies and environmental degradation. Under seismic loading, the situation becomes more critical, as the response of retaining systems depends on both the amplitude and frequency content of ground motion, which are inherently unpredictable. Deterministic analyses such as the limit equilibrium method (LEM), finite element method (FEM), and finite difference method (FDM) have been widely used to evaluate slope and wall stability (Griffiths & Lane, 1999; Duncan & Wright, 2005). While these methods provide a factor of safety (FoS) as a measure of structural adequacy, they fail to account for the variability of input parameters. As shown by Griffiths and Fenton (2004), two slopes with identical

https://doi.org/10.63898/HVIB5938
*Corresponding Author

FoS values can exhibit vastly different probabilities of failure when variability in soil properties is taken into account. This highlights a critical limitation of deterministic design, its inability to quantify the actual risk of failure. In contrast, probabilistic methods provide a rigorous framework for quantifying the uncertainty associated with geotechnical design. These methods model input parameters as random variables or spatially varying fields and evaluate the probability of failure (P_f) or the reliability index (β) through analytical or simulation-based techniques. Early developments in this domain include the first order second-moment (FOSM) method and the first-order reliability method (FORM), which offer computational efficiency but are limited in handling highly nonlinear problems. Monte Carlo Simulation (MCS), though more general, often requires thousands of model evaluations to achieve statistical convergence, making it computationally intensive (Harr, 1987; Baecher & Christian, 2003). Recent research has addressed these challenges through the use of surrogate models, such as response surface methods (RSM), support vector machines, and multivariate adaptive regression splines (MARS), which approximate the performance function and significantly reduce computational demands (Fenton & Griffiths, 2008; Cho, 2010). Other developments, such as the stochastic response surface method (SRSM) and transformation techniques like the Fourth-Moment Normal Transformation (FMNT), have further improved the efficiency and applicability of probabilistic slope stability analysis (Luo et al., 2016; Agarwal & Pain, 2022).

Despite these advances, the adoption of probabilistic approaches in routine geotechnical practice remains limited. The reasons are multifold: lack of standardized guidelines, perceived computational complexity, limited understanding of probabilistic metrics among practitioners, and insufficient integration of probabilistic tools within commercial design software (El-Ramly et al., 2002). These barriers highlight the need for research that not only advances methodology but also emphasizes clarity, usability, and relevance to practical engineering problems. This keynote aims to contribute toward that goal through the following objectives:

1. To provide a structured synthesis of the literature on the probabilistic stability analysis of reinforced soil retaining structures, with particular emphasis on developments over the past 15 years.
2. To summarize and contextualize our contributions, which span rigorous deterministic formulations, surrogate-based probabilistic frameworks, and advanced numerical techniques tailored for seismic conditions.
3. To demonstrate the practical utility of probabilistic analysis through a real-world case study involving seismic loading, illustrating that conventional design based on FoS may overlook significant risks that are more accurately captured using probabilistic methods.

To set the foundation for this discussion, we begin with a comprehensive review of key studies in probabilistic slope stability analysis, categorized by methodology and application context. We then present our own contributions, including models based on the pseudo-static (PS) and modified pseudo-dynamic (MPD) frameworks, surrogate-assisted probabilistic simulations, and efficiency-optimized algorithms for conditions where data are sparse, or distributions are unknown. Finally, we provide key reflections, practical guidance, and future directions, emphasizing the growing role of reliability-based methods in modern geotechnical engineering practice.

BRIEF LITERATURE REVIEW

Table 1 offers a summary of selected studies from the past 15 years on the probabilistic analysis of slope stability. The purpose of this review is not to include every paper in the field but to identify key developments and shifts in direction over time. The chosen studies reflect representative trends from well-regarded journals and conferences, giving readers a sense of evolution of field, both in terms of methodology and application.

Table 1: Thematic literature review of probabilistic slope stability (2008–2025)

S No.	Study	Methodology	Seismic	Backfill
1	Sayed et al. (2008a)	RFEM via FEM	No	Dry
2	Sayed et al. (2008b)	FOSM, PEM, FORM	Yes	Dry
3	Basha & Babu (2010a, b)	Reliability-based design optimization (RBDO)	Yes	Dry
4	Cho (2010)	MCS + random LEM	No	Saturated

5	Wang et al. (2010)	Subset simulation	No	Dry
6	Zevgolis & Bourdeau (2010a)	MCS + response surface	Yes	Dry
7	Zevgolis & Bourdeau (2010b)	MCS	No	Dry
8	Basha & Babu (2012)	RBDO framework	Yes	Dry
9	Johari et al. (2013)	Joint random variables	No	Dry
10	Otálvaro & Cordão-Neto (2013)	FEM / LEM / FOSM	No	Partially saturated
11	Javankhoshdel et al. (2016)	RFEM + RLEM	No	Dry
12	Jiang & Huang (2016)	RSM + subset	No	Dry
13	Lin et al. (2016)	FLAC-based RSM	No	Dry
14	Luo et al. (2016)	RFEM	No	Dry
15	Javankhoshdel & Bathurst (2017)	MCS	No	Dry
16	Yu & Bathurst (2017)	RSM	No	Dry
17	Hamrouni et al. (2018)	RSM, genetic algorithm	No	Dry
18	Santos et al. (2018)	Ant colony + FORM	No	Dry
19	Bozorgzadeh et al. (2019)	MCS	No	Dry
20	Burgess et al. (2019)	Random field FEM	Yes	Dry
21	Gao et al. (2019)	Generalized subset simulation	No	Dry
22	Le et al. (2018)	FEM + MCS	No	Partially saturated
23	Luo & Bathurst (2018)	RFEM	No	Dry
24	Zhang et al. (2018)	Bayesian updating	No	Partially saturated
25	Johari & Mousavi (2019)	Joint random variables	No	Dry
26	Kang et al. (2019)	GP regression + MCS	No	Dry
27	Nguyen & Likitlersuang (2019)	RFEM	No	Partially saturated
28	Wang et al. (2019)	Smooth particle hydrodynamics	No	Dry
29	Bathurst et al. (2020)	Closed-form spreadsheet MCS	No	Dry
30	Hamrouni et al. (2020)	MCS	No	Dry
31	Jiang et al. (2020)	Bayesian structural reliability	No	Saturated
32	Jiang et al. (2020)	Surrogate modeling	No	Partially saturated
33	Peng et al. (2020)	Total probability rule	No	Dry
34	Agarwal et al. (2021b)	Genetic Programming	Yes	Dry
35	Dastpak et al. (2021)	Random LEM	No	Dry
36	Huang et al. (2021)	Bayesian RFEM	No	Partially saturated
37	Liu & Wang (2021)	FEM + material point method	No	Partially saturated
38	Pain & Agarwal (2021)	FORM + MCS	No	Dry
39	Peng et al. (2021)	Probability density evolution	Yes	Dry
40	Agarwal et al. (2022b)	MCS	Yes	Dry
41	Agarwal et al. (2022a)	FORM + surrogate MCS	Yes	Dry
42	Falae et al. (2022)	Multivariate regression machine learning (ML)	No	Dry
43	Agarwal et al. (2023a)	Fourth-moment normal transformation (FMNT)	No	Dry
44	Agarwal et al. (2023b)	ML multivariate regression	Yes	Dry
45	Agarwal et al. (2024)	Stochastic response-surface	Yes	Partially saturated
46	Johari & Maroufi (2024)	RFEM	Yes	Dry
47	Kuili & Jakka (2024)	RFEM using Slope/W	Yes	Dry
48	Mustafa et al. (2024)	Ensemble ML	Yes	Dry
49	Luo & Agarwal (2024)	RFEM	Yes	Dry
50	Raghuram & Basha (2024)	Second-order RM	No	Partially saturated
51	Agarwal and Pain (2025)	Surrogate-based MCS	No	Dry
52	Nandi and Ghosh (2025)	Latin hypercube sampling with dependence	Yes	Dry

53	Sharma and Pain (2025)	FE-based lower bound limit analysis	No	Dry
54	Wu et al. (2025)	3D Convolutional Neural Network	No	Dry

Several clear patterns emerge from the literature. Early research focused mainly on conventional techniques such as MCS and FORM/FOSM. Over time, more advanced and efficient approaches like subset simulation, response surface methods, and stochastic collocation have gained popularity, especially for problems involving many uncertain parameters. There has also been increasing use of RFEM which is particularly useful for accounting for spatial variability in soil properties, a major source of uncertainty in geotechnical engineering. More recently, machine learning and surrogate modeling have been explored as ways to reduce computational time while maintaining accuracy, especially in high-dimensional problems.

