VOLUME 1 No. 1
January, 1964

BULLETIN

INDIAN SOCIETY
OF
EARTHQUAKE TECHNOLOGY
ROORKEE, U.P. (INDIA)

Price Rs. 5.00
Foreign $1.50

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CONTENTS

INDIAN SOCIETY OF EARTHQUAKE TECHNOLOGY
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The Society ("ISET") has been formed with the purpose of disseminating the knowledge in the field of Earthquake Technology dealing with Seismological, Geological Geophysical and Engineering aspects.

The first bulletin is now being issued. It is proposed to issue bulletins of "ISET" every six months. Technical papers concerned with earthquake technology are invited for publication. The last date for submission of papers is as follows.

(a) For January issue - 15th. December of previous year.

(b) For June issue - 15th. May of the same year.

The membership of the Society is open to all individuals, associated with or interested in Earthquake Technology. In addition to this, he must have the following qualifications:—

(a) A recognised Engineering Degree or its equivalent;

(b) A Master's Degree in any branch of Science or Technology or its equivalent.

The Subscription for Individual Membership is Rs. 15/- per year and for Institution Membership is Rs. 100/- per year. The subscription is payable in advance on the 1st January for the calender year.

Membership form and other details are available from the Secretary of the Society.

SECOND SYMPOSIUM ON EARTHQUAKE ENGINEERING

UNIVERSITY OF ROORKEE

November 1962

The behaviour of soils and structures under dynamic loads is a relatively new problem and has been taken cognizance of only in recent years. In the past, the designs of many constructional works like dams, bridges, tall buildings, water towers and other structures in the country were based mainly on the equivalent static forces acting on them. But the increasing tempo of irrigational and multipurpose projects and construction works like power houses, office buildings, living quarters, factories and industrial structures in seismic zones has necessitated a thorough study of the dynamic forces expected to come into play, and increasing attention is being paid to this aspect. The starting in 1960 of the School of Research and Training in Earthquake Engineering at the University of Roorkee has been a step in the right direction. With a view to apprising the work done by the group of research workers at this school to other interested workers and to exchange the experience of workers engaged in similar or related work elsewhere the university organized in 1959 the First Symposium on Earthquake Engineering and this was followed by the Second Symposium held at the university during 10-12 November 1962.

Forty-one papers were presented to the symposium which was attended by about 200 delegates; papers were also contributed by workers from USA, England and Japan. The symposium was conducted in five technical sessions of half-day each. The five sessions dealt with the following subjects:

DESIGN OF DAMS IN SEISMIC ZONES

1. "Effect of Earthquake on Pore Pressure in Earth Dam" by V.J. Patel and S.D. Bokil.
2. "The Seismic Stability Analysis of Earth Dams (Part II)" by N.N. Ambraseys.
3. "Earthquake Resistant Design of Earth Dams" by Jai Krishna.
4. "Effect of Seismic Forces on Various Power Houses in Uttar Pradesh" by Ravi Datta and P.S. Nigam.
5. "Water Pressure on Overflow Dams During Earthquakes" by V.G. Palnitkar.
6. "Some Aseismic Design Aspects of Hydro-electric Structures" by Y.K. Murthy, K. Madhavan and K K. Rao:

VIBRATION STUDIES OF SOILS

7. "Dynamic Behaviur of soils" by Shamsher Prakash and R. Gupta.
8. "Earthquake Resistant Design of Retaining Walls" by I.P. Kapila.
9. "Vibratory Response of a Cohesive Soil in Uniaxial Compression" by R.L. Kondner,
10. "Behaviour of Soils During Seismic Disturbances" by R.S. Mithal and L.S. Srivastava.
11. "A Simplified Method for Computing Resonant Frequency of Square Footings" by H.A. Balakrishna Rao.

SEISMOLOGY AND INSTRUMENTATION FOR VIBRATION STUDIES

(a) SEISMOLOGY

12. "Seismology in India" by A.N. Tandon.
13. "Assesment of Maximum Seismic Intensity and Ground Acceleration" by A.N. Tandon.
14. "Seismic Regionalisation of India" by S.K. Guha.
15. "On predominant Period of the Surface Layer in the Assam Plateau" by B.P. Saha.
16. "Vibration of Elastic Constants with Frequency and Under Different Physical Conditions" by V.K. Gaur

17. "Preliminary Investigations of the Problem of Predicting Earthquake by Measurment of the Magnetic Field in the Region" by V.K. Gaur.

18. "Study of Earthquakes Recorded by Gravimeter" by S. Balakrishna.

(b) INSTRUMENTS FOR VIBRATION STUDIES

19. "Design of Shock Vibration Table" by Jai Krishna and A.R. Chandrasekaran

20. "Design of Structural Response Recorders" by Jai Krishna and A.R. Chandrasekaran

21. "Design of a Vibration Measuring Device" by A.R. Chandrasekaran and S.N. Pande.

ANALYSIS AND DESIGN OF STRUCTURES FOR EARTHQUAKE FORCES

22. "Earthquake Stresses in Tall Buildings with Set Backs" by Glen V. Berg.

23. "Study on the Dynamical Response of An Actually Designed Structure Against Severe Earthquake" by Hajime Umemura.

24. "Effect of Joint Rotation on the Dynamics of Multi-Storeyed Frames" by A.R. Chandrasekaran.

25. "Vibrations of Catenary Arch" by K.V. Mital and J.S. Tomer.

26. "Response of Structures to Artificially Generated Explosion Waves in Earth" by S.K. Guha, G.V. Rao. R. Nath, B.B. Khade and R.P. Belgal.

27. "Structural Damping of Brick Masonry in Different Mortars" by D.V. Mallick.

28. "Building Construction in Seismic Zones of India" by Jai Krishna and A.S. Arya.

29. "Settlement Studies on a Model Footing Under Dynamic Loads" by A. Sridharan.

30. "Japanese Mode of Thinking of Earthquake Resistant Design" by R.N. Joshi.

31. Vibration Problems in Hydraulic Structures" by Satish Chandra.

GEOLOGICAL STUDIES FOR ENGINEERING PROJECTS IN SEISMIC ZONES

32. "Significance of the Moradabad Fault in the Indo-Gangetic Basin and Other Faults in the Sub-Himalayas, in Relation to the Ramganga River Project, U.P." by V.S Krishnaswamy.

33. "A Geological Evaluation of the Thrusts and Faults in the Vicinity of the Beas-Sutlej Link Project and the Uhl Hydel Project, Himachal Pradesh," by B.M. Hukku.

34. "Probable Correlation of the Structural and Tectonic Features of the Punjab Himachal Pradesh Tertiary Re-Entrant with the Patterns of Seismicity of the Region," by V.S. Krishnaswamy.

35. "Geological studies Relating to the Seismicity of the Sarpduli-Dhikala Thrust, Ramganga River Project, District Garhwal, U.P." by R.S. Varma

36. "Seismic Phenomena in North Eastern India" by P.C. Hazra and D.K. Ray.

37. "Regional Geology and Seismicity of the Yamuna Hydel Project, Dehradun District, U.P." by P.N. Mehta.

38. "Studies Relating to the Seismiciry of the Satlitta Thrust, Beas Dam Project." by S.P. Jalote.

39. "The Origin and Significance of the Vindhyan Scarp," by F. Ahmad.

40. "Seismicity of the Area around Baraumi, Bihar," by R.S. Mithal and L.S. Srivastava.

41. "Development of Concentric Shear Planes and the Related Seismic Phenomena in the Gangetic Synclinorium", by P. Kumar.

Presentation of Papers in each session was followed by lively discussions. All the papers along with discussion have been printed in the proceeding of the symposium. The bound Proceedings (575 pp.) are available for Rs. 12/- ($ 3.50 for foreign orders) from the O.C. Book Depot, University of Roorkee, Roorkee U. P. (INDIA).

THIRD WORLD CONFERENCE ON EARTHQUAKE ENGINEERING.

Jan.–Feb. 1965

The Third World Conference on Earthquake Engineering will be held at Auckland and Wellington (New Zealand) from Friday, January 22, 1965 to Monday February 1, 1965. The conference is being organised by the International Association for Earthquake Engineering.

The New Zealand National Committee has issued an invitation to Scientists and Engineers from the Seismic Countries of the World to attend the forthcoming Conference, to take part in the technical sessions and to join in the post conference tours.

Papers are also invited by the Organizing Committee for oral presentation at the Conference. The last date for submission of papers is **30th June, 1964**. The official language of the Conference is English. The technical papers will be separated into five different Themes or Sessions. These are to be classified broadly as follows:

(1) Soil and Foundation Conditions Related to Earthquake Problems.

(2) Analysis of Structural Response ; Instruments.

(3) Seismicity and Earthquake Ground Motion.

(4) Earthquake Resistant Design, Construction and Regulations.

(5) Recent Strong Motion Earthquakes and Resulting Damage.

Each Session will have a General Reporter, whose function will be to summarise the important features of all papers of that Session.

A session will commence with a summary by the General Reporter. Authors will then be allocated approximately ten minutes' presentation time. In this time, he would be expected to high-light principal features of his or her paper or to introduce new data not previously available. After the introduction of papers, major portion will be devoted to discussion.

Each session will close with a summing-up by the General Reporter.

An Exhibition has also been planned on the occasion of this conference. Any item connected with earthquakes or earthquake engineering would be acceptable. Exhibits may include photographs, films, drawings, models, instruments, equipment and publications.

A Book shop will also operate to sell earthquake engineering publications on behalf of the organisations or individuals.

Enquiries regarding submission of papers, participation in the Conference Exhibition and Book-shop should be made to :

The Administrative Secretary,
Third World Conference on Earthquake Engineering,
P.O. Box. 5180,
WELLINGTON, New Zealand.

INDIAN SOCIETY OF EARTHQUAKE TECHNOLOGY

ROORKEE, (U.P.) INDIA.

Dear member,

We feel great pleasure in presenting the First Bulletin (Vol. 1, No: 1) in your hands. Since the inception of the Society in November 1962, we had been considering to institute a publication. A large part of the year 1963 was devoted to printing of the Proceedings of the Second Symposium on Earthquake Engineering organised by the School of Research and Training in Earthquake Engineering. The Proceedings are available from the O.C. Book Depot, University of Roorkee, Roorkee (UP), for Rs. 12/- each excluding postage etc.

Immediately after completing this work, we invited contributions from members and others for this Bulletin and we were glad at the encouraging response from various contributors, for which we are extremely thankful to them.

The Society has evoked enough interest not only in India, but in other countries also. We have already received two papers for the Second Bulletin (Vol. 1, No: 2) from the U.S.A. The Bulletin will be issued in June 1964. The dead line for submission of papers is 15th December and 15th May for the January and June issues respectively.

Contributions are welcome on the following topics:—

1. Analysis and Design of Structures for Earthquake Forces.
2. Design of Dams in Seismic zones.
3. Dynamic Loading of Soils and Foundations.
4. Geological Studies for Engineering Projects in Seismic Zones.
5. Instruments concerned with Seismology and Earthquake Engineering Studies.
6. Earthquake Records and Reports.
7. General Topics Having a Bearing on the Subject of Earthquake Technology.

Depending upon the response in future, we will consider to increase the frequency of publication.

We have not so far planned any meeting of the Society, mainly because of lack of funds. A statement of the budget for the year 1963 will shortly be forwarded to you. Despite shortage of funds we have kept its price about two-thirds of the actual cost, so that more people are encouraged to buy the same. We are sure that, with your cooperation, we will have enough funds soon. We, therefore, take this opportunity to remind you that your subscription for the year 1964 is due and you are, therefore, requested to remit the money as early as possible.

