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INDIAN SOCIETY OF EARTHQUAKE TECHNOLOGY

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INDIAN SOCIETY ON EARTHQUAKE TECHNOLOGY

Roorkee (U.P.) India

OFFICE BEARERS

A REQUEST

The First Bulletin (Vol. I, No. 1) published in January 1964 has been sold ou Querries for its availability are still pouring in especially from abroad. The Society will fe grateful to the members who can spare their copy gratis for the use of others. The Societ will record with appreciation this gesture of goodwill by the members.

Secretary

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INDIAN SOCIETY OF EARTHQUAKE TECHNOLOGY

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EDITORIAL

Dear Member,

We feel great pleasure in presenting the Third Bulletin (Vol. II No. 1) as per schedule. The response of the authors to contribute papers has been very encouraging. We did consider the question of increasing the frequency of the Bulletin to four per year instead of two at present. We had to defer the question for a later date because of the lack of funds.

Although the membership, as on February 10, 1965, stood as follows:—

Individual Members	—	120,
Institution Members	—	16,

the funds realized are just sufficient for two bulletins. We have been trying to improve the quality of printing also, but it needs more money. On financial grounds, purely, we had to defer the issue of the booklet containing messages from various high dignatories. It is hoped we will be able to surmount these difficulties which are usual for every new organisation.

It will be a matter of pleasure for all the members to note that our Society has been registered with the Registrar of Societies and Firms, Lucknow vide registration No. 845 of 16.12.1964.

The papers published in this as well as the previous Bulletin (Vol I, No. 2) are open for discussion upto 31st May 1964. The discussion alongwith author's replies will be included in the Fourth Bulletin (Vol. II No. 2).

Contributions are invited on the following topics for the subsequent issues of the Bulletin :—

1. Analysis and Design of Structures for Earthquake forces.
2. Design of Dams and other Appurtenant Works in Seismic Zones.
3. Dynamic Loading of Soils and Foundations.
4. Tectonic Features Inflencing Occurrence of Earthquakes.
5. Instruments Concerned with Seismology and Earthquake Engineering Studies.
6. Earthquake Records and Reports.
7. Wave Propogation and Energy Attenuation in Different Strata.
8. General Topics Having a Bearing on the Subject of Earthquake Technology.

INDIAN SOCIETY OF EARTHQUAKE TECHNOLOGY
ROORKEE (U.P.) India

PROPOSAL FOR MEMBERSHIP

I propose that..
be admitted as a Member of the Society. For full details see below.

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(Member ISET)
No......

I second the above proposal of admission of..
as a Member of Society.

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DECLARATION BY THE NOMINEE

I hereby declare that I will abide by the Statutes and Regulations of the Society and offer my cooperation in promoting its objects.

Date..........................1965 Signature..............................

1. Name in full (Block letters)..
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6. Societies & institution of which already a member (i)...
   (ii)..

Notes :—1. Mail to the Secretary, Indian Society of Earthquake Technology, Roorkee U.P. India.

2. Use blank paper if proforma is not available.

Paper No. 21

A NOTE ON THE MAGNITUDE DETERMINATION OF EARTHQUAKES

Umesh Chandra*

SYNOPSIS

The various methods for the determination of earthquake magnitudes put forward during the last few decades, have been briefly reviewed and followed by a critical examination of the procedure involved. The task of extending the magnitude formula, from a knowledge of the absorption coefficient k for different periods and paths, for surface waves using periods deviating considerably from 20 sec., has been discussed. Further, in view of the revised work of Gutenberg and Richter (1956 a, equation 18) the magnitude formula for body waves needs a revision. An accurate magnitude determination for deep focus earthquakes also requires a more systematic investigation, than is yet available. An extension of the magnitude scale to other waves, especially the various waves encountering the core would be desirable. However, some problems, not yet solved theoretically, arise. It is suggested that a magnitude scale based on the measurements of a few (but not all) crests and troughs on either side of the maxima, rather than the maxima alone, would yield more accurate results: To some extent it obviates the difficulties due to the interference of waves and also takes account of the particular amplitude and period spectrum of a particular earthquake. However, the proposed idea requires a critical examination before being put to some practical use. The necessity of developing magnitude formulae and associated tables to Indian regions, similar to Southern California, is emphasized.

INTRODUCTION

For many purposes in theoretical and practical seismology, it is desirable to have a scale for rating the various earthquakes in terms of the energy released in them which would be independent of the local effects produced at any particular point of observation. Following the suggestion by Wood, Richter (1935) devised such a scale called the, 'magnitude scale'. The magnitude thus assigned is characteristic of the earthquake as a whole and as such it differs from the intensity which varies from point to point of the affected area. The investigations of Gutenberg and Richter have simplified the practical determination of magnitude to such a straight forward procedure that it can be very easily applied in routine bulletin work for a given station. To day, many seismological stations, all over the world, regularly report magnitudes.

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MAGNITUDE FROM LOCAL SHOCKS

If we consider two shocks of different magnitudes originating from the same focus, all other circumstances being identical in both cases, a seismograph at a particular station should be expected to write two records one of which should be very closely an enlarged copy of the other. The ratio of this enlargement should be the same, independent of the recording seismograph, provided that we assume the instrumental constants to remain unaltered during the two events, and that the response of the registering apparatus is linear. Clearly this ratio may be taken to be a measure of the relative magnitudes of the two shocks. Richter (1935) studied a large number of earthquakes and for each case, he plotted the logarithm of the trace amplitude against epicentral distance. It was found that all the points for any shock tend to lie on a certain smooth curve, and these curves (corresponding to different shocks) were seen to be roughly parallel; in accordance with the proportional amplitude hypothesis. Following the general pattern of the curves, a curve parallel to these and passing through an arbitrarily selected point (epicentral distance=100 km., and maximum trace amplitude=1 micron) was taken to be a standard shock of Zero magnitude, and a table giving the logarithm (to the base 10) of the computed maximum trace amplitude in millimeter, with which the standard short-period torsion seismometer ($T_0=0.8$ sec., $V_s=2800$, $h=0.8$) should register the shock of zero magnitude at various distances, was prepared. Accordingly, Richter (1935) gave the following definition of the magnitude scale, for shocks of Southern California.

"The magnitude of any shock is taken as the logarithm of the maximum trace amplitude expressed in microns, with which the standard short-period torsion seismometer ($T_0=0.8$ sec., $V_s=2800$, $h=0.8$) would register that shock at an epicentral distance of 100 kilometers."

The definition may be expressed as follows :

$$M_1 - M_2 = \log B_1 - \log B_2 \qquad (1)$$

Where, M_1 and M_2 are the magnitudes of two earthquakes and, B_1 and B_2 are their corresponding maximum trace amplitudes recorded on a standard torsion seismograph at an epicentral distance, $\triangle$:

This definition applies strictly only for $\triangle = 100$ km. The zero of the scale is fixed by setting $M=3$ when $b=1$ mm. at the standard distance of 100km. Further, tne definition is applicable only for local shocks of Southern California with a depth of focus of about 16 km., it being assumed that no special structure or material is involved either at the focus or at the recording station or in the wave path.

Reductions, of the maximum trace amplitudes at different distances, to the standard distance of 100 km. involves an empirically determined table for the logarithm of the maximum trace amplitude, for a shock of magnitude zero, as a function of epicentral distance (Table 1, Richter 1935, or table 1, Gutenberg and Richter 1956a, revised and extended to cover the range 0 to 25 km., also). Nordquist (1942) designed a nomogram by which these tables are more conveniently represented and magnitude, from the trace amplitude in mm. of a standard torsion seismometer, is determined directly. Station corrections (see table 3, Gutenberg and Richter 1956 a) are applied to magnitudes so determined.

The initial success of Richter (1935) is attributable to a set of fortunate circumstances. The method was developed for magnitude determinations in Southern California region, where the focal depths are fairly constant. Uncertainties arising from this factor are thus largely eliminated. The Wood-Anderson Torsion Seismometer is quite stable and writes a very legible record for small or moderately large shallow shocks. The prevailing periods corresponding to maximum waves for shocks in this magnitude range are such that the magnification of the torsion instrument is nearly constant. However, for large shocks difficulties arise and use is made of other methods to be described later.

The magnitude scale just described was applied by Richter (1935) to earthquakes recorded in California region within an epicentral distance of 600 km. For magnitude determination of distant shocks (i.e. for teleseisms), the method was extended by Gutenberg and Richter (1936), Gutenberg (1945 a) and M. Bath (1952), who set up further empirical tables whereby observations made at distant stations and on seismographs other than the standard torsion seismograph ($T_0=0.8$ Sec., $V_s=2800$, $h=0.8$) could be reduced to correspond with the standard conditions in Richter's definition. Gutenberg (1945 b, c) produced further empirical tables to cover earthquakes of significant focal depth from amplitude and period measurements of body waves (P, PP, S).

MAGNITUDE FROM SURFACE WAVES (DISTANT EARTHQUAKES)

For magnitude determinations on the same scale from distant earthquake records Gutenberg and Richter (1936) also made use of the trace amplitude of surface waves, of course restricted to shallow shocks only. In this case magnitude is expressed as

$$M=\log A-\log B+C+D \tag{2}$$

where,

A = The total horizontal component of ground movement in microns caused by surface waves having periods of about 20 seconds,

B = A quantity similar to A corresponding to a shock of zero magnitude. B depends only on the distance of the station from the epicentre, for a given focal depth; log B being always negative, and

C = a constant for each station, correcting for the effects of the special conditions of the ground near the station and of the instrumental equipment (Table 1, Gutenberg 1945 a).

D = A factor depending upon the depth of focus, the original distribution of energy in various azimuths, the absorption of waves, and on the effect of irregularities lying in the path of the wave.

According to Gutenberg and Richter (1936), maximum trace amplitude b (measured in mm.), corresponding to epicentral distances greater than 20°, as recorded by a standard Wood-Anderson torsion seismograph is given as

$$\log b = \log B - 2.5 \qquad (3)$$

The formula does not hold for epicentral distances less than 20° and no attempt has been made to find the values of B for these distances. The zero of scale is fixed by taking log B=−5.04 (Gutenberg 1945 a, previously it was taken as log B=—5.0 for $\triangle = 90°$, see Gutenberg and Richter 1936). For $\triangle > 20°$, corresponding to a shock of magnitude zero, the maximum horizontal ground amplitude B of surface waves having periods of about 20 sec. may be obtained,

(i) From a table of observed values giving −log B as a function of △,

(ii) From the formula

$$-\log B = 5.04 + \tfrac{1}{2}\,[48.25\,k\,(\triangle - 90) + \log \sin \triangle + \tfrac{1}{3}\,(\log \triangle - 1.954)] \qquad (4)$$

Where k is the absorption factor per kilometer for surface waves with periods of about 20 sec.

or (iii) more easily from the empirically determined formula

$$-\log B = 1.818 + 1.656 \log \triangle \qquad (5)$$

(for △ between 15° and 130°)

The revised values, which were finally adopted, of −log B as a function of △ are given in table 4 (Gutenberg 1945 a) and hold for an average depth of about 20–25 km.

Seeking to investigate the effect of attenuation of surface waves due to absorption and the variation of velocity along the wave path, Gutenberg (1945 a) has concluded that for shocks arriving in Southern California along the critical azimuths, e.g., from Southern Japan and from Eqador-Peru, a term at least as big as 0.5 should be added to the calculated magnitudes. If the surface waves have crossed the Pacific Basin without being tangent to its boundary, 0.1 or 0.2 should suffice, whereas for paths completely outside the Pacific Basin 0.1 or 0.2 should be subtracted. Special researches on regional characteristics are needed to determine the corresponding corrections for waves reaching other stations.

Markus Bath (1952) has developed the following magnitude formula from the vertical component of surface waves with periods of about 20 seconds from the records obtained at Pasadena,

$$M = \log A_z - \log B + \bar{\delta}(h) + m_r + C(M_o - M_{calc}) \qquad (6)$$

where,

A_z=the vertical component of the maximum ground movement in microns for surface waves of about 20–sec. period. This term takes account of the amplitude.

B=A quantity similar to A_z corresponding to a shock of magnitude zero, and is here assumed to be the same as for the horizontal component (the constant term in the expression of B is different, but that is taken into account by $\bar{\delta}(h)$). This term accounts for the amplitude variation with epicentral distance.

$\bar{\delta}(h)=0.0082\ h$ (determined empirically) is a function of focal depth down to 100 km.

m_r=regional correction, depending upon the properties of the path of surface waves.

$C(M_o\text{-}M_{calc})$=a correction term, which corrects for the variation of the ratio (A_z/A_h) of the vertical to the horizontal amplitudes of surface waves with magnitude (M).

It has been shown by Bath (1952) that because of certain factors such as the earthquake mechanism, distribution of energy in different azimuths etc., it is not possible to obtain the magnitude from the records at one station to an accuracy higher than $\pm$ 1/4.

The use of vertical component of ground amplitude in surface waves is advantageous, as it is necessary to measure only one record, whereas the total horizontal component requires two records (N-S and E-W) corresponding to two mutually perpendicular directions in the horizontal plane. From the theoretical view point also the vertical component has an advantage, as it represents the vertical component of Rayleigh waves alone, whereas the horizontal component generally is a combination of both Rayleigh and Love waves.

On the other hand disadvantages in the use of vertical components are the following: (1) that the number of stations supplying data for this component is smaller than those supplying the horizontal component records, and (2) that often the accuracy of the constant seems to be less than that for the horizontal instruments.

The above magnitude formulae are applicable to shallow earthquakes only. In order to correct for the depth h a term approximately equal to $0.01\ h-0.2$ is added (Gutenberg and Richter, 1956 a).

MAGNITUDE FROM BODY WAVES

Gutenberg (1945 b, c) further developed methods for the determination of magnitudes of distant deep focus shocks from the amplitude and period measurements of body waves (P, PP, S).

From the original theory of Zoeppritz, Geiger and Gutenberg (1912), the expression for the ground displacement, in terms of epicentral distance $\triangle$, during a single body wave is given as follows : (see Appendix : Gaur and Chandra 1964).

$$u\ (\text{or } w)=K\ T\ N\sqrt{E} \tag{7}$$

where
$$N=Z\sqrt{(F_1F_2......F_n)\ a\ \frac{\cos i\ di/d\triangle}{\sin e\ \sin\triangle}} \tag{8}$$

K=a constant, its value is dependent on the fraction of energy E passing into the wave under consideration, and is different for P, SH, and SV waves.

T=Period of the observed wave.

U and W=The horizontal and vertical components of N.

Z=ratio of the ground displacement to the incident amplitude, having different values for the horizontal and vertical components (u and w, respectively) of the ground displacement, being a function of e and Poisson's ratio just below the surface.

F=ratio of transmitted or reflected energy to incident energy at each point where the wave has encountered a discontinuity in density or velocity or both. It's value at a given discontinuity depends upon the angle of incidence thereon, the densities and wave velocities on both sides of the discontinuity and the type of wave-longitudinal (P) or transverse (SV in the plane of propagation, or SH with vibrations perpendicular to the plane of propagation only).

$a=e^{-kd}$ gives the effect of absorption, k is the absorption factor, d the distance.

i=angle which the ray leaving the focus makes with level surface through it.

e=angle between the emergent ray and the surface.

Δ=the angular distance from its source to its point of emergence. From equation (7) we get

$$\tfrac{1}{2}\log E=\log u-\log K-\log T-\log U \tag{9}$$

and similarly for the vertical component.

Again from Gutenberg and Richter (1942, equation 35, pp 180),

$$\log E=11.3+1.8\ M \tag{10}$$

Assuming that the duration t of a given phase increases with the distance Δ proportionally with T, we get

$$E=q\ t\ E_1/T=q\ t_0E_1/T_0 \tag{11}$$

The subscript o refers to the source and q, assumed to be constant, is the fraction of energy imparted to the phase under consideration. Assuming, further, that t_0/T_0 does not depend appreciably on magnitude, we find from a combination of equations (9), (10) and (11) that for a given earthquake,

$$L=0.9\ M-\log u+\log T+\log U \tag{12}$$

should have a nearly constant value for all waves starting as P, another for all starting as SV, and a third for those starting as SH. It will be sufficient here to assume that there is one constant C in all shocks for P waves and another for all S waves (combined SV and SH) given by

$$C=M-\log u+\log U-0.1\ (M-7)+\log T. \tag{13}$$

$$\therefore\quad A=C-\log U=M-\log u-0.1\ (M-7)+\log T \tag{14}$$

The value of C was found by Gutenberg to be 6.3.

$$M = A + 0.1\ (M - 7) - \log T + \log u \text{ (or } \log w) \tag{15}$$

Gutenberg gave the value of A (1945 b, for shallow shocks table 4, and 1945 c, for deep focus shocks see figures 2, 3 and 4, the same has been revised by Gutenberg and Richter 1956 b, table 2, figures 3, 4 and 5) as a function of epicentral distance Δ and depth of focus h. The value of A together with their ground amplitudes in microns (total horizontal u, vertical w, considering the station corrections given in table 2, 1945 b) and their periods T give the magnitude M from equation (15). For all longitudinal waves in great shocks or shocks of magnitude less than 6½, a tentative additional correction +0.1 (M−7) is applied.

Thus we see that so for three imperfectly consistent magnitude scales have been developed for current use. These are,

1. M_L, determined from the maximum trace amplitudes of local earthquake records, according to the original definition of Richter (1935).

2. M_S, determined from the maximum ground amplitudes (Horizontal components vectorially combined, Gutenberg 1945 a, and vertical component, Bath 1952) corresponding to 20 second period for shallow distant shocks.

3. M_B, calculated from the maximum of the ratio of amplitude and period (A/T) for body waves (P, PP and S) for distant earthquakes (shallow shocks: Gutenberg 1945 b, shocks of any depth Gutenberg 1945 c).

Finally, Gutenberg and Richter (1956 b) introduced a 'unified magnitude' m, whose formal definition may be phrased as follows :

$$m - 7.0 = q \tag{16}$$

at a distance of 90° for normal shallow focal depth, where $q = \log w/T$ refers to PZ, and the station ground conditions. It has the following relations to M_L, M_S and M_B,

$$m = 1.7 + 0.8\, M_L - 0.01\, M_L^2 \tag{17}$$

$$m = M_S - 0.37\,(M_S - 6.76) \tag{18}$$

$$m = M_B \quad \text{(without correction)} \tag{19}$$

The unified magnitude is placed on a self consistent and independent basis as satisfactory for teleseisms as that of M_L for local earthquakes, and with the great advantage of being applicable directly to seismograms recorded on instruments of all types and at all stations. The relation (18) is based on a large body of data, but since the relation of M_L to m is not yet on a definitive basis, Gutenberg and Richter (1956 b) suggest that the 'Richter Scale' as defined in 1935 be retained for determining magnitudes of local shocks. They have preferred and strongly recommended the use of the unified magnitude scale m, for teleseisms. It appears possible, in very near future, to express the entire range of observed magnitudes in terms of the unified magnitude m.

