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2-D DEFORMATION OF EARTH INDUCED BY A BLIND VERTICAL DIP-SLIP FAULT

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ABSTRACT

A solution of the plane strain deformation of a two layered Earth model consisting of a homogeneous, elastically isotropic layer (EIL) of finite thickness lying on a homogeneous, elastically isotropic half-space (EIHS) induced by two-dimensional faulting placed in the EIHS is obtained. The Airy's stress function method is used to obtain the deformation field in the integral form induced by a blind dip-slip faulting. The linear combination of exponential terms occurring in the denominator of integral expressions is estimated as a finite sum of exponential terms (FSET) employing least square method. Then the integrals are evaluated analytically. The displacement field is computed analytically and numerically for two different models of Earth. The stresses can be computed similarly.

MSC 2010: 86A17, 74B05

KEYWORDS: Plane Strain Deformation; Elastically Isotropic Layer; Elastically Isotropic Half-Space; Continental Crust; Oceanic Crust

INTRODUCTION

The foci of most of the earthquakes lie in the crust of the Earth. Recently, Jamtveit et al. (2018), Menegon et al. (2017) and Petley-Ragan et al. (2018) have demonstrated that during an organic process, dynamic ruptures occur at an early stage in the structural and metamorphic development of the lower crust. Microstructural evidence suggests that seismic slip in highly stressed rocks may be initiated by brittle deformation and the rock damage caused by dynamic ruptures play a significant role in deep crustal earthquakes. In this paper an attempt is made to know the deformation when the fault exists in the lower ductile crustal layer.

A two layered model of Earth consisting of a layer over a half-space representing the lithosphere-asthenosphere is considered by several investigators assuming the elastic layer with the elastic/viscoelastic half- space. Singh et al. (1997) derived the displacements analytically by approximating the denominator as FSET, for the static deformation of a homogeneous, elastically isotropic layer of finite thickness welded with an elastic isotropic half space. Verma et al. (2016) examined the plane strain deformation induced by dip-slip faulting in a poroelastic stratum lying on an elastic half space using stress function approach. Kumari and Madan (2022) obtained analytic stresses caused by a vertical dip-slip fault embedded in the upper half-space, considering a two phase medium consisting of homogeneous, elastically isotropic half-space joined with an orthotropic half space.

A detailed solution for the lithospheric plane strain deformation of a two layered composition consisting of a homogeneous, elastically isotropic stratum of finite thickness lying on a homogeneous, elastically orthotropic substratum produced by two-dimensional faulting placed in the isotropic stratum has been obtained by Soni and Rani (2023). They used the least square method to calculate the integrals analytically and elaborated the deformation field for a vertical dip-slip line dislocation. Cao and Mavroeidis (2024) investigated the spatial and temporal characteristics of near-fault ground strains and rotations from actual earthquakes with predominantly dip-slip mechanisms. They performed forward ground-motion simulations of the 1994 Mw 6.7 Northridge, the 1989 Mw 6.9 Loma Prieta, and the 1985 Mw 8.1 Michoacan earthquakes using finite-fault rupture models.

In this paper, the deformation of upper crust due to a normal fault situated in elastic lower crust has been studied. Our aim is to find the analytical solution for a two layer model consisting of a homogeneous, isotropic, elastic layer welded with a homogeneous, isotropic, elastic half space. A dip-slip line source is situated in the elastic half-space overlain by an elastic layer. A line dislocation is a point source continuously located along a line. The Airy's stress function technique has been used to obtain the deformation field in the integral form. Employing the least squares method (Scarborough, 1955), the linear combination of exponential terms that occur in the denominator of the integral expressions of the deformation field is approximated as FSET (Sneddon, 1951). Then the integrals are solved analytically using standard integral tables. The displacement field is computed numerically for a vertical dip-slip line dislocation (VDLD) located at the interface of the two phase medium for the Earth's oceanic crust and the continental crust models (Ben Menahem and Gillon, 1970).

MATHEMATICAL FORMULATION

In the Cartesian coordinate system (x, y, z), for a two-dimensional approximation the displacement components $\vec{u} = (u_x, u_y, u_z)$ are independent of one coordinate, say x, therefore $\frac{\partial}{\partial x} \equiv 0$. Hence, the plane strain problem $(u_x = 0)$ and the anti-plane strain $(u_y = u_z = 0)$ are split up and can be computed independently. We consider the plane strain problem in the current paper. Let a two layered medium is made up of a homogeneous EIL of finite thickness H lying over a homogeneous EIHS $(z \geq H)$. The z-axis is taken vertically downwards with origin at the free surface of the layer (Figure 1).

The stress strain relation for a homogeneous, elastically isotropic medium characterized by the elastic constants: Poisson ratio (ν) and shear modulus (μ) is given by

$$\begin{gathered} p_{yy} = 2\mu\left(e_{yy} + \frac{\nu}{1-2\nu}\theta\right), p_{zz} = 2\mu\left(e_{zz} + \frac{\nu}{1-2\nu}\theta\right), \\ p_{yz} = 2\mu e_{yz}, \quad p_{xx} = \nu(p_{yy} + p_{zz}), \quad p_{xy} = p_{zx} = 0. \end{gathered} \tag{1}$$

where $p_{ij}\,(i, j = x, y, z)$ denote the stress components and $e_{ij}\,(i, j = x, y, z)$ denote the strain components and

$$\theta = \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \tag{2}$$

is the dilatation. For an isotropic media, the plane strain problem in terms of Airy's stress function (Sokolnikoff, 1956) U satisfying the equations of equilibrium, in the yz-plane is formulated as:

$$p_{yy} = \frac{\partial^2 U}{\partial z^2}, p_{yz} = -\frac{\partial^2 U}{\partial y \partial z}, p_{zz} = \frac{\partial^2 U}{\partial y^2} \tag{3}$$

satisfying

$$\nabla^2 \nabla^2 U = 0 \tag{4}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{5}$$

