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ADVANCED DYNAMIC SOIL-PILE INTERACTION: CHALLENGES IN ELUCIDATING THE EFFECTS OF NONLINEARITIES ON FREQUENCY DEPENDENCY OF DYNAMIC STIFFNESS AND ITS MODELING

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ABSTRACT

This paper presents new findings related to the dynamic characteristics of soil-pile interaction in terms of “impedance functions (IFs)”. IFs play an important role to represent the dynamic characteristics of soil-pile systems supporting the superstructure. In general, IFs have frequency dependent characteristics, where the stiffness and the damping change with the excitation frequencies. On the other hand, nonlinearities such as behavior of soil in the vicinity of piles and slippage and separation at the interface between the soil and the pile largely dominates the IFs when a large amplitude of excitation is applied to the pile head. Although both characteristics affect the dynamic response of structural systems, the relationship between the frequency-dependent behavior and the nonlinear behavior of soil-pile systems has seldom been thoroughly investigated. Furthermore, although several lumped parameter models representing the frequency-dependent IFs has been proposed over the past few decades, no model accounting for the nonlinearities of soil-pile systems has yet been developed. To improve the accuracy of the dynamic response estimations in structures, an appropriate model considering both effects is desired. This paper reviews recent experimental results obtained by the author's team, which demonstrate a convergence trend in frequency-dependent IFs across varying loading amplitudes. Additionally, the author presents an advanced lumped parameter model that incorporates both frequency dependency and nonlinear effects, which is also discussed.

KEYWORDS: Impedance Functions, Pile Foundations, Nonlinearitiies, Frequency Dependencies, Gyro-Lumped Parameter Models

INTRODUCTION

In recent seismic design codes and provisions, the elongation of the natural period and the increase of the damping in structural systems due to a so-called “soil-structure interaction (SSI)” are generally considered. The detailed treatment of SSI in these provisions is based on numerous significant contributions by pioneers in the fields of earthquake and geotechnical engineering. Research on SSI, initiated by E. Reissner in 1936, has a long history of elucidating the dynamic characteristics of rigid and pile foundations through theoretical, numerical, and experimental approaches. To the author’s best knowledge, one of the most important discoveries through these longstanding studies is the identification of “frequency dependency” in soil-foundation systems. Frequency-dependent behavior can be observed in various aspects such as the subgrade reaction of soil along the embedded sidewall, the displacement transmission functions between piles, and the integrated dynamic stiffness at the head of soil-foundation systems. This frequency dependency arises from wave propagation in the soil medium, where different types of waves, depending on the frequency, lead to variations in the spring and damping characteristics of the system. Design formulas quantifying these effects for rigid foundations (e.g., Pais and Kausel (1988)) and pile foundations (e.g., Mylonakis and Roumbas (2001), Dobry et al. (1982), Gazetas and Dobry (1984a, b)) are typically expressed as functions of frequency, highlighting the frequency-dependent nature of dynamic spring and damping characteristics. The application of these formulas basically assumes that the soil and foundation behave elastically. On the other hand, it is well known that the soil nonlinearity affects the static restoring force characteristics of foundations. Research on this traditional soil-foundation nonlinear behavior predates studies on SSI, and the methods for calculating the subgrade reaction and the integrated force characteristics including bearing capacity have long been established.

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Looking back at the history of both fields above, it becomes evident that there is no clear connection between the dynamic force-displacement relationship and the static one accounting for nonlinearities, despite both targeting the same objective: an appropriate representation of soil-foundation systems. When considering the representation of a soil-pile system in practice, one is a so-called “impedance functions (IFs)”, expressed as a complex number where the real and imaginary parts express the stiffness and mass effects, and the damping effect, respectively. On the other hand, the static force-displacement relationship at the pile cap is calculated using an analytical model consisting of the pile shaft where nonlinear subgrade reaction springs are connected along the depth. These approaches are based on the dynamic and static concepts mentioned earlier, respectively. This leads to an important question: which approach is more suitable for designing a structural system? Few studies have focused on the effect of nonlinearities on frequency dependency while both methodologies are widely used in practice. As a result, designers often face the challenge of choosing between the two approaches with limited reference material to guide their decision.