ENERGY MAGNITUDE RELATIONS

The following magnitude energy relations have been given by Gutenberg and Richter (1956 a).

$$\text{Log } E = 9.4 + 2.14\, M - 0.054\, M^2 \tag{20}$$

For M ranging from 1 to 8.7, this is numerically equivalent to

$$\log E = 9.1 + 7\,M/4 + \log(9-M). \quad (21)$$

The most reliable connection between the total energy E (in ergs) of seismic waves and the 'unified magnitude m', is given as (Gutenberg and Richter 1956 b),

$$\log E = 5.8 + 2.4\,m \quad (22)$$

Hence, substituting (22) in equations (17), (18) and (19), the total energy released in the form of elastic waves following a certain earthquake may be given in terms of magnitudes determined from local shocks (M_L), body waves for distant shocks of any depth (M_B) and surface waves (M_S) for distant shallow shocks, as follows :

$$\log E = 9.9 + 1.92\,M_L - 0.024\,M_L^2 \quad (23)$$

$$\log E = 5.8 + 2.4\,M_B \quad (24)$$

$$\log E = 11.8 + 1.5\,M_S \quad (25)$$

By employing completly different methods and different material, M. Bath (1958) obtained the following energy formula.

$$\log E = 12.24 + 1.44\,M_S \quad (26)$$

which is in very good agreement with the results of Gutenberg and Richter (compare with equation 25 above).

However, owing to the effect of absorption, which in the case of body waves may account for a factor of approximately 20 in total energy, any determination of energy obtained from P and S waves alone can not be expected to yield an absolute value. Hence an exact determination of absorption and its consideration is important in energy computations.

It appears that energy computations made by measuring surface waves, for which the absorption can fairly easily be determined, may give more reliable results.

REAPPRAISAL OF THE METHODS OF MAGNITUDE DETERMINATION

Richter's magnitude scale (1935) involves a reduction of observed amplitudes at different distances to the expected amplitude at the standard distance of 100 km. and is accomplished by means of a tabulation of amplitude as a function of distance for a standard shock (magnitude zero). The method is based on the assumption that the ratio of amplitude at two given distances is the same for all shocks and in all azimuths. This does not strictly hold good, since the amplitude ratio, at two given distances, depends on various factors, which are in general not the same for different earthquakes. These are, (1) depth of focus, predominant period and fraction of energy passing into the wave under consideration (in general, for near shocks SH has the maximum trace amplitude), (2) variation of absorption factor with period and path (for body waves, the absorption factor is more or less constant, whereas for surface waves it varies considerably with period), (3) unequal distribution of energy in different azimuths due to anisotropy and the particular mechanism of energy release, and, (4) linear extent of faulting, its rate of progression.

Bath (1955 b) has presented a method for the computation of A_{20} corresponding to 20 sec. period from the amplitude A of surface waves with periods T different from 20 sec. He states that, "the underlying idea should be to define A_{20} such that the corresponding energy is the same for A, rather than to put their velocities equal", and that "we have to take account not only of the period difference but also of the difference in the absorption coefficient (k) for different periods". Equating two energies in a given time interval, he obtained the following expression.

$$\frac{A^2_{20}}{20} e^{k(20)\Delta} = \frac{A^2}{T} e^{k(T)\Delta} \tag{27}$$

$$\text{or} \quad \log A_{20} = \log A + 1/2 \log 20/T + 24.13 \Delta [k(T) - k(20)] \tag{28}$$

Instead of equating energies in a given time interval, one could, as an alternative, equate energies per wave length. This would mean to equate $A^2 e^{k\Delta}$ instead of $A^2/Te^{k\Delta}$. The numerical difference between these two methods would not be of much significance for magnitude determinations, if $10 \leq T \leq 40$ sec., which in fact covers all periods of importance in this case.

One could naturally argue which of these two methods of equating energies is the correct one. However, the essential thing is to equate energies and doing it one way or the other is mainly a matter of defining the corresponding A_{20}.

The investigations of Gutenberg and Richter (1936) and Gutenberg (1945 a), for the determination of magnitudes using maximum ground amplitudes during surface waves, make use of the formula;

$$\frac{A_2}{A_1} = \frac{T_2}{T_1} e^{-k(\Delta_2 - \Delta_1)/2} \sqrt{\frac{\sin \Delta_1}{\sin \Delta_2}} \sqrt[6]{\frac{\Delta_1}{\Delta_2}} \tag{29}$$

Originally developed by Jeffreys (1925). A_1 and A_2 are the amplitudes at distances Δ_1 and Δ_2 respectively; T_1, T_2 are the corresponding periods, k is the absorption coefficient. In developing the formula (29), the absorption coefficient k has been assumed constant along the whole path and also for the period range under consideration.

A little consideration of equation (29) and (4) shows that in the expression for-log B on the right hand side in equation (4), the term $\log (T_{90}/T_\Delta)$ has been omitted. T_{90} was taken to be 20 sec., and $\log (T_{90}/T_\Delta)$ can vanish only if $T_\Delta = 20$ sec. It is perhaps not necessary to give complete details here, as they are contained in the original papers by Gutenberg and Richter (1936) and Gutenberg (1945 a).

It is evident that the magnitude formulae and the associated tables (in Gutenberg 1945 a, Gutenberg & Richter 1936) hold only for surface waves with maximum ground amplitude corresponding to periods of about 20 sec. Hence there may be two ways of extending the magnitude formula for surface waves using periods deviating considerably from 20 sec.

1. From the energy equality postulated between surface waves of period T and surface waves of period 20 sec., we can calculate what A_{20} should be, given A_T. Knowing A_{20} (e.g. from eq. 28), we can proceed as usual, that is with the tables and graphs which Gutenberg has given. This method would only require a knowledge of k (T).

2. We may use A_T directly, that is not to pass over A_{20}. This would require new tables and graphs for each period to be considered.

Obviously, the first method would be simpler in practice, although the outcome of the two should be equivalent. The main trouble is that we still have too meagre information on k(T). It may appear to be an interesting piece of investigation to reverse the procedure mentioned above, that is, combine known M with measured A_T and calculate k(T) for a given range of periods.

The magnitude determination from body waves (Gutenberg 1945 b, c) is based on the equation (8), which has been derived by means of several simplified assumptions such as a spherically symmetrical earth, zero depth of focus, the validity of the ray theory for the calculation of energy flux, equal distribution of energy in all directions from the source, conditions of perfect elasticity, deviations from which are quite significant in many cases. The value of L in equation (12) and hence of C in equation (13) depends on the fraction of energy distributed among the fundamental types of waves and thus is likely to differ from one shock to another. The value C = 6.3, found by Gutenberg (1945b), represents some sort of average condition. In arriving at the final expression (equation 15) for magnitude, equation (10) has been used for magnitude energy relationship. In view of the revised work of Gutenberg and Richter (1956 a, equation 18), this needs a revision. The simplified form of equation (11) may be responsible for some differences in the values of residuals.

In magnitude determination for deep focus earthquakes (Gutenberg 1945 c), the magnitude is defined in such a way that the energy released in two shocks of the same magnitude become equal, regardless of the focal depth. Contrary to the case of shallow shocks, it is not possible to find the residuals of the magnitude M calculated from the amplitudes of body waves, when compared with the magnitude found from that of the surface waves. The assumption has been made that the average magnitude found from the theory, assuming C=6.3 and using data from various distances is substantially correct. From the average value of M thus obtained the residuals were plotted as a function of epicentral distance △ and focal depth h; and corrections to the calculated values of U and W as a function of △ have been obtained. At present, it is not possible to evaluate the errors which affect M as the focal depth h increases. A more systematic investigation, is thus needed for an accurate evaluation of the magnitude of deep focus earthquakes.

The magnitude determinations from body waves has so far been confined to P, PP and S only. The extension of the method to other waves, especially the various core waves, would be desirable. The results of a very systematic study by Kazim Ergin (1953) for the ratios of (displacement/period) of P_c P, P_c S, S_c S, and S_c P to that of the corresponding incident wave e.g.,

$\left(\frac{\text{displacement}}{\text{period}}\right)_{P_c P} \Big/ \left(\frac{\text{displacement}}{\text{period}}\right)_{P}$, using intermediate and deep focus earthquake seismograms, indicate that the observed ratios of the horizontal component of the waves that are reflected as P waves (i.e., P_c P/P and S_c P/S) and that of the vertical component of the waves that are reflected as the S waves (i.e., ScS/S and PcS/P) at the mantle-core boundary are considerably larger than the theoretical ones, whereas the observed ratios of the vertical component of the first group and that of the horizontal component of the second group are in fairly good agreement with the theoretical values. Further, he finds that the behaviour of the direct P and S waves is in accord with theory, but the vibration of the ground is not in the direction of propagation for Pc P and Sc P, and is not perpendicular to the direction of propagation for PcS and ScS as expected. From the foregoing, it is obvious that unless the difficulties, just mentioned, are resolved, it would be premature to attempt for any development of the magnitude scale based on these waves.

All the magnitude determinations have been made with the help of formulae which are essentially far more simplified than what actually occurs in case of the earth. The plane wave theory and point source assumption has been used throughout. In shocks of large magnitudes, which are generally characterised by a large linear extent of faulting and involve large dimensions of crustal blocks, the point source theory does not hold and line source assumption would be expected to yield better results although the resulting theory would be inevitably involved. In large shocks the change in elastic properties may be significant. Factors such as diffraction, scattering and internal friction do not seem to have been adequately taken into account owing of course to the mathematical difficulties in formulating these phenomena, as they depend on the particular geological structures and crustal irregularities involved. The effects of dispersion and observed oscillatory character of motion, appear to be quite significant, though here again a satisfactory theory is not available for the explanation of oscillatory character of motion on seismograms attending the body waves. Only the dispersion of surface waves has to some extent lent itself to mathematical formulation, but the effect does not seem to have been considered in developing the magnitude formulae. Further details are given by Gaur & Chandra (1964).

It is felt that a consideration of the complete spectrum of amplitude and periods rather than the maxima alone (maximum trace amplitude on torsion seismometer for local shocks, maximum ground amplitude corresponding to 20 sec. period for surface waves and maximum amplitude/period ratio for body waves P, PP and S) would result in a quantity more representative of the shock as a whole. The foregoing idea suggests that the energies computed by the complete integration of seismogram would give more precise result. The method was applied by Bath (1955 a, 1958). But a further complication arises. The onset of any new phase follows interference with the preceding waves and thus far it has not been possible to evaluate its effect. From a study of seismograms it appears that, in general, just before the onset of a new phase, the preceding wave has significant amplitude, and neglect of the effect of interference may not be quite reasonable. It appears that the measurement of a few (but not all) crests and troughs on either side of the maxima would yield more useful information. Of course, the idea needs a critical examination before it may be put to some practical use. To a certain extent, it takes care of the particular energy distribution as represented by the amplitude and period spectrum, and also the effect of interference may be expected to die out or become negligible at this time i.e., the time by which the said maxima is reached.

However, the detailed statistical studies, the results of which are incorporated in terms such as station correction, regional correction, residuals of magnitude etc., eliminates in some cases, and averages out in others, the uncertainties due to many factors.

No systematic study, for the magnitude determinations from the records of Indian observatories, seems to have been undertaken so far. Some investigators report magnitude values for certain earthquakes recorded at Indian stations, but the formulae used by them are those which were originally developed by Gutenberg and Richter for the Southern California region. Further, the magnitude determinations involve station correction, corrections for the path and regional corrections. This requires a detailed statistical study in the absence of which all the available reports would appear to be misleading. Unfortunately the number of recording stations is very small in this part of the world, but from whatever is available it would be desirable to develop magnitude relations for this region as has been done for Southern California. This can be done by comparison with earthquake magnitudes which have been determined sufficiently accurately from the records of standard observatories, such as Pasadena etc.

ACKNOWLEDGEMENTS

I am thankful to Dr. M.Bath for some valuable discussions, through correspondence, on this problem. My thanks are also due to Dr. Jai Krishna and Dr. V.K. Gaur for certain discussions during the preparation of this paper and the pains they took in going through the manuscript.

REFERENCES

Bath, M. (1952), "Earthquake Magnitude Determination from the Vertical Component of Surface Waves", Trans. Amer. Geophys. Union., 33, (1), 81-90.

Bath, M. (1955 a), "The Relation between Magnitude and Energy of Earthquakes", Trans. Amer. Geophys. Union., 36, (5), 861–865.

Bath, M. (1955 b), "The Problem of Earthquake Magnitude Determination", Ass. Seism. Phys., l' Interieur. Terre., 19, 5-93.

Bath, M. (1958), "The Energies of Seismic Body Waves and Surface Waves", "Contributions in Geophysics: In Honor of Beno Gutenberg", 1-16.

Gaur, V.K., and U. Chandra (1964), "Ground Displacement and Acceleration Caused by Earthquakes", Indian Society of Earthquake Technology, 1, (2), 91–109

Giorgi (M) e Valle (P.E.) (1948), "Contributo allo Studio delle Onde ((M))," Ann. di Geofisica., 1, (1), 87–102.

Gutenberg, B. (1945 a), "Amplitudes of Surface Waves and Magnitudes of Shallow Earthquakes", Bull. Seism. Soc. Amer., 35, (1), 3-12.

Gutenberg, B. (1945 b), "Amplitudes of P, PP, and S and Magnitudes of Shallow Earthquakes," Bull. Seism. Soc. Amer., 35, (2), 57–69.

Gutenberg, B. (1945 c), "Magnitude Determination for Deep Focus Earthquakes". Bull. Seism. Soc. Amer., 35, (3), 117-130.

Gutenberg, B. and Richter, C.F. (1936), "On Seismic Waves" (third paper), Gerlands Beitr. Z. Geophysik., 47, 73–131.

Gutenberg, B. and Richter, C. F. (1942), "Earthquake Magnitude, Intensity, Energy, and Acceleration (First paper)", Bull. Seism. Soc. Amer., 32, (3), 163–191.

Gutenberg, B. and Richter C.F. (1956 a), "Earthquake Magnitude, Intensity, Energy, and Acceleration (Second Paper)", Bull. Seism. Soc. Amer., 46, (2), 105–145.

Gutenberg, B. and Richter, C. F. (1956 b), "Magnitude and Energy of Earthquakes", Ann. Geofis., 9, (1), 1–15.

Jeffreys, H. (1925), "On the Surface Waves of Earthquakes", Mon. Not. Roy. Astron. Soc., Geophys. Suppl., 1, p. 257.

Kazim Ergin, (1953), "Amplitude of Pc P, PcS, ScS, and ScP in Deep Focus Earthquakes", Bull. Seism. Soc. Amer., 43, (1), 63–83.

Richter., C.F. (1935), "An Instrumental Earthquake magnitude Scale". Bull. Seism. Soc. Amer., 25, (1), 1–32.

Paper No. 22

A TRANSISTORISED AMPLIFIER FOR SEISMIC SIGNALS

T. P. S. Bajwa*

SYNOPSIS

A transistorised directly coupled amplifier for the amplification of weak seismic signals is described. A low value of drift in the amplifier is obtained by using the two matched transistors in a differential configuration.

INTRODUCTION

To improve the recording of very weak earthquakes, or micro-seisms, an electronic amplifier is used between the seismometer and the recorder as shown in Fig. 1. As the frequencies to be amplified are very low, of the order of 0-25 c/s, a directly coupled amplifier is used. The d. c. stability of the amplifier is affected by the poor regulation of power supplies and changes in ambient temperature. To obtain high d. c. stability, the differential configuration is employed with matched transistors and circuit elements.

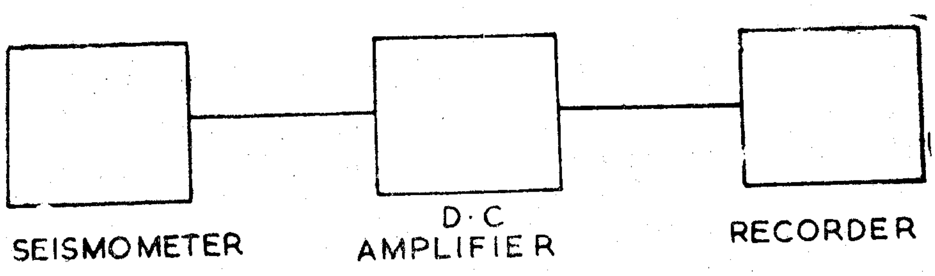

Fig. 1

To understand how differential amplifier provides a high d. c. stability, the general circuit of a differential amplifier as shown in Fig. 2 will be discussed first. In Fig. 2, neither of the input and the output terminals, is grounded, so the amplifier has floating input and output.

Let us assume that the amplifier of Fig. 2 is symmetrical and balanced i.e. $R_{1a}=R_{1b}$, $R_{2a}=R_{2b}$ and $\beta a=\beta b$ where β is the common emitter current gain.

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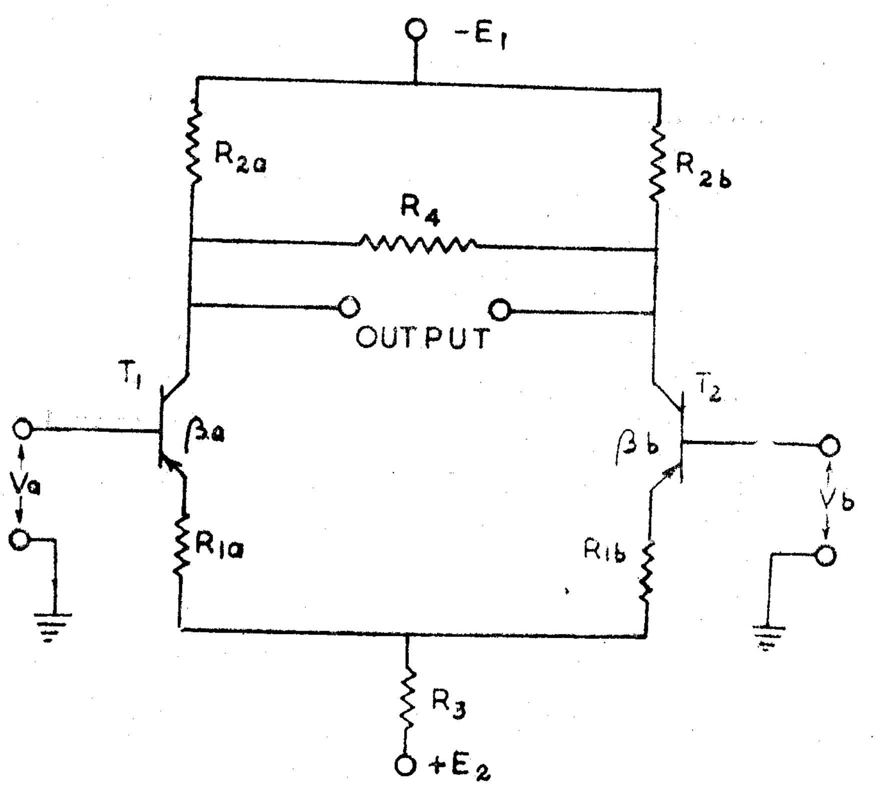

Fig. 2

If the two inputs 'Va' and 'Vb' are equal in magnitude and phase, then they are called 'inphase inputs' or 'common mode input signals'. As the circuit is balanced, the collector currents and collector voltages are same, and thus R_4 is connected between two homologous points. No current flows in R_4 and current in common element R_3 is twice that present in one side. Thus in a balanced circuit, common mode inputs do not produce any output. When 'Va' and 'Vb' are not equal in magnitude they can be replaced by their common mode equivalent input signal $Vc = \frac{Va+Vb}{2}$. If Va=Vb then Vc = Va = Vb. The common mode input signal is defined as the average of the signal voltages of the two input terminals with respect to ground.