At the same time, some challenges remain. Although seismic conditions are being considered more often now (especially in recent studies like Agarwal et al., 2024 and Burgess et al., 2019), most analyses still assume dry or fully saturated soil, with relatively few addressing partially saturated backfills. Many surrogate or AI-based models have shown promise, but questions remain about their performance across different sites and loading conditions. Also, many studies are based on simplified assumptions about input data, soil conditions, or boundary behavior, which may not reflect the complexity of real-world problems. While the field has advanced significantly on the academic side, its practical use in everyday geotechnical design remains limited. This is due in part to a lack of standardized procedures, limited integration into commercial software, and the perception that probabilistic methods are too complex for routine use.

These gaps raise an important question: what needs to change for probabilistic methods to become a regular part of engineering design? We believe future research should focus on making these methods easier to use and interpret, developing tools that work within existing design processes, and showing their practical value through real-world case studies. This challenge may be better addressed by placing greater emphasis on improving the accessibility, transparency, and practical relevance of existing models, rather than solely developing new methods that may be difficult to implement in real-world settings.

While several of our studies are included in the literature review for their methodological contributions, we do not view them as complete solutions to the broader challenges identified. Rather, they represent focused attempts to address specific aspects such as computational efficiency, seismic considerations, or partial saturation, that continue to limit the adoption of probabilistic methods in practice. In the following sections, we discuss these contributions in greater detail, not as definitive answers, but as part of the field's ongoing efforts to bridge the gap between research and implementation.

ADVANCING PROBABILISTIC SLOPE STABILITY ANALYSIS: SELECTED CONTRIBUTIONS

1. Improving Seismic Representations Through Rigorous Deterministic Formulations

Before advancing into probabilistic formulations, it is essential to revisit deterministic slope stability analysis, not as a standalone design method, but as the foundation on which most probabilistic simulations are built. Since probabilistic analyses typically involve thousands of deterministic evaluations under varying input realizations, the reliability and physical realism of the underlying deterministic framework directly influence the quality of probabilistic outcomes. Deterministic approaches remain central to geotechnical design practice, especially for geosynthetic-reinforced soil retaining structures (GRSRS). These analyses typically assume soil properties, loading conditions, and reinforcement strengths as fixed mean values and compute a single safety margin in the form of a factor of safety. Such analyses form the basis of many existing design codes and guidelines for geosynthetic-reinforced soil structures, such as BS8006, AASHTO LRFD Bridge Design Specifications, and FHWA guidelines (Agarwal et al., 2025a). However, when considering seismic effects, the limitations of conventional deterministic models become evident.

A large portion of past studies (e.g., Table 1) both academic and applied, have relied on the pseudo-static (PS) approach, which assumes that earthquake-induced inertial forces can be simplified into constant horizontal accelerations. While convenient, this simplification neglects the transient and depth-dependent nature of seismic loading. Early enhancements to the PS model led to the development of pseudo-dynamic formulations (e.g., Nimbalkar et al., 2006), which attempted to capture time-varying accelerations but introduced boundary inconsistencies and omitted damping effects. To address these shortcomings, Bellezza (2014) introduced the modified pseudo-dynamic (MPD) method, which includes time and depth variation of acceleration and satisfies boundary conditions and incorporates damping. This method has since been applied in several advanced studies (e.g., Pain et al., 2017; Sharma et al., 2020) and forms a key component of our work. In both deterministic and probabilistic settings, we integrated PS and MPD formulations to evaluate their performance under static and seismic loading conditions.

Given the widespread industry adoption of LEMs, our work also emphasizes deterministic models that remain compatible with design practice while advancing physical realism. Within LEMs, our preference is toward formulations that satisfy all three equilibrium conditions, force and moment balance, such as the Spencer's or Morgenstern-Price methods. For internal stability assessments, we employed the horizontal slice method (HSM), which divides the reinforced zone into horizontal layers and computes reinforcement forces by satisfying equilibrium within each slice and the entire failure wedge. Unlike vertical slice-based methods, HSM offers the advantage of capturing variation in seismic acceleration along the wall height. This makes it especially relevant for pseudo-dynamic or MPD settings, where inertial forces vary with depth. We employed a modified form of the HSM which improves upon the traditional approach by reducing underlying assumptions. We enhanced and applied the most rigorous form of this method, known as the (5*N*-1) formulation (Nouri et al., 2006), which solves for horizontal, vertical, and moment equilibrium in each slice (Figure 1), while keeping assumptions about interslice forces to a minimum. This enhances both accuracy and reliability, particularly when the results serve as inputs to probabilistic analyses.

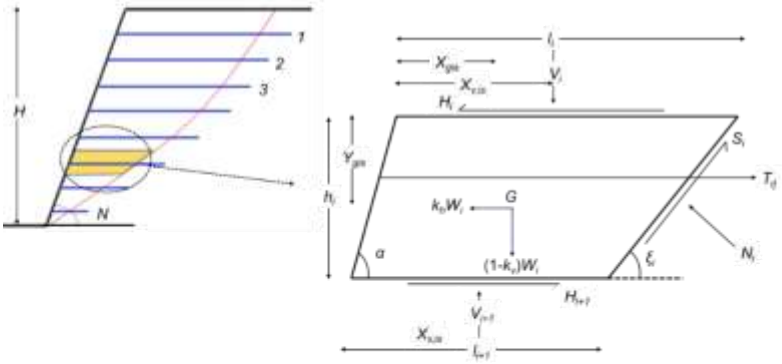

Fig. 1 Schematic of GRSRS and horizontal slice-based internal stability framework (adapted from Agarwal and Pain, 2021b)

The governing equations (Eq. 1, 2, 3) for the i^{th} horizontal slice are provided below for the readers' reference. For more detailed formulations and derivations, readers may refer to Agarwal and Pain (2021b):

$$\sum F_y = V_{i+1} - V_i - (1 - k_v)W_i + S_i \sin\alpha_i + N_i \cos\alpha_i = 0 \tag{1}$$

$$\sum F_x = T_j + S_i \cos\alpha_i - N_i \sin\alpha_i - k_h W_i + H_{i+1} - H_i = 0 \tag{2}$$

$$\sum M_o = V_{i+1}(R_{i+1}\cos\theta_{i+1} - L_{i+1} + X_{v,is+1}) + T_j(Y_{r,j} + R_o \sin\theta_o) - V_i(R_i \cos\theta_i - L_i + X_{v,is}) + H_{i+1}(R_{i+1}\sin\theta_{i+1}) + (S_i \sin\alpha_i + N_i \cos\alpha_i)(X_{nss}) - H_i(R_i \sin\theta_i) + (S_i \cos\alpha_i - N_i \sin\alpha_i)(Y_{nss}) - (1 - k_v)(W_i)(R_i \cos\theta_i - L_i + X_{gis}) - k_h(W_i)(R_i \sin\theta_i + Y_{gis}) = 0 \tag{3}$$

where, H_i and V_i denote the horizontal and vertical inter slice forces, respectively, on the upper portion of the slice; S_i and N_i are the tangential and normal forces acting on the failure surface, respectively; W_i is weight of slice; k_h and k_v are the horizontal and vertical seismic acceleration coefficients; $X_{v,is}$ is the horizontal distance between the slope face and the point of application of V_i; ω_i is the inclination angle of the base of the i_{th} slice; γ is the unit weight; X_{gis}, Y_{gis} denote the coordinates of the centroid calculated from the upper left corner of the slice; T_j is the total mobilised tensile force in j^{th} reinforcement layer; $Y_{r,j}$ is the depth of the j^{th} reinforcement; and X_{nss}, Y_{nss} are coordinates of point of application of N_i and S_i, respectively.

These equations are solved iteratively along the entire height of the wall to determine the distribution of tensile forces and critical reinforcement demand (K). Validation of this method against closed-form solutions showed good agreement (Figure 2) (Agarwal and Pain, 2022), with variations under 5% for typical design cases.

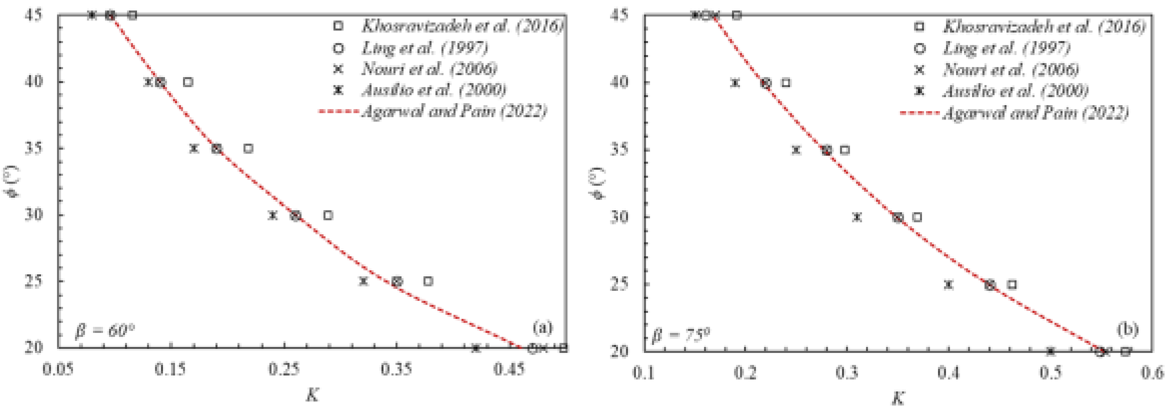

Fig. 2 Validation of employed HSM against closed-form solutions (adapted from Agarwal and Pain, 2022)

In terms of external stability, our focus was on sliding, considered the most critical failure mode in GRSRS (Basha and Babu, 2009). The reinforced mass is modeled as a rigid wedge interacting with the retained soil, forming a two-part sliding mechanism (Figure 3). The critical reinforcement length, l_s, is calculated by determining the force balance for the reinforced wedge, accounting for seismic inertia and soil-reinforcement interaction. This approach is aligned with simplified design routines but extended here to integrate MPD effects and variable reinforcement properties.