After integrating the constitutive Equations (1), we get the displacements which are:

$$2\mu u_y = -\frac{\partial U}{\partial y} + (1-\nu)\int \nabla^2 U dy, \tag{6}$$

$$2\mu u_z = -\frac{\partial U}{\partial z} + (1-\nu)\int \nabla^2 U dz, \tag{7}$$

Using Fourier transform of (4) with respect to y and solving the corresponding ordinary differential equation for the elastic layer $\left(0 \le z \le H\right)$ having a line source parallel to the x-axis acting at $(0,0,h)$, a suitable solution of (4) is as follows:

$$U^{(1)} = \int_0^\infty \left[\left(C_1 + C_2 kz\right)e^{-kz} + \left(C_3 + C_4 kz\right)e^{kz}\right]\begin{pmatrix}\sin ky \\ \cos ky\end{pmatrix} dk \tag{8}$$

where $C_i \left(i = 1,2,3,4\right)$ are the unknown constants. From equations (3), (6)-(7) and (8), the stresses and the displacements for the isotropic layer are computed.

Let there be a line source parallel to $x-$axis passing through the point $(0,0,h)$ in the EIHS $(z \ge H)$ (Figure1).The Airy's stress function U_0 for a line source parallel to the $x-$axis located at the point $(0,0,h)$ in the EIHS is given by Rani and Singh (2007):

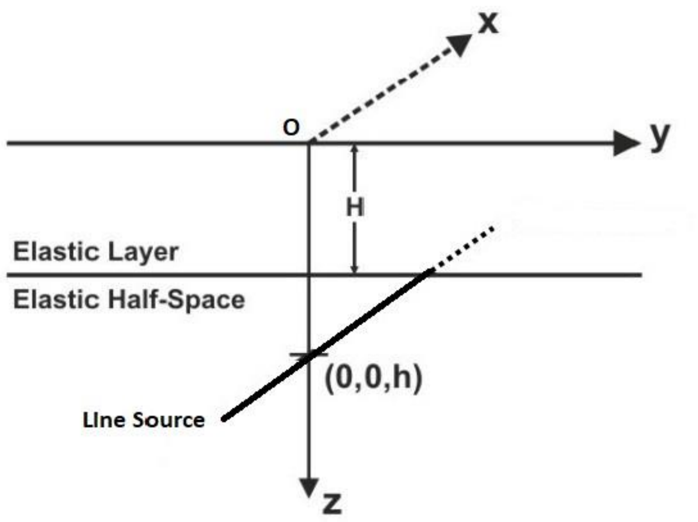


Fig. 1 An Earth model consisting of an EIL of finite thickness H $\left(0 \le z \le H\right)$ lying over an isotropic half-space $(z \ge H)$ with a line source parallel to $x-$axis situated at the point $(0,0,h)$ in the half-space

$$U_0 = \int_0^\infty \left(S_1 + S_2 \varepsilon k Z_1\right) e^{-\varepsilon k Z_1} \begin{pmatrix}\sin ky \\ \cos ky\end{pmatrix} \frac{dk}{k} \tag{9}$$

where

$$Z_1 = z - h, \quad \varepsilon = \pm 1 \tag{10}$$

the upper sign stands for $Z_1 > 0$ and the lower sign for $Z_1 < 0$ and S_1, S_2 are the source coefficients given in Table 1 for vertical dip slip fault and dip slip fault dipping at 45° explained by Soni and Rani (2023). Let the half-space is characterized by the elastic constants: Poisson ratio $\left(\nu'\right)$ and shear modulus (μ') and quantities related to half-space are denoted by dashed notation. Again, solving (4), the Airy stress function for the EIHS $(z \ge H)$ with a line dislocation, is as follows:

$$U' = U_0 + \int_0^\infty \left(A_1 + A_2 kz\right) e^{-kz} \begin{pmatrix}\sin ky \\ \cos ky\end{pmatrix} dk \tag{11}$$

where A_1 and A_2 are the arbitrary constants. From (3), (6)-(7) and (11), the stresses and the displacements are obtained for the half-space.

Table 1: Source coefficients for the Vertical dip slip and Dip slip fault dipping at 45° **, where** F_{23}, F'_{23} **are the moments for the corresponding sources**

Source	S_1	S_2	**Solution**
Vertical dip slip	0	$\frac{\varepsilon F_{23}}{2\pi(1-\nu')}$	Upper
Dip slip fault dipping at 45°	0	$\frac{F'_{23}}{2\pi(1-\nu')}$	Lower

BOUNDARY CONDITIONS

The surface of the layer $z=0$ is supposed to be stress free which gives

$$p_{yz} = p_{zz} = 0 \tag{12}$$

at $z=0$. Also, the elastic layer is in welded contact with the EIHS along $z=H$, hence the boundary conditions are:

$$\begin{aligned} p_{yz} &= p'_{yz}, \quad p_{zz} = p'_{zz}, \\ u_y &= u'_y, \quad u_z = u'_z, \end{aligned} \tag{13}$$

at $z=H$. We notice that the source components S_1 and S_2 varies for $Z_1 > 0$ and $Z_1 < 0$. Let S'_1, S'_2 be the values of S_1 and S_2 for $Z_1 < 0$. Using the boundary conditions (12)-(13), the system of six equations in six unknowns $C_i, (i=1,2,3,4)$ and $A_i (i=1,2)$ is obtained. Applying Cramer's rule to evaluate the system of equations, we find the unknowns constants $C_i, (i=1,2,3,4)$ and $A_i (i=1,2)$ given in Appendix A.