If the two inputs 'Va' and 'Vb' are equal in magnitude, but opposite in phase, i.e. Va = − Vb, then they are called 'antiphase input' or 'differential mode input signals'. As the circuit is balanced, the collector currents of the two transistors, rise and fall equally, hence, the collector voltages of the two transistors are not equal. It means, R_4 is not connected between two equipotential points, and a virtual ground exists at the centre of R_4. The change in current flowing in R_3, due to two transistors, is equal and opposite, so no current flows in R_3 due to the input signals. Hence, in a balanced circuit the differential mode input signals produce differential output. When 'Va' and 'Vb' are not equal in magnitude, then they can be replaced by their differential mode equivalent input signal. $Vd = \frac{Va - Vb}{2}$. If Va= —Vb, then Vd = Va = —Vb. The differential mode input signal is defined as half the difference of the signal voltages of the two input terminals with respect to ground.

Thus, CM signal is prevented from producing any output, by the negative feedback voltage developed across R_3. For DM signal, no current flows in R_3 and the emitters of two transistors are at virtual ground. The inherent discrimination of the circuit of Fig. 2 against common mode, in favour of differential mode input signals, leads to the name of 'differential amplifier':

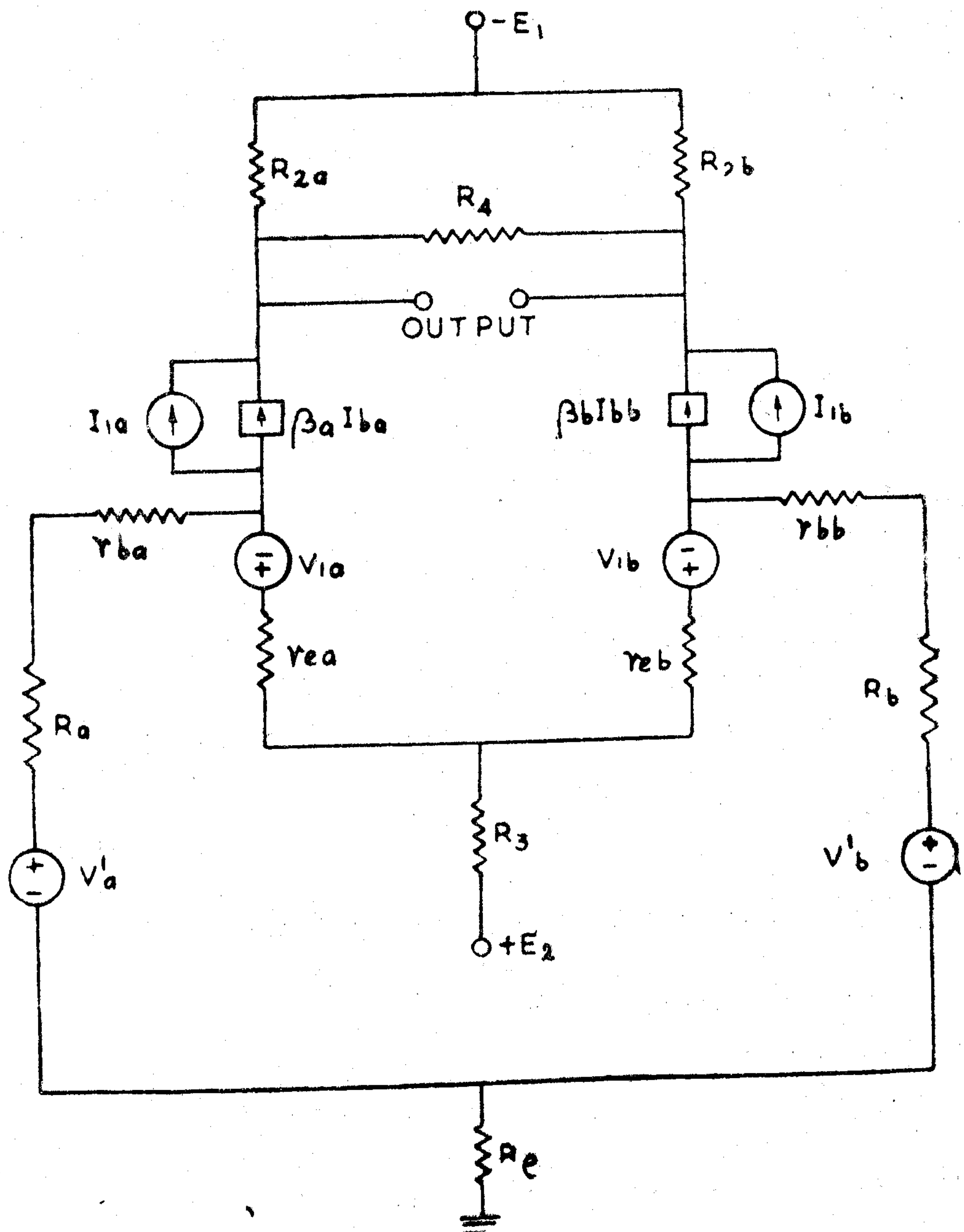

Fig. 3

Effects of Circuit Unbalances

In Fig. 3, the transistors are replaced by their common emitter T equivalent circuits. V_{1a} and V_{1b} are the forward biased base-emitter junction voltage drops, I_{1a} and I_{1b} the open base reverse saturation currents, r_{ea} and r_{eb} the internal emitter resistances, and r_{ba} and r_{bb} are the internal base resistances of the two transistors T_1 and T_2.

Next we consider the effects of unbalances in the circuit of Fig. 2. Let us suppose that all the homologous points of the circuit are balanced but R_{2a} is not equal to R_{2b}. Now, if a pure CM signal (i.e. Va=Vb=Vc) is applied to the two input terminals of the two sides, the currents at the collectors of T_1 and T_2 will be equal because $\beta a = \beta b$. But the voltage drops in R_{2a} and R_{2b} will not be equal producing unequal voltages at the collectors of T_1 and T_2. This will make a differential output to be present. Hence unbalance between R_{2a} and R_{2b} has given a cross-coupling effect, whereby a CM input signal has produced a DM output. By analogous argument, a DM input signal will produce a CM output.

Similar cross-coupling effects will be produced, if unbalances are present in (1) R_{1a} and R_{1b} (2) βa and βb, (3) V_{1a} and V_{1b} and (4) I_{1a} and I_{1b}.

The base-emitter voltage drops V_{1a} and I_{1b} can be split into their CM and DM equivalent voltages V_{1c} and V_{1d}. Similarly reverse saturation current I_{1a} and I_{1b} can be replaced by their CM and DM components.

Hence $$V_{1c} = \frac{V_{1a}+V_{1b}}{2} \qquad V_{1d} = \frac{V_{1a}-V_{1b}}{2}$$

$$V_{1c} = \frac{I_{1a}+I_{1b}}{2} \qquad V_{1d} = \frac{I_{1a}-I_{1b}}{2}$$

The CM output of the amplifier is produced by the following sources (1) CM input signal in the presence of emitter degeration due to R_3 (2) The two external CM supplies E_1 and E_2 (3) Internal generators V_{1c} and I_{1d} (4) DM input signal Vd (5) Internal DM generators V_{1d} and I_{1d}.

Sources (4) and (5) will produce CM output only in the presence of circuit unbalances due to cross-coupling effects.

The DM output of the amplifier is produced by the following sources (1) External DM input signal Vd (2) The internal DM sources V_{1d} and I_{1d} (3) CM sources V_c, E_1, E_2, V_{1c} and I_{1c} in the presence of unbalances in the circuit.

Expressions for the Different Parameters (Middle brook)

As we are interested in DM output due to external DM input signal, and complete suppression of the CM input signals, the important parameters for this purpose are those for which the expressions are given below by equations (1) to (7) :

$$\text{CM voltage gain } A_{cc}=\frac{\alpha \,.\, R_2}{R_1+2R_3} \tag{1}$$

$$\text{DM voltage gain } A_{dd}=\frac{\alpha}{R_1}\cdot\frac{R_2}{1+2R_2/R_4} \tag{2}$$

Common mode rejection factor (CMR) Hc is given by

$$\frac{1}{Hc}=\frac{R_1}{R_1+2R_3}\left(\frac{\delta R_2}{R_2}-\frac{\delta R_1}{R_1}+\frac{1}{1+\beta}\cdot\frac{\delta\beta}{\beta}\right) \tag{3}$$

CMR is defined as the ratio of CM input voltage to the DM input voltage to give same output voltage. It is a measure of how best a CM input is prevented from producing a DM output. In a balanced circuit value of Hc is infinity.

CM input admittance of the amplifier

$$Y_{cc}=\frac{1}{(\beta+1)(R_1+2R_3)} \tag{4}$$

DM input admittance of the amplifier

$$Y_{dd}=\frac{1}{(\beta+1)\,.\,R_1} \tag{5}$$

CM to DM transfer input admittance

$$Y_{dc}=-\frac{1}{(1+\beta)(R_1+2R_3)}\cdot\left(\frac{\delta R_1}{R_1}+\alpha\,.\,\frac{\delta\beta}{\beta}\right) \tag{6}$$

DM to CM transfer input admittance

$$Y_{cd}=-\frac{1}{(1+\beta)\,.\,R_1}\left(\frac{R_1}{R_1+2R_3}\cdot\frac{\delta R_1}{R_1}+\alpha\,.\frac{\delta\beta}{\beta}\right) \tag{7}$$

In an unbalanced circuit Y_{dc} will be present producing DM output due to CM input. Similarly, Y_{cd} produces CM output due to DM input. In a balanced circuit both will be zero.

In the above equations (1) to (7) the elements have following values;

$$R_1=\frac{R_{1a}+R_{1b}}{2}, \qquad \delta R_1=\frac{R_{1a}-R_{1b}}{2}$$

$$R_2=\frac{R_{2a}-R_{2b}}{2}, \qquad \delta R_2=\frac{R_{2a}-R_{2b}}{2}$$

$$\beta=\frac{\beta_a+\beta_b}{2}, \qquad \delta\beta=\frac{\beta_a-\beta_b}{2}$$

and $\alpha=\dfrac{\beta}{\beta+1}$

Effects of Signal Source Impedance (Middle brook)

In the derivation of the above equations, the impedance of the signal source was assumed to be zero. A differential amplifier whose two floating input terminals are at least partially isolated from ground, is necessarily driven from a three terminal signal source, even if the source impedance to ground is only stray coupling. A basic form of signal source with finite impedances is shown in Fig. 4, in which arbitrary source voltages 'V^1_a' and 'V^1_b' appear in series with source resistances 'Ra' and 'Rb'. Resistance Rc, common to both branches will be present, if no point of the source is grounded. The connection of the source with the input of the amplifier is shown in Fig. 3. Signal source resistances shown in Fig. 4 can be broken into its CM and DM components as follows:

CM source impedance $Z^1_{cc} = R + 2R_c$ (8)

DM source impedance $Z^1_{dd} = R$ (9)

DM to CM source transfer impedance $Z^1_{cd} = \delta R$ (10)

CM to DM source transfer impedance $Z^1_{dc} = \delta R$ (11)

where $R = \dfrac{R_a + R_b}{2}$ (12)

and $\delta R = \dfrac{R_a - R_b}{2}$ (13)

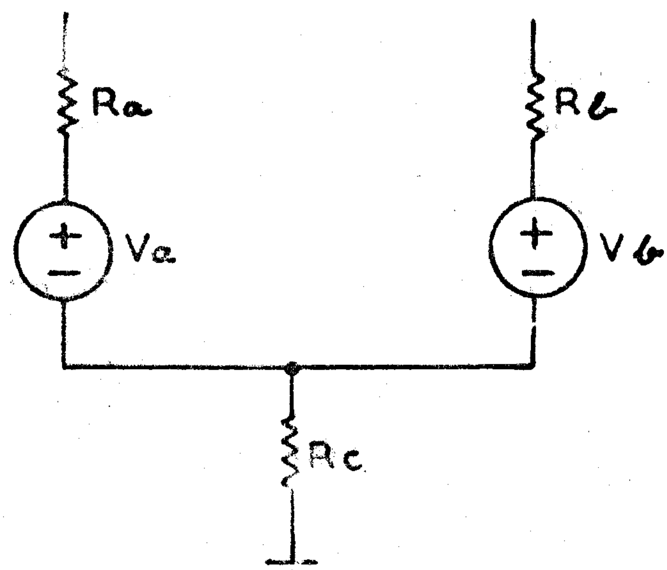

Fig. 4

By relating the amplifier input voltages to the source voltages, the following equations for the three important source parameters can be obtained:

$$(A_{dd})_s = \frac{1}{1 + Z^1_{dd} \cdot Y_{dd}} \quad (14)$$

$$(A_{cc})_s = \frac{1}{1+Z^1_{cc} \cdot Y_{cc}} \tag{15}$$

$$\left(\frac{1}{H_c}\right)_s = -\frac{Z^1_{dc} \cdot Y_{cc}+Z_{1dd} \cdot Y_{dc}}{1+Z^1_{cc} \cdot Y_{cc}} \tag{16}$$

In the equation (14), (15) and (16) subscript 's' corresponds to source. Z^1_{dd}, Z^1_{cc} and Z^1_{dc} are source impedances and Y_{dd}, Y_{cc} and Y_{dc} are the amplifier input admittance given by equations (4), (5) and (6).

Hence the total performance parameters of the amplifier and the signal source are:

$$(A_{cc})_t = (A_{cc})_s \cdot A_{cc} \tag{17}$$

$$(A_{dd})_t = (A_{dd})_s \cdot A_{dd} \tag{18}$$

$$\left(\frac{1}{H_c}\right)_t = \left(\frac{1}{H_c}\right)_s + \frac{(A_{cc})_s}{(A_{dd})_s \cdot H_c} \tag{19}$$

where subscript 't' stands for the total performance.

The adverse effects of the poor regulation of power supplies E_1 and E_2 on the d.c. stability of the amplifier can be reduced by having a high value of CMR.

Reducing the circuit unbalances or getting two transistors of nearly same β is in our hand, but we have no control over the reverse saturation currents and base-emitter drops of two transistors. These two quantities are affected by ambient temperature changes, producing extraneous signals in the output. By selecting silicon transistors which have very low saturation currents, the effect of the saturation currents on the amplifier performance can be made very small. The two transistors T_1 and T_2 are strapped together in a clip so that variation in ambient temperature affects both the transistors in the same way, thus producing CM equivalent input signals at both the input basis. These signals can be prevented from producing any significant output by marking CMR very high.

The effect of CM equivalent input V_{1c} corresponding to the base-emitter voltage drops, of the two transistors can be reduced by having high CMR but the DM equivalent V_{1d} will produce the undesirable DM output, which is not due to the input signals Va and Vb. Hence, the performance of the amplifier is limited by the mismatch of the base-emitter voltage drops of the two transistors T_1 and T_2.

The high value of CMR can be obtained by making R_3 as large as possible as seen from equation (3) but this will necessitate large value of E_2 to keep the same d.c. biasing conditions. The value of E_2 can be kept small if R_3 is replaced by a grounded base transistor whose high output impedance will act as R_3, without affecting the d.c. biasing conditions. For good performance β of T_1 and T_2 should be as high as possible because it comes in the denominator of most of the equations of the parameters.

Designed Circuit:

The circuit shown in Fig. 5 was designed for collector currents of T_1 and T_2 each equal to half milli-amp., so the collector current of T_3 was slightly more than one milli-amp. The two transistors T_1 and T_2 were kept in contact with an aluminium clip, to keep the effects of ambient temperature changes on them as nearly equal as possible. Potentiometer P is used to get zero output for zero input at any one particular temperature generally room temperature. In the circuit following values of the components were used:

T_1, T_2 and T_3 = Mullard silicon transistors OC203

$E_1 = -12$ V, $E_2 = +9$ V

$R_{2a} = 15470$ ohms, $R_{2b} = 15500$ ohms

$$\delta R_2 = \frac{R_{2a} - R_{2b}}{2} = 15 \text{ ohms}$$

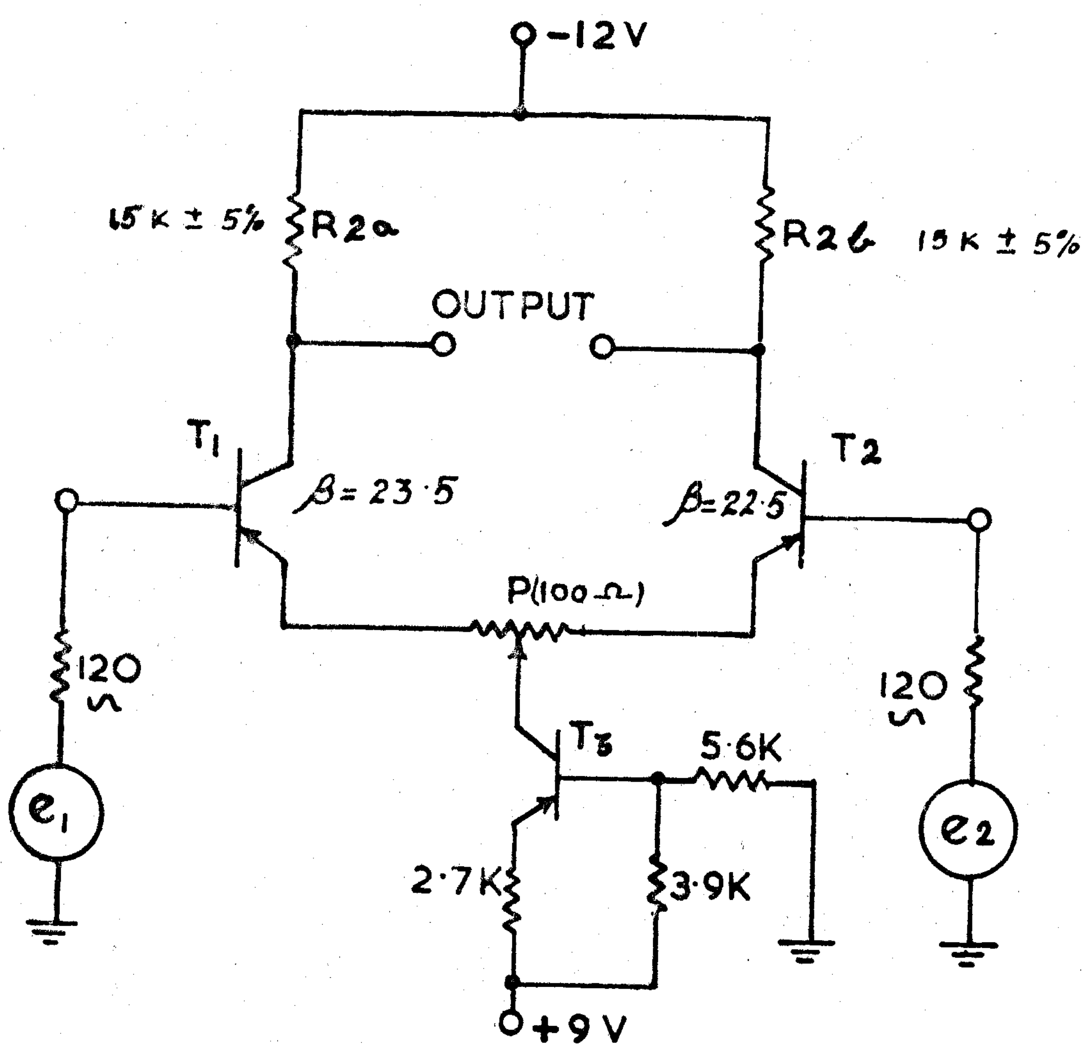

Fig. 5

$$\beta_a = 23.5, \qquad \beta_b = 22.5, \qquad \delta\beta = \frac{\beta_a - \beta_b}{2} = 0.5$$

$$\beta = \frac{\beta_a + \beta_b}{2} = 23$$

$V_{1a} = 525$ mv, $V_{1b} = 540$ mv

$$V_{1c} = \frac{V_{1a} + V_{1b}}{2} = 532.5 \text{ mv}, \quad V_{1d} = \frac{V_{1b} - V_{1a}}{2} = 7.5 \text{ mv}$$

$P = 100$ ohms potentiometer

$R_{1a} = R_{1b} =$ Internal emitter resistance of T_1 or $T_2 + P/2$

$$= \frac{25}{\text{emitter current in mA}} + 100/2$$

$= 100$ ohms.