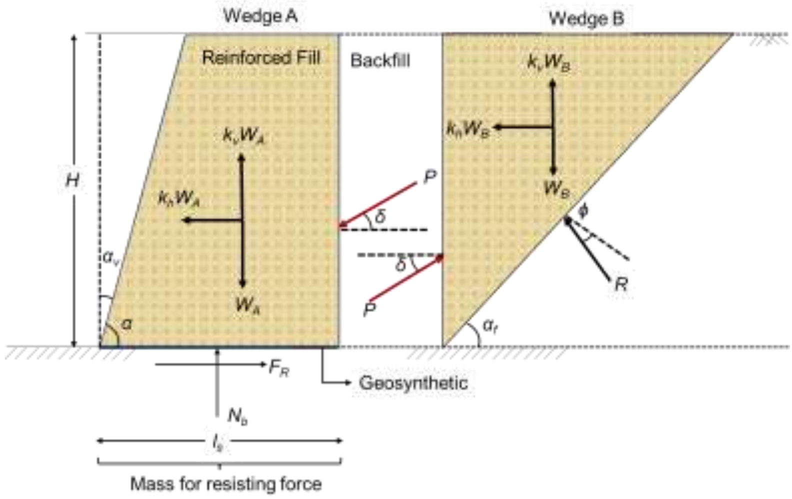

Fig. 3 Direct sliding mechanism and two-wedge external stability model (adapted from Agarwal and Pain, 2021b)

By embedding realistic seismic loading into both internal and external stability models, we attempted to offer a bridge between theory and practical application. These deterministic models do not merely serve as design tools on their own, they form the computational backbone of the probabilistic simulations discussed in later sections.

2. Surrogate Modeling for Uncertainty Management

One of the most persistent challenges in applying probabilistic methods in geotechnical engineering is the sheer computational effort involved. When analyzing the stability of GRSRS, estimating the probability of failure typically requires running thousands of simulations each based on a detailed deterministic evaluation. For problems that involve nonlinear soil behavior, reinforcement interaction, and dynamic forces, these evaluations can be expensive, both in terms of time and computational resources. This often discourages their practical use, even when uncertainty clearly plays a critical role in safety.

Surrogate modeling offers a promising solution. Rather than relying entirely on computationally intensive simulations for every realization, surrogate models approximate the relationship between input uncertainties and structural response using a reduced set of well-chosen simulations. Once trained, these models can be evaluated rapidly, allowing probabilistic methods like MCS to be used efficiently, even for complex problems. In our work, we focused on two surrogate techniques that complement the deterministic formulations discussed earlier. The first, multivariate adaptive regression splines (MARS), is a flexible regression-based method that partitions the input space into regions and fits piecewise linear functions to model nonlinear interactions. It's particularly effective when dealing with high-dimensional input spaces involving variables such as friction angle, unit weight, and seismic acceleration coefficients (k_h, k_v). The second approach, the stochastic response surface method (SRSM), uses a polynomial chaos expansion (Eq. 4) to build a global approximation over collocation points sampled from the input distributions. This method is valuable for incorporating correlations between variables, something that's often overlooked but frequently relevant in geotechnical settings.

$$Y \approx c_o + \sum_{a=1}^{n_{rv}} c_a X_a + \sum_{a=1}^{n_{rv}} c_{aa}(X_a^2 - 1) + \sum_{a=1}^{n_{rv}} c_{aaa}(X_a^3 - 3X_a) + \sum_{a=1}^{n_{rv}-1} \sum_{b>a}^{n_{rv}} c_{ab} X_a X_b + \sum_{a=1}^{n_{rv}} \sum_{\substack{b=1 \\ b \neq a}}^{n_{rv}} c_{abb}(X_a X_b^2 - X_a) + \sum_{a=1}^{n_{rv}-2} \sum_{b>a}^{n_{rv}-1} \sum_{c>b}^{n_{rv}} c_{abc} X_a X_b X_c \tag{4}$$

where, Y is the random output vector, c denotes the unknown coefficients (constant numbers), X is the standard Gaussian random variable, and n_{rv} is the number of random variables.

What distinguishes our approach is the integration of these surrogate models within a physically rigorous deterministic framework that uses both PS and MPD formulations. The MPD method, as noted in the previous section, accounts for depth-varying seismic accelerations and damping, making it better suited for dynamic loading than the traditional PS approach. However, MPD formulations are rarely used in probabilistic studies because of their complexity and the difficulty in resolving their governing equations over many iterations. By pairing MPD with surrogate models, we were able to bridge this gap, offering both realism and efficiency. The overall surrogate modeling workflow (Figure 4) that we followed involves:

1. Defining the input variables and their statistical distributions,
2. Generating training data using sampling techniques such as Latin Hypercube Sampling or collocation methods,
3. Performing deterministic simulations (based on PS or MPD frameworks) to evaluate performance functions,
4. Constructing surrogate models (MARS or SRSM) from the simulation results,
5. Validating the surrogate against test data to ensure reliability,
6. Estimating failure probabilities and reliability indices using efficient sampling, and
7. Interpreting the outcomes with respect to both individual and system-level failure modes.

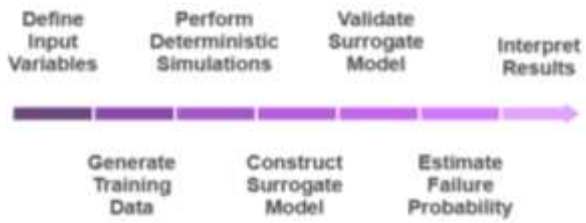

Fig. 4 Generalized workflow for surrogate-based probabilistic slope stability analysis

This process enabled us to examine a wide range of parameters and loading scenarios that would be impractical to explore through conventional simulation alone. For example, we looked at the influence of normalized frequency ($\frac{\omega H}{V_s}$) on system reliability, where ω is the angular frequency of seismic motion, H is wall height, and V_s is shear wave velocity. Although normalized frequency is well-understood in seismic wave propagation, it is rarely incorporated into probabilistic slope stability models due to the need for time-domain seismic formulations like MPD. Our results show a clear trend; higher normalized frequencies lead to lower reliability indices (Figure 5), for tensile (β_t), pullout (β_{po}), and system (β_{sys}) failure modes. Moreover, the influence of k_v becomes more pronounced, as an increase in k_v leads to a higher tensile force, further reducing the reliability indices. This is physically intuitive, more dynamic excitation increases reinforcement demand, but difficult to capture without a method that includes depth-varying accelerations.

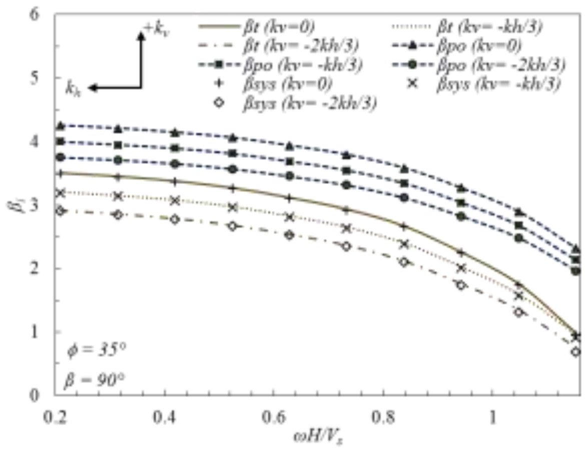

Fig. 5 Influence of $\frac{\omega H}{V_s}$ on reliability indices (adapted from Agarwal et al., 2021a)

Another often-overlooked factor is damping (ξ). Many conventional models either ignore it or use fixed, arbitrary values. We observed that as damping ratio increases, reliability indices improve noticeably (Figure 6). This suggests that incorporating site-specific damping estimates, even approximately, can meaningfully enhance the accuracy of probabilistic assessments.