Secretary

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PROBLEM OF EARTHQUAKES

Jai Krishna*

The most uncontrollable force that nature unleashes on mankind is that through earthquakes which occur without warning and which are unpredictable. Science has not been able to help either in knowing about their occurrence before hand or preventing them in future. The man, therefore, looks helpless in the wake of this destructive force which results in tremendous loss of life and property in different areas of the world from time to time. The feeling of an earthquake is most awful since the very earth to which we look up for stability, begins to tremble and this naturally causes fear psychologically and quite often a very real one. In the last 4 years several devastating shocks have occurred starting from Agadir in Morocco followed by Lar in Iran, the five big shocks of Chile coming one after the other within 24 hours in 1960, heavy damage in Libya, the devastation of a great part of Tehran last year and the latest one resulting in destruction of the town of Scopje in Yogoslavia. Luckily for India we have not had any major shock since the great earthquake of 1950 in Assam, but we had more than our share in the last 60 to 70 years. The 1950 Assam earthquake was preceded by Bihar earthquake of 1934, (11,000 people killed) Kangra valley earthquake of 1905 and perhaps the greatest earthquake of Assam in 1897 and many other smaller ones which may not have left a deep scar on the history of the country but did do considerable damage locally. These four earthquakes were perhaps the biggest shocks experienced anywhere in recent history, and therefore, India is susceptible to considerable loss due to this force of nature on which we have no control. Since, for many years no big shock has occurred in Western Himalayas, one should be expected in the near future. It is, however, fortunate that in the last six years there have been several small shocks which may lessen the intensity of the bigger shock to follow. Similar is the situation in Gauhati-Shillong axis. The earth as a whole experiences about 800 to 900 shocks a year which can cause damage locally. Most of them occur either in sea or in unpopulated regions. We, therefore, hear only of those which shock the thickly populated areas. Following are the biggest shocks of the twentieth century:

Year	Country	Human Beings Killed
1905	Kangra (India)	19,000
1908	Italy	75,000
1915	Italy	30,000
1920	China	1,80,000
1923	Japan	1,00,000
1932	China	70,000
1934	Bihar (India)	11,000
1935	Quetta (Pakistan)	30,000
1939	Turkey	23,000
1950	Assam (India)	1,526
1957	Iran	2,500
1960	Agadir	12,000
1960	Chile	5,700
1962	Iran	10,000
1963	Skopje	2,000

It has not been possible so far to give a precise scientific explanation of how the earthquakes actually occur. Several theories have, however, been put forward from time to time, the most reliable among them being based on the principle that volumetric changes take place inside the earth mass continuously because the temperature and pressure of the material inside the earth are very high and some readjustments connected with the "origin of Earth" continue to take place. These changes result in the earth material being strained and, when the strains become too big rupture takes place. This rupture

*Director, School of Research & Training in Earthquake Engineering, University of Roorkee, Roorkee, U.P. (India)

COMPARATIVE STUDY OF ENERGY RELEASED BY THE EARTHQUAKES OF DIFFERENT MAGNITUDES M

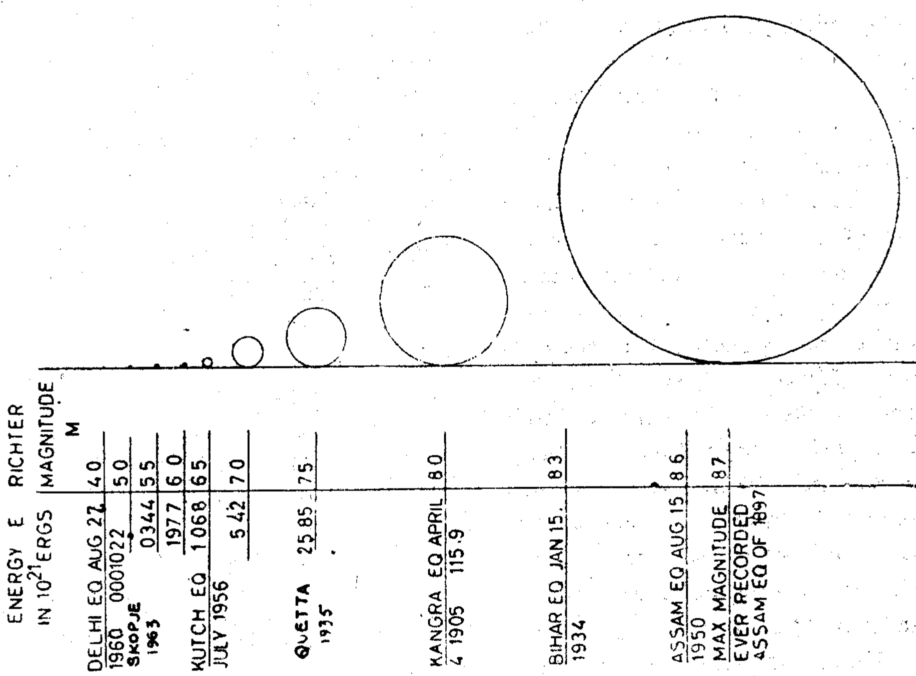


Fig. 1

Fig. 3. A Typical Station

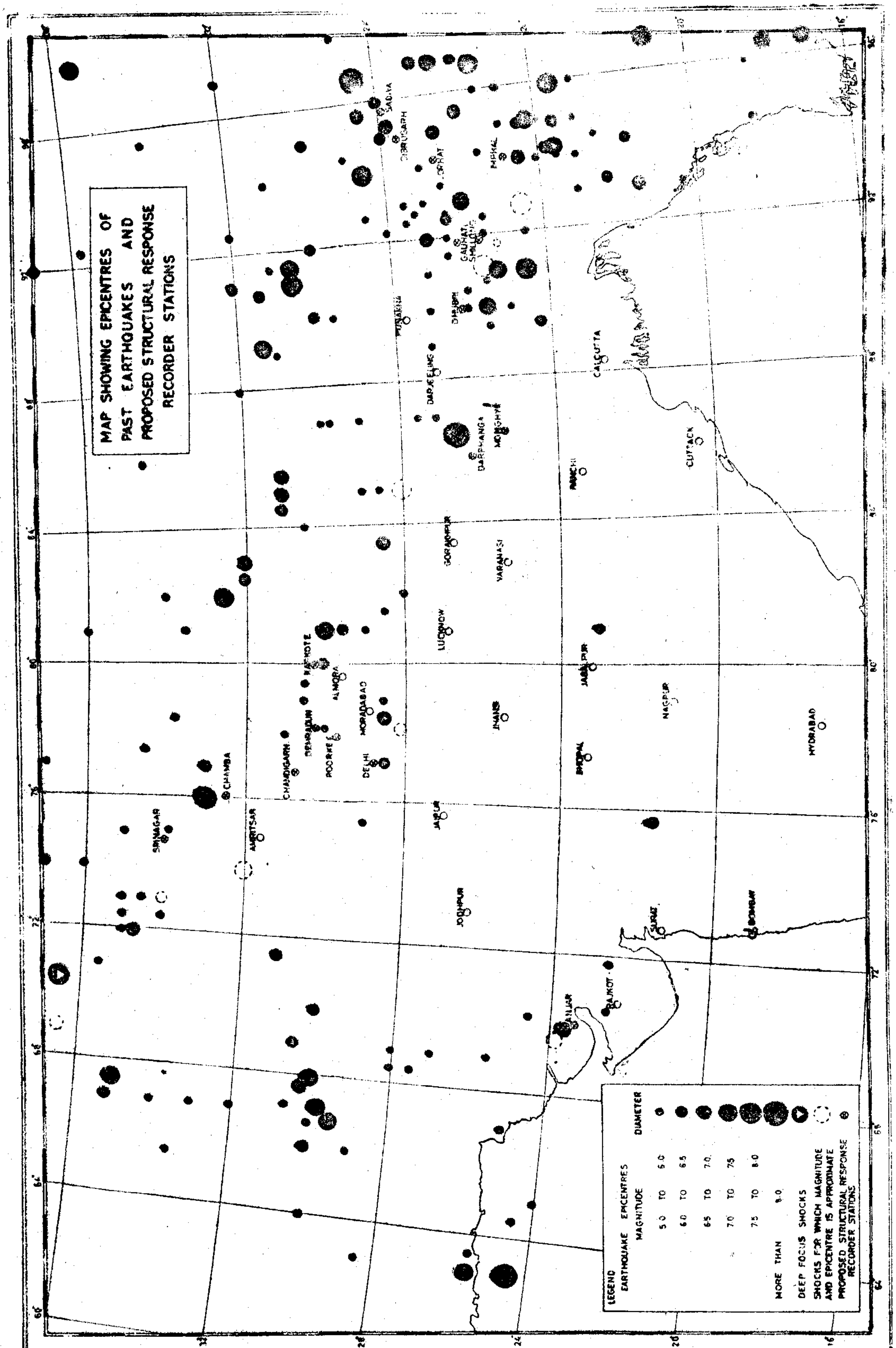
may occure over many miles and the bursting process may involve hundreds of cubic miles of earth mass. It would thus be seen that tremendous amount of energy is released which traveis in the form of waves to the surface shaking everything that came in their way. It is only when these waves reach the surface that we feel the occurrence of an earthquake. Anything standing on ground is vibrated to and fro and if the buildings and other structures are not designed for earthquake forces they collapse resulting in demage to life and property. Sometimes the travel of the waves causes loud frightening noise but that by itself is not anything dangerous. It is the excessive vibrations which can be dangerous. Perhaps the greatest loss of life occurred in the China earthquake of 1920 and the Tokyo Earthquake of 1923, where over 180,000 and 100,000 people respectively were known to have died, although these earthquakes in terms of energy released, were not bigger than our giant earthquakes of Bihar and Assam but, since the areas where these occurred were thickly populated, the loss of life was much greater. Energy released during an earthquake is so huge that some times big dams crack and are washed away. Even the smallest earthquake of consequence generates much more energy than the biggest man-made nuclear bomb. For comparison series of spheres shown in Fig. 1 indicate the measure of the energy released in different major earthquakes.

Since it is not possible to prevent the occurrence of earthquakes nor is it possible even to predict them, the only alternative available to engineers is to study the science of strengthening the buildings, dams and other structures in such a manner that when earthquakes occur they remain safe. This, of course, poses a difficult economic and psychological problem. Earthquakes do not occur every day in one region. Perhaps it may not occur even once in 50 years and, the public memory being short, there is always a tendency to ignore them or minimise their danger. Some designers would argue that an earthquake may or may not occur in the life time of the building and we might as well economise by ignoring these forces. Even when an engineer decides upon designing a building against earthquake forces, the problem before him is to decide as to which size of an earthquake he should design it for. The force that a building is subjected to in an earthquake depends upon the place where the earthquake originates i.e. the distance of the origin from the building, and the size of the earthquake. The greater is the force for which a building is designed, the greater is its initial cost. The designer has thus to make a compromise between two requirements. The guidance to this decision should come from the more familiar case of taking out an insurance policy. One always insures himself according to his financial ability and the importance of his life to his dependants. Another instance with which we are more familiar is that of the design of a bridge. Rivers, in India carry water whose quantity varies very greatly from one season to the other and from one year to the other. A designer has always got to take a decision as to what size of flood he should design the bridge for, and this would depend upon its importance and the effect on the initial cost. In the case of earthquake resistant design also this decision has to be made by the designer on the same basis taking into account the known history of the place and assuming that in future also earthquakes of similar sizes will take place, and the financial implications of the decision. He must, of course, realise that, whatever he does, he is not in a position to guarantee that the structure was safe for all possible earthquakes that nature was capable of unleashing if the epicentre happens to be within a few miles of the structure. He can, however satisfy himself because the probability of such an occurrence is very small and, on that basis, whatever safety he provides for, is likely to prove adequate against failure. But this uncertainty about the future should not thwart our efforts to understand the problem, carry out research and use the existing knowledge to the best of our ability. Nor should this make us take the position that we take no notice of them and look to the immediate gain by not taking the earthquake forces into account. The price of this attitude may have to be paid very dearly later. The recent occurrences in India and outside are a clear pointer to this.

The map of India and the surrounding region (Fig.2) indicates the positions of the epicentres of the past earthquakes in the present century. Many more must have occurred earlier. Bulk of these regions were unpopulated so far but with the rapid pace of industrialisation large townships, oil refineries, many industrial concerns of large and small sizes, dams and irrigation works, bridges and numerous other structures are being planned in these regions and the construction of some of them has already been taken in hand. The question of determining the cheapest methods of strengthening these buildings and other structures against earthquake forces is of paramount importance to this country. Unfortunately very little scientific data regarding the behaviour of structures in these regions is available, and for a long-term study, steps would have to be taken to collect this data. In the meantime, laboratory study will provide considerable information that would be helpful in design work.

Anticipating these requirements of industry and other engineering projects, the Council of Scientific and Industrial Research sanctioned the establishment of a School of Research and Training in Earthquake Engineering at the University of Roorkee in 1960. The School has since built up the necessary facilities and has embarked upon the programme of research on the methods of strengthening buildings, dams, water towers and other structures against earthquake forces. Some work has already been published. As a part of long-term work, Simple Pendulum Recorders have been designed and it is proposed to instal them at different stations shown on the map. In future earthquakes these instruments will record more precise information about the behaviour of buildings and other structures, so that the future designer could strengthen these structures more scientifically. A typical station is shown in Fig. 3. India has, therefore, gone ahead to meet the designers' future need. Another important step already taken by the Indian Standards Institution is to draft a code of practice for "Earthquake Resistant Design" and it has recently been published. This provides adequate guidance for the design of simple structures.

Although research will yield much information in course of time to make the task of the designer more precise and the structures consequently more economical and safe, a good deal of information already exists which can be made use of for insuring our buildings and other structures against failure to a great extent for normal earthquakes. There is absolutely no need to fall back on "Fate", and suffer the consequences. At the same time complacency in areas where a shock has not occurred in the last 50 years is also dangerous. Prevention in this case too is far better than cure. Past experience has shown that the buildings designed for earthquake forces have stood the shocks much better than others not designed for them. The konwledge has further advanced in the last 25 years enabling even safer construction. Further work is in progress which will cut down losses to a minimum.