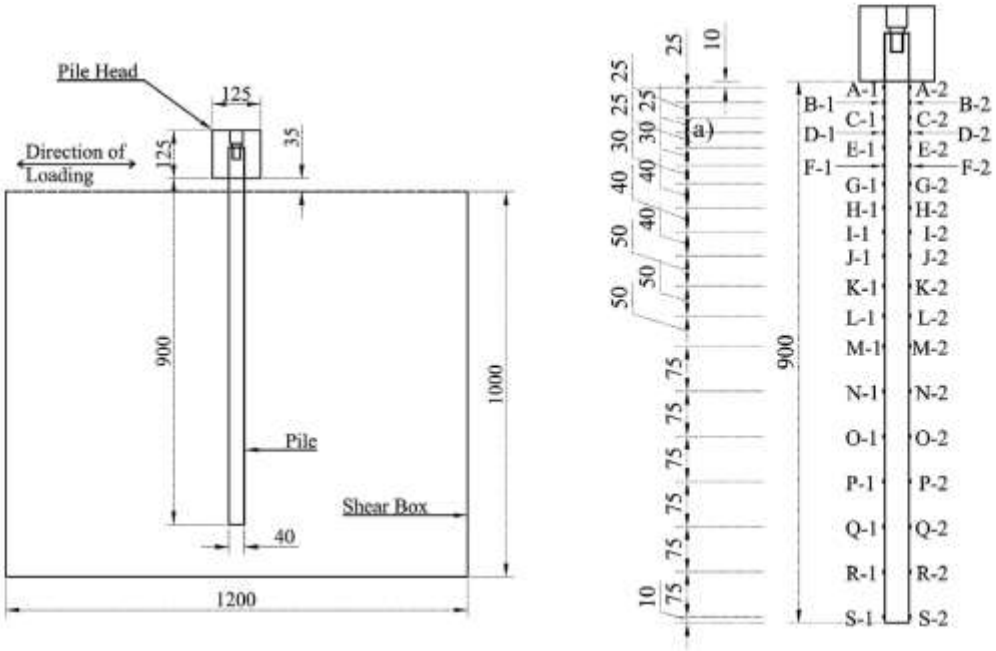
The author and the team at Saitama university have been addressing the problem described above in the recent decades (e.g., Goit et al. (2013, 2014)). Recently, the team made a groundbreaking discovery through model experiments involving a scaled single pile and a 2×2 pile group, revealing the relationship between frequency dependency and nonlinearities in the system. Moreover, an analytical model that simultaneously simulates both characteristics was developed by the author. This keynote lecture paper presents a summary of these studies.

IMPEDANCE FUNCTIONS OF SINGLE PILES UNDER EXTREME LARGE AMPLITUDES OF LOADING

Shrestha et al. (2021) investigated the characteristics of IFs at the head of a single pile embedded in dry, homogeneous, cohesionless Gifu sand, with a relative density of 78 % and a friction angle of 40.7°. The experimental setup is shown in Figure 1. In the study, the scaling law derived by Kokusho and Iwatate (1979) was applied, with a geometric scaling ratio of 0.05 between the model and the prototype under 1*g* conditions. A solid cylindrical pile made of a polyoxymethylene homopolymer (POM-H), with a diameter of 40 mm and a length of 900 mm, was used. The pile head was excited with a wide range of frequencies and subjected to small to large amplitude harmonic loadings. In addition, static monotonic loadings were applied to obtain the force-displacement relationship at the head (hereinafter referred to as the “back-bone curve”). Shrestha et al. (2021) reported that the real part of the IFs exhibited clear frequency dependency and a decreasing trend with increasing loading amplitude. In their study, a significant reduction in dynamic stiffness (the real part of the IFs) was observed below the resonant frequency, primarily due to the combined effect of yielding of the surrounding soil and the interface nonlinearity between the soil and the pile. Moreover, even near and above resonance, where pile-head displacements were relatively small, a decrease in stiffness and a downward shift in resonant frequency were also evident, likely reflecting the strain-dependent characteristics of the soil. An intriguing observation was also made regarding the dynamic stiffness and the stiffness of the back-bone curve, ultimately revealing the relationship between frequency dependency and nonlinearities in the system.