VERTICAL DIP-SLIP LINE DISLOCATION

According to Maruyama (1966), for the vertical dip-slip fault, the moment is equivalent to

$$F_{23} = \mu b ds \tag{14}$$

where b represents the magnitude of the slip and ds represents the width of the line source in the z-direction. Hence, from Table 1, the source components for a VDLD are

$$S_1 = S'_1 = 0, \quad S_2 = \frac{\varepsilon \mu' b ds}{2\pi(1-\nu')}, \; S'_2 = \frac{-\mu' b ds}{2\pi(1-\nu')} \tag{15}$$

On substituting the values of $S'_1, S'_2, C_i, (i=1,2,3,4)$ and $A_i (i=1,2)$ in the displacements and stress field for the EIL and for the EIHS ,we obtain the solution in the integral form. Here, the expressions for the displacements for EIL $(0 \le z \le H)$ are given below:

$$
\begin{aligned}
u_y = \frac{mbds}{\pi\lambda_1\lambda_2}\int_0^\infty \Big[& \left\{\lambda_3 - 2(1-\nu)\lambda_2 + k\left(2\lambda_3 H + \lambda_1 h - \lambda_2 z\right)\right\} e^{-k(h-z)} \\
& + \left\{-\lambda_3 - 2(1-\nu)\lambda_1 + k[\delta_1 z + (3-4\nu)(2\lambda_3 H + \lambda_1 h)] - 2k^2 z\left(2\lambda_3 H + \lambda_1 h\right)\right\} e^{-k(h+z)} \\
& + \left\{\lambda_3 - 2(1-\nu)(m-1) + k(m-1)[z - h - 4(1-\nu)Z_2] + 2k^2(m-1)(z-H)Z_2\right\} e^{-k(2H+h+z)} \\
& + \Big\{2(1-\nu)\left(2\lambda_3 - m + 1\right) - \lambda_3 + k[(m-1)\left(h - z - 4(1-\nu)H\right) + 2\lambda_3 z] \\
& - 2k^2 H(m-1)\left\{z + (3-4\nu)Z_2\right\} - 4k^3 zH(H-h)(m-1)\Big\} e^{-k(2H+h-z)} \Big] \frac{\cos ky}{\Delta'} dk
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
u_z = \frac{mbds}{\pi\lambda_1\lambda_2}\int_0^\infty \Big[& \left\{\lambda_3 + (1-2\nu)\lambda_2 + k\left(2\lambda_3 H + \lambda_1 h - \lambda_2 z\right)\right\} e^{-k(h-z)} \\
& + \left\{\lambda_3 - (1-2\nu)\lambda_1 - k[\lambda_1 z - (3-4\nu)(2\lambda_3 H + \lambda_1 h)] + 2k^2 z\left(2\lambda_3 H + \lambda_1 h\right)\right\} e^{-k(h+z)} \\
& - \left\{\lambda_3 + (1-2\nu)(m-1) + k(m-1)[z - h + 2(1-2\nu)Z_2] + 2k^2(m-1)(z-H)Z_2\right\} e^{-k(2H+h+z)} \\
& - \Big\{(1-2\nu)\left(2\lambda_3 - m + 1\right) + \lambda_3 + k[(m-1)\left(z - h - 2(1-2\nu)H\right) - 2\lambda_3 z] \\
& + 2k^2 H(m-1)\left\{z - (3-4\nu)Z_2\right\} + 4k^3 zH(H-h)(m-1)\Big\} e^{-k(2H+h-z)} \Big] \frac{\sin ky}{\Delta'} dk
\end{aligned}
\tag{17}
$$

where

$$\Delta' = 1 + \left(A + Bk^2H^2\right)e^{-2kH} + Ce^{-4kH},$$

$$A = \frac{\lambda_1(\lambda_1 - \lambda_2) + (\lambda_1 + \lambda_2)(m-1)}{\lambda_1\lambda_2}, B = \frac{4(m-1)}{\lambda_1}, \quad C = \frac{(1-m)(\lambda_2 - \lambda_1 - m + 1)}{\lambda_1\lambda_2}, \tag{18}$$

On substituting $h = H$ in Equations (16)-(17), the fault lies on the interface of the two layered medium. The displacement field of the layer agrees with the corresponding displacements given by Singh *et.al.* (1997) at the interface. To obtain the displacements in the closed-form, the integrals are to be evaluated analytically. For this, the linear combination of exponential terms $1/\Delta'$ is replaced as FSET so that the error is minimum. Expanding

$$\frac{1}{\Delta'} = \left[1 + \left(A + Bk^2H^2\right)e^{-2kH} + Ce^{-4kH}\right]^{-1} \tag{19}$$

by Binomial theorem and approximating as

$$\frac{1}{\Delta'} \approx 1 - \left(A + Bk^2H^2\right)e^{-2kH} + \left\{C' + \alpha(kH)^n\right\}e^{-\beta kH} \tag{20}$$

where, C' is a constant independent of kH; α, β, n depending upon kH, m, ν, ν' are to be computed employing the method of least squares. The asymptotic approximation is used to find the value of C' by taking the limit as $kH \to 0$ which gives

$$C' = \frac{A^2 + AC - C}{A + C + 1} \tag{21}$$

for all $n = 0, 1, 2, 3.....$ In our calculations, we choose $n = 2$. Therefore,

$$\frac{1}{\Delta'} = 1 - \left(A + Bk^2H^2\right)e^{-2kH} + \left(C' + \alpha k^2H^2\right)e^{-\beta kH} \tag{22}$$