MAP SHOWING EPICENTRES OF PAST EARTHQUAKES AND PROPOSED STRUCTURAL RESPONSE RECORDER STATIONS
LEGEND
EARTHQUAKE EPICENTRES
MAGNITUDE
DIAMETER
5.0 TO 6.0
6.0 TO 6.5
6.5 TO 7.0
7.0 TO 7.5
7.5 TO 8.0
MORE THAN 8.0
DEEP FOCUS SHOCKS
SHOCKS FOR WHICH MAGNITUDE AND EPICENTRE IS APPROXIMATE
PROPOSED STRUCTURAL RESPONSE RECORDER STATIONS
SADIYA
DIBRUGARH
JORHAT
IMPHAL
GAUHATI
SHILLONG
DHUBRI
DARJEELING
CALCUTTA
DARBHANGA
MONGHYR
CUTTACK
RANCHI
GORAKHPUR
VARANASI
LUCKNOW
JABALPUR
NAGPUR
HYDRABAD
JHANSI
BHOPAL
ALMORA
MORADABAD
DEHRADUN
ROORKEE
DELHI
CHANDIGARH
CHAMBA
SRINAGAR
AMRITSAR
JAIPUR
JODHPUR
SURAT
BOMBAY
RAJKOT
ANJAR

VIBRATION ANALYSIS OF STRUCTURES BY THE ENERGY METHOD

Dinabandhu Mukherjee*

SYNOPSIS

This paper deals with the Energy method for determining natural frequencies of structures. The frequency or the period of vibration is calculated by comparing the potential energy of the deflected structure with its Kinetic energy as it passes through its normal static position. Rayleigh's principle, the static deflection method or a special case of Rayleigh's method, Ritz's development and Southwell Dunkerley extension of Rayleigh's principle are discussed.

A method, called the method of "Dynamic Convergence", for solution of multistoreyed framed structures, is explained. A 10–storey reinforced concrete building frame, which has been originally designed by the author for an earthquake allowance of 20% g for comparing the increase in cost due to earthquake resistant design, is analysed for its period of vibration by applying the method of Dynamic Convergence and the Energy principle. Any desired accuracy can be obtained by this method and therefore it is comparable to the other classical methods.

INTRODUCTION

In the design of an earthquake proof structure, the basic seismological data required are the natural period of vibration of the structure and the natural period of vibration of the ground. The degree to which the natural period of vibration of a structure synchronizes with the ground movements may determine whether or not the building will successfully withstand the earthquake. Again, study of vibration characteristics of a building may also be required in areas other than the earthquake zones. Thus the housing of a machine having reciprocating motion in an existing building or the design of a new building for such use might well be conditioned by a vibration study of the building. Different methods are available for calculating period of vibration of structures. In this paper Energy method for determining the period of vibration of multi-storeyed frame is discussed.

Energy Method:

The differential equation of motion for a mass m performing simple Harmonic Motion can be written as–

$$m\ddot{x} + Kx = 0 \text{ or } \ddot{x} + p^2x = 0 \ldots\ldots\ldots (1) \left[p^2 = \frac{K}{m}\right]$$

where K is the spring constant or the restoring force. This equation is satisfied for

$$x = c_1 \cos pt + c_2 \sin pt$$

whence we can see that the time period $T = \frac{2\pi}{p}$ or the frequency $f = \frac{p}{2\pi}$

Natural frequencies can sometimes be advantageously calculated by using the law of conservation of energy provided that damping is negligible. Our discussion will be based on a study of the potential and the Kinetic energies of a system in motion and their simple relation with the system's natural frequency parameter.

If we multiply equation (1) by $\dot{x}$, we have $\dot{x}\ddot{x} + \frac{k}{m} x\dot{x} = 0$. This expression lends itself to a direct integration, viz.

$$\frac{1}{2}\dot{x}^2 + \frac{1}{2}\frac{K}{m}x^2 = C$$

$$\text{or} \quad \frac{1}{2}m\dot{x}^2 + \frac{1}{2}Kx^2 = Cm \quad (2)$$

The first term of this expression is seen to give the instantaneous Kinetic energy of the motion of the mass, and the second term represents the instantaneous potential energy content of the linear restoring element with reference to the potential energy level required by the static equilibrium position of the system. When the displacement x is a maximum X, the velocity $\dot{x}$ is zero and all the dynamic energy of the system is potential. Similarly,

* Lecturer in Applied Mechanics, Bengal Engineering College, Howrah.

when the displacement x is zero, the velocity $\dot{x}$ is a maximum $\dot{X}$ and all the dynamic energy of the system is Kinetic. The energy equation of a vibrating system can therefore be written as

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}Kx^2 = \frac{1}{2}Kx^2 = \frac{1}{2}m\dot{x}^2 = Cm = \rho \quad (3)$$

If the characteristic deflection curve of a structure, i.e. the maximum displacement of different points along its length is known, the work done upon the structure by the oscillating masses which is the potential energy of the deflected structure can be calculated. By the law of conservation of energy, the Kinetic energy of the system as it passes through its figure of rest becomes known. Since the masses are known and the Kinetic energy has been determined, the period may be deduced.

The formal requirements of the deflection curve are that it must sattisfy the end conditions and that in between these end conditions, it must coincide with the actual dynamic deflection curve. Rayleigh (Rayleigh, 1945) has shown that the fundamental natural frequency as calculated from the assumed shape of a dynamic deflection curve of a system, will be equal to or higher than the system's true natural frequency.

Static deflection curve has got a wide application for this purpose. This is the deflection that the system would undergo under static condition if the gravity action is supposed to be acting on the masses of the system.

Better approximations in calculating the fundamental frequency and also the frequencies of higher modes of vibration can be obtained by "Ritz's method" which is a further development of Rayleigh's method (Ritz, 1911). In using this method, the equation of the deflection curve representing the mode of vibration is to be taken with several parameters, the magnitudes of which should be chosen in such a manner as to reduce to a minimum the frequency of vibration.

To apply Ritz's method in calculating the frequency f of the fundamental type of vibration, the first step is to choose a suitable expression for the deflection curve. Let $Q_1(x)$, $Q_2(x)$, be a series of function satisfying the end conditions and suitable for rep esentation of y. Then

$$y = a_1 Q_1(x) + a_2 Q_2(x) + a_3 Q_3(x) +$$

represents suitable deflection curve of the vibrating system. Taking a finite number of terms in this expression means superimposition of certain limitation on the possible shapes of the deflection curve and due to this fact the calculated frequency is usually higher than the exact value of this frequency. In order to obtain the approximation as close as possible, Ritz proposed to choose the coefficients a_1, a_2, a_3... so as to make the frequency a minimum. Thus, by equating to zero the partial derivative of the expression for frequency with respect to a_n, a system of equations homogeneous and linear in a_1, a_2, a_3 ... is obtained, the number of which is equal to the number of coefficients a_1, a_2, a_3 .... Such a system of equations can yield for a_1, a_2, a_3 ... solution different from zero only if the determinant of these equations is equal to zero. This condition brings us to the "frequency equations" from which the frequencies of the various modes of vibration can be calculated.

If a composite system can be split up into several isolated systems, we can get a lower limit to the frequency of the composite system as compared with an upper limit given by the Rayleigh approximation. This extension of Rayleigh's principle is due to Southwell and Dunkerley. In the first case, if the composite system is such that it is possible to express the total Kinetic energy in the form of one integral while the potential energy remains the sum of several integrals or terms, then the sum of the squares of the true isolated frequencies would be either less than or equal to the square of the true frequency of the composite system (Southwell 1941). In the second case, where the potential energy is contained in one term and the Kinetic energy is contributed by various inertia elements, the sum of the squares of the reciprocals of the isolated frequencies furnishes an upper limit of $1/f^2$ and consequently a lower limit of f^2, i.e. the square of the true requency of the composite system (Dunkerley, 1894). The true fundamental frequency is obtained from the higher limit given by a Rayleigh approximation and the lower limit given by a Southwell-Dunkerley approximation.

Application to Building frames :

The dynamic characteristic of a 10-storey reinforced concrete building frame is obtained by applying the energy method. The frame forms a part of an R.C.C. framed structure and has been designed by the author for Dead load, Live load, and earthquake force of 20% g using the standard code of practice given by the Indian Standard specification. The frame consists of two bays of 25′ each. The floor heights have been kept as 12′-0″. The frames have been placed 12′-0″ centre to centre.

In this problem, as before, the important part is to determine the characteristic deflection curve. To obtain the deflection curve and the dynamic forces which produce the characteristic deflection curve, a process called the "Method of Dynamic Convergence" is employed. This method is due to J.E. Goldberg and is similar to Stodola-Vianello method (Bleich, 1950).

Determination of the Deflection Curve:

Considering the bent or frame as a whole, it is clear that the shape of the deflected structure, i.e. the characteristic shape of the deflection curve of the vibrating structure, is determined by the forces exerted upon the structure by the oscillation of the various particles or masses, these forces being proportional to the products of these masses times their respective amplitudes.

In determining the shape of the deflection curve to which the structure oscillates under free vibration, use is made of physical fact that, irregular though the forces may be which initially disturb the structure from its figure of rest, the structure seeks immediately to adjust its figure to the natural deflection curve of free oscillation and with each succeeding oscillation approaches the configuration of the natural curve more closely until, finally, the natural curve is accurately matched and the previous adjusting oscillations are succeeded by self-sustaining free vibration. This fact may be proved experimentally or analytically. We may, therefore determine the shape of the natural curve by producing analytically a chain of successive circumstances closely resembling the successve aspects of the structure during its transition from the irregular ngure induced by the initial disturbance to the final deflection curve of free oscillation.

Thus we may begin by assuming an initial deflection curve for the disturbed structure which, for practical reasons, should be as correct an estimate of the final free oscillation curve as we can make conveniently. In the absence of specific data on the exact shape of the free oscillation curve, it would be convenient to assume some simple curve for the initial aspect of the disturbed and deflected structure as for example, a straight line variation of deflection with height.

For the sake of simplicity and convenience in the consideration of the specific case of building frames, it will be assumed that the masses of all the bodies and elements which make up the mass of a storey act as a single mass, concentrated at the level of the floor system.

The force exerted by a mass, m, moving with simple Harmonic motion is, at the limit of its deflection

$$\text{Force} = Cm\triangle \qquad (4)$$

wharein C is a constant which applies generally to all masses of the system. A form somewhat more convenient for our purpose is obtained by transcribing equation (4)

$$\text{Relative force} = m\triangle \qquad (5)$$

Assuming that each mass has exerted a relative force equal to mass times its respective initial displacements, the first adjusted deflection curve is obtained for the structure under the action of this group of assumed forces by the use of the following slope–Deflection formulae for the diflections of building frames under transverse loads. Each formula is applied, in turn, to each storey of the bent to obtain the adjusted deflection curve.

$$\theta_n = \frac{M_n + M_o}{12 \Sigma K_{gn}} \quad \cdots \quad \cdots \quad (6)$$

$$R_n = \frac{M_n}{6 \Sigma K_{en}} + \frac{\theta_m + \theta_n}{2} \quad \cdots \quad (7)$$

Where

$K_c = 2\ EI/L$ = Stiffness of column

K_g = Stiffness of girder

M = Shear force × Storey height

= Moment acting at a joint

θ = Angular rotation at a joint
R = Slope at any storey from the vertical
D = Relative diflection
Δ = Actual deflection

The above expressions for θ_n and R_n are approximate formulae. But they are simple and can be conveniently used for the present purpose. The frequency or the time period is not appreciably changed due to this approximation. Equations (6) and (7) are used for all storeys above the first. For the first storey, equations (6a) and (7a) are used assuming full restraint against rotation at the bases of the first storey columns.

$$\theta_1 = \frac{M_1+M_2}{12\Sigma K_{g1}+\Sigma K_{c1}} \quad \ldots (6a)$$

$$R_1 = \frac{M_1}{6\Sigma K_{c1}} + \frac{\theta_1}{2} \quad \ldots (7a)$$

The actual linear deflections are given by

$$D_n = (R_n)(h_n) \quad \ldots \quad \ldots (8)$$

$$\Delta_n = \sum_{i=1}^{n} D_i \quad \ldots \quad \ldots (9)$$

On the basis of the first adjusted deflection curve or transition curve thus determined, a new set of relative forces is computed by means of equation (5). Again using equation (6) to (9), a second adjusted deflection curve is obtained. The procedure is repeated until a deflection curve of the desired accuracy is obtained, i.e., until the free oscillation curve is determined with the desired degree of accuracy. The degree of accuracy may be tested by comparison of the successive deflection curves.