Figure 2 presents a comparison between the real part of the IFs (K_{hh}) with the secant stiffness (K_{stat}) extracted from the back-bone curve at the corresponding displacements where the IFs were evaluated. The values were normalized by the product of small-strain elastic modulus of soil (E_s) and the pile diameter (d). The figure shows that, under small loading amplitudes, the dynamic stiffness (K_{hh}) and the static stiffness (K_{stat}) differ due to frequency dependency. However, as the amplitude of loading increases, particularly approaching the maximum load of the excitation system (referred to as the “Max-case”), the dynamic stiffness converges towards the static stiffness.

Based on these results, the study concluded that the dynamic stiffness exhibits a characteristic trend of converging to the secant static stiffness from the back-bone curve as the loading amplitude increases. This convergence diminishes the frequency dependency as the soil-pile system undergoes yielding.



(a) (b)

Fig. 1 a) Cross-sectional layout of the single pile embedded in dry Gifu sand within a shear box and b) instrumented model pile used under 1g conditions (not to scale; dimensions in mm). The experimental setup, developed by Shrestha et al. (2021), was designed to investigate frequency- and amplitude-dependent impedance functions at the pile head under a wide range of harmonic loadings

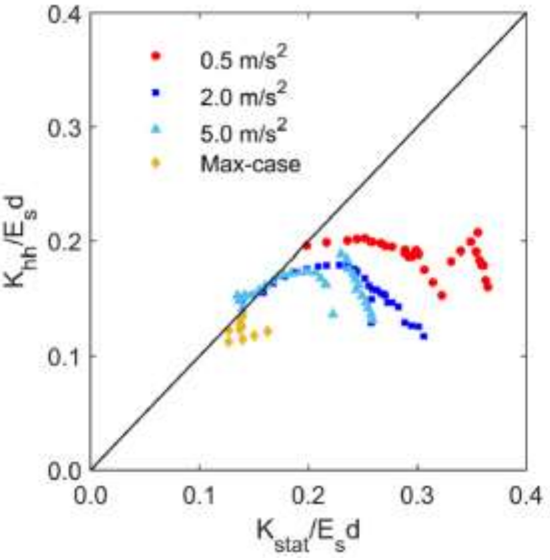


Fig. 2 Comparison between the real part of the impedance function K_{hh} and the secant stiffness K_{stat} for the single pile, obtained from the back-bone curve at corresponding displacement amplitudes. Both quantities are normalized by the product of the small-strain elastic modulus of the soil E_s and the pile diameter d . The figure illustrates the convergence of K_{hh} toward K_{stat} with increasing loading amplitude, indicating a reduction in frequency dependency due to yielding of the soil-pile system (adapted from Shrestha et al. (2021))

IMPEDANCE FUNCTIONS OF 2×2 PILE GROUP UNDER EXTREME LARGE AMPLITUDES OF LOADING

Shrestha et al. (2023) extended their study on single piles to pile groups in a straightforward manner. The experimental setup is shown in Figure 3, where a floating 2×2 pile group was investigated. The scaling ratio, as well as the soil and pile properties, were identical to those used in the single pile experiment mentioned earlier. The heads of the piles were connected to a rigid cap, which was excited in the same way as the single pile.