We consider two models of Earth: continental crust model and oceanic crust model [1970]. For the continental crust model,

$$\nu = \nu' = 0.25, \quad \mu'/\mu = 2.22 \tag{23}$$

and for oceanic crust model,

$$\nu = \nu' = 0.3, \quad \mu'/\mu = 1.76 \tag{24}$$

Employing the least square method for the nonlinear case given by Scarborough (1955), the sum of squares of deviations given by

$$\sum_i \left(a_0 \frac{\partial f}{\partial \alpha} + b_0 \frac{\partial f}{\partial \beta} + R_i \right)^2 \tag{25}$$

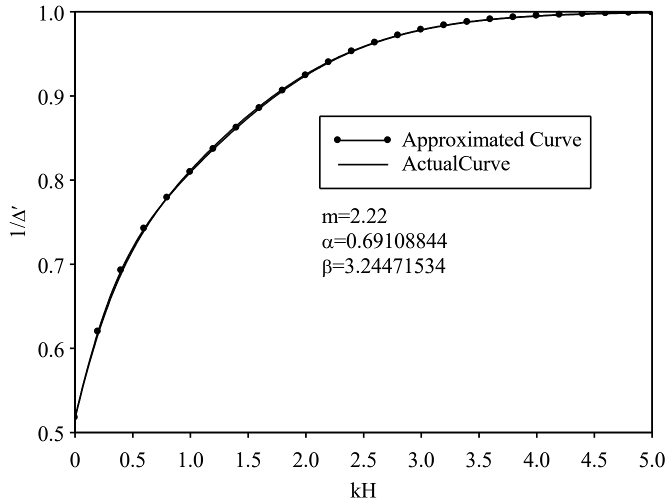
should be minimized, where a_0, b_0 are the corrections to the approximated values of α, β respectively and

$$R_i = \left(\frac{1}{\Delta'}\right)_{apprimated} - \left(\frac{1}{\Delta'}\right)_{actual} \tag{26}$$

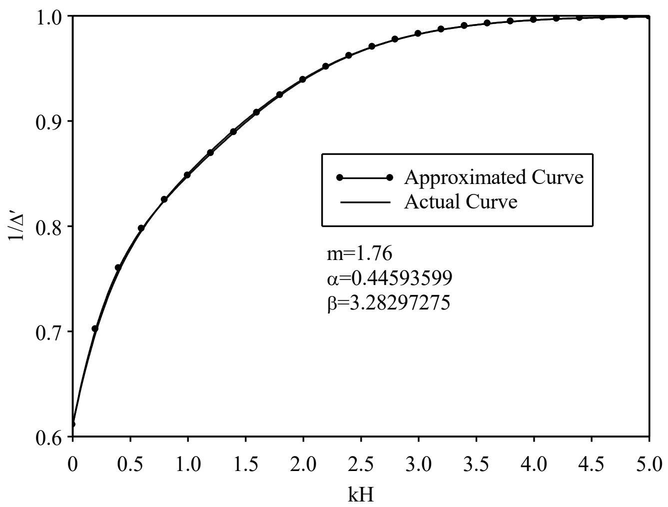
are the residuals and $f(kH, \delta_1, \delta_2, \beta)$ is the right side of Equation (22). Beginning with the initial approximation of α, β using iterative process, the best fitted values of α, β given in (22) are obtained, listed in Table 2. Figure 2(a)-2(b) shows the comparison of actual $\frac{1}{\Delta'}$ (Equation (19)) and the approximated $\frac{1}{\Delta'}$ (Equation (22)) with kH for the continental case and for the oceanic case respectively.

Table 2: Best fitted values of α and β for continental crust model and oceanic crust model

Crust Models	ν	ν'	α	β
Continental $\mu'/\mu = m = 2.22$	0.25	0.25	0.69108844	3.24471534
Oceanic $\mu'/\mu = m = 1.76$	0.3	0.3	0.44593599	3.28297275



(a)



(b)

Fig. 2 Comparison of the actual curve with the approximated curve of $1/\Delta'$ for (a) continental crust model (b) oceanic crust model

On substituting the values of α and β from Table 2 in the expression of $\frac{1}{\Delta'}$ given in (22), the displacements for the EIL given in (16)–(17) can be expressed as a linear combination of integrals, and these integrals are solved using the standard integrals (Gradshteyn and Ryzhik [2014]). The expressions for the displacements u_y and u_z for the EIL are given below:

$$u_y = \frac{mbds}{\pi\lambda_1\lambda_2}\left\{\phi_1 - A\phi_2 - BH^2\phi_3 + C'\phi_4 + \alpha H^2\phi_5\right\} \tag{27}$$

$$u_z = \frac{mybds}{\pi\lambda_1\lambda_2}\left\{\psi_1 - A\psi_2 - BH^2\psi_3 + C'\psi_4 + \alpha H^2\psi_5\right\} \tag{28}$$

where

$\phi_i (i = 1,2,3,4,5)$, $\psi_j (j = 1,2,3,4,5)$ are given in Appendix B. The stresses can be computed similarly. On taking the limit $H \to \infty$, the displacements for a vertical dip-slip line source in a uniform half-space coincide with the displacements given by Rani *et.al*. (1991).