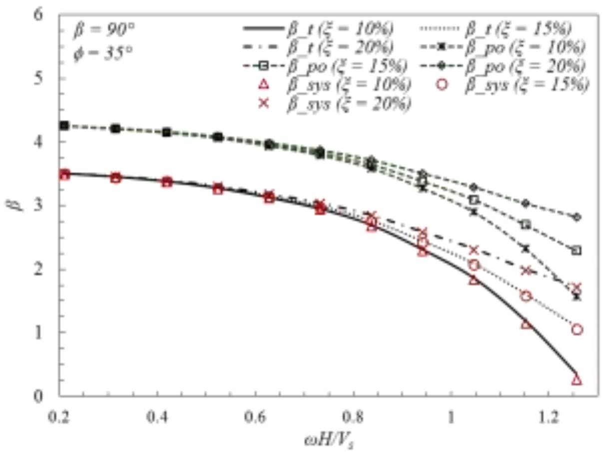

Fig. 6 Influence of ξ on reliability indices (adapted from Agarwal et al., 2021a)

We also analyzed the role of uncertainty itself, particularly the coefficient of variation (CoV) of critical parameters such as the soil friction angle. As CoV increases, the reliability index decreases sharply (Figure 7), reinforcing the importance of high-quality site investigation and characterization in reducing design uncertainty. This is a reminder that we require good data along with good models to achieve robust design.

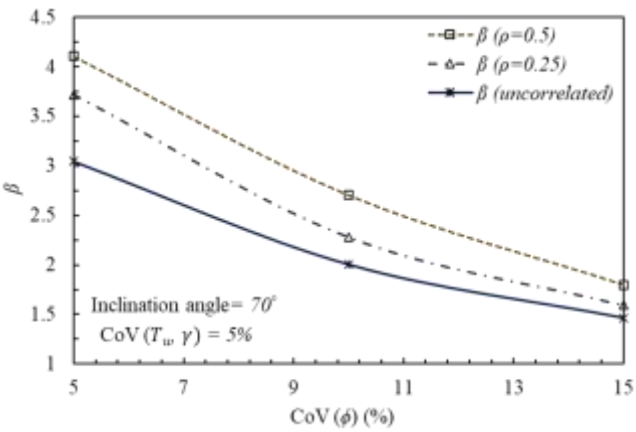

Fig. 7 Influence of uncertainty on reliability index (adapted from Agarwal and Pain, 2021a)

By using surrogate models, we were also able to evaluate system-level behavior (Figure 8). Rather than looking at internal and external stability modes in isolation, we analyzed their interaction in a unified framework to focus on the critical reliability index which happens to be the system reliability index. This broader view is essential when considering real-world failures, which rarely result from a single mode acting alone.

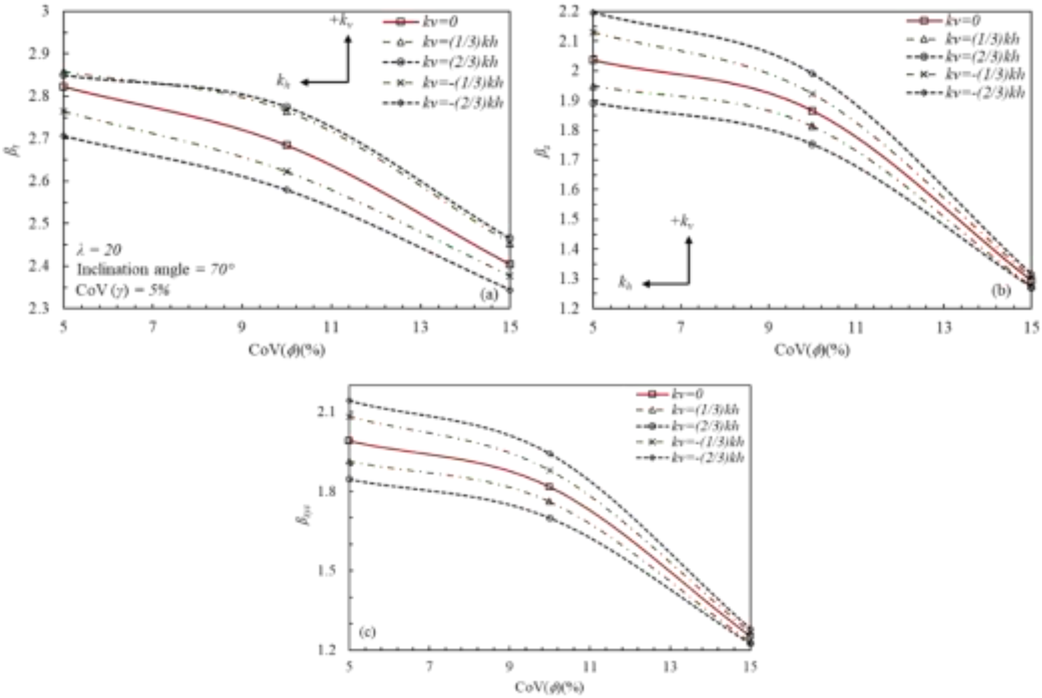

Fig. 8 Influence of extent of uncertainty on reliability index (adapted from Agarwal and Pain, 2021a)

Taken together, these findings suggest that surrogate modeling is a way to make rigorous, dynamic, and probabilistic assessments truly accessible. The key isn't in replacing detailed analysis, but in enabling it to scale. When embedded within a robust deterministic framework and applied thoughtfully, surrogate models help shift probabilistic slope stability analysis from research to practice.

3. Stochastic Algorithms for Sparse or Uncertain Data

Practical implementation of probabilistic analysis often hinges on the availability of well-characterized input distributions. This is a significant barrier, particularly in regions with limited instrumentation, historical infrastructure, or sparse geotechnical records. Many traditional methods, such as MCS or FORM, presume the availability of precise probability distribution functions (PDFs) and cumulative distribution functions (CDFs), which may not be available in many practical settings. To address this, we explored the use of the Fourth-Moment Normal Transformation (FMNT), a technique that requires only the first four statistical moments of input parameters, mean, standard deviation, skewness, and kurtosis, to approximate distributions. This approach is particularly valuable when empirical data are limited, as it avoids making strong assumptions about the form of the underlying distribution. FMNT transforms input variables into a standard normal space using a cubic transformation, offering a robust foundation for reliability analysis even in data-poor conditions. Its simplicity and analytical clarity make it well suited for field implementation.

We extended the FMNT method to both analytical and numerical domains. On the analytical side, we used FMNT to generate transformed variables and integrated them with FORM to estimate probability of failure for reinforced soil slopes. When performance functions were highly nonlinear, FMNT also proved effective in modeling the distribution of output responses directly, thus avoiding the need for repeated simulations.

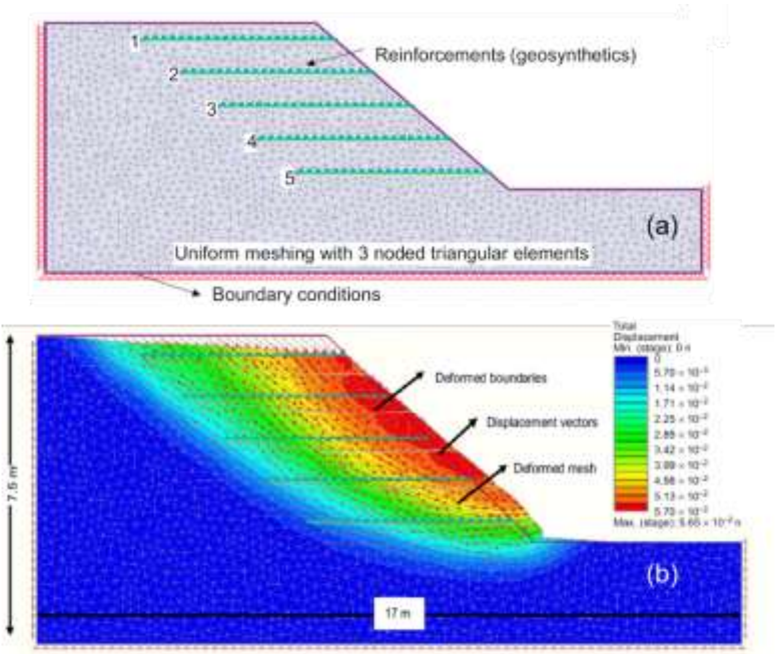

Fig. 9 (a) Reinforced slope generated in RS2; (b) Slope settlement contours for ϕ=30° (adapted from Agarwal and Pain, 2022)

For numerical implementation, we coupled FMNT with finite element analysis in RS2 (Figure 9). This integration allowed us to evaluate failure probabilities using as few as 11 simulations compared to thousands typically required in Monte Carlo approaches. Despite the need for some manual steps, the time savings were substantial. While a full probabilistic run using RS2 and traditional MCS would take over 100 hours, the FMNT-based process completed in under an hour, offering nearly a 180-fold improvement in computational efficiency. Importantly, these gains were not achieved at the expense of accuracy. Our results aligned well with published benchmarks and captured expected trends (Figure 10). The method demonstrated flexibility across various distributions and maintained robustness in both reinforced and unreinforced slope scenarios.