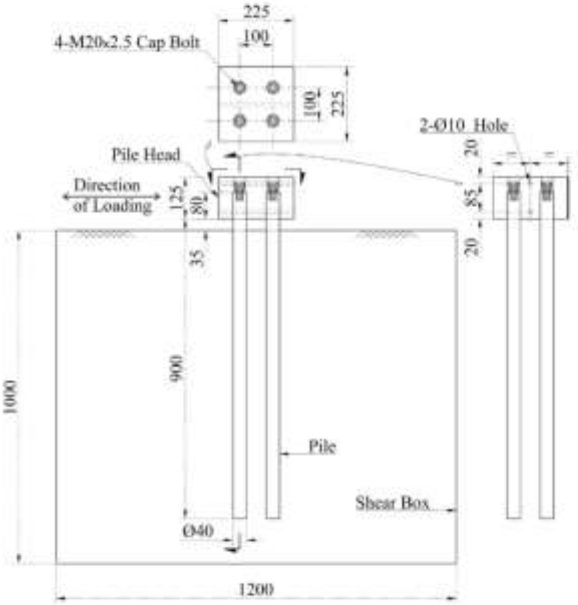


Fig. 3 Cross-sectional layout of the 2×2 pile group embedded in dry Gifu sand within a shear box (not to scale, dimensions in mm). The experimental setup, adapted from Shrestha et al. (2023), was designed to study frequency- and amplitude-dependent impedance functions at the rigid pile cap under a wide range of harmonic loadings

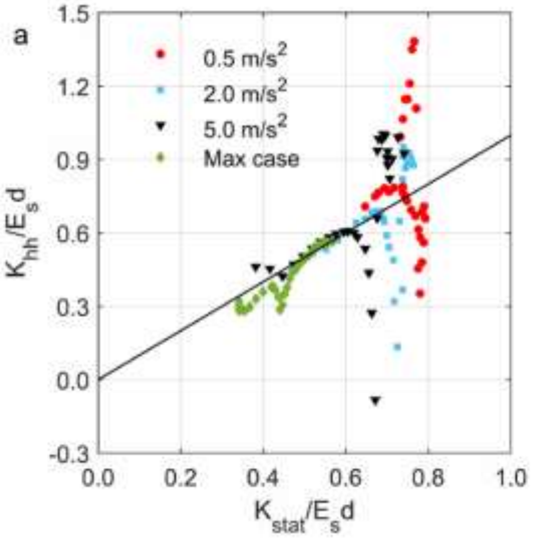


Fig. 4 Comparison between the real part of the impedance function K_{hh} and the secant stiffness K_{stat} for the 2×2 pile group, obtained from the back-bone curve at corresponding displacement amplitudes. Both quantities are normalized by the product of the small-strain elastic modulus of the soil E_s and the pile diameter d. As the loading amplitude increases, the dynamic stiffness K_{hh} gradually converges toward the static stiffness K_{stat}, indicating a reduction in frequency dependency due to yielding in the soil-pile system. This behavior is similar to that observed in the single pile case (adapted from Shrestha et al. (2023))

The study found that the pile group exhibited a similar convergence trend to that observed in the single pile, as shown in Figure 4. While there was a notable discrepancy between the dynamic stiffness and the secant static stiffness of the pile group, the dynamic stiffness tended to converge towards the secant static stiffness of the back-bone curve as the loading amplitude increased, thereby reducing its frequency dependency as the soil-pile system underwent yielding.

Consequently, this convergence behavior of IFs can be considered as a general trend, a finding that has long been sought after in the geotechnical earthquake engineering field.

PHENOMENOLOGICAL MODEL OF FREQUENCY-DEPENDENT IMPEDANCE FUNCTIONS WITH NONLINEARITIES IN SOIL-PILE SYSTEMS

Based on these experimental results, the author recently proposed a phenomenological model to reproduce the frequency dependency of IFs along with their nonlinear convergence trend (Saitoh, 2022). To capture the experimentally observed decrease in dynamic stiffness with increasing loading amplitude, the model incorporates a nonlinear yielding mechanism. Furthermore, to account for both the reduction in stiffness and the downward shift in resonant frequency, the model introduces frequency-dependent behavior by allowing certain parameters to vary with the amplitude of loading, thereby reflecting the strain-dependent characteristics of the soil. While the full details of the model are provided in the cited paper, a brief overview is presented below.