NUMERICAL RESULTS

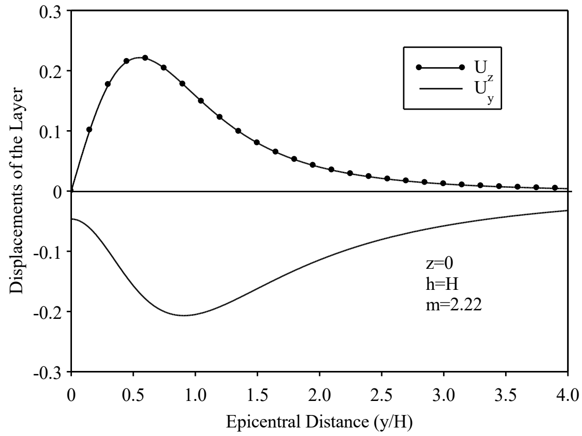
We compute the displacement field numerically for the continental crust model and oceanic crust model using the parameters given in Table 2. Let the VDLD lies on the interface of the two layered medium at $h = H$ taking the fault location at the material discontinuity. For numerical computations, the following quantities are normalized as:

$$U_y = \frac{u_y H}{bds}, \quad U_z = \frac{u_z H}{bds}, \quad Y = \frac{y}{H}, Z = \frac{z}{H} \tag{29}$$

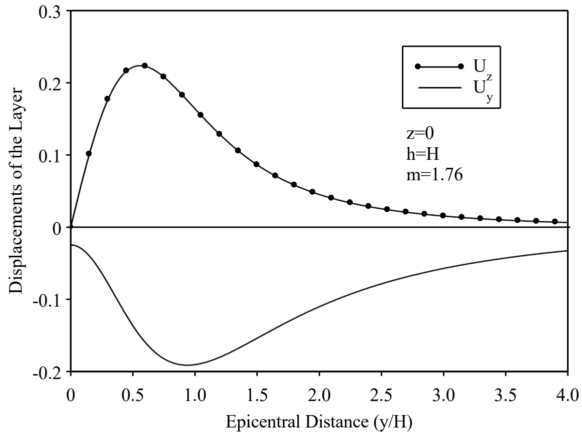
where U_y, U_z are the normalized displacements of the layer, Y is the normalized epicentral distance and Z is the normalized focal depth. Therefore, the displacements $u_i, (i = y, z)$ are computed in units of $\frac{bds}{H}$.

Figure 3(a)-3(b) depict the behavior of horizontal displacement U_y and the vertical displacement U_z at the surface of the layer $z = 0$ with epicentral distance y/H for continental crust and the oceanic crust, respectively. We find that the vertical displacement U_z is more significant than the horizontal displacement U_y. Also U_y and U_z exhibit reverse behavior.

Figure 4(a)-4(b) demonstrate the contour map of the horizontal displacement in units of $U_y \times 10^2$ in the layer for continental crust and Oceanic crust. The displacement has a significant variation near the fault location $(0, H)$. The isolines are shown in the plot and the negative values of U_y are denoted by dashed lines. Figure 5(a)-5(b) displays the contour map of the vertical displacement in units of $U_z \times 10^2$ in the layer for continental crust and Oceanic crust. The displacement U_z has a remarkable variation near the fault. Solid lines denote the positive values of U_z and dashed lines denote negative values. Also, there is a symmetry about the line $y = 0$.

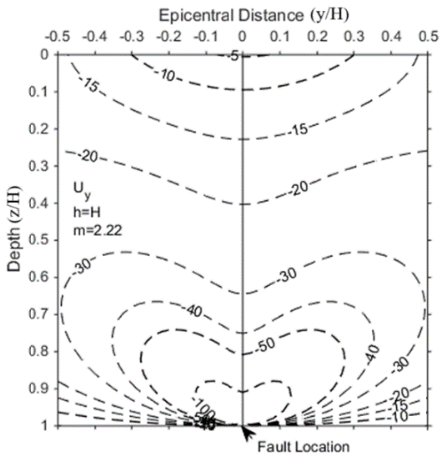


(a)

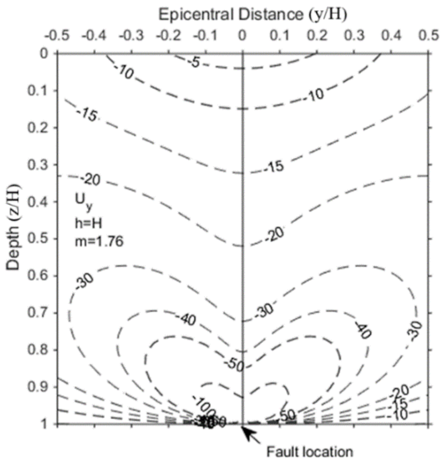


(b)

Fig. 3 Graph showing the variation of the dimensionless horizontal displacement U_y and vertical displacement U_z of the stratum with normalized horizontal distance from a VDLD located at the interface on the surface $z = 0$ for (a) continental crust model (b) oceanic crust model