Determination of the period of vibration :

Having definitely determined the deflection curve to which the freely oscillating structure deflects and, incidentally, having determined the forces at each storey which have induced this deflection, it now becomes possible to deduce a value for the natural period of vibration. At the instant the structure has reached the end of its swing, i.e., it is fully deflected, the instantaneous velocity of each particle is zero. The total energy of the system is, therefore, entirely potential at that instant and is equal to the work done upon the structure by the decelerating particles or, what amounts to the same thing, by the relative forces previously determined.

Again, at the instant of passing through its figure of rest or normal static position, the entire energy of the structure is Kinetic and is equal to the summation of the Kinetic energies of the individual particles or masses of the system. Neglecting the internal losses due to friction and other losses of similar nature, these two quantities of energy must be equal by the law of conservation of energe.

$$\text{Total work} = \Sigma\,(\text{average force} \times \Delta) = \Sigma\left(\frac{F\Delta}{2}\right) \quad (10)$$

where, the F and Δ values used are the accepted final and correct values.

$$\text{Total Kinetic Energy} = \Sigma\left[\frac{m}{2}\left(\frac{2\pi\Delta}{T}\right)^2\right] \ldots (11)$$

Equating (10) and (11) and solving for T gives

$$T = 2\pi\sqrt{\frac{\Sigma\,(m\Delta^2)}{\Sigma\,(F\Delta)}} \quad \ldots \quad \ldots (12)$$

An alterative formula, taking into account the motion of only a single mass, may be obtained as

$$T = 2\pi\sqrt{\frac{m\Delta}{F}} \quad \ldots \quad \ldots (13)$$

Being simpler than equation (12) and entirely satisfactory for normal use, equation (13) is particularly useful as a means of checking. Equation (12), however, is more easily modified for other than the normal motions and has the further capacity of minimizing irregularities and the errors resulting thereform.

Applying the above equations, the natural period of vibration of the 10-storey reinforced concrete frame, already described, is determined analytically. A dimensioned sketch of the frame is given in Fig. (1), on which the moments of inertia (I) of the external and internal columns and of the girders are shown. The figures within bracket are corresponding values of K=2EI/L. Data and calculations are tabulated, all elements of the calculations being tabulated progressively in the order in which they are computed. The calculations for the fifth storey, whicn is typical, are carried out in detail below. The items are numbered in accordance with the numbering of the column heading of the table and will serve to explain each step of the procedure.

(1) K_c=2EI/L for each column of the designnated storey. Tabulated stiffness values are in units of 10^6lb-ft. At the fifth storey the stiffness of the two exterior

columns is 77.5 (x10^6) lb. ft. and that of the interior column is 149 (x10^6) lb. ft.

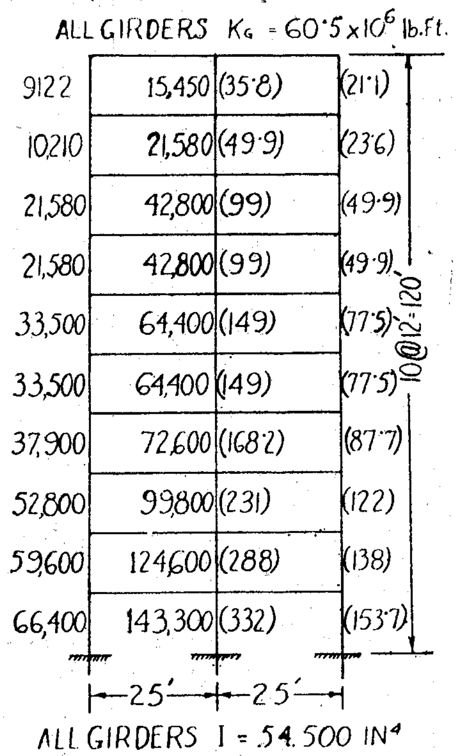


Fig. 1

(2) For the two exterior and one interior columns of the fifth storey, 6 ΣK_c5 = 6 x (2 x 77.5 + 149) = 1824 (x 10^6) lb. ft.

(3) The stiffness of each girder of the frame is K_g = 60.5 × 10^6 lb. ft.

Therefore, for the two girders at each storey

12 ΣK_g = 12 × 2 × 60.5 = 1452 (x 10^6) lb. ft.

Note that at the first storey,

12 ΣK_{g1} + ΣK_{c1} = 1452 + 639 4 = 2091

is tabulated for subsequent use in equation (6a), whereas the other items in this column are for use in equation (6).

(4) Mass is the mass assumed to be concentrated at the top, i.e., at the level of the girders, of the storey in question and includes the floor system, one half of the columns, walls, partitions, both above and below the floor level. The total weight of these elements divided by the acceleration of gravity is the mass for that storey.

(5) For the initial estimate of the deflection curve, a deflection of 1.00 ft. is assumed at the top of the frame with the deflection at all other points in proportion to their elevation. At the fifth storey = 1.00 × 60/120= 0.500 ft.

(6) By equation (5) the force assumed to be acting at the top of the fifth storey is relative force $=m_5\Delta_5$ = 3790 × 0.500= 1895 lbs.

(7) The external shear at the fifth storey is the total of all the forces from the top of the frame down to and including the force at top of the fifth storey.

$Shear_5$ = $Shear_6$ + force 5 = 14,592 + 1895 = 16,487 lbs.

(8) M_5=$Shear_5$×storey height=16,487×12=197, 844 lb. ft.

(9) By eqn. (6) $\theta_5/2$ = (197844 + 175,104/2× (1452) = 128.4 Radian/10^6

(10) $M_5/6\ \Sigma K_c5$ = 197,844/1824 = 108.5 Radian/10_6

(11) By equation (7) R_5 = 108.5 + 142.0 + 128.4 = 378.9 Radian/10^6

(12) By equation (8) D_5 = 378.9 × 12 = 4547 ft./10^6

(13) The total deflection of any storey, by equation (9), is the running total of item (12) from the first storey upto and including the storey in question. Thus = 15854 + 4547 = 20,401 ft/10^6

(14) The deflections thus determined are scaled to a value of 1.00 ft. at the top of the frame. Thus at the fifth storey, relative deflection = 20401/36219 = 0.563. This item is useful chiefly in progressively checking the accuracy of the successive deflection curves.

(15) to (23) Comprise the second approximation or, in other words, determine the shape of the second transition curve. They are a repetition of the processes of (6) to (14). The second approximation corresponds very closely to the final free deflection curve and may, therefore, be used for all ordinary purposes. However, for complete convergence and a final check on the accuracy of the work, a third approximation is made.

(24) to (32) Comprise the third approximation which is made in the same manner as the first and second

Total	1	2	3	4	5	6	7	8	9	10		STOREY
	639.4	564.0	475.0	343.6	304.0	304.0	198.8	198.8	97.1	78.0	1	ΣK_c 10^6 lb. ft.
	3836	3384	2850	2062	1824	1824	1193	1193	583	468	2	$6 \Sigma K_c$ 10^6 lb. ft.
	2091	1452	1452	1452	1452	1452	1452	1452	1452	1452	3	$12 \Sigma K_g$ 10^6 lb. ft.
	3790	3790	3790	3790	3790	3790	3670	3670	3670	3510	4	Mass
	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000	5	Δ, assumed, ft.
	379	758	1137	1516	1895	2274	2569	2936	3303	3510	6	Force lbs.
	20277	19898	19140	18003	16487	14592	12318	9749	6813	3510	7	Shear lbs.
	243324	238776	229680	216036	197844	175104	147816	116988	81756	42120	8	M lb. ft.
	115.5	161	153.5	142	128.4	111.2	91.2	68.4	42.6	14.5	9	$\theta/2$ radians/10^6
	63.5	70.5	80.5	104.7	108.5	96	124	98	140.4	90.1	10	$M/6 \Sigma K_c$
	179	347	395	400.2	378.9	335.6	326.4	257.6	251.4	147.2	11	R radians/10^6
	2148	4164	4740	4802	4547	4027	3917	3091	3017	1766	12	D ft./10^6
	2148	6312	11052	15854	20401	24428	28345	31436	34453	36219	13	Δ ft./10^6
	0.059	0.174	0.305	0.438	0.563	0.675	0.782	0.869	0.950	1.000	14	Relative Δ
	224	660	1156	1660	2135	2560	2870	3190	3485	3510	15	Force lbs.
	21450	21226	20566	19410	17750	15615	13055	10185	6995	3510	16	Shear lbs
	257400	254712	246792	232920	213000	187380	156660	122220	83940	42120	17	M lb. ft.
	122.5	172.7	165	153.7	137.9	118.5	96	71	43.4	14.5	18	$\theta/2$ radians/10^6
	67	75.3	86.5	113	116.9	102.7	131	102.3	144	90.1	19	$M/6 \Sigma K_c$
	189.5	370.5	424.2	431.7	408.5	359.1	345.5	269.3	258.4	148	20	R radians/10^6
	2274	4446	5090	5180	4902	4309	4146	3232	3101	1776	21	D ft./10^6
	2274	6720	11810	16990	21892	26201	30347	33579	36680	38456	22	Δ ft./10^6
	0.059	0.175	0.307	0.441	0.570	0.682	0.789	0.873	0.954	1.000	23	Relative Δ
	224	663	1163	1672	2160	2582	2895	3200	3495	3510	24	Force lbs.
	21564	21340	20677	19514	17842	15682	13100	10205	7005	3510	25	Shear lbs.
	258768	256080	248124	234168	214104	188184	157200	122460	84060	42120	26	M lb. ft.
	123	173.5	166.1	154	138.5	119	96.3	71.1	43.5	14.5	27	$\theta/2$ radians/10^6
	67.4	75.7	87.2	113.5	117.5	103.2	131.9	102.5	144.5	90.1	28	$M/6 \Sigma K_c$
	190.4	372.2	426.8	433.6	410	360.7	347.2	269.9	259.1	148.1	29	R radians/10^6
	2285	4466	5122	5203	4920	4328	4166	3239	3109	1777	30	D ft./10^6
	2285	6751	11873	17076	21996	26324	30490	33729	36838	38615	31	Δ ft./10^6
	0.059	0.175	0.307	0.441	0.570	0.682	0.789	0.873	0.954	1.000	32	Relative Δ
24.14	0.02	0.17	0.54	1.11	1.83	2.63	3.42	4.18	4.99	5.25	33	m Δ^2
623.33	0.51	4.47	13.80	28.55	47.50	68.00	88.20	108.00	128.80	135.50	34	F Δ

$$T = 2\pi \sqrt{24.14/623.33} = 2\pi \sqrt{.0387} = 2\pi \times .1966 = 1.235 \text{ sec.}$$

$$T_5 = 2\pi \sqrt{(3790 \times .021997)/2160} = 2\pi\sqrt{.0386} = 2\pi \times .1964 = 1.235 \text{ secs.}$$

approximations. The perfect agreement between the respective quantities of (23) and (32) may be noted.

(33) The next step in the procedure is to compute the energy quantities for use in equation (12). Using the actual deflection listed in (31), again taking the fifth storey as an example,

$$m_5 \Delta_5^2 = 3790 \times (0.021996)^2 = 1.83$$

$$\Sigma (m\Delta^2) = 24.14$$

(34) Correspondingly,

$$F_5 \Delta_5 = 2160 \times .021996 = 47.50$$

$$\text{and } \Sigma (F\Delta) = 623.33$$

The procedure to use in any case is briefly :

(a) A simple deflection curve is assumed, linear, parabolic etc., depending upon the character of the structure and the distribution of the masses.

(b) The dynamic forces are calculated by means of equation (5).

(c) The deflections induced by the assumed dynamic forces are determined using the appropriate deflection calculating method.

(d) The structure is passed through the necessary number of cycles of dynamic convergence, to determine the true free vibration deflection curve, by repeating steps (b) and (c)

(e) The period is calculated by means of equation (12)

CONCLUSION

(i) From the above discussion it may be seen that the method of Dynamic convergence with Energy principle can be used, with great advantage in the analysis of multistoreyed building frames for dynamic loading.

(ii) Instead of the Dynamic convergence method, the static deflection curve can also be used. But then the dynamic character of the problem is not acknowledged. Because, actually the dynamic forces which produce the deflection curve are proportional not simply to the various masses but to the product of mass and acceleration of each element of the structure.

(iii) Compared to other methods of frequency determination, Energy Method is more general and can be made to suit any given condition by incorporating different losses that may come in actual structures.

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ON THE VIBRATION BEAM AND SLAB BRIDGES

K. T. Sundara Raja Iyengar* and K. S. Jagadish**

SYNOPSIS

The beam and slab bridge is a complex structure and the calculation of its natural frequency is quite involved. A simplified procedure has been given in this paper for such a calculation. A Fourier series expansion in terms of plate-eigenfunctions has been used to express the deflection of the bridge, the bridge being considered as a plate resting on beams. It has been found that a one-term approximation gives satisfactory results and numerical work has been carried out for various bridge aspect-ratios based on a one-term approxmation. Graphs have been given to facilitate a rapid estimate of the frequency.