Figure 5 illustrates the conceptional behavior of the model, depicting the force-displacement relationship of the system. The stiffness of the model varies from the static stiffness K_0 depending on the excitation frequency due to the frequency-dependent characteristics when the loading level is small. However, beyond a certain threshold force, the frequency dependency diminishes, converting toward the back-bone curve and emulating the convergent behavior observed in the experimental results.

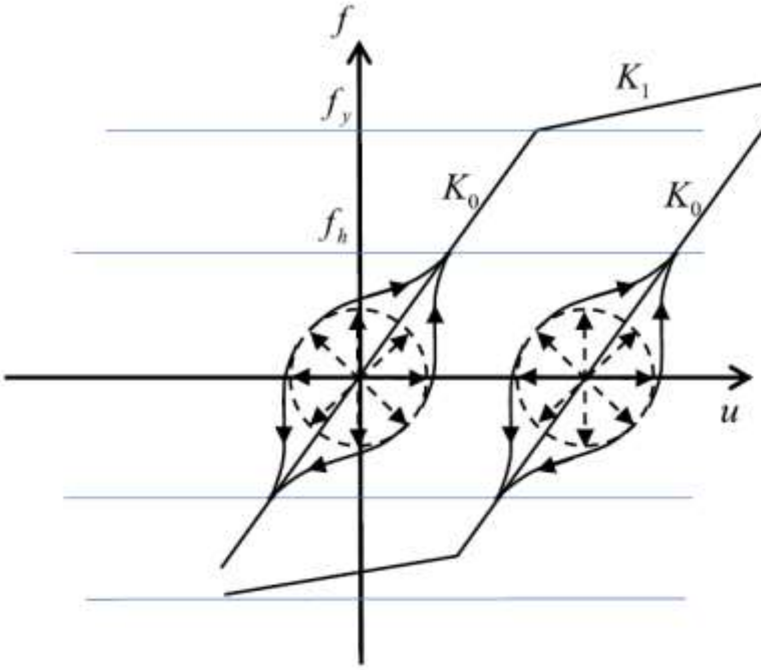


Fig. 5 Conceptual illustration of the hysteretic force–displacement behavior in the proposed phenomenological model, which incorporates both frequency- and amplitude-dependent characteristics of impedance functions. At low loading amplitudes, the model exhibits frequency-dependent stiffness, deviating from the static stiffness. As the loading amplitude increases beyond a certain threshold, the frequency dependency diminishes, and the response converges toward the backbone curve, emulating the nonlinear convergence behavior observed in experiments (adapted from Saitoh (2022))