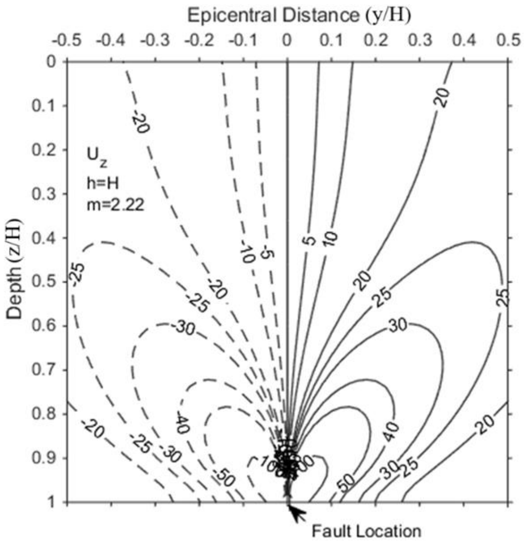


(a)

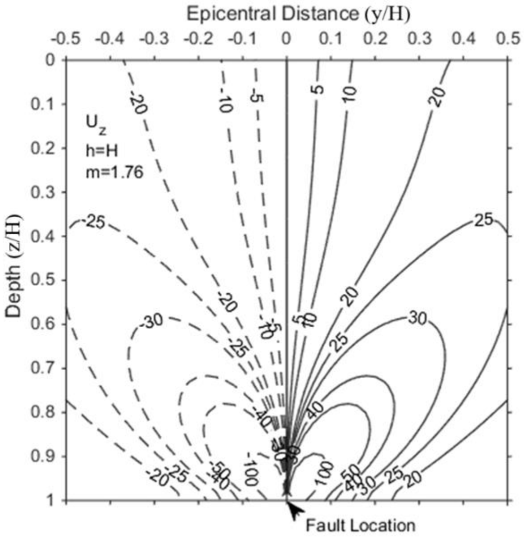


(b)

Fig. 4 Contour map of the normalized horizontal displacement U_y for a VDLD located at $h = H$ of the stratum for (a)continental crust model(b) oceanic crust model



(a)



(b)

Fig. 5 Contour map of the normalized horizontal displacement U_z for a VDLD located at $h = H$ of the stratum for (a) continental crust model (b)oceanic crust model

CONCLUSIONS

- We have obtained the analytical solution for a two-layered composition with a homogeneous EIL lying on a homogeneous EIHS. A dip-slip line source is situated in the elastic half-space overlain by an elastic layer.
- The Airy's stress function technique has been used to obtain the deformation field in the integral form.
- The least squares method is employed to approximate the linear combination of exponential terms that occur in the denominator of the integral expressions of the deformation field as FSET in the numerator and then integrals are solved analytically using standard integral tables.
- The displacement field is computed numerically for a VDLD located at the interface of the two phase medium for two Earth models considering oceanic crust and the continental crust.
- Near the source location, we observe that the displacements vary considerably. In comparison to the horizontal displacement, the vertical displacement has a larger magnitude.

RESEARCH HIGHLIGHTS

- The plane strain deformation of a two layered Earth model produced by blind dip-slip faulting is obtained.
- The Airy's stress function method is used.
- The linear combination of exponential terms occurring in the denominator is estimated as a finite sum of exponential terms employing least square method.
- The integral expressions of the displacement field are evaluated analytically. The displacement field is computed numerically for two different models of Earth.

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APPENDIX A

$$C_1=\frac{4(1-\nu')e^{-kh}}{k\Delta}\Big[S_1'\left\{\lambda_1+(m-1)(1-2kH)e^{-2kH}\right\}+S_2'\left\{\lambda_3+k\left(2\lambda_3H+\lambda_1h\right)\right\}$$
$$-S_2'e^{-2kH}\left\{\lambda_3-(m-1)\left(kh+2k^2HZ_2\right)\right\}\Big]$$

$$C_2=\frac{4(1-\nu')e^{-kh}}{k\Delta}\Big[2S_1'\left\{\lambda_1+(m-1)e^{-2kH}\right\}$$
$$-S_2'\left\{\lambda_1-2k\left(2\lambda_3H+\lambda_1h\right)\right\}-S_2'e^{-2kH}(m-1)\left(1+2kZ_2\right)\Big]$$

$$C_3=-C_1$$

$$C_4=\frac{4(1-\nu')e^{-kh}}{k\Delta}\Big[-4S_1'kH(m-1)e^{-2kH}+S_2'\lambda_2$$
$$-S_2'e^{-2kH}\left\{2\lambda_3-(m-1)\left(1+2kH+4k^2HZ_2\right)\right\}\Big]$$

$$A_1=\frac{e^{-kh}}{k\Delta}\Big[-S_1'\left(e^{-2kH}+e^{2kH}\right)\lambda_1(1-m)(1-2kH)$$
$$+S_1'\left\{\lambda_1^2+(m-1)^2+4kH[(\lambda_1+m-1)\lambda_3-(m-1)^2]+4(m-1)^2k^2H^2(1-2kH)\right\}$$
$$-S_2'\left(e^{-2kH}+e^{2kH}\right)\left\{\left(\lambda_2-m+1\right)\lambda_3+\lambda_1(1-m)\left(kh+2k^2HZ_2\right)\right\}$$
$$+S_2'\Big\{2\left(\lambda_2-m+1\right)\lambda_3-kh\left(\lambda_1^2+(m-1)^2\right)+4k^2Hh[(\lambda_1+m-1)\lambda_3-(m-1)^2]$$
$$+4k^2H^2\left(\lambda_3^{\,2}+(\lambda_3-m+1)^2\right)+4(m-1)^2\left(k^3H^2h+2k^4H^3Z_2\right)\Big\}\Big]$$