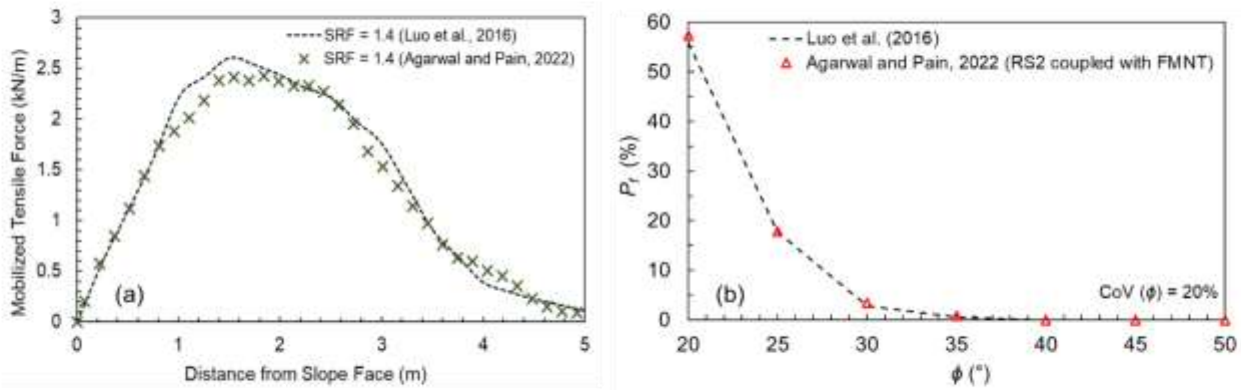

Fig. 10 (a) Deterministic validation using RS2; (b) Probabilistic validation of RS2 coupled with FMNT (adapted from Agarwal and Pain, 2022)

By enabling probabilistic design without requiring detailed input distributions or intensive computation, FMNT provides a practical alternative that bridges the gap between theory and real-world feasibility. As probabilistic slope stability analysis moves toward broader field adoption, such efficient and adaptable approaches can play a crucial role, especially in settings where data limitations are the norm rather than the exception.

4. Addressing Partial Saturation in Backfill

A significant limitation in conventional stability analysis of GRSRS is the treatment of soil conditions as either fully saturated or completely dry (see Table 1 for literature). In reality, particularly for backfills exposed to climatic variation or seismic activity, soils often remain partially saturated for extended periods. This intermediate condition introduces matric suction and variable hydraulic conductivity, both of which affect shear strength and deformation behavior, yet remain largely overlooked in routine design. While the deterministic models incorporating unsaturated soil mechanics have been proposed, they often simplify complex soil-water interactions and rely on fixed input values. Moreover, extending such models into the probabilistic domain has proven especially challenging. The presence of highly nonlinear performance functions arising from suction stress and water retention relationships, complicates the use of traditional reliability methods. Random field techniques are useful, but their computational complexity and sensitivity to scale of fluctuation parameters limit broader adoption.

To address this, we developed a probabilistic formulation that explicitly accounts for partial saturation through the inclusion of suction stress and unsaturated hydraulic conductivity in both static and seismic scenarios. At the deterministic level, this required modifying our HSM approach to incorporate environmental parameters such as suction head profiles. These modified formulations served as the training backbone for a surrogate model using the collocation-based stochastic response surface method (CBSRSM). CBSRSM closely represented the behavior of unsaturated soils, making it useful for infrastructure in regions with variable rainfall, seasonal drying, or seismic-induced desaturation.

To illustrate the impact of this formulation, we presented a comparative plot showing the deterministic and probabilistic stability margins of a reinforced slope under partially saturated conditions (Figure 11). The probabilistic model captured the variability in matric suction and hydraulic conductivity, resulting in a broader distribution of safety margins than deterministic estimates would suggest. The results reinforced the importance of incorporating partial saturation not only for accurate risk estimation but also for the development of resilient, climate-sensitive geotechnical infrastructure.

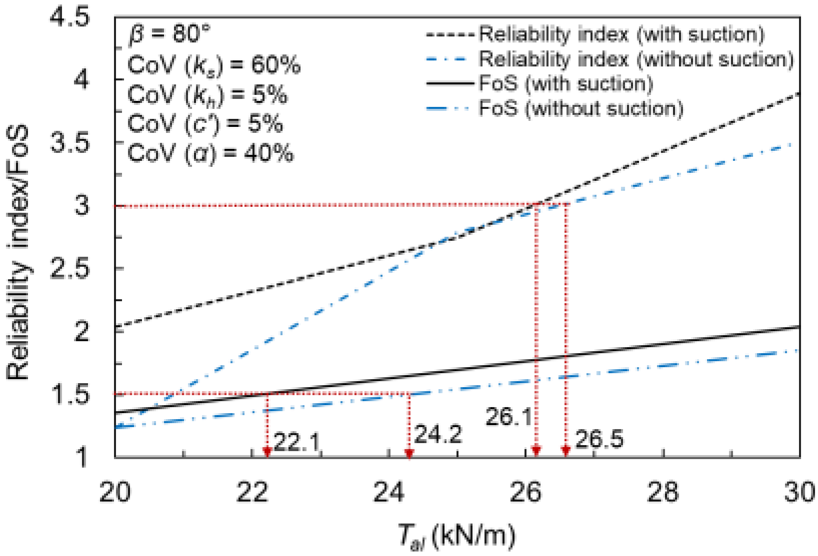

Fig. 11 Deterministic vs. probabilistic approach for the cases of suction and no-suction (adapted from Agarwal et al., 2024)

By combining advanced unsaturated soil modeling with a surrogate-based probabilistic framework, this approach offers a path toward more realistic, computationally tractable slope stability assessments. It also emphasizes the growing need for probabilistic models that are robust to environmental uncertainties, particularly as climate variability increasingly influences geotechnical performance.

CASE STUDY: NORTHRIDGE APPLICATION

Probabilistic methods are rarely applied in real-world geotechnical design, even though they can offer better insight into safety and reliability. Most field projects still rely on deterministic approaches, which do not fully capture the uncertainty present in soil properties, loading conditions, or seismic effects. This gap limits the practical impact of recent advances in probabilistic analysis. To help close this gap, we used the stochastic response surface method in a field-based case study using seismic records from the 1994 Northridge earthquake in California. The goal was to show that surrogate-based probabilistic analysis can be used with real seismic data to support design decisions, such as selecting reinforcement strength based on target safety levels.

In practice, seismic parameters can be derived through two primary routes: predictive models like the ground motion prediction equations (GMPEs), or directly from site-specific ground motion recordings. In this study, we opted for the latter, using acceleration time-history and Fourier amplitude data from the Northridge event (Figure 12) to extract two key inputs: the peak ground acceleration (PGA), measured as 0.09845g, and the mean period (0.56156), from which a dominant frequency of 1.78 Hz was derived.

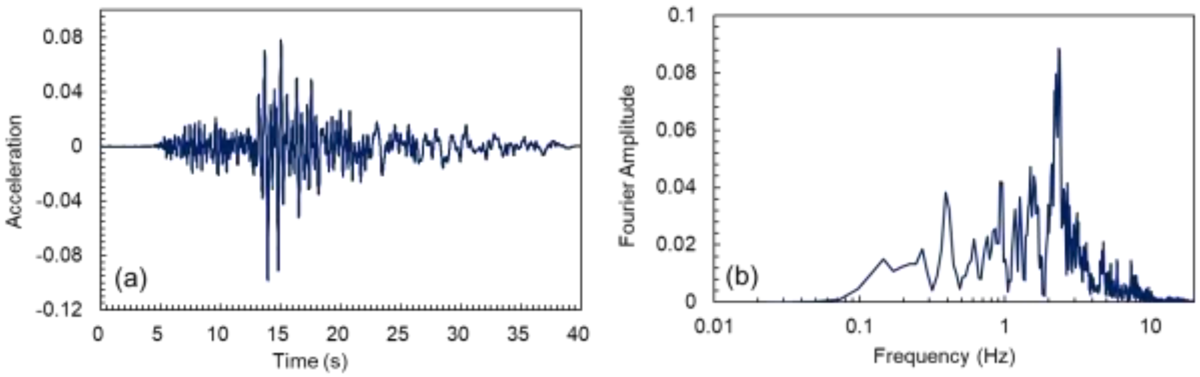

Fig. 12 Plot of: (a) Acceleration vs. time; (b) Fourier amplitude vs. frequency for Northridge earthquake (adapted from Agarwal et al., 2021c)

These seismic inputs were combined with the geotechnical characteristics of a typical reinforced slope, considering the effect of foundation soil depth in this case, which is often overlooked in design. Shear wave velocities for backfill and foundation soils were taken as 180 and 210 m/s, respectively and a damping ratio of 10% was also assumed, consistent with typical site conditions for granular fills. To explore design sensitivities, we varied the ultimate tensile strength of the reinforcement comparing two scenarios: 20 kN/m and 25 kN/m. All input variables were assigned a CoV of 5% to account for natural variability. The SRSM-based probabilistic analysis yielded failure probabilities of 3.35% and 0.008% for the two cases, respectively. The results indicate that a modest increase in reinforcement strength can lead to a substantial improvement in reliability, reducing the probability of failure to near-negligible levels. This analysis highlights a critical insight; a failure probability of 3.35% may seem small on paper, but in geotechnical terms, it signals a reliability index close to the lower bound of acceptable performance for infrastructure in seismic zones. More importantly, SRSM can use field-based seismic data to estimate reliability, which is especially helpful after an earthquake or in places where detailed design data is not available. This case study shows that probabilistic models, when combined with field data, allow engineers to design not only for safety but also for site-specific performance, cost, and long-term resilience. Including such evaluations in routine design processes makes probabilistic metrics practical tools for everyday engineering decisions, not just theoretical concepts.