To get zero output for zero input the setting of 'P' was 14 ohms away from the centre making $R_{1a} = 50 + 64 = 114$ ohms and $R_{1b} + 50 + 36 = 86$ ohms. Therefore

$$\delta R_1 = \frac{114 - 86}{2} = 14 \text{ ohms.}$$

The output resistance of T_3 at the operating point $Vce = -6$v and $Ie = 1.1$ mA was found to be nearly 70 K ohms.

The internal base-resistance $r_{ba} = r_{bb}$

$= 20 \times$ (internal emitter resistance of transistor) approx.

$$= 20 \times \frac{25}{.5\text{mA}}$$

$= 1000$ ohms.

In the experiment the impedance of each source of Fig. 5 was 120 ohms. The sources are in series with the internal base resistances of T_1 and T_2, one end of both the sources was grounded. For the source impedances we have

$R_a = R_b = 1000 + 120 = 1120$ ohms

and $R_c = 0$

From equation (8), (9), (12) and (13)

$$R = \frac{R_a + R_b}{2} = 1120 \text{ ohms}, \quad Z_{cc} = 1120 \text{ ohms}$$

$Z_{dd} = R = 1120$ ohms.

From equations (18) and (19), putting $R_4 =$ infinity

Overall DM voltage gain $(A_{dd})_t = 100$ (20)

Overall CMR $(H_c)_t = 4000$ (21)

From equations (2) and (3)

DM voltage gain of amplifier alone $A_{dd}=144$ (22)

CMR of amplifier alone HC=9,300 approx. (23)

The value of overall total DM gain (A_{dd}) t was in full agreement with the experimentally measured value. For higher DM gain the amplifier should have more than one stage.

Frequency Response

It was found to be flat from zero to 10 Kc/s and 3 db point was at 35 Kc/s. Frequency response curve is given in Fig. 6. Break in the curve is made in order to compress the curve in a similar space. Zero 'db' corresponds to a voltage gain of 100.

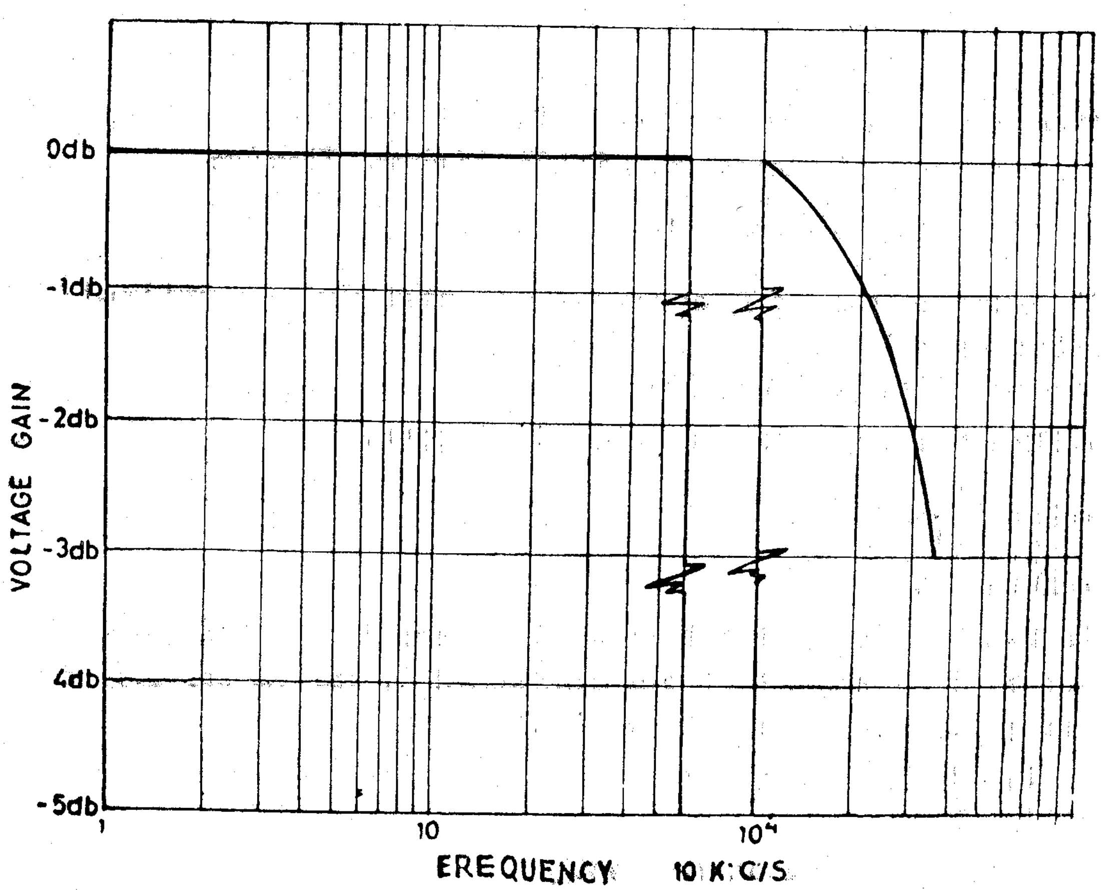

Fig 6

Input and Output

The amplifier can amplify a minimum signal of 2 mv. It can give a maximum output of 8 volts without distortion, thus giving a linear input-output characteristic up to the maximum output of 8 volts.

The base to collector reverse saturation current flows through the source resistance, thus causing an undesired input signal. In silicon transistors, the reverse saturation current is very small, permitting use of a source having resistance up to few thonsands ohms, without producing any appreciable undesired input. Hence, the seismograph with high source resistance (e. g. Milne-Shaw Seismograph which has coil resistance of 1400 ohms), can be directly connected to the amplifier without the necessity of matching.

Drift

The drift was measured over a temperature range of 25°C to 75°C. Drift referred to input was found to be between 20 μv to 30 μv per degree centigrade rise of temperature.

CONCLUSIONS

The performance parameters of the amplifier were found in the presence of the actual unbalances in the circuit. The author did not assume that the circuit is balanced. From equations (20) , (21) , (22) and (23) it is seen that if the source resistance is higher the DM voltage gain and value of CMR go down. Thus a seismograph producing a higher source resistance will degrade the performance of the amplifier more than that producing a lower source resistance.

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REFERENCES

Middlebrook, R. D. "Differential Amplifiers", John Wiley and Sons, London.

The Engineering staff of Texas Instruments Inc., "Transistor Circuit Design", Mc Graw-Hill New York.

Paper No. 23

DYNAMIC TEST OF A CONCRETE FRAME

A. R. Chandrasekaran*

Synopsis

Static and dynamic testing was performed on a four storeyed reinforced concrete frame. Theoretical analysis compared favourably with the experimental values. Experiments indicate a decrease in damping in higher modes.

Introduction

Natural periods of vibration and damping are the two important parameters which determine the behaviour of structures under dynamic loads. Damping could only be determined experimentally. Periods could be, theoretically determined, provided the mass and stiffness distributions are known. Mass distribution could be, relatively, easily estimated. Experimental determination of periods is necessary to verify the assumptions made in arriving at stiffness distribution.

Reinforced concrete was used as a construction material for the model, as this is the material generally used for construction of multi-storeyed frames of medium height. Static loads were applied horizontally to the frame, corresponding deflections were measured and the spring constants of the system have been, thereby, evaluated. The frame has been pulled horizontally, then let go and the consequent free vibrations have been measured to determine the natural period of the structure and damping in that system. The base of the structure was subjected to steady state sinusoidal excitations at various frequencies and amplitudes of vibration at various storey levels have been measured. Damping was obtained from experimental records. Theoretical analysis was made to obtain periods of vibration and these have been compared with values obtained experimentally.

Size of the Model

A shaking table of size 6′0″×4′0″ with a head room of 7′6″ above the table was available for doing experimental work. A four-storeyed reinforced concrete frame of

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dimensions as shown in Figure 1 was chosen such that it could be conveniently accommodated on the table.

The size of the column, $1\frac{1}{2}'' \times 1\frac{1}{2}''$ was the minimum that could be conveniently cast in situ. The thickness of the slab had to be increased at the supports to provide proper cover to top and bottom reinforcements. This model could be construed to be a geometrically similar model, with a length scale of eight, of a conventional four-storeyed frame.

Free Vibration Behaviour

Even neglecting the stiffening provided by the slab, the ratio of moment of inertia per unit length of girder to that of column works out to be four. Static deflection tests indicate that floor system acts as a rigid unit. The model, therefore, could be assumed to be represented by a four-degree of freedom system as shown in inset of Fig. 1

The equation of motion for free undamped vibration of the system is given by

$$[M]\{\ddot{x}\} + [K]\{x\}=0 \qquad (1)^*$$

where [M] and [K] are as defined below.

$$[M] = \begin{bmatrix} m & & & \\ & m & & O \\ & & m & \\ O & & & m \end{bmatrix} \quad \text{and} \quad [K] = \begin{bmatrix} k & -k & 0 & 0 \\ -k & 2k & -k & 0 \\ 0 & -k & 2k & -k \\ 0 & 0 & -k & 2k \end{bmatrix}$$

The solution of the above equation results in the following frequencies and mode shapes.

$$p_1 = 0.3473\sqrt{\frac{k}{m}} \qquad (2)$$

$$p_2 = 1.000\sqrt{\frac{k}{m}} \qquad (3)$$

$$p_3 = 1.5321\sqrt{\frac{k}{m}} \qquad (4)$$

$$p_4 = 1.8794\sqrt{\frac{k}{m}} \qquad (5)$$

TABLE 1

No.	1st Mode	2nd Mode	3rd Mode	4th Mode
i	$\phi_1^{(1)}$	$\phi_1^{(2)}$	$\phi_1^{(3)}$	$\phi_1^{(4)}$
1	0.6565	0.5774	0.4285	0.2280
2	0.5774	0.0000	−0.5774	−0.5774
3	0.4285	−0.5774	−0.2280	0 6565
4	0.2280	−0.5774	0.6565	−0.4285

* Notations are defined as they first appear in text.

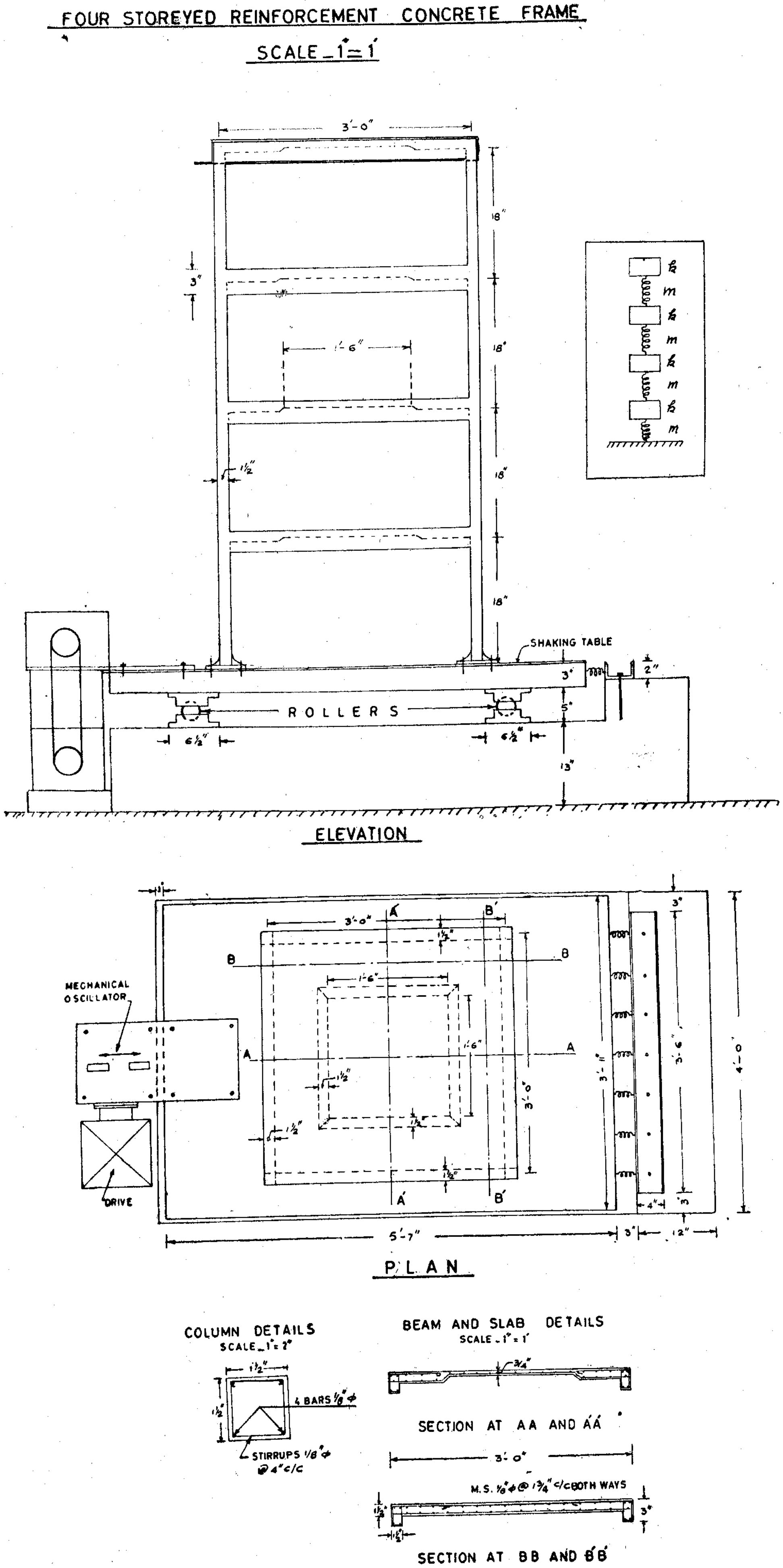
FOUR STOREYED REINFORCEMENT CONCRETE FRAME
SCALE _ 1" = 1'
3'-0"
18"
18"
18"
18"
3"
1'-6"
1½"
SHAKING TABLE
ROLLERS
6½"
6½"
3"
2"
5"
13"
ELEVATION
MECHANICAL OSCILLATOR
DRIVE
3'-0"
1'-6"
1'-6"
3'-0"
3'-11"
3'-6"
4'-0"
5'-7"
3"
12"
4"
3"
PLAN
COLUMN DETAILS
SCALE _ 1" = 2"
1½"
4 BARS ⅛" ϕ
STIRRUPS ⅛" ϕ @ 4" c/c
BEAM AND SLAB DETAILS
SCALE _ 1" = 1'
¾"
SECTION AT AA AND A'A'
3'-0"
M.S. ⅛" ϕ @ 1¾" c/c BOTH WAYS
3"
SECTION AT BB AND B'B'

Fig. 1

$\phi_i^{(r)}$'s have been calculated such that $\sum_{j=1}^{n} m_j(\phi_j^{(r)})^2 = 1$

The weight of each floor including the weight of columns works out to be 200 lbs.

Discussion of Results of free Vibration Tests

For the purpose of estimation of stiffness k, static deflections were measured for known horizontal loads. Horizontal loads were applied to the topmost mass and horizontal deflections at various floor levels were measured. For small loads, (refer Table 2 and Fig. 2) load deflection curves were sensibly linear. In this range, k of each floor works out to be 6667 lbs/in. This value of k corresponds to a modulus of elasticity, E_1 equal to 1.92×10^6 psi assuming moment of inertio. I, to be evaluated on the basis of gross cross-sectional area of concrete and an effective length of column equal to distance between centre to centre of floors. Using the above value of k and m in the frequency equation, $p=0.3473\sqrt{k/m}$, the fundamental frequency of vibration works out to be 6.27 cps. The experimentally determined fundamental frequency of vibration, from free vibration tests, is 6.25 cycles per second.

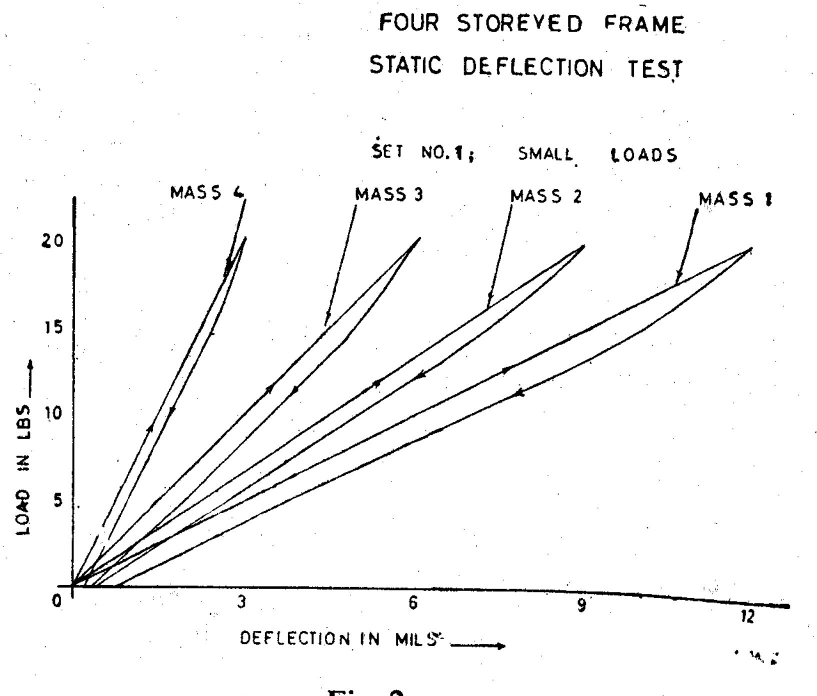

Fig. 2

When large loads are applied, (refer Table 3 and Fig. 3) static deflection tests indicate deflections increasing more rapidly with increase in load. The increased deflection would indicate that k decreases with increase in load. This means that E also decreases. This behaviour is typical of concrete. When k decreases, frequency also should decrease. This is also borne out by free vibration tests in which large initial displacement were given.