$$A_2=\frac{e^{-kh}}{k\Delta}\Big[2S_1'\left(e^{-2kH}+e^{2kH}\right)\lambda_1(m-1)+2S_1'\left\{\lambda_1^2+(m-1)^2+4(1-m)^2k^2H^2\right\}$$
$$+S_2'\left(e^{-2kH}+e^{2kH}\right)\lambda_1(1-m)\left(1+2kZ_2\right)-S_2'\Big\{(1-2kh)\left(\lambda_1^2+(m-1)^2\right)$$
$$+4kH[(\lambda_1+m-1)\lambda_3-(m-1)^2]-4(1-m)^2k^2H^2\left(1+2kZ_2\right)\Big\}\Big]$$

where

$$\Delta = -\lambda_1\lambda_2 - \left\{-2\lambda_1\lambda_3 + (\lambda_1+\lambda_2)(m-1) + 4\lambda_2(m-1)k^2H^2\right\}e^{-2kH} - (1-m)(2\lambda_3 - m + 1)e^{-4kH},$$

$$\lambda_1 = 1 + (3-4\nu)m, \quad \lambda_2 = 3 - 4\nu' + m, \quad 2\lambda_3 = \lambda_2 - \lambda_1, \quad Z_2 = H - h, \quad m = \frac{\mu'}{\mu}$$

APPENDIX B

$$\phi_1 = |X_1|_{n=0} + |X_2|_{n=2}$$

$$\phi_2 = |X_1|_{n=2} + |X_2|_{n=4}$$

$$\phi_3 = |X_3|_{n=2} + |X_4|_{n=4}$$

$$\phi_4 = |X_1|_{n=\beta} + |X_2|_{n=\beta+2}$$

$$\phi_5 = |X_3|_{n=\beta} + |X_4|_{n=\beta+2}$$

$$\psi_1 = |Y_1|_{n=0} + |Y_2|_{n=2}$$

$$\psi_2 = |Y_1|_{n=2} + |Y_2|_{n=4}$$

$$\psi_3 = |Y_3|_{n=2} + |Y_4|_{n=4}$$

$$\psi_4 = |Y_1|_{n=\beta} + |Y_2|_{n=\beta+2}$$

$$\psi_5 = |Y_3|_{n=\beta} + |Y_4|_{n=\beta+2}$$

where

$$X_1 = \frac{\{\lambda_3 - 2(1-\nu)\lambda_2\}q_n}{Q_n^2} - \frac{2\lambda_3 H + \lambda_1 h - \lambda_2 z}{Q_n^2}\left(1 - \frac{2q_n^2}{Q_n^2}\right) - \frac{\{\lambda_3 + 2(1-\nu)\lambda_1\}p_n}{P_n^2}$$
$$- \frac{\{\delta_1 z + (3-4\nu)(2\lambda_3 H + \lambda_1 h)\}}{P_n^2}\left(1 - \frac{2p_n^2}{P_n^2}\right) + \frac{4z(2\lambda_3 H + \lambda_1 h)p_n}{P_n^4}\left(3 - \frac{4p_n^2}{P_n^2}\right)$$

$$X_2 = \frac{\{\lambda_3 - 2(1-\nu)(m-1)\}p_n}{P_n^2} - \frac{(m-1)\{z - h - 4(1-\nu)Z_2\}}{P_n^2}\left(1 - \frac{2p_n^2}{P_n^2}\right) - \frac{4(m-1)(z-H)Z_2 p_n}{P_n^4}\left(3 - \frac{4p_n^2}{P_n^2}\right)$$
$$+ \frac{\{2(1-\nu)(2\lambda_3 - m + 1) - \lambda_3\}q_n}{Q_n^2} - \frac{\{(m-1)(h - z - 4(1-\nu)H) + 2\lambda_3 z\}}{Q_n^2}\left(1 - \frac{2q_n^2}{Q_n^2}\right)$$
$$+ \frac{4H(m-1)\{z + (3-4\nu)Z_2\}q_n}{Q_n^4}\left(3 - \frac{4q_n^2}{Q_n^2}\right) - \frac{24zHZ_2(m-1)}{Q_n^4}\left(1 - \frac{8q_n^2}{Q_n^2} + \frac{8q_n^4}{Q_n^4}\right)$$

$$X_3 = \frac{2\{-\lambda_3 + 2(1-\nu)\lambda_2\}q_n}{Q_n^4}\left(3 - \frac{4q_n^2}{Q_n^2}\right) + \frac{2\{\lambda_3 + 2(1-\nu)\lambda_1\}p_n}{P_n^4}\left(3 - \frac{4p_n^2}{P_n^2}\right)$$
$$+ \frac{6(2\lambda_3 H + \lambda_1 h - \lambda_2 z)}{Q_n^4}\left(1 - \frac{8q_n^2}{Q_n^2} + \frac{8q_n^4}{Q_n^4}\right) + \frac{6\{\delta_1 z + (3-4\nu)(2\lambda_3 H + \lambda_1 h)\}}{P_n^4}\left(1 - \frac{8p_n^2}{P_n^2} + \frac{8p_n^4}{P_n^4}\right)$$
$$- \frac{48z(2\lambda_3 H + \lambda_1 h)p_n}{P_n^6}\left(5 - \frac{20p_n^2}{P_n^2} + \frac{16p_n^4}{P_n^4}\right)$$

$$X_4 = -\frac{2\{\lambda_3 - 2(1-\nu)(m-1)\}\,p_n}{P_n^4}\left(3 - \frac{4p_n^2}{P_n^2}\right) + \frac{6(m-1)\{z - h - 4(1-\nu)Z_2\}}{P_n^4}\left(1 - \frac{8p_n^2}{P_n^2} + \frac{8p_n^4}{P_n^4}\right)$$