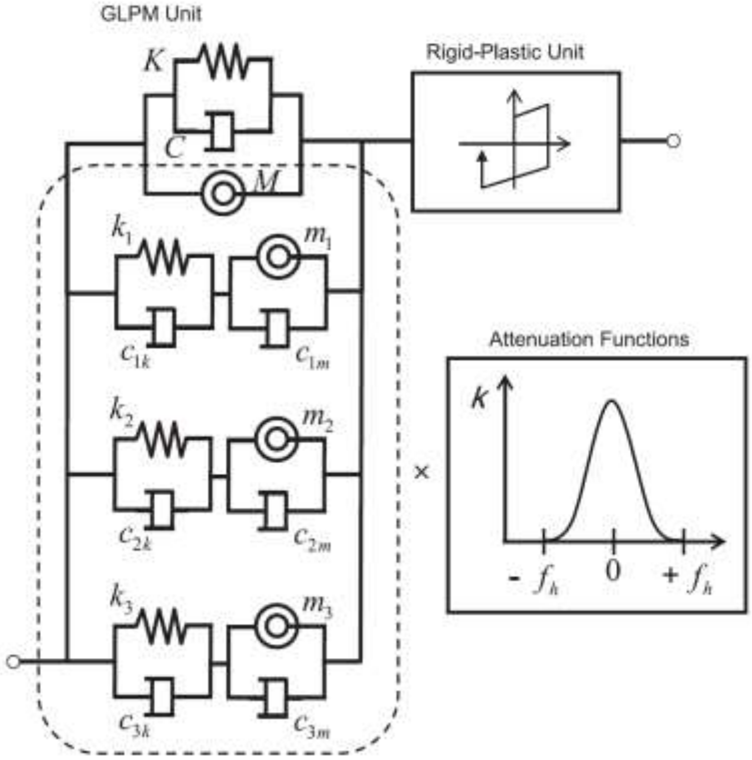


Fig. 6 Configuration of the proposed extended gyro-lumped parameter model (eGLPM), consisting of a GLPM unit with attenuation functions and a rigid–plastic unit. The GLPM unit incorporates gyro-mass elements to simulate frequency-dependent impedance behavior, while the attenuation functions modulate this dependency according to loading amplitude. The rigid–plastic unit represents the nonlinear hysteretic response using a simplified bilinear approximation of the backbone curve (adapted from Saitoh (2022))

Figure 6 shows the mechanical expression of the behavior presented in Figure 5. This mechanical model consists of a gyro-lumped parameter model (GLPM) with attenuation functions and a unit representing the nonlinear hysteretic behavior. The GLPM was originally proposed by Saitoh (2007) and further generalized by Saitoh (2012). A key feature of the GLPM is the use of a “gyro-mass”, where the reaction force is proportional to the relative acceleration of the two nodes in which the element is placed. Instead of an ordinary mass, using the gyro-mass allows for the reproduction of intricate frequency-dependent variations in IFs with fewer elements. Attenuation functions are introduced to represent the variation in stiffness that depends on both frequency and amplitude. These functions are applied as multiplicative factors to the frequency-dependent components of the GLPM, gradually reducing the degree of frequency dependence as the loading amplitude increases. Through these functions, the model is capable of reproducing both the observed convergence trend and the downward shift in resonant frequency associated with strain-dependent soil behavior. In Saitoh (2022), three types of attenuation functions were considered: a linear triangle function, a Gauss peak function, and a Modified Gaussian-Lorentzian cross function (MGL function). Their performance was validated through comparison with the experimental results of single pile. The detailed mathematical expressions of these functions are omitted here for brevity and can be found in the original publication. The second unit is associated with the back-bone curve, incorporating a hysteretic characteristic in the system. For simplicity, the static hysteretic curve is replaced by a standard bilinear model. In the study, this proposed model was referred to as the ‘‘extended GLPM”, or briefly, ‘‘eGLPM”, and further details of modeling and its validation and verification are provided in that study.