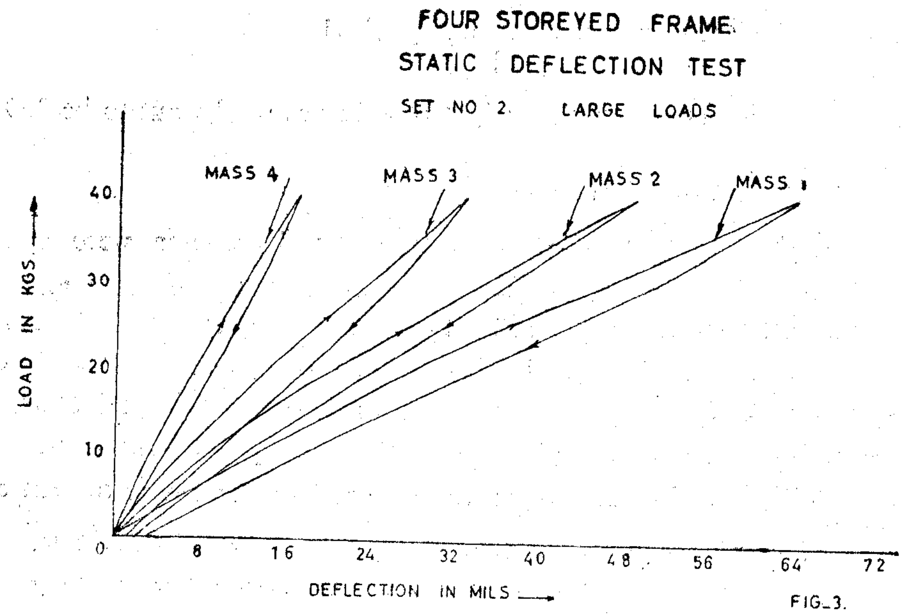

Fig. 3

Fig. 4 shows the free vibration records of the four storeyed frame. The fundamental frequency of vibration (f_1) and damping factor (ζ_1) could be obtained from the records.

Free vibration tests indicate that the experimental value of fundamental frequency of vibration is in accordance with the theory. Average values of damping factor ζ_1 as obtained from free vibration studies, is indicated in Fig. 4.

Forced Vibration Behaviour

To study the behaviour under forced vibration, the frame was subjected to steady state sinusoidal excitation. The frame was cast on top of a platform and the platform was subjected to sinusoidal excitation.

Analysis

Consider the building frame to be subjected to a sinusoidal ground motion represented by $\ddot{y} = \ddot{y}_b \times \sin wt$.

The equation of motion for such a system is given by

$$[M]\{\ddot{z}\} + [C]\{\dot{z}\} + [K]\{z\} = -[M]\{\ddot{y}\} \tag{6}$$

where [M] and [K] are as defined by equation 1.

Z_i = displacement of the i^{th} mass, relative to a coordinate system fixed in the base.

y(t) = displacement of the base, relative to a 'fixed' frame of reference.

[C] = damping matrix.

It will be assumed that [C] would be such that the undamped model columns do remain

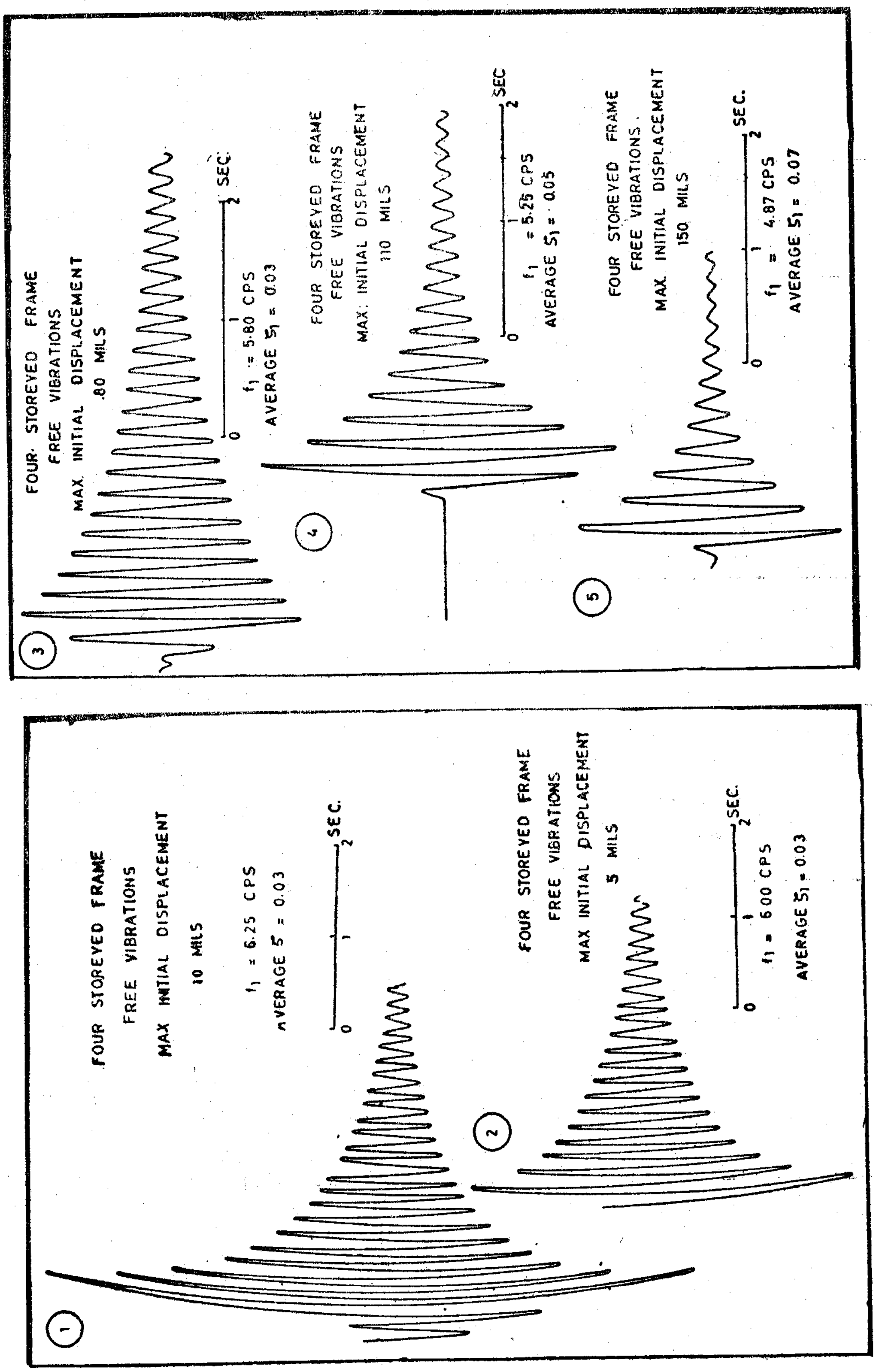
FOUR STOREYED FRAME
FREE VIBRATIONS
MAX INITIAL DISPLACEMENT
10 MILS
f_1 = 6.25 CPS
AVERAGE ξ = 0.03
0
1
2
SEC.
1
FOUR STOREYED FRAME
FREE VIBRATIONS
MAX INITIAL DISPLACEMENT
5 MILS
0
1
2
SEC.
f_1 = 6.00 CPS
AVERAGE ξ_1 = 0.03
2
FOUR STOREYED FRAME
FREE VIBRATIONS
MAX. INITIAL DISPLACEMENT
80 MILS
0
1
2
SEC.
f_1 = 5.80 CPS
AVERAGE ξ_1 = 0.03
3
FOUR STOREYED FRAME
FREE VIBRATIONS
MAX. INITIAL DISPLACEMENT
110 MILS
0
1
2
SEC
f_1 = 5.25 CPS
AVERAGE ξ_1 = 0.05
4
FOUR STOREYED FRAME
FREE VIBRATIONS.
MAX. INITIAL DISPLACEMENT
150 MILS
0
1
2
SEC.
f_1 = 4.87 CPS
AVERAGE ξ_1 = 0.07
5

Fig. 4

STATIC DEFLECTION TEST

TABLE 2

Load		Deflection in Mils							
Lbs.	Kgs.	Mass no. 1 Loading	Mass no. 1 Un-Loading	Mass no. 2 Loading	Mass no. 2 Un-Loading	Mass no. 3 Loading	Mass no. 3 Un-Loading	Mass no. 4 Loading	Mass no. 4 Un-Loading
0	0	0.0	0.75	0.0	0.50	0.0	0.40	0.0	0.25
5	2.27	3.0	3.75	2.25	2.75	1.5	2.0	0.75	1.00
10	4.54	6.0	7.0	4.50	5.0	3.0	3.5	1.50	1.75
15	6.80	9.0	10.0	6.75	7.25	4.5	5.0	2.25	2.50
20	9.07	12.0		9.0		6.0		3.0	

TABLE 3

Load in kgs.	Deflection in Mils							
	Mass no. 1 Loading	Mass no. 1 Un-Loading	Mass no. 2 Loading	Mass no. 2 Un-Loading	Mass no. 3 Loading	Mass no. 3 Un-Loading	Mass no. 4 Loading	Mass no. 4 Un-Loading
0	0.0	3.0	0.0	2.0	0.0	1.50	0.0	1.00
10	13.5	18.0	9.0	12.5	6.75	9.50	3.50	5.00
20	28.0	34.0	21.0	24.5	14.50	18.0	7.50	9.50
30	46.5	51.0	35.0	37.0	24.00	26.25	12.25	13.50
40	64.0		49.0		33.0		17.00	

valid. It has been shown (Rayleigh, 1945) that [C] then should be a linear combination of mass [M] and stiffness [K] matrices. It would be further assumed that the percentage of critical damping in each mode of vibration remains the same. It is possible to choose a damping matrix that will satisfy the above assumption.

If the ground excitation is sinusoidal and is expressed as $\ddot{y} = \ddot{y}_b \sin wt$, then the steady state solution of equation 6 is given by

$$Z_i = - \sum_{r=1}^{n} B_i^{(r)} \cdot \frac{\ddot{y}_b}{p_r^2} \cdot \mu_r \cdot \text{Sin}\,(\omega t - \theta_r) \tag{7}$$

where $B_i^{(r)}$ = mode factor in the r^{th} mode of vibration

$$= \frac{\phi_i^{(r)} \cdot \sum_{j=1}^{n} m_j \phi_j^{(r)}}{\sum_{j=1}^{n} m_j (\phi_j^{(r)})^2}$$

$\ddot{y}_b$ = maximum amplitude of ground acceleration

p_r = natural frequency of vibration in the r^{th} mode.

μ_r = a dimensionless parameter denoting the dynamic amplification factor in the r^{th} mode

$$= \frac{1}{[(1-\eta_r^2)^2 + (2\eta_r\zeta_r)^2]^{\frac{1}{2}}}$$

$\eta_r = \omega/p_r$ = frequency ratio

ζ_r = damping factor in the r^{th} mode.

θ_r = phase angle in the r^{th} mode.

$$= \tan^{-1} \frac{2\eta_r \zeta_r}{1-\eta_r^2}$$

The steady state acceleration $\ddot{Z}_i$ will be given by, differentiating equation (7),

$$\ddot{Z}_i = \sum_{r=1}^{n} B_i^{(r)} \cdot \ddot{y}_b \cdot \eta_r^2 \, \mu_r \cdot \text{Sin}\,(\omega t - \theta_r) \tag{8}$$

The absolute acceleration $\ddot{x}_i$ of any mass i, is equal to $(\ddot{Z} + \ddot{y})$

$$\ddot{x}_i = \ddot{y}_b \left[\sin \omega t + \sum_{r=1}^{n} B_i^{(r)} \cdot \eta_r^2 \cdot \mu_r \cdot \sin\,(\omega t - \theta_r) \right] \tag{9}$$

$$\frac{(\ddot{x}_i)_{max}}{\ddot{y}_b} = \left[\left\{ 1 + \left(\sum_{r=1}^{n} B_i^{(r)} \eta_r^2 \, \mu_r \cos \theta_r \right) \right\}^2 + \left\{ \sum_{r=1}^{n} B_i^{(r)} \cdot \eta_r^2 \cdot \mu_r \cdot \sin \theta_r \right\}^2 \right]^{\frac{1}{2}} \tag{10}$$

The values of $B_i^{(r)}$ for the four storeyed frame is given in Table 4 below. (These have been obtained by making use of Table 1)

TABLE 4

Mass i	Mode Factor $B_i^{(r)}$			
	1st Mode	2nd Mode	3rd Mode	4th Mode
1	1.2411	—0 3333	0.1199	—0.0277
2	1.0914	0.0000	—0.1615	0.0700
3	0.8101	0.3333	—0.0638	—0.0797
4	0.4310	0.3333	0.1836	0.0520

Figure 5 shows a theoretical relationship between $(\ddot{x}_1)_{max.}/\ddot{y}_b$ Vs. η_1 assuming that ζ_r remains constant for all r's and equal to 0.10.

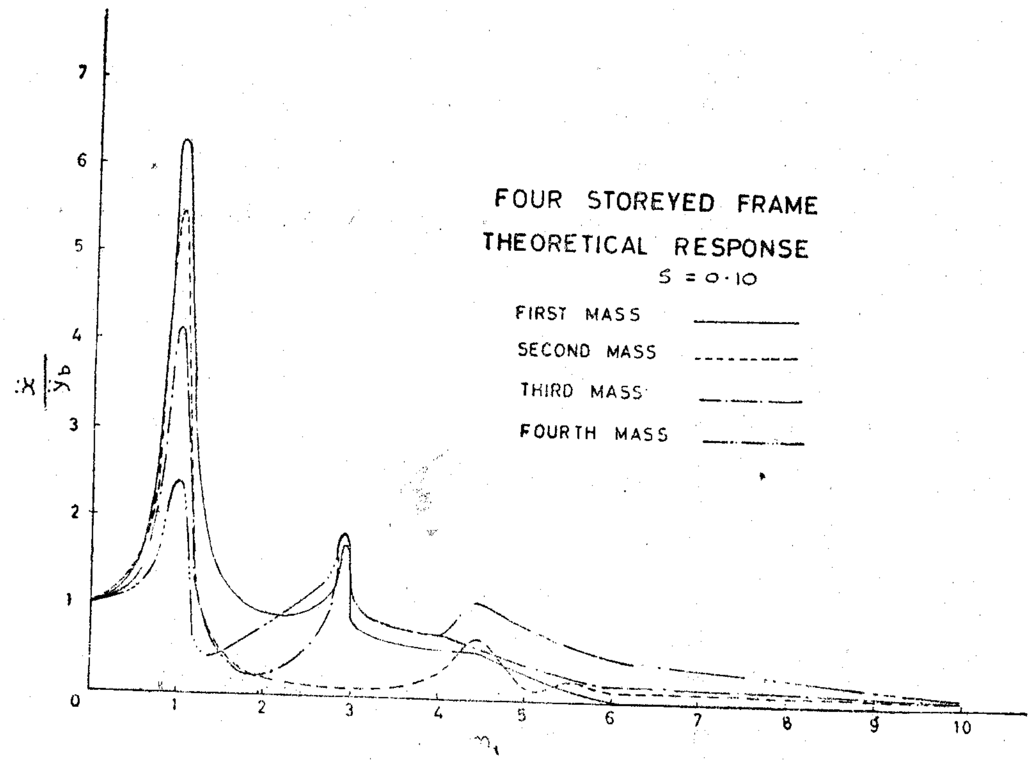

Fig. 5

Experimental Set Up

The four storeyed frame was cast in situ on a platform which rests on rollers such that it was free to move in a horizontal direction. Lazan Mechanical Oscillator was firmly attached to the platform. The oscillator was driven by a Graham's variable speed transmission drive. This set up is capable of giving a steady state sinusoidal excitation to the table.

The oscillator utilises the centrifugal force of unbalanced masses to generate a variable alternating force. It consists of two eccentrices (unbalanced masses) mounted in such a way that they produce a harmonic force in one direction only. There is also an arrangement to vary the amplitude of excitation by adjusting the position of eccentrices.

The frequency of excitation is varied by means of the transmisssion drive which use the compound planetary gear system except that the non-rotating member is a traction ring which engages tapered rollers at varying diameters. The traction ring is moved lengthwise of the transmission to change the speed.

The acceleration of the base (platform) and that of the various floors of the frame were measured by Miller accelerometers, which are variable air gap inductance pick-ups. The pickup is connected to a bridge circuit and the output of the bridge is amplified by a Brush Universal Amplifier and recorded on a Brush ink writing oscillograph.

An experimental run consists of measuring the acceleration of the base and that of a floor for various exciting frequencies. Acceleration of all the floors were measured.

Discussion of Results

Table 5 and Figure 6 give the response of various floors, obtained experimentally. The

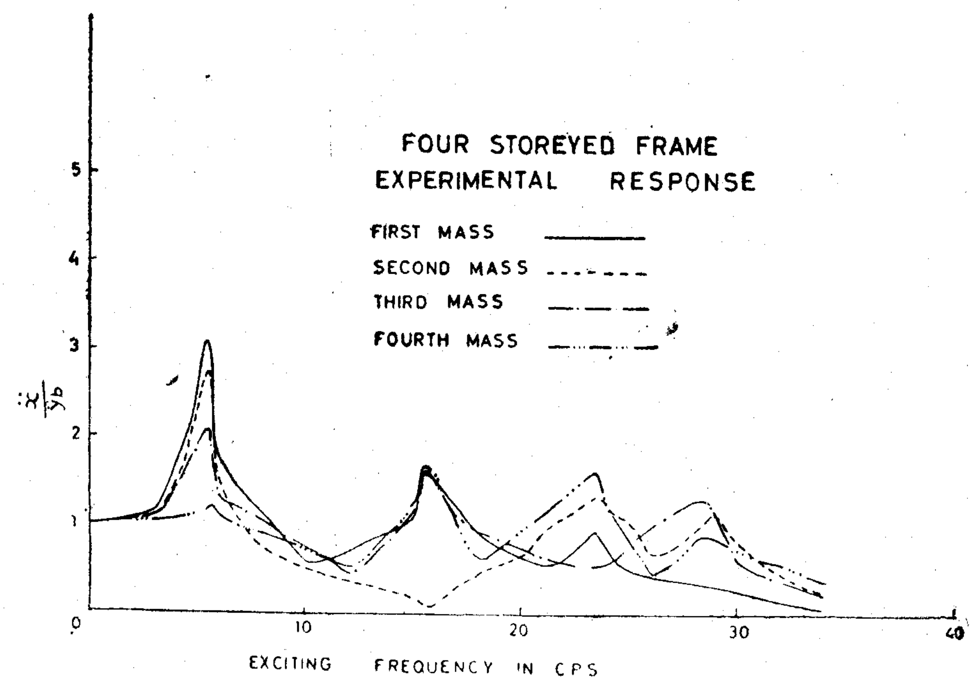

Fig. 6

resonant frequencies correspond to that of the natural frequencies as obtained theoretically for a four degree freedom system. However, the experimental response curves (Fig. 6) do not resemble the theoretical response curves. The theoretical response curves were evaluated for a constant damping factor in all modes. In practice, this is not the case.

TABLE 5

Four Storeyed Frame

Forced Vibration by Steady state Sinusoidal Excitation

RESPONSE OF MASSES

Exciting Frequency 'f' in C.P.S.	RESPONSE			
	$\ddot{x}_1 / \ddot{y}_b$	$\ddot{x}_2 / \ddot{y}_b$	$\ddot{x}_3 / \ddot{y}_b$	$\ddot{x}_4 / \ddot{y}_b$
3.0	1.15	1.10	1.10	1.05
4.5	1.96	1.85	1.70	1.10
5.5	3.10	2.75	2.08	1.20
6.0	1.80	1.50	1.30	1.10
7.5	1.26	0.86	1.12	0.94
8.5	1.05	0.67	0.95	0.88
10.0	0.60	0.52	0.76	0.74
12.0	0.72	0.38	0.46	0.55
14.0	0.94	0.27	0.87	0.95
15.0	1.10	0.18	1.15	1.25
15.5	1.60	0.10	1.65	1.70
17.0	1.20	0.32	1.12	1.05
18.0	0.95	0.47	0.95	0.64
20.0	0.65	0.67	0.77	0.98
21.0	0.57	0.95	0.64	1.13
22.0	0.64	1.14	0.58	1.35
23.5	0.98	1.35	0.55	1.64
24.0	0.73	1.22	0.60	1.24
25.0	0.52	1.08	0.80	0.83
26.0	0.48	0.70	1.00	0.48
27.0	0.42	0.78	1.15	0.60
28.5	0.37	1.05	1.31	0.92
30.0	0.28	0.86	0.61	0.73
32.0	0.12	0.47	0.42	0.60
34.0	0.08	0.29	0.21	0.40

Note: Subscripts 1, 2, 3 and 4 refers to masses, numbered serially from top downwards. $\ddot{y}_b$ represents acceleration of base.