INTRODUCTION

The natural frequencies of a structure, especially of the graver modes, form important basic data necessary in evaluating its response under a variety of dynamic loads. These frequencies are of interest whether one wishes to determine the dynamic deflection of a bridge under the action of a moving load or its behaviour under blast loadings and strong-motion-earthquakes. An easy procedure to calculate the fundamental natural frequency, without sacrificing the accuracy of the result, would then be eminently desirable.

The dynamic behaviour of bridges has been studied extensively by various authors such as Inglis (1934) and others, but, they have confined themselves mostly to the study of Railway bridges. The natural frequencies of such bridges can be obtained easily by the theory of beam-vibrations. The situation is more complex when one considers a beam and slab bridge. The problem is essentially two dimensional and a one-dimensional approximation is not reliable unless the span is very large when compared with the width of the bridge. A rational analysis of the problem then requires, a consideration of the bridge in the light of the plate theory.

Stiffened Plate structures have been studied in the literature, most often by the orthotropic plate theory. Considering the beam and slab bridge as an orthotropic plate one arrives at a transcendental equation for the bridge frequency (Hoppmann and Huffington, 1958). This approach has been utilised by Naruoka and Yonezawa (1958) for the frequency analysis of the beam bridge. But the solution of the transcendental equation for various bridge dinamensions is quite cumbersome and is not easily amenable for engineering purposes.

In this paper the bridge has been considered as a plate resting on beams. It has been assumed that there is no restraint against slippage between the beam and the slab and that the beam offers only vertical forces of reaction against the plate. Torsional resistance of the beams and the internal damping in the bridge have been neglected. The consideration of restraint against slippage leads to very involved equations and is not amenable to a rapid calculation. The influence of restraint against slippage tends to increase the potential energy of deformation of the structure and its effect may also be considered by increasing the stiffness of the beam on the basis of the effective width concept. The question of effective width in dynamic problems has not been well understood and this consideration merits more detailed theoretical and experimental investigations.

THE PLATE-EIGEN FUNCTIONS

It is well known that the normal modes of a vibra-

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ting plate can be described in terms of known functions when its two opposite edges are simply supported. A slab bridge can be considered as a plate simply supported on two opposite edges and free at the other two (Fig. 1).

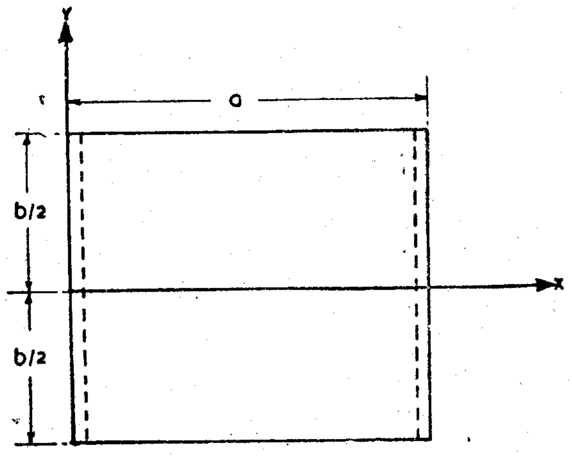

Fig. 1.

Considering the classical plate equation

$$\nabla^4 W + \frac{\rho}{D}\frac{\partial^2 W}{\partial t^2} = 0 \quad \dots (1)$$

where W is the deflection of the plate, ρ the mass of the plate per unit area, D the flexural rigidity of the plate, one can take for the m-n[th] mode of vibration

$$W_{mn} = Y_{mn}(y)\,\text{Sin}\frac{m\pi x}{a}\,\text{Cos}\,P_{mn}t \quad \dots (2)$$

where 'a' is the span of the slab bridge, P_{mn} the circular frequency of vibration for the m-n[th] mode, m and n being any two integers. Y_{mn} will now take the form,

$$Y_{mn}(y) = A_1\,\text{Cosh}\frac{\alpha_{mn}y}{b} + A_2\,\text{Sinh}\frac{\alpha_{mn}y}{b} + A_3\,\text{Cos}\frac{\beta_{mn}y}{b} + A_4\,\text{Sin}\frac{\beta_{mn}y}{b} \quad \dots (3)$$

where b is the width of the slab bridge,

$$\alpha_{mn} = b\sqrt{\sqrt{\frac{\rho p^2_{mn}}{D}} + \frac{m^2\pi^2}{a^2}} \quad \dots (4a)$$

$$\beta_{mn} = b\sqrt{\sqrt{\frac{\rho p^2_{mn}}{D}} - \frac{m^2\pi^2}{a^2}} \quad \dots (4b)$$

and $$\alpha^2_{mn} - \beta^2_{mn} = 2m^2\pi^2\frac{b^2}{a^2} \quad \dots (4c)$$

The free edge boundary conditions at $y = \pm\frac{b}{2}$ may be expressed as

$$\left.\frac{d^2Y_{mn}}{dy^2} - v\frac{m^2\pi^2}{a^2}Y_{mn}\right|_{y=\pm\frac{b}{2}} = 0 \quad \dots (5a)$$

$$\left.\frac{d^3Y_{mn}}{dy^3} - (2-v)\frac{m^2\pi^2}{a^2}\frac{dY_{mn}}{dy}\right|_{y=\pm\frac{b}{2}} = 0 \quad (5b)$$

where v is the Poisson's ratio.

Considering only modes symmetric in y we obtain the frequency equation to satisfy these boundary conditions for the slab bridge as

$$\tan\frac{\beta_{mn}}{2} = -\frac{\alpha_{mn}}{\beta_{mn}}\left(\frac{\beta^2_{mn} + vm^2\pi^2\frac{b^2}{a^2}}{\beta^2_{mn} + (2-v)m^2\pi^2\frac{b^2}{a^2}}\right)^2 \times \tanh\frac{\alpha_{mn}}{2} \quad \dots (6)$$

This equation has to be solved together with (4c) to obtain the frequency parameters α_{mn} and β_{mn}. These equations will have an infinite number of roots for each value of m. The eigenfunction corresponding to the n[th] root may now be taken as

$$Y_{mn}(y) = \frac{\text{Cosh}\frac{\alpha_{mn}y}{b}}{\text{Cosh}\frac{\alpha_{mn}}{2}} + \frac{\alpha^2_{mn} - v m^2\pi^2\frac{b^2}{a^2}}{\beta^2_{mn} + vm^2\pi^2\frac{b^2}{a^2}} \times \frac{\text{Cos}\frac{\beta_{mn}y}{b}}{\text{Cos}\frac{\beta_{mn}}{2}} \quad \dots (7)$$

The first roots of the equations corresponding to the fundamental mode of vibration have been obtained by an iteration procedure given in Appendix I. The values of the roots are given in Table 1 for $v=0$ and $v=0.2$.

It can be shown that integrals of the type

$$\int_{-b/2}^{+b/2}\int_0^a Y_{mn}Y_{ij}\,\text{Sin}\frac{m\pi x}{a}\,\text{Sin}\frac{i\pi x}{a}\,dx\,dy = 0 \quad \dots (8)$$

whenever $i \neq m$ or $j \neq n$. It may thus be seen that the functions $Y_{mn}\,\text{Sin}\frac{m\pi x}{a}$ form an orthogonal set in the domain $x = 0$ to a and $y = -\frac{b}{2}$ to $+\frac{b}{2}$.

This property of the plate-eigenfunctions allows a Fourier series expension of an arbitrary function in terms of these functions. A knowledge of the value of the integral

$$\frac{1}{ab}\int_{-b/2}^{+b/2}\int_{o}^{a} Y^2_{mn}\ \mathrm{Sin}^2\frac{m\pi x}{a}\,dx\,dy = K_{mn} \quad \dots \quad (9)$$

is essential for such a Fourier series expansion. The values of this integral have been given in Table 2 for $m = 1$ and $n = 1$

ANALYSIS

A typical beam and slab bridge with k beams disposed symmetrically about the x–axis, the free edges being parallel to the x-axis (Fig. 2) may now be considered. The analysis of free vibration of this structure will now be based on the following assumptions.

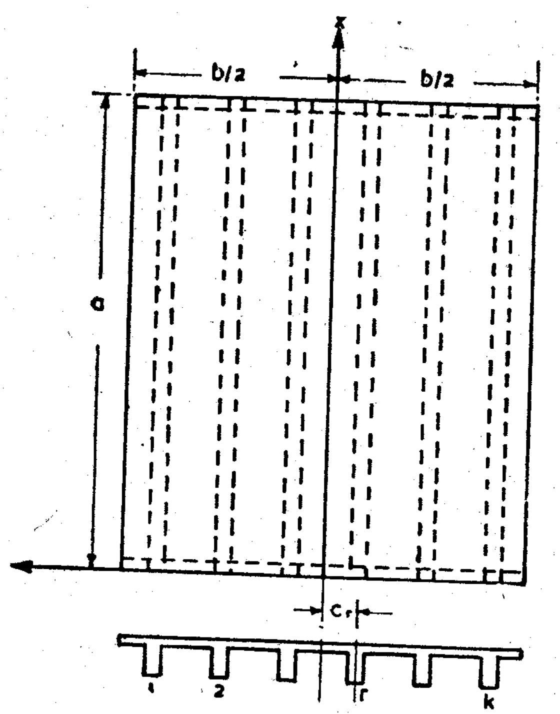

Fig. 2.

(i) There is no restraint against slippage between the beams and the slab,

(ii) The torsion of the beams may be neglected and

(iii) The internal damping in the bridge may be neglected.

Though the first assumption is rarely realised in practice, the inaccuracies arising from this assumption may be compensated for by suitably increasing the stiffness of the beam based on the effective width concept. Such an engineering approach to simplification is necessary since the equations would otherwise become very complicated. The second assumption is besed on the fact that the rotations of plate cross-sections will be small in the first mode of vibration.

The motion of the slab may be described by the equation

$$D\nabla^4 W - \rho p^2 W = -\sum_{r=1}^{k} \delta(c_r)\, f_r(x) \quad \dots \quad (10)$$

after separating the time variable, where $y = c_r$ denotes the location of the r^{th} beam, $f_r(x)$ is the amplitude of the vertical force of interaction between the r^{th} beam and the slab and $\delta(c_r)$ is the Dirac-delta function at $y = c_r$. The deflections of the beams are given by the equations

$$EI\frac{d^4\overline{W}_r}{dx^4} - \gamma p^2 \overline{W}_r = f_r(x) \quad \dots \quad (11)$$

$$r = 1, 2, \dots k$$

where EI is the stiffness of any beam, γ is the mass per unit length of any beam and $\overline{W}_r(x) = W(x, c_r)$.
W is now expanded in terms of the plate eigenfunctions discussed previously and one can write

$$W = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} Y_{mn}\, \mathrm{Sin}\frac{m\pi x}{a} \quad \dots \quad (12)$$

It may be noted that the functions used in the above expansion satisfy all the boundary conditions exactly. The coefficients A_{mn} now remain to be chosen so as to satisfy the equations (10) and (11). Expanding

$\sum_{r=1}^{k} \delta(c_r) f_r(x)$ by another Fourier series,

$$\sum_{r=1}^{k} \delta(c_r)\, f_r(x) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} Y_{mn}\, \mathrm{Sin}\frac{m\pi x}{a} \quad (13)$$

where

$$a_{mn} = \frac{1}{K_{mn}\,ab}\sum_{r=1}^{k} Y_{mn}(c_r)\int_{o}^{a} f_r(x)\, \mathrm{Sin}\frac{m\pi x}{a}\,dx \quad (14)$$

Using (13) and (12) in (10),

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\frac{\rho p^2_{mn}}{D} Y_{mn}\operatorname{Sin}\frac{m\pi x}{a} - \frac{\rho p^2}{D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} Y_{mn}\operatorname{Sin}\frac{m\pi x}{a} = -\frac{1}{D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} Y_{mn}\operatorname{Sin}\frac{m\pi x}{a} \quad \dots (15)$$

where p_{mn} is the circular frequency for the m$-$n[th] mode of vibration of the slab without beams.