CRITICAL REFLECTION, PRACTICAL GUIDELINES, AND FUTURE DIRECTIONS

Even though researchers have made great progress in using probabilistic methods for slope stability, these tools are still not commonly used in real-world projects. One major reason is that it can be difficult to apply these techniques outside the lab or academic setting. This section offers practical insights and reflections based on both the literature and recent field-oriented research. A common issue engineers face is choosing the right input values, especially the CoV. In many real projects, we simply don’t have enough field data to calculate CoV accurately. This is especially true for older infrastructure or sites in remote or developing regions. In such cases, engineers either make assumptions that are too optimistic (and unsafe) or too conservative (and expensive). A better approach is to fix CoV at small values like 5-15% for reliable parameters like unit weight or reinforcement strength and allow more flexibility for uncertain properties like cohesion, friction angle, or Young’s modulus. The key idea is to base CoV on the data available, not on fixed rules.

Choosing the right probabilistic method is also critical. MCS is widely trusted, but it takes a lot of computing time when used with numerical models like FEM. Methods like FORM are faster, but they rely on simplifying assumptions that may not work for all types of problems. Surrogate models such as

SRSM and MARS have become popular. These helps reduce the number of simulations needed while still capturing the main behavior of the system. They are especially useful in cases where models are complex such as slopes with unsaturated soils or those subjected to seismic loads. Surrogates also make it easier to include less common but important parameters like damping ratio or frequency, which are often ignored in simpler methods. Our own work and others' show that these surrogate models can handle tough situations well. That said, surrogates aren't perfect. Their accuracy depends on how many data points we use and where those points are placed. If the model has sharp nonlinear behavior or sudden changes, the surrogate might miss important details unless it's trained very carefully.

Another area gaining attention is the use of machine learning. Neural networks and other learning models can be trained on real or synthetic data to predict complex behaviors in geotechnical systems. These methods are powerful and flexible, but they need a lot of data and are sometimes hard to interpret. Still, they hold promise specifically for problems that involve many uncertain variables or where traditional models fall short.

Software choice also matters. While tools like ABAQUS are widely used for standard analysis, they don't directly support full probabilistic workflows. To get around this, we can couple them with external models like FMNT (Agarwal et al., 2026, accepted) or SRSM. This kind of setup drastically reduces the number of simulations needed, making probabilistic analysis more practical for everyday use.

Even with all this progress, probabilistic methods are still underused in real projects. Many engineers see them as too complex or time-consuming. One way to fix this is to make the methods easier to understand and apply. Step-by-step guides, clear explanations of assumptions, and use of real-world case studies can help bridge the gap between research and practice. For example, the Northridge earthquake case study we discussed earlier shows that these methods can work with actual seismic data to support better design decisions. Probabilistic methods are not just academic tools; they are becoming essential for designing safer and smarter infrastructure.

Looking ahead, there are a few areas where more work is needed to make probabilistic design easier and more useful in practice. First, we need simple, open-source tools that engineers can use without needing to write complex code or rely on expensive software. These tools should fit easily into everyday design work. Second, it would help to build shared reference datasets that list typical CoV values for common soils and materials. This would make it easier for engineers to make realistic assumptions when field data is limited. Another important direction is to combine traditional physics-based models with machine learning (for example, Pei and Luo, 2025; Agarwal et al., 2025b, accepted). This could make our models both faster and smarter, especially when dealing with limited or noisy data. We also need to go beyond just checking whether a design is likely to fail. Probabilistic analysis should also cover post-failure response, long-term structural behavior, and reliability under changing conditions such as heavy rainfall or seismic events. There is a growing need to develop partial safety factors based on large-scale datasets and rigorous FEM simulations, so that design codes can better reflect real-world variability. Another promising area is the use of random field-based methods, which are designed to account for the spatial variability of soil properties more realistically than random variable approaches. While they offer clear advantages in theory and have been explored in several studies, their application in routine design remains limited due to high computational cost and modeling complexity. Our work focuses on random variable-based methods for their simplicity and ease of implementation, but there is a strong need for continued research aimed at simplifying and streamlining random field approaches so they can be used more widely in practice. When used the right way, probabilistic methods don't make design harder, they make it clearer. By accepting the fact that uncertainty is part of every project, we can create infrastructure that is stronger, more reliable, and better prepared for the real world.

REFERENCES

1. Agarwal, E., Luo, N., Liu, F., and Zheng, L. (2026). "Automated Probabilistic Slope Stability Assessment Using Four-Moment Normal Transformation Integrated with ABAQUS", *In: 21st International Conference on Soil Mechanics and Geotechnical Engineering (ICSMGE)*, *Vienna, Austria* (accepted).
2. Agarwal, E., Luo, N. and Liu, K. (2025a). "Probabilistic Analyses of Geosynthetic-Reinforced Pile-Supported Embankments Using Design Methods and 3D Finite Element Models Considering Soil Variability", *Geotextiles and Geomembranes*, Vol. 53, No. 2, pp. 559-576.

3. Agarwal, E., Pei, T. and Luo, N. (2025b). "Towards Robust and Adaptive Bearing Capacity Prediction for Dry Shallow Foundations Using Physics-Informed Machine Learning", *In: Geo-Extreme 2025*, pp. 107-117. *Long Beach, CA.*
4. Agarwal, E., Pain, A., Mukhopadhyay, T., Metya, S. and Sarkar, S. (2021a). "Efficient Computational System Reliability Analysis of Reinforced Soil-Retaining Structures under Seismic Conditions Including the Effect of Simulated Noise", *Engineering with Computers*, pp. 1-23.
5. Agarwal, E. and Pain, A. (2021a). "Efficient Probabilistic Stability Analysis of Geosynthetic Reinforced Slopes Using Collocation-Based Stochastic Response Surface", *International Journal of Geomechanics*, Vol. 21, No. 10, pp. 04021189.
6. Agarwal, E. and Pain, A. (2025). "External Stability Analysis of Reinforced Wall Using Surrogate Based Probabilistic", *Soil Dynamics and Computational Geomechanics: Select Proceedings of 8IYGEC 2021*, pp. 428, 347.
7. Agarwal, E. and Pain, A. (2021b). "Probabilistic stability analysis of geosynthetic-reinforced slopes under pseudo-static and modified pseudo-dynamic conditions", *Geotextiles and Geomembranes*, Vol. 49, No. 6, pp. 1565-1584.
8. Agarwal, E., Verma, A.K., Pain, A. and Sarkar, S. (2021b). "Probabilistic Analysis of a Geosynthetic Reinforced Soil Retaining Wall Under Seismic Conditions Using Genetic Programming", *In Indian Geotechnical Conference*, pp. 217-226. Singapore: Springer Nature Singapore.
9. Agarwal, E., Pain, A. and Sarkar, S. (2021c). "Stochastic Stability Analysis of Geosynthetic Reinforced Slopes Subjected to Harmonic Base Shaking", *Transportation Geotechnics*, Vol. 29, pp. 100562.
10. Agarwal, E. and Pain, A. (2022). "Reliability Assessment of Reinforced Slopes with Unknown Probability Distribution", *Geosynthetics International*, Vol. 30, No. 4, pp. 337-349.
11. Agarwal, E., Pain, A. and Sarkar, S. (2022a). "Probabilistic Analysis of Reinforced Soil Retaining Structures Using FORM and Surrogate-Based Monte Carlo Simulation", *In Ground Improvement and Reinforced Soil Structures: Proceedings of Indian Geotechnical Conference 2020*, Vol. 2. pp. 733-742. Springer Singapore.
12. Agarwal, E., Sharma, M. and Pain, A. (2022b). "Probabilistic Stability Analysis of Helical Soil Nailed Wall using the Monte-Carlo Simulation", *In Geohazard Mitigation: Select Proceedings of VCDRR 2021*, pp. 221-232. Springer Singapore.
13. Agarwal, E., Pain, A. and Zhou, A. (2023a). "Probabilistic Assessment of Reinforced Soil Walls using Fourth Moment Normal Transformation", *In Proceedings of the TMIC 2022 Slope Stability Conference (TMIC 2022)*, Vol. 13, p. 55. Springer Nature.
14. Agarwal, E., Sharma, M. and Pain, A. (2023b). "Efficient Surrogate Model Based Probabilistic Analysis of Helical Soil Nailed Wall under Seismic Conditions", *European Journal of Environmental and Civil Engineering*, Vol. 27, No. 3, pp. 1263-1284.
15. Agarwal, E., Pain, A., Annapareddy, V.R. and Zhou, A. (2024). "Probabilistic Seismic Stability Analysis of Reinforced Soil Retaining Structures in Partially Saturated Conditions", *International Journal of Geomechanics*, Vol. 24, No. 8, pp. 04024169.
16. Baecher, G.B. and Christian, J.T. (2005). "Reliability and statistics in geotechnical engineering", *John Wiley & Son*s.
17. Babu, G.S. and Singh, V.P. (2009). "Reliability Analysis of Soil Nail Walls", *Georisk*, Vol. *3*, *No.* 1, pp. 44-54.
18. Basha, B.M. and Babu, G.S. (2010). "Optimum Design for External Seismic Stability of Geosynthetic Reinforced Soil Walls: Reliability Based Approach", *Journal of geotechnical and geoenvironmental engineering*, Vol. 136, No. 6, pp. 797-812.
19. Basha, B.M. and Babu, G.S. (2010). "Reliability Assessment of Internal Stability of Reinforced Soil Structures: A Pseudo-Dynamic Approach", *Soil Dynamics and Earthquake Engineering*, Vol. 30, No. 5, pp. 336-353.
20. Basha, B.M. and Babu, G.S. (2012). "Target Reliability-Based Optimisation for Internal Seismic Stability of Reinforced Soil Structures", *Géotechnique*, Vol. 62, No. 1, pp. 55-68.