At resonance, the amplification is essentially due to vibration in the mode correspon-ding to the resonent frequency. By considering the amplification at resonant frequencies, the damping factor works out as follows :

Resonant Frequency Corresponds to	*Damping Factor.*
1st mode.	0.20
2nd mode.	0.10
3rd mode.	0.05
4th mode.	0.03

It appears as though the damping as obtained from steady state vibration records is somewhat higher than that of those from free vibration records. This might be due to inade-quacy of the speed control unit which cannot give very small increments to forcing frequency. There is a possibility that the structure has not been excited at true resonance. Nielsen (1964) has reported that a very sensitive speed control unit is required to excite a structure resonance.

CONCLUSIONS

Free vibration tests indicate that the experimental values of fundamental frequency is in accordance with the theory. The resonant frequencies as obtained from forced vibration tests had a good correspondance with the various natural frequencies calculated by theory. Damping depends on the amplitude of vibration, larger the amplitude more the damping. An average value of damping, for this structure, is 5%. Steady state forced vibration tests indicate that damping corresponding to forced vibration is more than that obtained from free vibration tests. This might be due to the speed control unit being not very sensitive. It was observed that damping was maximum in the first mode and it decreases in higher modes.

ACKNOWLEDGEMENT

The author is thankful to the staff of Earthquake Research School, University of Roorkee for assistance rendered in the experimental work and to Dr. Jai Krishna for his valuable guidance in this research project.

REFERENCES

Nielsen, N.N. (1964), "Dynamic Response of Multistorey Buildings", Earthquake Engineering Research Laboratory, Pasadena, California.

Rayleigh B. (1945), "The Theory of Sound", Vol. 1, Dover Publications, New York, N.Y.

Paper No. 24

SEISMIC DESIGN OF MULTISTOREYED CONCRETE FRAMES WITH ANALYSIS OF COST

Dinabandhu Mukherjee,*

SYNOPSIS

The Author presents the results of his study of the increase in cost of multistoreyed concrete frames due to incorporation of seismic factor in the design.

Very little work has been done, uptil now, in connection with the relationship between the seismic factor and cost of multistoreyed framed structures. Particularly no work seems to have been done for concrete frames.

This paper deals with the study of multistoreyed reinforced concrete frames ranging from one to ten storeys in height. The frames are designed for varying degree of lateral forces corresponding to different earthquake intensities. This provision for earthquake allowance involves additional expenditure. The relationship between the increase in cost and the seismic factor is examined.

INTRODUCTION

In the regions frequented by earthquakes, structures need special designing against seismic forces. In the case of an earthquake, the movement of the foundation of a building is transmitted to the superstructure. Since the superstructure has to accelerate from rest to motion, inertia forces act on the superstructure in a direction opposite to that of the earth movement. The shearing forces in each storey due to dynamic loading result from the inertia forces of all the masses above that storey. Seismic design consists of making the structure strong enough to resist the dynamic loading.

It is not practicable to use the dynamic equation for every design as the computations are extremely complex on account of the large number of factors involved. Moreover, the dynamic behaviour of most structures is not fully known. Usual practice, therefore, is to formulate an equivalent statical method of design. The procedures followed by different building codes are to assume the shape of the shear force distribution and then derive an empirical formula for the equivalent horizontal static force co-efficient.

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Since earthquake waves may strike a building at any angle, buildings should be strong enough to resist lateral forces in any direction. However, the waves can be resolved into two components parallel to the major and minor axes of the building and therefore it is sufficient to investigate its strength in the two perpendicular directions only. In a framed building the longer frames along the major axis and the shorter frames along the minor axis have to be individually made strong for lateral seismic loading. The shorter frames are more susceptible to lateral loads. For the investigation we take up a single shorter frame along the minor axis of the building and consider the lateral strength in its own plane.

Every building has its own natural period of vibration. If the period of vibration of an earthquake coincides with the natural period of vibration of the building, excessive stresses are produced in the structure due to resonance.

For earthquake resistant design of multistoreyed buildings, the design formula, according to Indian standard specification (I S 1892–1962) is

$$V=\frac{0.35\,S}{N+0.9\,(S-8)}\,CW$$

where V=Total horizontal shear force

S=Total number of storeys in the building (It shall be taken as 13 when the number of storeys is 13 or less)

N=No. of storeys above the one under consideration.

C=Seismic Coefficient.

W=Weight of the structure above the storey under consideration.

For buildings having not more than 13 storeys this formula simplifies to the form suggested by Jaikrishna (1958).

$$V=\frac{4\,5}{N+4\,5}\,CW$$

Earthquake resistant design of a structure involves additional cost and a compromise has to be drawn between the increase in cost and the additional safety it ensures. Therefore, a systematic study of the relation between the seismic coefficient of design and the increase in cost for different types of building frames should prove helpful in deciding whether it would be practicable to incorporate seismic loading in the standard design practice of our country. In the present study a number of R. C. C. frames with two bays and different storey heights were designed at first without considering seismic loading and then considering seismic loading. The increase in cost due to seismic design over usual design was calculated. The relationship of this increase in cost with the number of storeys and the seismic factors was obtained.

DESIGN OF FRAMES

The present study is restricted to R. C. C. framed structures of the following description.

The frames consist of two bays of 25 feet each so that the floor space can be divided into two rows of rooms (Fig. 1) one being a row of large rooms and the other one being of small rooms. The two rows of rooms will be separated by a corridor formed in the bay containing the row of small rooms. The wall for the corridor is assumed to be having its own footing, so that it does not affect the design of the frame. The floor heights have been kept as 12′-0″. The frames have been placed 12′-0″ centre to centre.

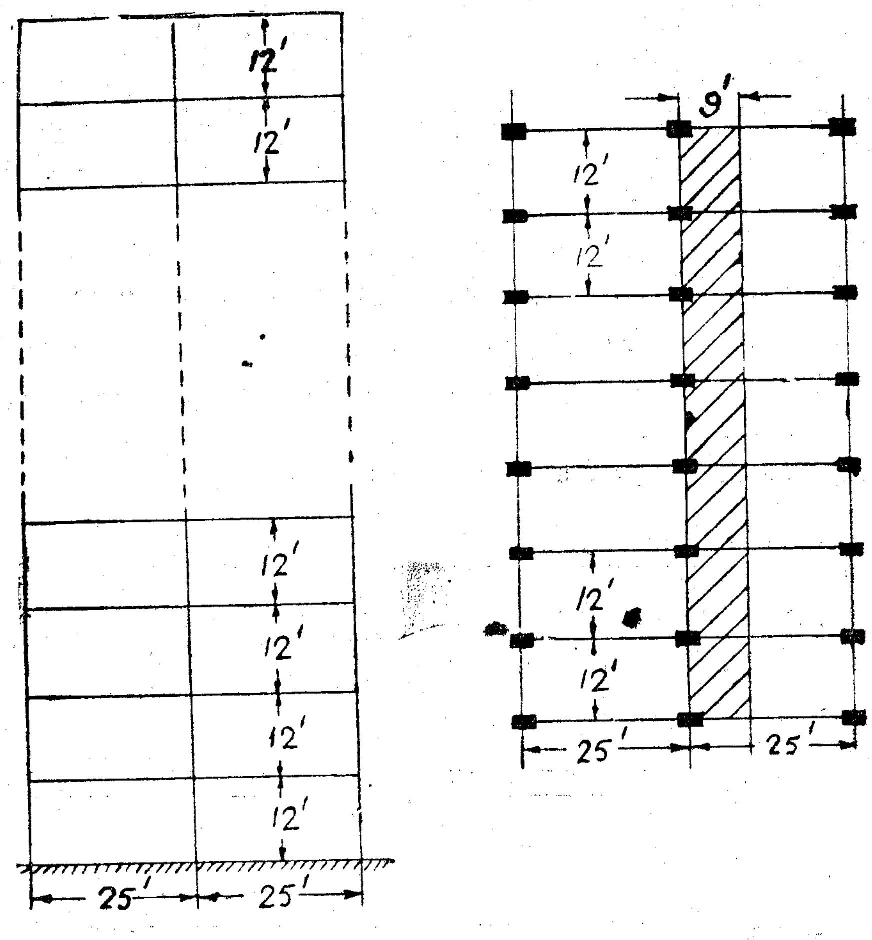

Fig. 1 Multistoreyed Framed Structure

Usual Design

The frames are designed for dead load, live load and wind load. The procedure adopted is as follows.

(i) Various loads are calculated making suitable assumptions for the self-weight of different members.

(ii) A preliminary design is worked out with the help of approximate methods. The column sizes are so chosen that the "Principles of Multiples" may be applied to the frame.

(iii) Dead load analysis is done by using "Kani's method" (Kani, 1957), an iterative process.

(iv) Live load analysis is carried out by "Substitute frame metood" (Jaikrishna & Jain 1960). Moments are calculated for a few suitable frames and the remaining moments are obtained by interpolation.

(v) Combined effect of dead and live loads was then examined, so that the stresses do not exceed the permissible limit.

(vi) Wind load analysis is carried out with the help of "Modified substitute Cantilever Method" (Kloucek, 1958).

(vii) The stresses caused due to the combined effect of dead loads, live loads, and wind loads are checked so that they do not exceed the permissible limit which is $33\frac{1}{3}\%$ in excess of those allowed in the code for normal loading.

(viii) The foundation is lastly designed as a reinforced concrete raft, to provide the necessary bearing area and also making it strong enough to resist the moments.

As a specimen design, the design of the 10-storeyed frame is shown. The other frames (8, 6, 4, 2 and 1 storeyed) were designed similarly. The final design of the 10-storeyed frame is shown in the Table 1.

The foundation is designed as shown is Fig. 2.

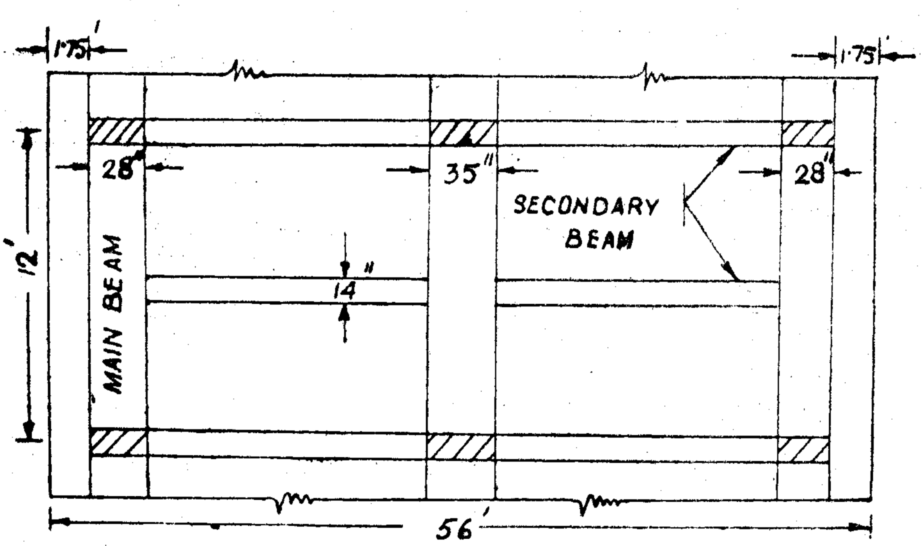

Fig. 2 Design of Foundation

The slab is made 8" thick. The secondary beams are 14"×28". The outer beams (main) are made 28"×31" and the central beams 35"×32", suitable reinforcements are provided in the slab and in the beam.

TABLE - 1

USUAL DESIGN OF 10 STOREYED FRAME

COLUMN STOREY	EXTERIOR	INTERIOR (MAX STRESS)
1st	14" × 28", 1"φ; CONCRET = 65.2 C.FT, STEEL = 2720 C.IN	14" × 35", $1\frac{1}{4}$"φ; CONCRETE = 40.8 C.FT, STEEL = 2130 C.IN
2nd	14" × 28, 1"φ; 65.7 C.FT, 2720 C.IN	14" × 35", 1"φ; 40.8 C.FT, 1360 C.IN
3rd	14" × 28", 1"φ; 65.2 C.FT, 1816 C.IN	14" × 35", 1"φ; 40.8 C.FT, 908 C.IN
4th	14" × 24", $1\frac{1}{8}$"φ; 56 C.FT, 1815 C.IN	14" × 30", $1\frac{1}{8}$"φ; 35 C.FT, 1152 C.IN
5th	14" × 24", 1"φ; 56 C.FT, 1815 C.IN	14" × 30", 1"φ; 35 C.FT, 680 C.IN
6th	14" × 24", 1"φ; 56 C.FT, 1360 C.IN	14" × 30", 1"φ; 35 C.FT, 452 C.IN
7th	14" × 20", 1"φ; 46.6 C.FT, 1815 C.IN	14" × 25", 1"φ; 29.2 C.FT, 452 C.IN
8th	14" × 20", 1"φ; 46.6 C.FT, 1815 C.IN	14" × 25", 1"φ; 29.2 C.FT, 452 C.IN
9th	14" × 16", 1"φ; 37.4 C.FT, 1815 C.IN	14" × 20", 1"φ; 23.3 C.FT, 452 C.IN
10th	14" × 16", 1"φ; 37.4 C.FT, 904 C.IN	14" × 20", 1"φ; 23.3 C.FT, 452 C.IN
BEAM SECTION	26" × 14", 3", 2.3", 3.51 in², 7.0 in²	CONCRETE = 1240 C.FT, STEEL = 63,000 C.IN

TOTAL VOLUME OF CONCRETE = 2104 C.FT

TOTAL VOLUME OF STEEL = 90573 CU IN = 52.4 C.FT

Dynamic Design

In this case, the wind forces are replaced by earthquake forces and the analysis is done by "Modified Substitute Cantilever method". Sections are designed for the combined moments and thrusts due to dead load, live load and earthquake load.

Earthquake forces are calculated according to the simplified formula (See Introduction)

$$\text{Sheer}=\frac{4.5}{N+4.5}\times CXW$$

Calculations of Earthquake forces for the 10-storeyed frame, corresponding to a seismic co-efficient of 20% g, are shown in the Table 2.

Table-2. Earthquake Forces.

		Shear (Kips)	Eqvt. force (Kips)
10th Storey	$\frac{4.5}{0+4.5}\times 0.2\times 113$	22.60	22.60
9th Storey	$\frac{4.5}{1+4.5}\times 0.2\times(113+118)$	37.80	15.20
8th Storey	$\frac{4.5}{2+4.5}\times 0.2\times(231\times 118)$	48.40	10.60
7th Storey	$\frac{4.5}{3+4.5}\times 0.2\times(349+118)$	56.00	7.60
6th Storey	$\frac{4.5}{4+4.5}\times 0.2\times(467+122)$	62.40	6.40
5th Storey	$\frac{4.5}{5+4.5}\times 0.2\times(589+122)$	67.40	5.00
4th Storey	$\frac{4.5}{6\times 4.5}\times 0.2\times(711+122)$	71.40	4.00
3rd Storey	$\frac{4.5}{7+4.5}\times 0.2\times(833+122)$	74.80	3.40
2nd Storey	$\frac{4.5}{8+4.5}\times 0.2\times(955+122)$	77.60	2.80
1st Storey	$\frac{4.5}{9+4.5}\times 0.2\times(1077+122)$	80.00	2.40

The design for 0.2g is shown in the Table 3.

Similarly, the earthquake designs for 0.15g, 0.1g, and 0.05g are carried out for the 10-storeyed frame.

TABLE - 3
SEISMIC DESIGN OF 10 STOREYED FRAME FOR 0.2g

COLUMN STOREY	EXTERIOR	INTERIOR
1st	14" × 28"; 12 in^2; CONCRETE = 62.2 c.ft; STEEL = 6900 c in^2	14" × 35"; 15 in^2; CONCRETE = 40.8 c.ft; STEEL = 4320 c.in
2nd	14" × 28"; 10 in^2; 4"; 65.2 c.ft; 5760 c.in	14" × 35"; 12 in^2; 40.8 c.ft; 3450 c.in
3rd	14" × 28"; 8 in^2; 62.5 c.ft; 4600 c.in	14" × 35"; 8 in^2; 40.8 c.ft; 2300 c.in
4th	14" × 24"; 10 in^2; 56 c.ft; 5760 c.in	14" × 30"; 10 in^2; 35 c.ft; 2,880 c.in
5th	14" × 24"; 8 in^2; 56 c.ft; 4,600 c.in	14" × 30"; 8 in^2; 35 c.ft; 2,300 c.in
6th	14" × 24"; 8 in^2; 56 c.ft; 4,600 c.in	14" × 30"; 8 in^2; 35 c.ft; 23.00 c.in
7th	14" × 20"; 10 in^2; 46.6 c.ft; 5,760 c.in	14" × 25"; 10 in^2; 29.2 c.ft; 2880 c in^2
8th	14" × 20"; 10 in^2; 46.6 c.ft; 5,760 c.in	14" × 25"; 10 in^2; 29.2 c.ft; 2880 c.in
9th	14" × 16"; 10 in^2; 37.4 c.ft; 5,760 c.in	14" × 20"; 10 in^2; 23.3 c.ft; 2880 c.in
10th	14" × 16"; 8 in^2; 37.4 c.ft; 4,600 c in	14" × 20"; 5 in^2; 23.3 c.ft; 1,440 c in
BEAM SECTION	26" × 14"; 18 in^2; 18 in^2	CONCRETE = 1240 c.ft; STEEL = 144,000 c.in

TOTAL VOLUME OF CONCRETE = 2104 C.FT
TOTAL VOLUME OF STEEL = 225,730 C.In = 130 C.F.T.

The design shown for the 10-storeyed frame was repeated for 8-storeyed, 6 storeyed 4 storeyed, 2 storeyed and 1 storeyed frames, obtaining the design in each case with respect to wind load, earthquake allowances of 0.2g, 0.15g, 0.1g, 0.05g over and above dead load and live load.

COST ANALYSIS

For the purpose of comparing the cost of buildings designed for different purposes, we calculate the cost of structural frame, the floors, the foundation and assume that other details of the buildings and the various electrical and sanitary fittings contribute about half of the total cost of the complete building.

Rates assumed* :—

Concrete in multistoreyed frame, excluding cost of steel but including cost of bending and placing of reinforcement	Rs. 4.00 per cu. ft.
Steel in reinforcement ...	Rs. 170.00 per cu. ft.
Cost of flooring ...	Rs. 3.00 per sq. ft.
Foundation concrete, including cost of steel	Rs. 6.00 per cu. ft.