Collecting the coefficient of each $Y_{mn}\operatorname{Sin}\frac{m\pi x}{a}$ in (15) and using (14)

$$A_{mn}\frac{\rho}{D}(p^2_{mn}-p^2) = -\frac{1}{K_{mn} Dab}\sum_{r=1}^{k} Y_{mn}(c_r)\int_0^a f_r(x)\operatorname{Sin}\frac{m\pi x}{a}dx \quad (16)$$

Since $\overline{W}_r=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} Y_{mn}(c_r)\operatorname{Sin}\frac{m\pi x}{a}$, equation (11) becomes, $f_r(x)$

$$=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} Y_{mn}(c_r)\times\left(EI\frac{m^4\pi^4}{a^4}-\gamma p^2\right)\operatorname{Sin}\frac{m\pi x}{a} \quad \dots (17)$$

Substituting (17) in (16)

$$A_{mn}\frac{\rho}{D}(p^2_{mn}-p^2) = -\frac{1}{K_{mn}ab}\sum_{r=1}^{k} Y_{mn}(c_r)\int_0^a\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} A_{ij} Y_{ij}(c_r) \operatorname{Sin}\frac{m\pi x}{a}\operatorname{Sin}\frac{i\pi x}{a}\left(\frac{EI}{D}\frac{i^4\pi^4}{a^4}-\frac{\gamma p^2}{D}\right)dx \quad \dots (18)$$

Since $\int_0^a \operatorname{Sin}\frac{i\pi x}{a}\operatorname{Sin}\frac{m\pi x}{a}dx = 0$ when $i\neq m$

$=\frac{a}{2}$ when $i=m$,

(18) reduces to

$$A_{mn}\frac{\rho b^4}{D}(p^2_{mn}-p^2) = -\frac{1}{2K_{mn}}\sum_{r=1}^{k}\sum_{j=1}^{\infty} A_{mj} Y_{mn}(c_r)\, y_{mj}(c_r)\left(\frac{EI}{Db}\frac{m^4\pi^4b^4}{a^4} - \frac{\gamma}{\rho b}\times\frac{\rho p^2 b^4}{D}\right)$$

$$n=1, 2, \dots \quad \dots (19)$$

Here one has an infinite set of homogeneous equations in A_{mj} for each value of m.

Putting $\lambda_{mn}=\frac{\rho p^2_{mn}b^4}{D}$, $\lambda=\frac{\rho p^2 b^4}{D}$, $K_1=\frac{EI}{Db}$ and $K_2=\frac{\gamma}{\rho b}$, the equation may be written as

$$A_{mn}(\lambda_{mn}-\lambda) = -\frac{1}{2K_{mn}}\sum_{r=1}^{k}\sum_{j=1}^{\infty} A_{mj} Y_{mj}(c_r) Y_{mn}(c_r)\times\left(K_1\frac{m^4\pi^4b^4}{a^4}-K_2\lambda\right)$$

$$n=1, 2, \dots \quad \dots (20)$$

For non trivial solutions of A_{mj} the determinant of the coefficients of A_{mj} should vanish. The roots of this determinant give the values of λ, the frequency parameter, for various modes.

CONVERGENCE OF SOLUTION

For purposes of numerical analysis it is necessary to consider a finite number of terms in the series, the number of terms depending upon the rapidity of the convergence of the solution. A typical case has been examined by taking $\frac{a}{b}=2.0$ and considering a three-beam bridge such that $c_1=\frac{3b}{8}$, $c_2=0$, and $c_3=-\frac{3b}{8}$. Putting m = 1, one obtains modes with no nodal lines in the x direction. Taking three terms in the series corresponding to A_{11}, A_{12} and A_{13} a third order determinant is obtained: The determinant is given below putting $K_1 = 2.0$ and $K_2 = 0.2$.

$$\begin{vmatrix} 749.0910-1.58871\lambda & -29.6167+0.48647\lambda & -4.3230+0.07100\lambda \\ -34.6162+0.56859\lambda & 15488.7-1.50991\lambda & -8.1135+0.13326\lambda \\ -5.1452+0.08451\lambda & -8.2618+0.1357\lambda & 91053.3-2.04541\lambda \end{vmatrix} = 0$$

The values of $\sqrt{\lambda}$ for the first mode corresponding to a one-term approximation, two-term approximation and three-term approximation have been determined and they are,

No. of terms	1	2	3
$\sqrt{\lambda}$	21.714	21.669	21.669

It may be noticed that the convergence is very rapid and that a one-term approximation is sufficiently accurate for engineering purposes. The rapidity of convergence stems from the fact that the diagonal terms are very large when compared when the others, and they increase very rapidly as the order of the diagonal term is increased. The same situation prevails in other determinants corresponding to various aspect-ratios of the bridge. The results of one-term approximation may therefore be used for a practical calculation of the bridge frequency.

ONE-TERM APPROXIMATION

The frequency analysis now becomes very much simplified since no determinantal equation need be solved. As the frequency of the first symmetric mode is the most important data, numerical work has been carried out only for this mode. Putting m=1 and taking the first term in the series corresponding to A_{11} the frequency equation becomes

$$2K_{11}\ \lambda_{11}+K_1\ \frac{\pi^4 b^4}{a^4} \sum_{r=1}^{k} Y^2_{11}\ (c_r)$$
$$= [2K_{11}+K_2 \sum_{r=1}^{k} Y^2_{11}\ (c_r)]\ \lambda \qquad \ldots\ (21)$$

The calculation of λ is now quite straight forward once the values of K_{11}, λ_{11}, α_{11} and β_{11} for each $\frac{a}{b}$ ratio are known. The values of α_{11}, β_{11}, and K_{11} are to be found in Tables 1 and 2. The values of λ_{11} are given in Table 3.

The frequency parameters for various bridge dimensions have been presented in the graphs (Figs. 3 to 14)* both for $\upsilon=0$ and $\upsilon=0.2$ to facilitate rapid frequency determination. The variation of $\sqrt{\lambda}$ with respect to K_1 is very nearly a straight line in the range of values of K_1 chosen and the graphs have been drawn as such. The graphs cannot be extended much beyond the value of $K_1 = 10$, since then the variation will not be linear, and also the accuracy of the one-term approximation is affected for large values of K_1. Nonetheless, the approximation gives satisfactory results in the range of bridge dimensions met with in practice.

CONCLUSIONS

The procedure outlined in the previous sections provides a rapid way of calculating the bridge frequency unlike the one using the orthotropic plate theory. The orthotropic plate theory necessitates the solution of a cumbersome transcendental equation for every frequency calculation. The present procedure owes its simplicity to the fact that the bridge deflection is expanded in terms of the plate-eigenfunctions and also to the orthogonal properties of these functions.

It may be noticed from the graphs that in general, except for $\frac{a}{b} = 1.5$, the bridge frequency is less than the frequency of the corresponding beamless slab, in the range of values of K_1 and K_2 considered. This indicates that the influence of the mass of the beams dominates over the influence of the stiffness of the beams in the above range.

The graphs also clearly demonstrate the influence of Poisson's ratio on the bridge frequency, which arises due to the presence of free edges. A decrease in Poisson's ratio is attended by an increase in the frequency. This is in conformity with the well known theorem due to Rayleigh that the introduction of constraints raises the frequency of a system. One may also notice that as the span/width ratio increases the Poisson's ratio effect is less pronounced. For span/width ratios beyond 3.0 the neglect of Poisson's ratio introduces only negligible errors. This mey be considered as being due to the predominently one-dimensional action of the bridge at large span/width ratios.

It is necessary to remark here that the assumption of absence of restraint against slippage leads to frequencies lower than the actual, since this absence of restraint is not realized in practice. This again follows from the well known theorem due to Rayleight (1945). The consideration of this restraint in the problem would render the equations sufficiently complicated as to make the numerical labour prohibitive. As mentioned earlier in the introduction this difficulty may be circumvented by suitably calculating the stiffness of the beam by taking a portion of the plate to act with the beam. This altered stiffness may then be used in the procedure developed in this paper to calculate the frequency. A similar problem has been encountered in the treatment of buckling of plate-beam systems (Timoshenko and

*Figures given at the end of this paper.

Gere. 1961). It is needless to emphasize that the width of the plate to be included in the stiffness calculation can only be determined by detailed theoretical and experimental investigations.

Table 1

FREQUENCY PARAMETERS FOR THE SLAB

$\frac{a}{b}$	$v=0$		$v=0.2$	
	α_{11}	β_{11}	α_{11}	β_{11}
1.5	5.9283	5.1353	5.8439	5.0376
2.0	5.4609	4.9886	5.4013	4.9233
2.5	5.2188	4.9069	5.1757	4.8610
3.0	5.0784	4.8576	5.0463	4.8241
3.5	4.9902	4,8261	4.9655	4.8005
4.0	4.9315	4.8049	4.9120	4.7848

Table 2

VALUES OF K_{11}

$\frac{a}{b}$	$v=0$	$v=0.2$
1.5	0.71599	0.67541
2.0	0.62231	0.60131
2.5	0.57881	0.56587
3.0	0.55501	0.54619
3.5	0.54056	0.53415
4.0	0.53113	0.52626

Table 3

VALVES OF λ_{11}

$\frac{a}{b}$	$v=0$	$v=0.2$
1.5	946.056	885.912
2.0	748.229	713.235
2.5	658.269	635.473
3.0	609.754	593 825
3.5	580.649	568.846
4.0	561.850	552.767

APPENDIX I

(i) Iteration Procedure to obtain α_{mn} and β_{mn}

The frequency may be written as

$$\tan\frac{\beta_{mn}}{2} = -\frac{\alpha_{mn}}{\beta_{mn}}\left(\frac{\beta^2{}_{mn}+v\dfrac{m^2\pi^2b^2}{a^2}}{\beta^2{}_{mn}+(2-v)\dfrac{m^2\pi^2b^2}{a^2}}\right)^2 \times \tanh\frac{\alpha_{mn}}{2}$$

First approximations to the values of α_{mn} and β_{mn} for various values of $\frac{b}{a}$ ratio have been given by Thein Wah (1961) using a graphical procedure. These values have been used as the initial values for the iteration. Substituting an initial value in the right hand side of the above equation, a new value of β_{mn} was obtained by equating $\tan\frac{\beta_{mn}}{2}$ to the calculated right hand side. Using this second approximation to β_{mn}, a second approximation to α_{mn} was obtained by the relation $\alpha^2{}_{mn} = \beta^2{}_{mn}+2m^2\pi^2\frac{b^2}{a^2}$. The procedure was then repeated using these second approximations. The iterations were repeated until the differences between successive approximations were negligible. The results of such an iteration are presented in Table 1.

(ii) Expression for K_{mn}

Substituting the expresson for Y_{mn} in the relation

$$K_{mn} = \frac{1}{ab}\int_{-b/2}^{+b/2}\int_{0}^{a} Y^2{}_{mn}\,\mathrm{Sin}^2\frac{m\pi x}{a}\,dx\,dy$$

$$= \frac{1}{2b}\int_{-b/2}^{+b/2} Y^2{}_{mn}\,dy$$

one obtains,

$$K_{mn} = \frac{1}{4\,\mathrm{Cosh}^2\frac{\alpha_{mn}}{2}} + \frac{\tanh\frac{\alpha_{mn}}{2}}{2\alpha_{mn}} + \frac{(\alpha^2{}_{mn}-vm^2\pi^2\frac{b^2}{a^2})^2}{(\beta^2{}_{mn}+vm^2\pi^2\frac{b^2}{a^2})^2} \times$$

$$\left(\frac{1}{4\,\mathrm{Cos}^2\frac{\beta_{mn}}{2}} + \frac{\tan\frac{\beta_{mn}}{2}}{2\beta_{mn}}\right) + \frac{2}{(\alpha^2{}_{mn}+\beta^2{}_{mn})} \times$$

$$\frac{(\alpha^2{}_{mn}-vm^2\pi^2\frac{b^2}{a^2})}{(\beta^2{}_{mn}+vm^2\pi^2\frac{b^2}{a^2})}\left(\alpha_{mn}\tanh\frac{\alpha_{mn}}{2}+\beta_{mn}\tan\frac{\beta_{mn}}{2}\right)$$

The values of K_{11} are given in Table 2.

APPENDIX II

NOTATION

a	Span of the bridge
b	Width of the bridge
c_r	y-co-ordinate of the r^{th} beam
D	Flexural rigidity of the slab
EI	Flexural rigidity of any beam
K_{mn}	A definite integral depending on m and n
k	No. of beams
K_1	$\frac{EI}{Db}$
K_2	$\frac{\gamma}{\rho b}$
$f_r(x)$	Amplitude of the reaction of the r^{th} beam against the slab
m,n,i,j	Integers
P_{mn}	Circular frequency of the slab alone for the m-n^{th} mode
p	Circular frequency of the beam and slab bridge
W	Deflection of the bridge
$\overline{W}_r$	$W(x, c_r)$ Deflection of the r^{th} beam
W_{mn}	Deflection of a slab bridge in the m—n^{th} mode of vibration
α_{mn}	Frequency perameter for the slab
β_{mn}	Frequency parameter for the slab
γ	Mass of beam per unit length
$\delta(c_r)$	Dirac-delta function in one dimension at $y = c_r$
λ_{mn}	$\frac{\rho p^2_{mn} b^4}{D}$ Frequency parameter of the slab
λ	$\frac{\rho p^2 b^4}{D}$ Frequency parameter of the beam and slab
υ	Poisson's ratio
ρ	Mass of slab per unit area.