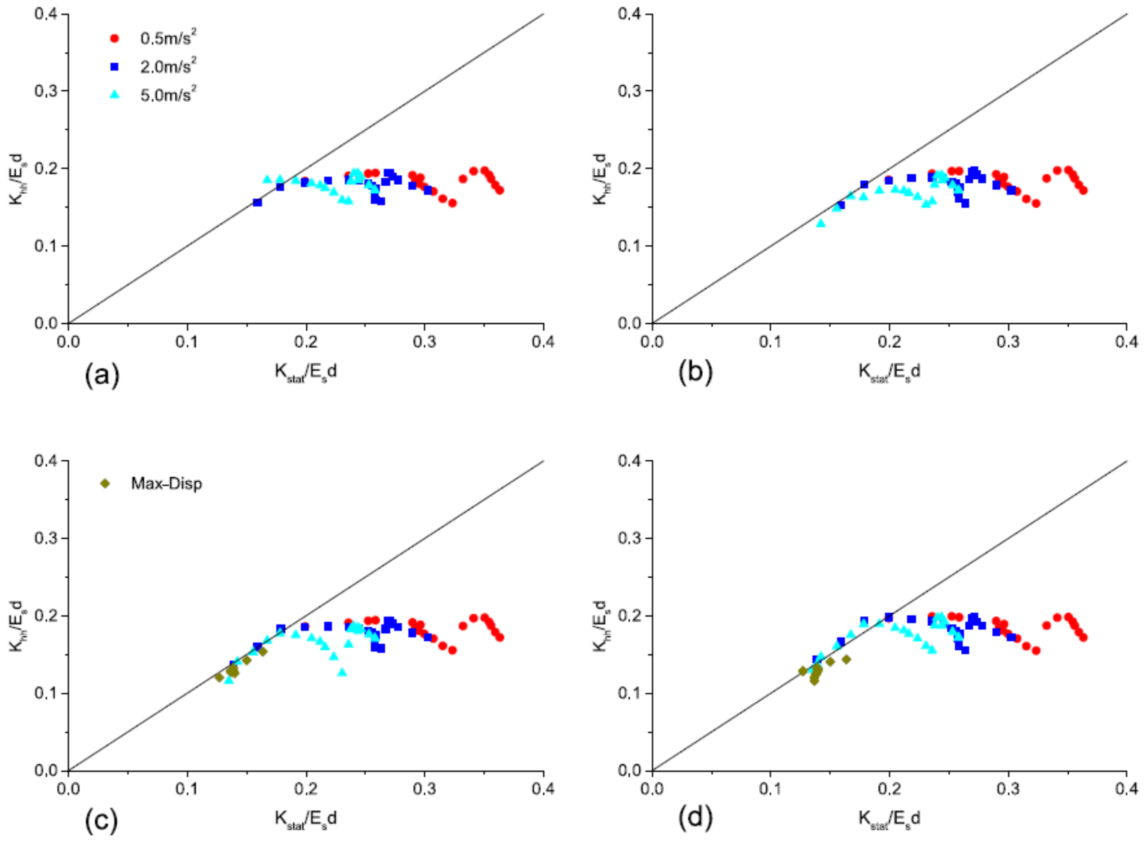


Fig. 7 Comparison between the dynamic stiffness K_{hh} obtained from the proposed model and the static secant stiffness K_{stat} from the backbone curve at corresponding displacement amplitudes, for various attenuation function cases: (a) linear triangle, (b) Gauss peak, (c) modified Gaussian–Lorentzian (MGL), and (d) no attenuation function. The figure illustrates how the dynamic stiffness K_{hh} converges toward the static stiffness K_{stat} with increasing loading amplitude, reproducing the nonlinear behavior observed in experiments. Among the evaluated cases, only the MGL function achieved results for the maximum amplitude, owing to better computational convergence (adapted from Saitoh (2022))

Figure 7 presents the relation between the dynamic stiffness obtained from the proposed models and the static secant stiffness of the back-bone curve at corresponding displacements for which the IFs are evaluated. The figure shows that under small loading amplitudes, the dynamic stiffness deviates from the static secant stiffness due to frequency dependence, while a convergence trend emerges in a manner closely resembling the experimental result shown in Figure 2. In particular, the data points at the maximum excitation amplitude align well with the equivalent stiffness line. Among the cases with attenuation functions, only the MGL function successfully yielded IFs at the maximum excitation amplitude, as computational convergence issues were encountered with the other functions. These issues can be attributed to discontinuities or rapid local variations inherent in the triangle and Gauss peak functions. In contrast, the MGL function exhibits a more gradual variation, making it more suitable for this model. Furthermore, as demonstrated in Saitoh (2022), although the model without attenuation functions cannot capture detailed high-frequency variations in IFs, it provides smooth and stable convergence under large loading amplitudes, indicating its potential as a practical tool for approximate simulations.

CONCLUSIONS

This keynote paper presented an overview of the recently discovered phenomena regarding frequency-dependent impedance functions of both single piles and pile groups affected by nonlinearities in the system as observed from model experiments conducted by the author and the team at Saitama university. In addition, a phenomenological model representing the convergence of dynamic stiffness toward the static secant stiffness from the back-bone curve was also discussed. Thus far, model verification has been limited to experimental results for single piles. Nevertheless, given that pile groups exhibit a similar convergence trend in impedance functions, the model is considered to have high applicability to such configurations. However, its behavior under different foundation types or soil conditions – particularly in cases lacking experimental validation – remains uncertain. Further research is essential to clarify convergence trends under a wide range of conditions. These studies are expected to improve the accuracy of dynamic response estimation in structural systems interacting with soil-foundation systems and will encourage further research, contributing to the ongoing development in this field.

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