$$+ \frac{48(m-1)(z-H)Z_2 q_n}{Q_n^6}\left(5 - \frac{20p_n^2}{P_n^2} + \frac{16p_n^4}{P_n^4}\right) - \frac{2\{2(1-\nu)(2\lambda_3 - m + 1) - \lambda_3\}\,q_n}{Q_n^4}\left(3 - \frac{4q_n^2}{Q_n^2}\right)$$

$$+ \frac{6\{(m-1)(h - z - 4(1-\nu)H) + 2\lambda_3 z\}}{Q_n^4}\left(1 - \frac{8q_n^2}{Q_n^2} + \frac{8q_n^4}{Q_n^4}\right)$$

$$- \frac{48H(m-1)\{z + (3-4\nu)(H-h)\}\,q_n}{Q_n^6}\left(5 - \frac{20q_n^2}{Q_n^2} + \frac{16q_n^4}{Q_n^4}\right)$$

$$+ \frac{480zHZ_2(m-1)q_n}{Q_n^6}\left(1 - \frac{18q_n^2}{Q_n^2} + \frac{48q_n^4}{Q_n^4} - \frac{32q_n^6}{Q_n^6}\right)$$

$$Y_1 = \frac{\{\lambda_3 + (1-2\nu)\lambda_2\}}{Q_n^2} + \frac{2(2\lambda_3 H + \lambda_1 h - \lambda_2 z)q_n}{Q_n^4} + \frac{\{\lambda_3 - (1-2\nu)\lambda_1\}}{P_n^2}$$

$$- \frac{2\{\lambda_1 z - (3-4\nu)(2\lambda_3 H + \lambda_1 h)\}\,p_n}{P_n^4} - \frac{4z(2\lambda_3 H + \lambda_1 h)}{P_n^4}\left(1 - \frac{4p_n^2}{P_n^2}\right)$$

$$Y_2 = -\frac{\{\lambda_3 + (1-2\nu)(m-1)\}}{P_n^2} - \frac{2(m-1)\{z - h + 2(1-2\nu)Z_2\}\,p_n}{P_n^4} + \frac{4(m-1)(z-H)Z_2}{P_n^4}\left(1 - \frac{4p_n^2}{P_n^2}\right)$$

$$- \frac{1}{Q_n^2}\{(1-2\nu)(2\lambda_3 - m + 1) + \lambda_3\} - \frac{2\{(m-1)(z - h - 2(1-2\nu)H) - 2\lambda_3 z\}q_n}{Q_n^4}$$

$$+ \frac{4H(m-1)\{z - (3-4\nu)Z_2\}}{Q_n^4}\left(1 - \frac{4q_n^2}{Q_n^2}\right) + \frac{96zH(H-h)(m-1)q_n}{Q_n^6}\left(1 - \frac{2q_n^2}{Q_n^2}\right)$$

$$Y_3 = \frac{-2\{\lambda_3 + (1-2\nu)\lambda_2\}}{Q_n^4}\left(1 - \frac{4q_n^2}{Q_n^2}\right) - \frac{2\{\lambda_3 - (1-2\nu)\lambda_1\}}{P_n^4}\left(1 - \frac{4p_n^2}{P_n^2}\right) - \frac{48(2\lambda_3 H + \lambda_1 h - \lambda_2 z)q_n}{Q_n^6}\left(1 - \frac{2q_n^2}{Q_n^2}\right)$$

$$+ \frac{24\{\lambda_1 z - (3-4\nu)(2\lambda_3 H + \lambda_1 h)\}\,p_n}{P_n^6}\left(1 - \frac{4p_n^2}{P_n^2}\right) + \frac{48z(2\lambda_3 H + \lambda_1 h)\}}{P_n^6}\left(1 - \frac{12p_n^2}{P_n^2} + \frac{16p_n^4}{P_n^4}\right)$$

$$Y_4 = \frac{24\{\lambda_3 + (1-2\nu)(m-1)\}\,p_n}{P_n^6}\left(1 - \frac{4p_n^2}{P_n^2}\right) - \frac{24(m-1)[z - h + 2(1-2\nu)Z_2]}{P_n^6}\left(1 - \frac{12p_n^2}{P_n^2} + \frac{16p_n^4}{P_n^4}\right)$$

$$- \frac{48(m-1)(z-H)Z_2}{P_n^6}\left(1 - \frac{12p_n^2}{P_n^2} + \frac{16p_n^4}{P_n^4}\right) + \frac{2\{(1-2\nu)(2\lambda_3 - m + 1) + \lambda_3\}}{Q_n^4}\left(1 - \frac{4q_n^2}{Q_n^2}\right)$$

$$+ \frac{24\{(m-1)(z - h - 2(1-2\nu)H) - 2\lambda_3 z\}q_n}{Q_n^6}\left(1 - \frac{2q_n^2}{Q_n^2}\right) - \frac{48H(m-1)\{z - (3-4\nu)Z_2\}}{Q_n^6}\left(1 - \frac{12q_n^2}{Q_n^2} + \frac{16q_n^4}{Q_n^4}\right)$$

$$- \frac{960zHZ_2(m-1)q_n}{Q_n^8}\left(3 - \frac{16q_n^2}{Q_n^2} + \frac{16q_n^4}{Q_n^4}\right)$$

$$P_n^2 = y^2 + p_n^2, \quad p_n = nH + h + z,$$

$$Q_n^2 = y^2 + q_n^2, \quad q_n = nH - h + z,$$

$$\left|f(n)\right|_{n=\beta} = f(\beta)$$