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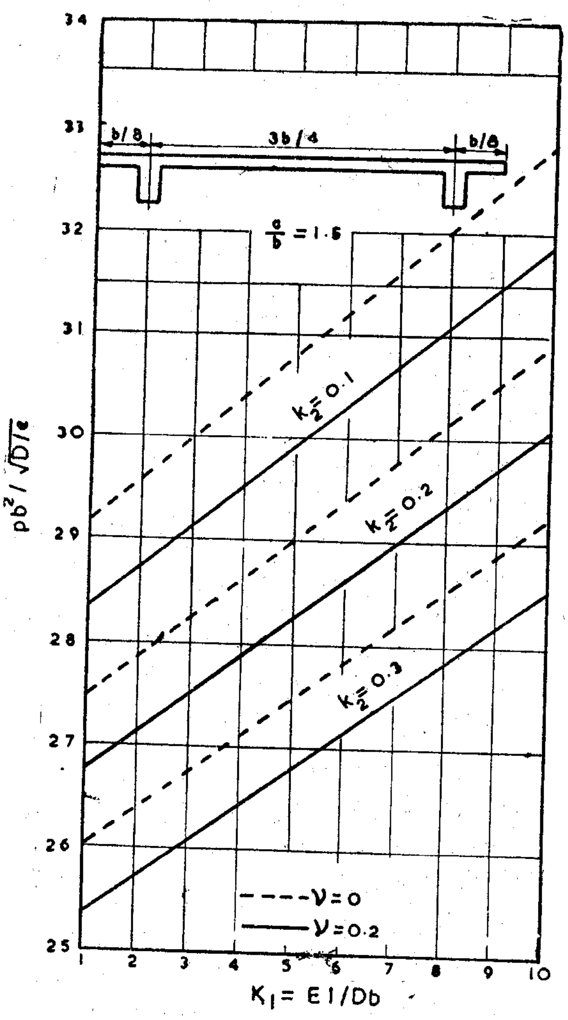

Fig. 3.

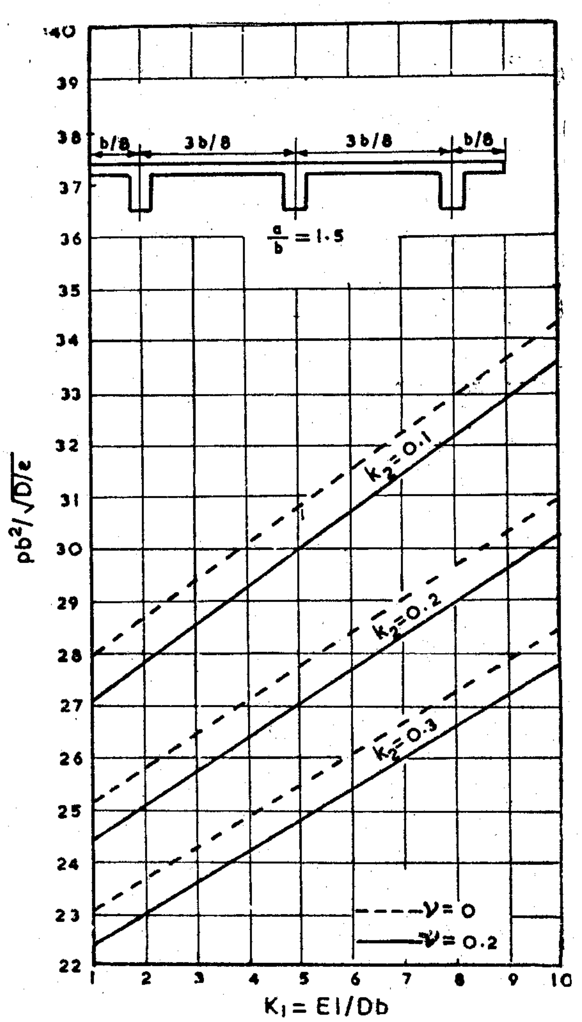

Fig. 4.

Values of $pb^2\sqrt{D/e}$ for a bridge with a/b=1.5 for the first symmetric mode.

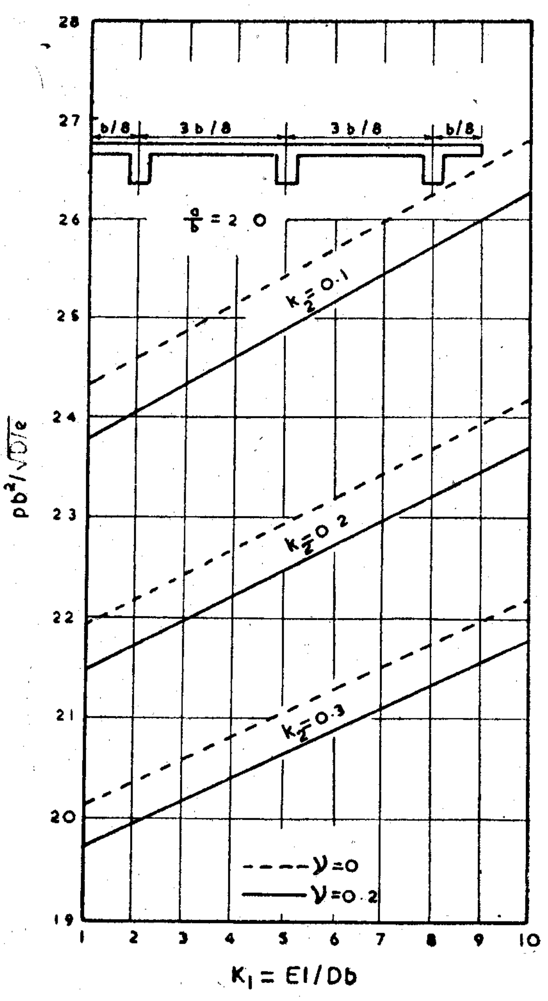

Fig. 5.

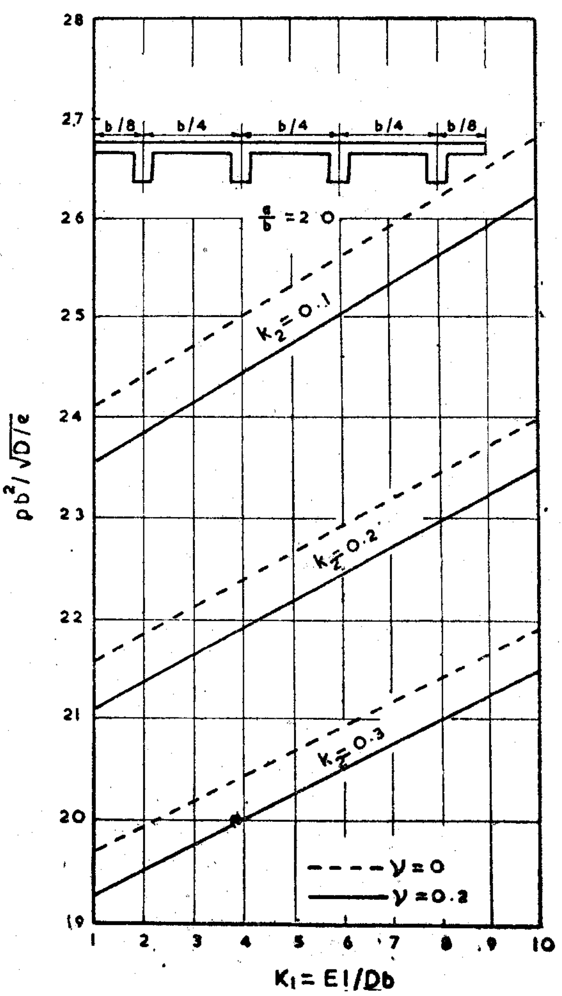

Fig. 6.

Values of $pb^2\sqrt{D/e}$ for a bridge with $a/b=2.0$ for the first symmetric mode.

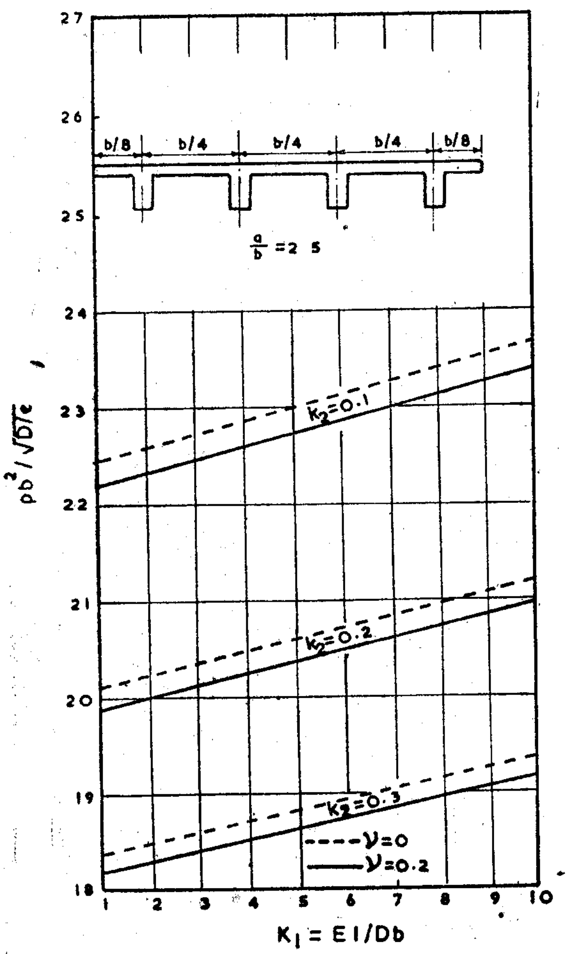

Fig. 7.

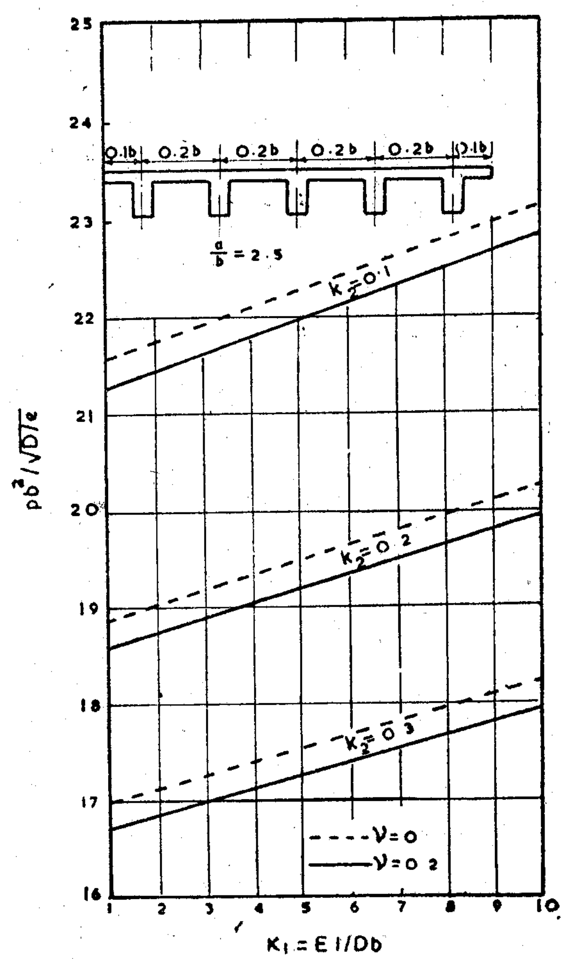

Fig. 8.

Values of $pb^2\sqrt{D/e}$ for a bridge with a/b=2.5 for the first symmetric mode.

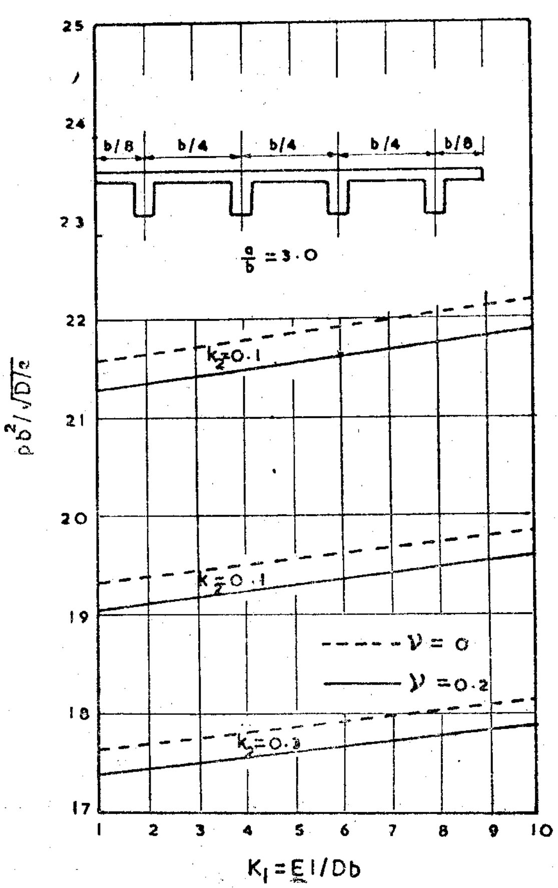

Fig. 9.