Sample calculation for 10-storeyed building :—

Initial cost $=(2104\times4+52.4\times170+6000\times3+1054\times6)\times2.0$
$=83{,}300$

For earthquake allowance of 0.2g, Increase in cost
$=(130-52.4)\times170=13{,}192$

Therefore, per cent increase in cost $=\dfrac{13{,}192}{83{,}300}\times100=15.8\%$

Per cent increase in cost for earthquake allowance of 0.15g

$$=\frac{(97.6-52.4)\times170}{83{,}300}\times100=9.2\%$$

Per cent increase in cost for Earthquake allowance of 0.1g

$$=\frac{(66.0-52.4)\times170}{83{,}300}\times100=2.8\%$$

Per cent increase in cost for Earthquake allowance of 0.05g is nil.

For different buildings, the initial cost, per cent increase in cost due to earthquake allowances given in non-dimensional form in the Table 4. The initial cost of one-storeyed building is calculated as Rs. 7056, which is assumed to be unity. The other costs are expressed as ratios to this cost of one storeyed building.

*Rates pertaining to the year base 1952.

Table—4

Cost of Buildings.

Building	Initial cost	Increase in cost due to Earthquake allowances of		per cent increase
		20% g	1.87	15.8%
		15% g	1.09	9.2%
10–Storeyed	11.80	10% g	0.327	2.8%
		5% g	nil	nil
		20% g	1.76	19.5%
		15% g	1.12	12.4%
8–Storeyed	9.05	10% g	0.555	6.1%
		5% g	nil	nil
		20% g	1.36	20.1%
		15% g	0.93	13.8%
6–Storeyed	6.75	10% g	0.438	6.5%
		5% g	0.014	0.21%
		20% g	0.91	20.6%
		15% g	0.62	14.1%
4–Storeyed	4.40	10% g	0.232	7.5%
		5% g	0.024	0.55%
		20% g	0.45	20.9%
		15% g	0.31	14.4%
2–Storeyed	2.15	10% g	0.17	7.8%
		5% g	0.024	1.1%
		20% g	0.178	17.8%
		15% g	0.14	14.0%
1–Storeyed	1.00	10% g	0.08	8.0%
		5% g	0.015	1.5%

Relationship between Seismic Factor, Cost and Number of Storeys

The percentage increases in cost of different buildings corresponding to various earthquake allowances are plotted and the graphs obtained thereby. Each curve in Fig. 3 shows the variation of percentage increase in cost due to different earthquake allowances for a certain building. The increase in cost is expressed as a percentage of the cost of the building not designed for earthquake forces but designed for wind forces. Each curve in Fig. 4 shows the variation of percentage increase in cost with the number of storeys of buildings for a certain fixed earthquake allowance.

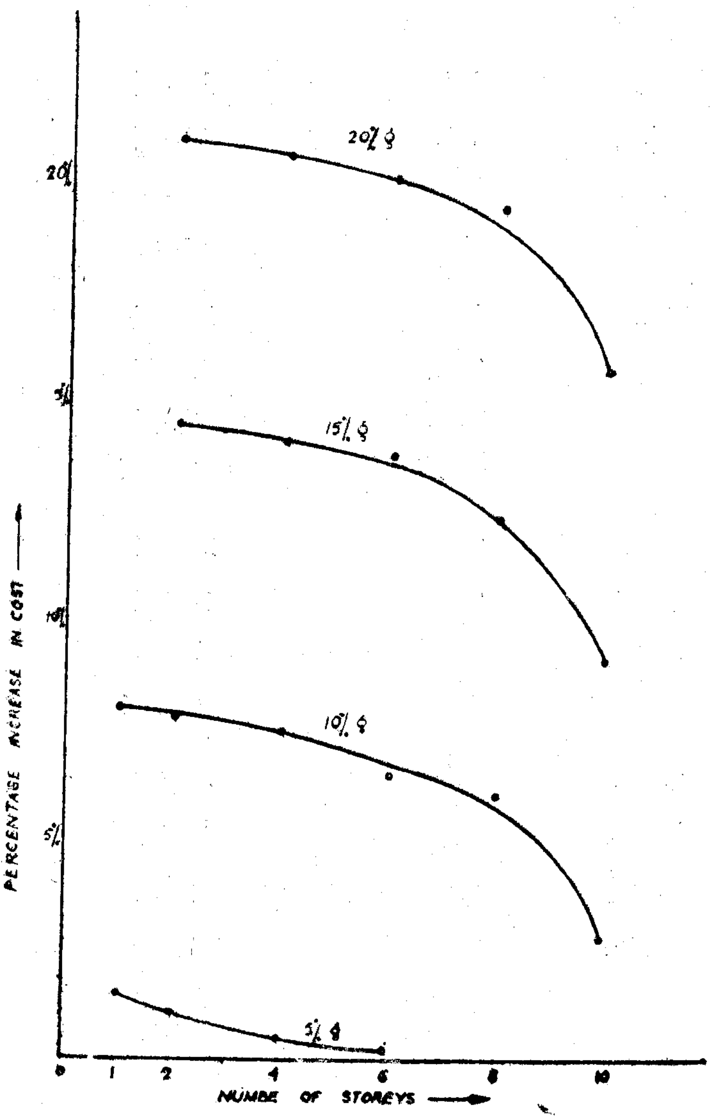

Fig. 3 Increase in Cost Vrs. Seismic Factor.

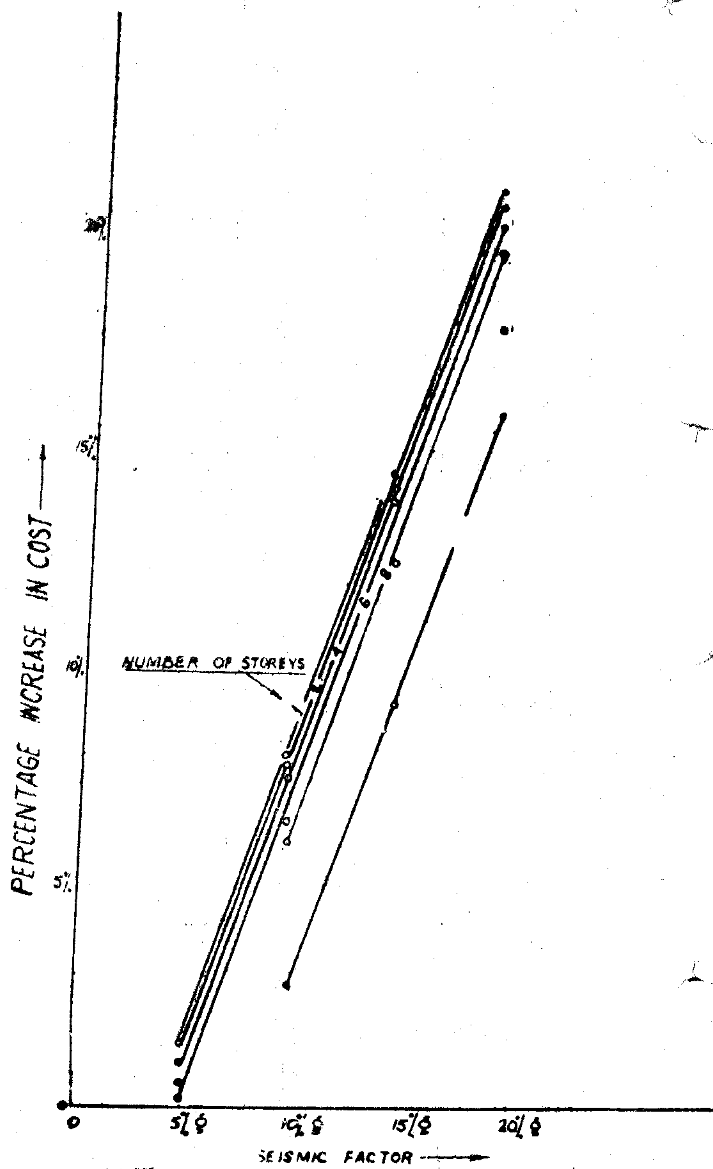

Fig. 4 Increase in Cost Vrs. Number of Storeys.

CONCLUSIONS

For the same height of building percentage increase in cost rises linearly with increase in seismic factore. If the same seismic factor is used for design of a number of buildings, the percentage increase in cost is less for taller buildings.

REFERENCES

Abbet, R. W., (1957), "American Civil Engineerin Practice" John Whiley and Sons, New York, Vol. 3.

Arya, A.S., (1958), "Sidesway of Multistoreyed Frames—A Direct Method of Analysis", Proceedings of the Symposium on Multistoreyed Office Buildings, National Buildings Organisation, New Delhi.

Indian Standards Institution, (1957), "Indian Standard Code of Practice for Plain and Reinforced Concrete for General Building Construction" I.S.I., New Delhi.

"Indian Standard Code of practice for Structural Safety of Buildings; Loading Standards", I.S.I., New Delhi.

"Indian Standard Recommendations for Earthquake Resistant Design of Structures", I.S.I., New Delhi.

Kani, G., (1957) "Analysis of Multistore yed Frames," Translated from the 5th German Edition by Charles J. Hyman; Fredrick Ungar Publishing Co. New York.

Krishna, J. (1958), "Earthquake Engineering Problems in India" Journal of the Institutions of Engineers (India), Vol. 39, No. 1, Part I, Sept. 1958.

Krishna, J. (1959) "Aseismic Design of Structures in India" Proceedings of the Symposium on Earthquake Engineering, University of Roorkee, Roorkee.

Krishna J. and O. P. Jain, (1960) "Plain and Reinforced Concrete" Vol. I. Nem Chand and Bros., Roorkee.

Krishna J. and O. P. Jain, (1961) "Plain and Reinforced Concrete" Vol. II. Mem Chand and Bros,, Roorkee.

Mathur. S. K. (1958) "A Review of Methods of Analysis for Wind Loads on tall Building Frames". Proceedings of the Symposium on Multistoreyed Office Buildings, National Buildings Organisation, New Delhi.

Rao, K. L.; "Reinforced Concrete Design", Pitman Publications, London.

Rloucek, V., (1958) "Distribution of Deformation", Translated from the Czech and German Editions by A. H. Waddel-Zalud and F. H. Zalud, Artia, Prague.

Sutherland H., and Cowman "An Introduction to Structural Theory and Design".

The Concrete Association of India, (1958). "Earthquake Resisting Buildings' Bombay.

Paper No. 25

ON THE APPLICATION OF THE RECIPROCAL THEOREM TO THE VIBRATION OF CONTINUOUS BEAMS

R. Narayana Iyengar*

SYNOPSIS

This paper illustrates the application of reciprocal theorem to the vibration of continuous beams. A few illustrative examples have been given.

INTRODUCTION

The vibration of continuous beams has been extensively studied in the past. The clasical approach for the study of the free vibrations is through the "Three moment" equation or the "slope-deflection" equations. Either of the methods results in a set of homogeneous equations with the support moments as the unknowns. Then the transcendental frequency equation is arrived at by the condition that the determinant formed by the coefficients of the support moments in the set of homogeneous equations must vanish for non-trivial solutions. The steady state forced vibrations of continuous beams has been studied by Gaskell (1952) by a moment balancing procedure analogus to the Hardy Cross method of moment distribution in statics. A serious limitation of this procedure is that the convergence of the solution is not assured for values of the exciting frequency greater than the fundamental natural frequency of the beam.

The steps taken by Saibel and D' Apolonia (1952) for the solution of the forced vibration problems are (a) Intermediate supports are removed leaving an ordinary simple beam for which the eigen values and eigen functions are known, (b) The deflection of the beam at any point is represented as an infinite series in terms of the eigen functions of the simple beam. (c) The constraints at the intermediate supports are introduced through undetermined multipliers. From the Lagrange equations of the motion and the conditions of constraint, equations are developed that yield the solution.

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In this paper a method based on Reciprocal theorem has been developed for the study of vibrations of the continuous beams. The method consists in essentially considering the system in two states, firstly under given loadings; secondly under a convenient fictitious loading and to apply the Reciprocal theorem which states that "if a body is considered, subject to two different states of forces then the work done by the forces of each state on the corresponding displacements of the other are equal."

The present method gives the same frequency equation as in the classical methods circumventing the formation of the simultaneous equations in the case of free vibrations and in the case of forced vibrations it has no problem of convergency, this being not a successive approximation method. Compared to the approach of Saibel and D' Apolonia (1952) the present method for the steady state solution does neither requires a knowledge of the eigen values and eigen functions of the simple beam nor encounters the solution of simultaneous equations. Hence the use of the method lies mostly in the solution of the forced vibration problems of continuous beams.

SOME USEFUL RESULTS

For a simply supported beam subjected to an end moment M Sinpt (Fig. 1) the equation of motion is

$$\frac{\partial^4 y}{\partial x^4}+\frac{m}{EI}\frac{\partial^2 v}{\partial t^2}=0 \qquad (1)$$

where m is the mass per unit length and EI is the flexural rigidity of the beam

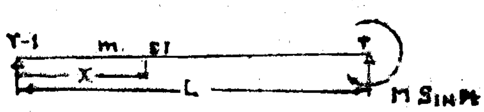

Fig. 1

with the Boundary conditions

$$y=0 \text{ at } x=0 \text{ and } L, \quad \frac{\partial^2 v}{\partial x^2}=0 \text{ at } x=0; \quad \frac{\partial^2 y}{\partial x^2}=M \sin pt \quad \text{at } x=L$$

The solution is

$$y\,(x,\,t)=\frac{M}{2EI\lambda^2}\left[\frac{\sin \lambda x}{\sin \lambda L}-\frac{\sinh \lambda x}{\sinh \lambda L}\right](\sin pt) \qquad (2)$$

where $\lambda^4=\frac{mp^2}{EI}$

The slope at the Left and Right ends are respectively

$$\theta_{r-1,\,r}=\frac{M}{2EI\lambda}\left[\frac{1}{\sin\lambda L}-\frac{1}{\sinh\lambda L}\right]\sin pt=\frac{ML}{6EI}\,\psi\sin pt$$

$$\theta_{r,\,r-1}=\frac{M}{2EI\lambda}\left[\coth\lambda L-\cot\lambda L\right]\sin pt=\frac{ML}{3EI}\,\theta\sin pt \qquad (3)$$

The dynamic carry over factor

$$=\frac{\theta_{r-1,\,r}}{\theta_{r,\,r-1}}=\frac{\psi}{2\theta} \qquad (4)$$

FREE VIBRATION OF CONTINUOUS BEAMS

Referring to Fig. 2 let it be required to find the natural frequencies of the beam on 'n' supports. The condition of the beam freely vibrating, and hence acted on by support moments M_r sinpt etc. is the I state for applying the Reciprocal theorem. The II state is the one in which the beam is acted on by opposite pulsating moments of unit amplitude applied at some support 'r'.

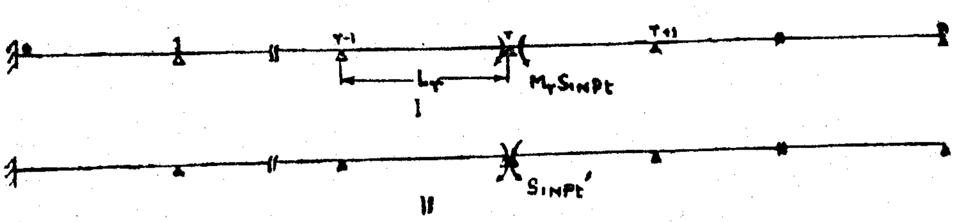

Fig. 2

The deflection of the span (r—1, r) in the I state can be written as

$$y_{rI}=X_{rI}\sin pt$$

and the force acting on the span

$$=\frac{m\partial^2 y_{rI}}{\partial t^2}=-mp^2X_{rI}\sin pt$$

Similarly the deflection in the II state is

$$y_{rII}=X_{rII}\,\text{sinpt}$$

Force acting $=-mp^2X_{rII}\,\text{sinpt}$

Let $\phi_{r,\,r-1}$ and $\phi_{r,\,r+1}$ be the amplitude of the slopes to the left and right of support 'r' in the II state.

From the Reciprocal Theorem

$$\sum_{r=1}^{n} \int_0^{Lr} m\frac{\partial^2 y_{rI}}{\partial t_2} y_{rII} dx + M_r \operatorname{Sinpt} (\phi_{r,\,r-1} + \phi_{r,r+1}) \operatorname{Sinpt}$$

$$= \sum_{r=1}^{n} \int_o^{Lr} m\frac{\partial^2 y_{rII}}{\partial t^2}\, y_{rI}\, d_x \qquad (5)$$

This simplifies to

$$\phi_{r,\,r-1} + \phi_{r,\,r+1} = 0 \qquad (6)$$

This is the frequancy equation for the beam. The tables of θ and ψ for values of λL within 2π, as given by G.L. Rogers greatly facilitate the solution of equation (6).

Example 1.

To find natural frequencies of the beam shown in fig. 3. m and EI remain constant throughout.

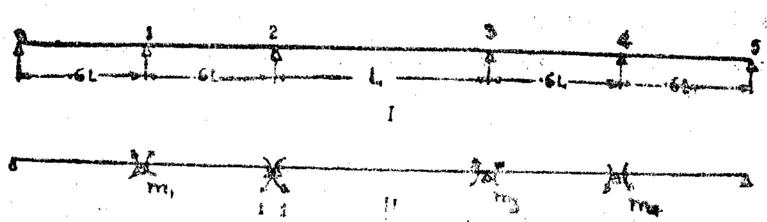

Fig 3

Moments with unit amplitude are applied at support '2' due to which moments of amplitude m_1, m_3 and m_4 are induced at 1, 3 and 4. They are found by successively applying Reciprocal theorem considering the beam to the left and right of support '2' as two separate continuous beams acted on by an end moment.

Henceforth in the figure for the II state only the amplitudes of the different quantities will be shown. Considering the condition of the portion 0–1–2 when acted on by a moment 'Sin pt' at '2' as the I state of loading and the condition in which opposite moments 'Sin pt' act at '1' as the II State from Reciporcal theorem.

$$2m_1 \left(\frac{0.6L}{3EI}\right) \theta_{0\cdot 6} = \left(\frac{0.6L}{6EI}\right) \psi_{0\cdot 6}$$

$$\text{i.e. } m_1 = \psi_{0\cdot 6}/4\theta_{0\cdot 6}$$

The subscripts for θ and ψ refer to the ratio of characteristic length (λL_r) of the span under consideration to that of span 2–3.

Next considering the portion 2–3–4–5 (Fig. 4) and applying Reciprocal theorem.

$$m_3 \left[\frac{L\theta}{3EI} + \frac{0.6L}{3EI}\,\theta_{0\cdot 6} - \frac{\psi^2_{0\cdot 6}}{4\theta_{0\cdot 6}}\left(\frac{0.6L}{6EI}\right)\right] = \frac{L}{6EI}\,\psi$$

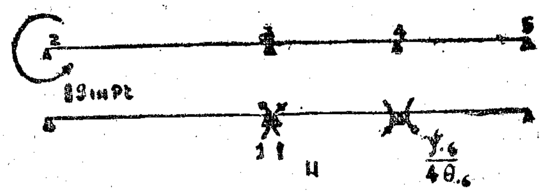

Fig. 4

Hence $m_3 = \left[\dfrac{\psi}{2\theta+1.2\theta_{0\cdot6}-0.15\,\dfrac{\psi^2_{0\cdot6}}{\theta_{0\cdot6}}}\right]$

The frequency equation referring to Fig. 3 is

$$\phi_{2,1} + \phi_{2,3} = 0$$

$$\text{i.e.}\quad \left(\frac{0.6L}{3EI}\,\theta_{0\cdot6} - m_1\,\frac{0.6L}{6EI}\,\psi_{0\cdot6}\right) + \left(\frac{L}{3EI}\,\theta - m_3\,\frac{L}{6EI}\,\psi\right) = 0$$

This simplifies to

$$0.6\,\theta_{0\cdot6} - 0.075\frac{\psi^2_{0\cdot6}}{\theta_{0\cdot6}} + \theta - \frac{\psi^2}{\left(4\theta+2.4\theta_{0\cdot6}-0.3\,\dfrac{\psi^2_{0\cdot6}}{\theta_{0\cdot6}}\right)} = 0$$

The solution of this equation leads to the different natural frequencies. The fundamental frequency is found to correspond to $\lambda L=3.38$.