21. Bathurst, R.J., Miyata, Y. and Allen, T.M. (2020). "Deterministic and Probabilistic Assessment of Margins of Safety for Internal Stability of As-Built PET Strap Reinforced Soil Walls", *Geotextiles and Geomembranes*, Vol. 48, No. 6, pp. 780-792.
22. Bellezza, I. (2015). "Seismic Active Earth Pressure on Walls using a New Pseudo-Dynamic Approach", *Geotechnical and Geological Engineering*, Vol. 33, pp. 795-812.
23. Bozorgzadeh, N., Bathurst, R.J., Allen, T.M. and Miyata, Y. (2020). "Reliability-Based Analysis of Internal Limit States for MSE Walls using Steel-Strip Reinforcement", *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 146, No. 1, pp. 04019119.
24. Burgess, J., Fenton, G.A. and Griffiths, D.V. (2019). "Probabilistic Seismic Slope Stability Analysis and Design", *Canadian Geotechnical Journal*, Vol. 56, No. 12, pp. 1979-1998.
25. Cho, S.E. (2010). "Probabilistic Assessment of Slope Stability that Considers the Spatial Variability of Soil Properties", *Journal of geotechnical and geoenvironmental engineering*, Vol. 136, No. 7, pp. 975-984.
26. Chowdhury, R.N. and Xu, D. W. (1992). "Reliability Index for Slope Stability Assessment—Two Methods Compared", *Reliability Engineering & System Safety*, Vol. 37, No. 2, pp. 99-108.
27. Dastpak, P., Jamshidi Chenari, R., Cami, B. and Javankhoshdel, S. (2021). "Noncircular Deterministic and Stochastic Slope Stability Analyses and Design of Simple Geosynthetic-Reinforced Soil Slopes", *International Journal of Geomechanics*, Vol. 21, No. 9, pp. 04021155.
28. Duncan, J.M. and Wright, S.G. (2005). "Soil Strength and Slope Stability", *John Wiley & Sons*.
29. El-Ramly, H., Morgenstern, N.R. and Cruden, D.M. (2002). "Probabilistic Slope Stability Analysis for Practice", *Canadian Geotechnical Journal*, Vol. *39, No.* 3, pp. 665-683.
30. Falae, P.O., Agarwal, E., Pain, A., Dash, R.K. and Kanungo, D.P. (2021). "A Data Driven Efficient Framework for the Probabilistic Slope Stability Analysis of Pakhi Landslide, Garhwal Himalaya", *Journal of Earth System Science*, Vol. *130*, pp. 1-15.
31. Fenton, G.A. and Griffiths, D.V. (2008). "Risk Assessment in Geotechnical Engineering", *John Wiley & Sons*.
32. Gao, G.H., Li, D.Q., Cao, Z.J., Wang, Y. and Zhang, L. (2019). "Full Probabilistic Design of Earth Retaining Structures using Generalized Subset Simulation", *Computers and Geotechnics*, Vol. 112, pp. 159-172.
33. Griffiths, D.V. and Fenton, G.A. (2004). "Probabilistic Slope Stability Analysis by Finite Elements", *Journal of geotechnical and geoenvironmental engineering*, Vol. 130, No. 5, pp. 507-518.
34. Griffiths, D.V. and Lane, P.A. (1999). "Slope Stability Analysis by Finite Elements", *Geotechnique*, Vol. 49, No. 3, 387-403.
35. Hamrouni, A., Dias, D. and Sbartai, B. (2018). "Reliability Analysis of a Mechanically Stabilized Earth Wall using the Surface Response Methodology Optimized by a Genetic Algorithm", *Geomechanics and Engineering*, Vol. 15, No. 4, pp. 937-945.
36. Harr, M.E. (1987). "Reliability-based design in civil engineering", *McGraw-Hill*.
37. Hamrouni, A., Dias, D. and Sbartai, B. (2020). "Soil Spatial Variability Impact on the Behavior of a Reinforced Earth Wall:, *Frontiers of Structural and Civil Engineering*, Vol. 14, pp. 518-531.
38. Huang, M.L., Sun, D.A., Wang, C.H. and Keleta, Y. (2021). "Reliability Analysis of Unsaturated Soil Slope Stability using Spatial Random Field-Based Bayesian Method", *Landslides*, Vol. 18, pp. 1177-1189.
39. Hu, H., Huang, Y. and Zhao, L. (2020). "Probabilistic Seismic-Stability Analysis of Slopes Considering the Coupling Effect of Random Ground Motions and Spatially-Variable Soil Properties", *Natural Hazards Review*, Vol. 21, No. 3, pp. 04020028.
40. Javankhoshdel, S. and Bathurst, R.J. (2017). "Deterministic and Probabilistic Failure Analysis of Simple Geosynthetic Reinforced Soil Slopes", *Geosynthetics International*, Vol. 24, No. 1, pp. 14-29.
41. Javankhoshdel, S., Luo, N. and Bathurst, R.J. (2017). "Probabilistic Analysis of Simple Slopes with Cohesive Soil Strength using RLEM and RFEM", *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, Vol. 11, No. 3, pp. 231-246.