FORCED VIBRATION OF CONTINUOUS BEAMS

When pulsating external loads are acting on a continuous beam, the Reciprocal theorem is applied to determine the support moments explicitly. Loads of different excitation frequency are to be considered separately and the effects superposed. Referring to Fig. 5 due to a load W Sinpt on span (r—1, r) oscillatory moments with amplitude Mo, M, etc. are induced at supports 0, 1 etc., considering another condition of the beam in which instead of external loads unit amplitude moments having the same frequency as the load are applied at support 'r'. From Reciprocal theorem

$$-W\delta + M_r\left(\phi_{r,r-1} + \phi_{r,r+1}\right) = 0 \qquad (7)$$

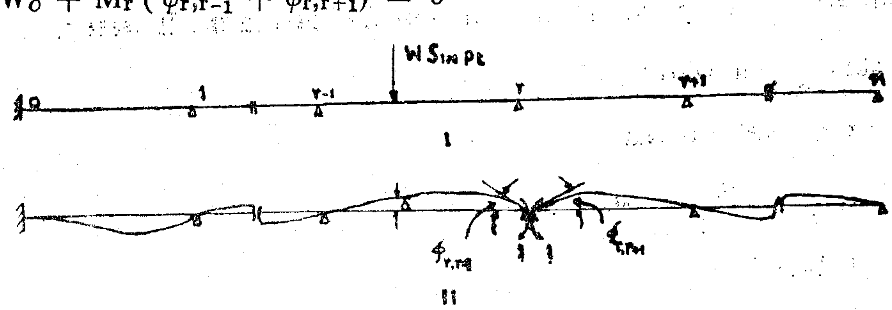

Fig. 5

where δ is the deflection amplitude at the point of application of the load in the II state.

From equation (7) M_r can be found out. The other support moments can be found by considering the beam to the left and right of support 'r' separately, acted on by one end moment M_r along with the given external loads. As the moments induced at the supports due to a moment at support 'r' will have been already calculated in the determination of M_r, the computation of the other support moments will not present much difficulty.

The problems which have been worked by the authors (Gaskell and Saibel & D'Apolonia) mentioned in the introduction are worked here to compare the numerical results.

Example 2.

To find the support moments of the beam shown in Fig. 6. m, EI are constant throughout.

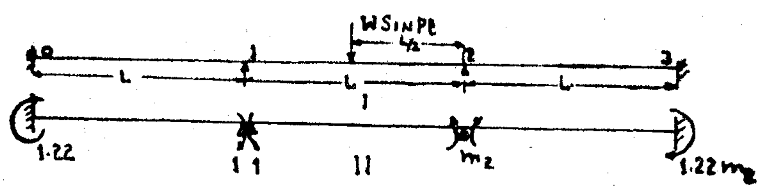

Fig. 6

The frequency of the load is such that $\lambda L = 3.3$.

for $\lambda L = 3.3$. $\theta = -2.3896$ $\psi = -5.8302$

The moment at '0' due to unit moment at '1' in the II state

$$= \frac{\psi}{2\theta} = 1.22$$

Considering the portion 1–2–3 separately in the II state we get

$$m_2 = 2.37$$

$$\phi_{10} = \left(\frac{\theta L}{3EI} - 1.22 \frac{\psi L}{6EI}\right) = 0.3865 \frac{L}{EI}$$

$$\phi_{12} = \left(\frac{\theta L}{3EI} - 2.37 \frac{\psi L}{6EI}\right) = 1.5035 \frac{L}{EI}$$

amplitude of deflection at the point of application of the load in the II state is

$$= 0.405 \frac{L^2}{EI} \text{ upwards}$$

Hence from the Reciprocal theorem

$$M_1 (\phi_{10} + \phi_{12}) - \frac{0.405}{EI} WL^2 = 0$$

i.e. $M_1 = 0.2142$ WL

The sign of M_1 indicates the direction assumed in the beginning for M_1 as shown in Fig. 5 is correct, a -ve sign would have indicated the opposited direction.

The values of moments are compared with those obtained by Gaskell by a 12-cycle moment balancing in the following table.

C.W. = clockwise. A.C.W. = Anti-clockwise

	M_{01} (C.W.)	$M_{10} = -M_{12}$ C.W.	$M_{23} = -M_{21}$ A.C.W.	M_{32} A.C.W.
Author	0.2615 WL	0.2142 WL	0.2142 WL	0.2615 WL
Gaskell	0.2630 WL	0.2160 WL	0.2170 WL	0.2630 WL

Example 3.

To find the deflection under the load for the beam in fig. 7. The frequency of the load is one fourth the fundamental frequency of the beam with support 1.

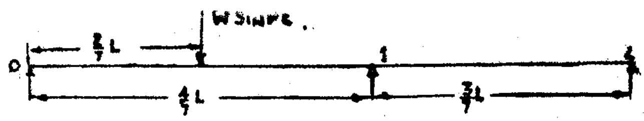

Fig. 7

$$P = \frac{\pi^4}{4}\sqrt{\frac{EI}{mL^4}} \text{ and } \lambda L = \frac{\pi}{2}$$

For Span 0—1

$$\lambda L_1 = \left(\frac{\pi}{2L}\right)\left(\frac{4L}{7}\right) = 0.897$$

For Span 1—2

$$\lambda L_2 = 0.625$$

Correspondingly in the II state of loading

$$(\phi_{10} + \phi_{12}) = \frac{0.333L}{EI}$$

and the deflection amplitude

$$\delta = \frac{2L^2}{EI\,\pi^2}\left[\frac{\text{Sin } 0.448}{\text{Sin } 0.897} - \frac{\text{Sinh } 0.448}{\text{Sinh } 0.897}\right] = \frac{0.021\,L^3}{EI}$$

From Reciprocal theorem

$$M_1\left(\frac{0.33L}{EI}\right) - W\left(\frac{0.021\,L^3}{EI}\right) = 0$$

$$\text{i. e. } M_1 = 0.063 \text{ WL}$$

The deflection of a simply supported beam of span 'L' subject to an external pulsating load at a distance 'c' from the left end is (4)

$$y(x, t) = \frac{2WL^3}{EI} \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi x}{L} \sin \frac{n \pi c}{L}}{n^4 \pi^4 \lambda^4 L^4} \text{ Sinpt}$$

'x' being measured from the left end. Treating the first span separately, the deflection below the load due to the external load only

$$= 0.00382 \frac{WL^3}{EI} \text{ Sinpt (downwards)}$$

Deflection due to moment M_1 Sinpt

$$= 0.001323 \frac{WL^3}{EI} \text{ Sinpt (upwards)}$$

Hence the resultant deflection below the load

$$= 0.002497 \frac{WL^3}{EI} \text{ Sinpt (downwards)}$$

The steady state deflection considering two terms in the infinite series solution given by Saibel and D' Apolonia is

$$= 0.002564 \frac{WL^3}{EI} \text{ Sinpt}$$

CONCLUSION

A method based on the Reciprocal theorem is presented for the analysis of free and forced vibrations of continuous beams. From the examples worked out it can be seen, that in problems of forced vibrations it is the easiest one among the existing methods.

ACKNOWLEDGEMENT

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REFERENCES

Gaskell, R.E., (1943), "On Moment Balancing in Structural Dynamics," Quarterly of Applied Mathematics pp. 237–249.

Rogers, G.L., "Dynamics of Framed Structurers," John Wiley & Sons, Inc. New Yark.

Saibel and D' Apolonia, (1952) "Forced Vibrations of Continuous Beams", Trans. A.S.C.E., p. 1075.

Timoshenko, S., "Vibration Problems in Engineering," D. Van Nostrand Company, Inc.

SEISMOLOGICAL NOTES

(India Meteorogical Department, New Delhi)

Earthquakes felt in and near about India during October-December, 1964.

Date 1964	Origin time (GMT) h. m. s.	Epicentre Lat. (°N)	Long. (°E)	Region	Approx.dep. (Km.)	Magnitude	Remarks
1	2 3 4	5	6	7	8	9	10
Apr. 15	16 35 56 (N.DLH)	21.8	88.5	South west of Calcutta	—	6.5 (N.DLH)	Felt at Calcutta (M.M. intensity VII), Kakdwip, Saugor Island, Mohan pur and some other parts of Midnapur Dist.
	16 35 57.5 (U.S.C.G.S.)	21.7	88.0	India East Pakistan border	36	5.5 + 0.4 (CGS)	(All M.M. intensity VII)
Apr. 16	—	—	—	—	—	—	Kakdwip, Saugor Island, Contai felt at about 09 hrs. 25 m. GMT (M. M. intensity V)
Apr. 16	—	—	—	—	—	—	Felt at Shillong at 11 hr. 10 mt. GMT (M.M. intensity V)
Apr. 22	14 56 53.1 (USCGS)	12.4	95.7	Andaman Island.	33	5.0 (CGS)	—
May 4	19 30 52 (N.DLH)	41Kms. NW of Delhi	—	—	—	3.7 (N.DLH)	—
May 7	17 41 39.8 (USCGS)	36.0	70.7	Hindukush	108	4.7 (CGS)	Recorded at a few Indian Observatories.
May 16	08 38 52 (Shillong)	36	71	Hindukush	—	—	Recorded at all Indian Observatories.
	08 38 54 (USCGS)	36.3	71.5	Hindukush	122	5.3 (CGS)	
May 17	—	—	—	—	—	—	Felt at Goalpara(Assam) at 20 hr. 25 mt. GMT (M.M. intensity V)
May 24	00 00 50.2 (USCGS)	30.1	82.1	Nepal	33	5.1 (CGS)	Recorded at a few Indian Observatories.
	00 00 49 (N.DLH)	—	—	—	—	5.3 (N.DLH)	

1	2	3	4	5	6	7	8	9	10
May 24		—		—	—	—	—	—	Felt at Mohanbari at 11 hr. 25 mt. GMT (M. M. intensity V)
May 28	06	14	27 (N.DLH)	11Kms. South-west of Delhi		—	—	2.6 (N.DLH)	Felt at Delhi (M.M. intensity III)
May 29	01	59	21 (N.DLH)	12Kms. away from Delhi	—	—	—	2.8 (N.DLH)	Felt at Delhi.
Jun. 3	02	49	15 (Shillong)	26	95	Naga Hills	—	6.1 (N.DLH)	Felt at Many places in Assam (M.M. intensity Shillong V, Mohanbari V, Lumding V, North Lakhimpur V)
	02	49	14.9 (USCGS)	25.9	95.8	Northern Burma	100	5.5 + 0.4 (CGS)	
Jun. 6	08	05	56.4 (N.DLH)	37.1	72.1	Hindukush	166	5.0 (CGS)	Recorded at a few Indian Observatories.
Jun. 9	12	33	25 (N.DLH)	21.7	87.7	South West of Calcutta	—	—	Felt at Calcutta, Kakdwipa, Saugor Island (M.M. intensity V)
Jun. 12		—		—	—	—	—	—	Felt at Srinagar at 05hr. 45 mt. GMT (M.M. intensity V)
Jun. 13	03	28	42 (N.DLH)	17Kms. South-East of Delhi	—	—	—	—	Felt at Delhi and Sonepat.
Jul. 3	14	10	33.0 (USCGS)	33.9	74.5	Kashmir	94	4 9 (CGS)	Recorded at a few Indian Observatories.
Jul. 7	21	12	33.6 (USCGS)	35.8	73.4	Hindukush	19	5.2 (CGS)	Recorded at a few Indian Observatories.
Jul. 12	20	16	00 (Shillong)	27	94	Northeast Assam	—	—	Recorded at all Indian Observatories.
	20	15	59 (USCGS)	24.9	95.3	Northwestern Burma	155	6 7	
Jul. 13	10	58	50 (Shillong)	24	94	Manipur-Burma Border	—	—	Recorded at all Indian Observatories.
	10	58	47.7 (USCGS)	23.7	94.7	Northwestern Burma	117	6.5 (CGS)	
Jul. 28	21	38	43.5 (USCGS)	14.3	96.2	Andaman Islands region	33	5.5 (CGS)	Recorded at a few Innian Observatories.

1	2 3 4	5	6	7	8	9	10
Jul. 28	22 46 34 (USCGS)	14.1	96.1	Andaman Islands region	14	5.6 (CGS)	Recorded at a few Indian Observatories.
Aug. 1	00 47 08 (USCGS)	36.7	70.3	Hindukush Region	149	—	Recorded at a few Indian Observatories.
Aug. 11	13 35 14 (USCGS)	6.3	97.3	Nicober Islands region	33	5.1 (CGS)	—
Aug. 12	04 07 35 (DELHI)	30Kms. North-west of Delhi	—	—	—	3.2 (DLH)	Felt at Delhi and Sonepat.
Aug. 16	03 29 33 (DELHI)	40Kms. North-west of Delhi	—	—	—	2.6 (DLH)	Felt at Sonepat.
Aug. 17	14 42 56.6 (USCGS)	24.2	94	India-Burma Border region	184	4.7 (CGS)	—
Aug. 28	13 21 13.5 (USCGS)	7.1	95.1	Nicober Islands Region	33	5.1 (CGS)	Recorded at a few Indian Observatories.
	13 22 05 (USCGS)	7.6	95.6	Nicober Islands Region	33	5.2 (CGS)	Recorded at a few Indian Observatories.
	13 36 50 (Shillong)	12	83	Off coast of Madras	—	—	Recorded at a number of Indian Observatories.
Aug. 30	02 35 05 (Shillong)	27	88	Nepal-Sikkim Border	—	—	Recorded at a number of Indian Observasories.
	02 35 08 (USCGS)	27.6	88.3	Sikkim	21	5.2 (CGS)	
Sep. 1	13 22 36 (Shillong)	27	92	Bhutan	—	5.7 (DLH)	Recorded at a number of Indian Observatories.
	13 22 36.6 (USCGS)	27.2	92.3	India-China Border	33	5.7 (CGS)	
Sep. 6	18 57 20.4 (USCGS)	7.1	93.7	Nicober Islands Region	46	5.2 (CGS)	
Sep. 15	15 29 40 (Shillong)	9	95	Andaman Islands Region	—	7.1 (DLH)	Recorded at all Indian Observatories
	15 29 32.2 (USCGS)	8.9	93.1	Nicober Islands Region	37	6.2 (CGS)	
Sep. 16	01 26 20 (Shillong)	11	94	Andaman Islands Region	—	—	Recorded at a few Indian Observatories.
	01 26 26.9 (USCGS)	10.9	93.1	Andaman Islands Region	47	5.7 (CGS)	
Sep. 19	00 39 10.7 (USCGS)	36.5	70	Hindukush	212	4.7 (CGS)	Recorded at a few Indian Observatories.
Sep. 26	00 05 55 (DELHI)	30	81	U.P.-Tibet Border	—	5.7 (DLH)	Recorded at all Indian Observatories. Felt Delhi Lucknow and other parts of Northern India.

1	2 3 4	5	6	7	8	9	10
Sep. 26	00 46 02.8 (USCGS)	30.1	80.7	Tibet-Indian Border Region	50	6 2 (CGS)	
Oct. 4	07 00 57.1 (USCGS)	27.9	69.2	India-West Pak. Border Region	14	4 8 (CGS)	Recorded at a few Indian Observatories.
Oct. 6	02 54 32.7 (USCGS)	30.3	94.6	Tibet	33	4.5 (CGS)	—
	20 19 35 (Shillong)	29	81	Nepal	—	5.4 (Delhi)	Recorded at all Indian Observatories.
Oct. 7	23 04 47.9 (USCGS)	32.7	83.9	Tibet	33	—	—
Oct. 13	23 02 26 (USCGS)	35.8	71.1	West Pakistan	120	5.8 (CGS)	Recorded at a few Indian Observatories.
Oct. 19	02 15 58.1 (USCGS)	31.4	79	Tibet-India Border Region	33	4.8 (CGS)	—
Oct. 21	17 23 33.7 (USCGS)	35.9	71.3	West Pakistan	181	4.4 (CGS)	—
	23 09 18.8 (USCGS)	28.1	93.8	India-China Border region	37	5.9 (CGS)	Recorded at all Indian Observatories.
Oct. 24	18 27 24 (Delhi)	18Kms. away from Sonepat	—	—	—	—	Felt at Sonepat.
Oct. 29	05 00 22.8 (Delhi)	40Kms. North-West of Delhi	—	—	—	—	Felt at Sonepat.
	13 30 44 (USCGS)	26.3	96.7	Burma	170	4.7 (CGS)	—
Nov. 4	15 20 23.5 (USCGS)	24.8	96.1	Burma	43	—	—
	19 46 06.2 (USCGS)	36.4	70.8	Hindukush Region	210	4.6 (CGS)	—
Nov. 7	22 03 18.8 (USCGS)	36.5	70.8	Hindukush Region	215	—	—
Nov. 9	16 12 50.6 (USCGS)	29.5	86	Tibet	33	4.7 (CGS)	—
Nov. 10	17 13 03.9 (USCGS)	29.8	92.2	Tibet	69	4.6 (CGS)	—
Nov. 12	10 15 47 (Delhi)	17Kms. away from Sonepat.	—	—	—	—	Felt at Sonepat.
Nov. 15	17 12 43.9 (USCGS)	36.5	70.9	Hindukush Region	220	5.0 (CGS)	—
Nov. 16	04 47 20 (Shillong)	37	70	Hindukush Region	200	—	Recorded at all Indian Observatories.

1	2	3	4	5	6	7	8	9	10
Nov. 18	07 (Delhi)	41	58	08Kms. away from Sonepat	—	—	—	—	Felt at Sonepat.
	11 (Delhi)	28	15	13Kms. away from Sonepat	—	—	—	—	Felt at Sonepat.
Nov. 25	08 (USCGS)	32	59	26.6	96.3	Burma	80	5.4 (CGS)	Recorded at a few Indian Observatories.
Nov. 27	11 (USCGS)	03	48	36.3	70.7	Hindukush Region	219	5.2 (CGS)	—
Nov. 30	12	24	09	6.2	93.7	Nicobar Islands Region	33	—	Recorded at all Indian Observatories.
Dec. 1	11 (USCGS)	45	21	10.6	93.4	Andaman Islands Region	33	4.7 (CGS)	—
Dec. 2	08 (USCGS)	21	43.3	29.5	81.3	Nepal	23	5.1 (CGS)	—
Dec. 19	17 (Delhi)	35	52.2	40Km. North West of Delhi	—	—	—	—	Felt at Sonepat.
Dec. 20	03 (USCGS)	31	36	29.5	81	Nepal	33	5.2 (CGS)	Recorded at all Indian Observatories.
Dec. 24	01	08	37.7	36.2	70.9	Hindukush Region	158	5.6 (CGS)	Recorded at all Indian Odservatories. Felt : Peshawar and Reasalpur

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