42. Jiang, S.H., Huang, J., Qi, X.H. and Zhou, C.B. (2020). “Efficient Probabilistic Back Analysis of Spatially Varying Soil Parameters for Slope Reliability Assessment”, *Engineering Geology*, Vol. *271*, pp. 105597.
43. Jiang, S.H. and Huang, J.S. (2016). “Efficient slope reliability analysis at low-probability levels in spatially variable soils”, *Computers and Geotechnics*, Vol. 75, pp. 18-27.
44. Jiang, S.H., Liu, X. and Huang, J. (2020). “Non-Intrusive Reliability Analysis of Unsaturated Embankment Slopes Accounting for Spatial Variabilities of Soil Hydraulic and Shear Strength Parameters”, *Engineering with Computers*, pp. 1-14.
45. Johari, A., Fazeli, A. and Javadi, A. A. (2013). “An Investigation into Application of Jointly Distributed Random Variables Method in Reliability Assessment of Rock Slope Stability”, *Computers and Geotechnics*, Vol. 47, pp. 42-47.
46. Johari, A. and Mousavi, S. (2019). “An Analytical Probabilistic Analysis of Slopes Based on Limit Equilibrium Methods”, *Bulletin of Engineering Geology and the Environment*, Vol. 78, pp. 4333-4347.
47. Johari, A. and Maroufi, M. (2024). “System Reliability Analysis of Geogrid Reinforced Retaining Wall using Random Finite Element Method”, *Transportation Geotechnics*, Vol. 48, pp. 101316.
48. Kang, F., Han, S., Salgado, R. and Li, J. (2015). “System Probabilistic Stability Analysis of Soil Slopes using Gaussian Process Regression with Latin Hypercube Sampling”, *Computers and geotechnics*, Vol. 63, pp. 13-25.
49. Kuili, S. and Jakka, R.S. (2024). “Probabilistic Assessment of the Seismic Stability of Gravity Retaining Wall”, *Japanese Geotechnical Society Special Publication*, Vol. 10, No. 15, pp. 489-494.
50. Le, T.M.H., Sanchez, M., Gallipoli, D. and Wheeler, S. (2019). “Probabilistic Study of Rainfall-Triggered Instabilities in Randomly Heterogeneous Unsaturated Finite Slopes”, *Transport in Porous Media*, Vol. *126*, pp. 199-222.
51. Lin, B.H., Yu, Y., Bathurst, R.J. and Liu, C.N. (2016). “Deterministic and Probabilistic Prediction of Facing Deformations of Geosynthetic-Reinforced MSE Walls using a Response Surface Approach”, *Geotextiles and Geomembranes*, Vol. 44, No. 6, pp. 813-823.
52. Liu, X. and Wang, Y. (2021). “Probabilistic Simulation of Entire Process of Rainfall-Induced Landslides using Random Finite Element and Material Point Methods with Hydro-Mechanical Coupling”, *Computers and Geotechnics*, Vol. 132, pp. 103989.
53. Luo, N. and Agarwal, E. (2024). “Seismic Stability and Failure Mechanisms of Gentle and Steep Slopes Considering Soil Rotated Anisotropy and Spatial Variability”, *Soil Dynamics and Earthquake.*
54. Luo, N. and Bathurst, R.J. (2018). “Probabilistic Analysis of Reinforced Slopes using RFEM and Considering Spatial Variability of Frictional Soil Properties Due to Compaction”, *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, Vol. 12, No. 2, pp. 87-108.
55. Luo, N., Bathurst, R.J. and Javankhoshdel, S. (2016). “Probabilistic Stability Analysis of Simple Reinforced Slopes by Finite Element Method”, *Computers and Geotechnics*, Vol. 77, pp. 45-55.
56. Mustafa, R., Samui, P. and Kumari, S. (2024). “Seismic Performance of Gravity Retaining Walls under Quasi-Static Approach using Probabilistic Analysis”, *Transportation Infrastructure Geotechnology*, Vol. 11, No. 2, pp. 612-649.
57. Nandi, S. and Ghosh, P. (2025). “Probabilistic Seismic Slope Stability Analysis Adopting Limiting Profile Approach”, *Engineering Geology*, pp. 108174.
58. Nguyen, T. S. and Likitlersuang, S. (2019). “Reliability Analysis of Unsaturated Soil Slope Stability under Infiltration Considering Hydraulic and Shear Strength Parameters”, *Bulletin of Engineering Geology and the Environment*, Vol. 78, pp. 5727-5743.
59. Nimbalkar, S.S., Choudhury, D. and Mandal, J.N. (2006). “Seismic Stability of Reinforced-Soil Wall by Pseudo-Dynamic Method”, *Geosynthetics International*, Vol. *13, No.* 3, pp. 111-119.
60. Nouri, H., Fakher, A. and Jones, C.J.F.P. (2006). “Development of Horizontal Slice Method for Seismic Stability Analysis of Reinforced Slopes and Walls”, *Geotextiles and Geomembranes*, Vol. 24, No. 3, pp. 175-187.

61. Otálvaro, I.F. and Cordão-Neto, M.P. (2013). "Probabilistic Analyses of Slope Stability under Infiltration Conditions", *Advances in Unsaturated Soils*, pp. 539-543.
62. Pain, A. and Agarwal, E. (2021). "Retaining Walls–Reliability-Based Approach", *In ICE Handbook of Geosynthetic Engineering: Geosynthetics and their applications*, pp. 105-114. *ICE Publishing*.
63. Pain, A., Choudhury, D. and Bhattacharyya, S.K. (2017). "Effect of Dynamic Soil Properties and Frequency Content of Harmonic Excitation on the Internal Stability of Reinforced Soil Retaining Structure", *Geotextiles and Geomembranes*, Vol. 45, No. 5, pp. 471-486.
64. Pei, T. and Luo, N. (*2025*) "Prediction of Ultimate Bearing Capacity of Shallow Foundations Using Physically Constrained Neural Networks", *In Geotechnical Frontiers*, pp. 11-20.
65. Peng, M., Sun, R., Chen, J.F., Zhang, L.M. and Yu, S.B. (2021). "Stochastic Seismic Analysis of Geosynthetic-Reinforced Soil Slopes Using the Probability Density Evolution Method", *Computers and Geotechnics*, Vol. 140, pp. 104485.
66. Peng, M., Sun, R., Chen, J.F., Rajesh, S., Zhang, L.M. and Yu, S.B. (2020). "System Reliability Analysis of Geosynthetic Reinforced Soil Slope Considering Local Reinforcement Failure", *Computers and Geotechnics*, Vol. 123, pp. 103563.
67. Raghuram, A.S.S. and Basha, B.M. (2024). "Rainfall-Induced Probability of Failure in Spatially Variable Soil Slopes and a Case Study of the Konkan Railway Slope Failure", *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, Vol. 10, No. 1, pp. 04023046.
68. Rahman, M.A., Tabassum, N. and Islam, M.R. (2021). "Different Aspects of Slope Failures Considering Large Deformation: Application of Smoothed Particle Hydrodynamics (SPH)", *Innovative Infrastructure Solutions*, Vol. 6, pp. 1-15.
69. Santos, M.G.C., Silva, J.L.D. and Beck, A.T. (2018). "Reliability-Based Design Optimization of Geosynthetic-Reinforced Soil Walls", *Geosynthetics International*, Vol. 25, No. 4, pp. 442-455.
70. Sayed, S., Dodagoudar, G.R. and Rajagopal, K. (2010). "Finite Element Reliability Analysis of Reinforced Retaining Walls", *Geomechanics and Geoengineering: An International Journal*, Vol. 5, No. 3, pp. 187-197.
71. Sayed, S., Dodagoudar, G.R. and Rajagopal, K. (2008). "Reliability Analysis of Reinforced Soil Walls under Static and Seismic Forces", *Geosynthetics International*, Vol. 15, No. 4, pp. 246-257.
72. Sharma, M., Choudhury, D., Samanta, M., Sarkar, S. and Annapareddy, V.R. (2020). "Analysis of Helical Soil-Nailed Walls Under Static and Seismic Conditions", *Canadian Geotechnical Journal*, Vol. 57, No. 6, pp. 815-827.
73. Sharma, P. and Pain, A. (2025). "Probabilistic Bearing Capacity of a Strip Footing Resting on a Spatially Varying Soil Slope Considering Nonstationary Characteristics", *European Journal of Environmental and Civil Engineering*, pp. 1-21.
74. Sjöberg, J. (2020). "Analysis of the Aznalcollar Pit Slope Failures—a Case Study", *In FLAC and numerical modeling in geomechanics*, pp. 63-70. *CRC Press*.
75. Wang, Y., Cao, Z. and Au, S.K. (2010). "Efficient Monte Carlo Simulation of Parameter Sensitivity in Probabilistic Slope Stability Analysis", *Computers and Geotechnics*, Vol. 37, No. 7-8, pp. 1015-1022.
76. Wang, Y., Qin, Z., Liu, X. and Li, L. (2019). "Probabilistic Analysis of Post-Failure Behavior of Soil Slopes Using Random Smoothed Particle Hydrodynamics", *Engineering Geology*, Vol. 261, pp. 105266.
77. Wu, C., Wang, Z.Z., Goh, S.H. and Zhang, W. (2025). "A Methodology for Three-Dimensional Slope Reliability Assessment Considering Spatial Variability", *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 151, No. 5, pp. 04025020.
78. Yu, Y. and Bathurst, R.J. (2017). "Probabilistic Assessment of Reinforced Soil Wall Performance Using Response Surface Method", *Geosynthetics International*, Vol. 24, No. 5, pp. 524-542.
79. Zevgolis, I.E. and Bourdeau, P.L. (2010a). "Probabilistic Analysis of Retaining Walls", *Computers and Geotechnics*, Vol. 37, No. 3, pp. 359-373.
80. Zevgolis, I.E. and Bourdeau, P.L. (2010b). "System Reliability Analysis of the External Stability of Reinforced Soil Structures", *Georisk*, Vol. 4, No. 3, pp. 148-156.

81. Zhang, L., Wu, F., Zheng, Y., Chen, L., Zhang, J. and Li, X. (2018). "Probabilistic Calibration of a Coupled Hydro-Mechanical Slope Stability Model with Integration of Multiple Observations", *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, Vol. 12, No. 3, pp. 169-182.