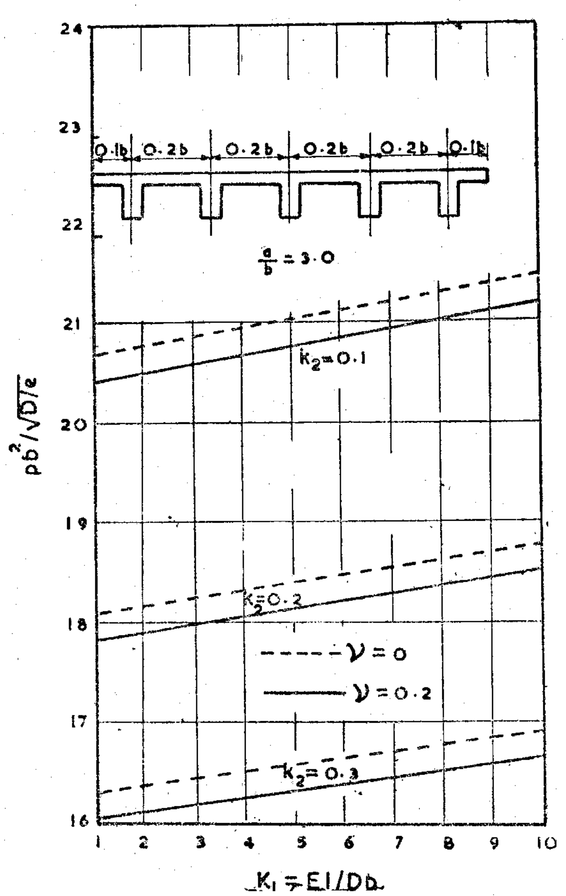

Fig. 10.

Values of $pb^2\sqrt{D/e}$ for a bridge with $a/b=3.0$ for the first symmetric mode.

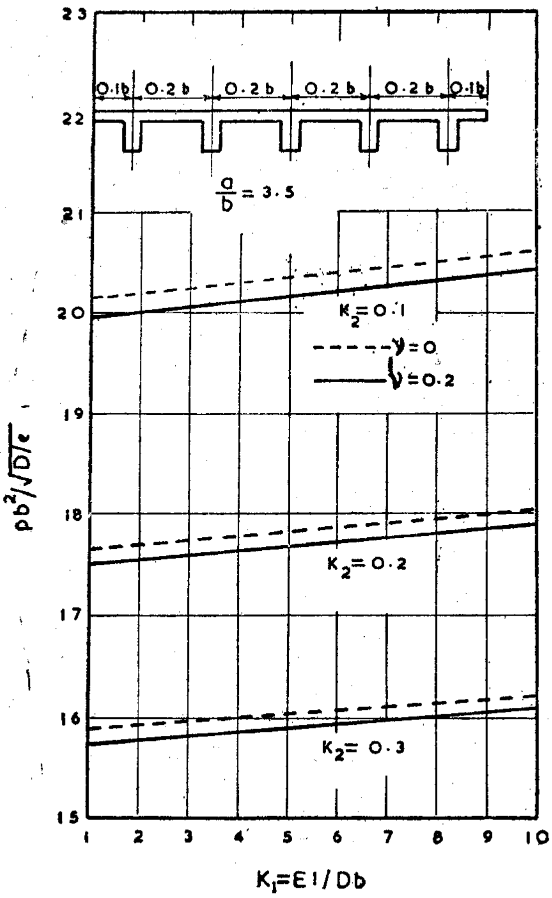

Fig. 11.

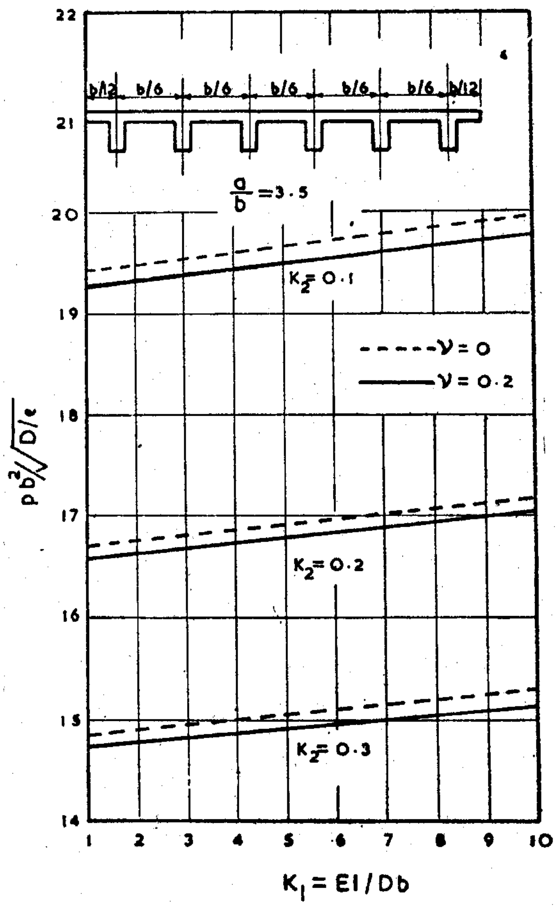

Fig. 12.

Values of $pb^2\sqrt{D/e}$ for a bridge with a/b=3.5 for the first symmetric mode.

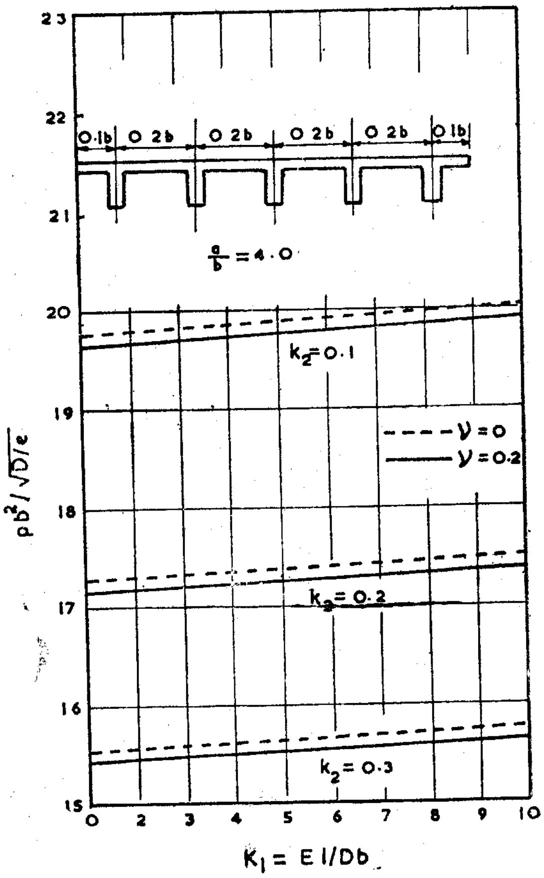

Fig. 13.

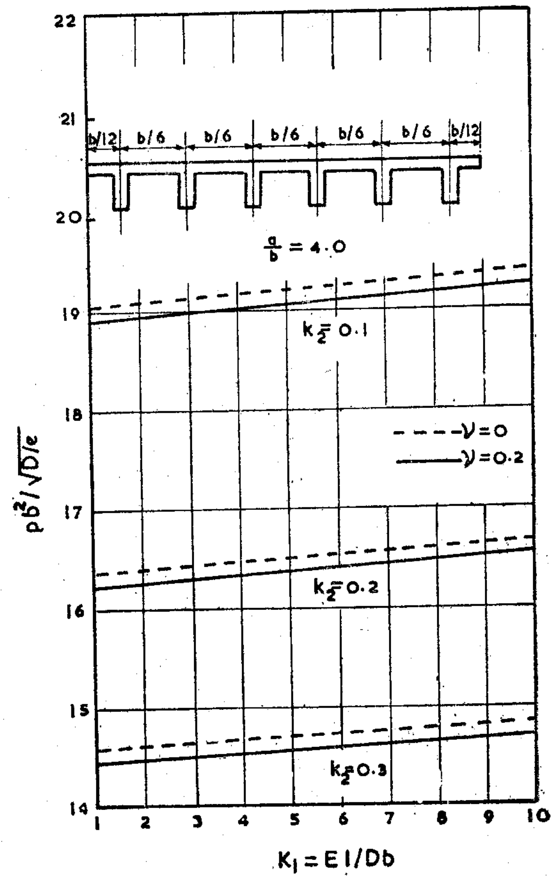

Fig. 14.

Values of $pb^2\sqrt{D/e}$ for a bridge with $a/b=4.0$ for the first symmetric mode.

EARTHQUAKE RESISTANT DESIGN OF AN ELEVATED WATER TOWER

Jai Krishna* and A. R. Chandrasekaran**

SYNOPSIS

This paper deals with the design of Water Towers in seismic zones with reference to a specific problem. The criterion for design suggested is that the system remains elastic under the effects of moderately strong ground motion which are expected to be more frequent in the particular zone, and under the effect of a severe ground motion, which may be expected there once in the life-time of the structure, the system may undergo plastic deformations.

INTRODUCTION

Liquid containing tanks and towers form a very important part of the life of a community as well as industrial undertakings. Many such structures have failed during earthquakes in the past resulting in considerable hardship and damage. The Earthquake Engineering Research School has consequently undertaken a programme to study the most economical methods of strengthening such structures in seismic zones of different intensities. A study has recently been made for a 50 ft. high, 50,000 gallons water tower proposed to be built at Rishikesh, a region which is not far from the main boundary fault running along the length of the Himalayas. The method employed in arriving at a suitable seismic coefficient and that of analysing the tower for earthquake forces have been explained in this paper. In estimating the future ground and the structural response, use has been made of the principles underlying the recommendations made by Jai Krishna (1960) and Housner (1959).

The design procedure recommended assures that under the vertical and horizontal loads the columns will deform as springs between two braces with the point of contraflexure at the mid-height of the panel. Further, the structure will stand the small earthquakes, which occur in the region quite often, remaining "elastic". During a major earthquake occurring not very far from the epicentre of 1905 earthquake, the structure will go "plastic", permit rotation at the junctions so that the entire resistance will come from the diagonal braces. It has been estimated that the structure will not deflect more than 6″ at the top even during such intense vibrations. The provision of diagonal braces in addition to the horizontal ones has been recommeded in view of the experience gained in Chile on account of 1960 shock. The tower at Rishikesh is founded on river deposits and, therefore, an eccentric reinforced concrete raft, that will resist torsion, small unequal settlements and excess pressures during earthquakes, has been recommended.

SEISMICITY OF THE REGION.

Rishikesh lies in the foot-hills of the Himalayas and is located within a few miles of the main Himalayan boundary fault which is seismically active. One of the major earthquakes that had occurred in the region was in 1905 with epicentres in Mussoorie and Kangra. The Mussoorie epicentre was about 25 miles from Rishikesh. There have been a number of earthquakes in the south east near about Almora and Kotdwar regions. The smaller earthquakes had their magnitude of about 6 and the major earthquake of 1905 had a magnitude of about 7.9. It is, therefore, essential that structures in this region should be designed for earthquakes. As far as a water tower is concerned, its safety is even more important because quite often an earthquake is followed by fire caused either by electrical short-circuiting or from the kitchen fires. The water storage, therefore, must

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remain in tact so that a fire could be dealt with. Keeping this in view, the ISI code (1893–1962) recommends somewhat higher values for design of water towers than for other structures. Both these factors have been kept in view in the design that follows :

DESIGN CRITERION

(a) According to Seismic Zoning Map [Fig. 1 page 7 of ISI Code (1893–1962)], Rishikesh is in zone IV. For soft soil, the horizontal seismic coefficient is equal to 0.08. As per clause 5.2.7.1. this factor is to be multiplied by 1.20 due to the tallness of the structure. The modified seismic coefficient is 0.096. The proximity of the boundary fault, however, necessitates special study.

(b) It is assumed that the boundary fault might be responsible for causing an earthquake of magnitude corresponding to N = 4 (The factor 'N' defines the intensity of earthquakes; N = 4 approximately corresponds to an earthquake of magnitude 8.0 at a distance of 25 miles or so from the epicentre).

(c) The structure is assumed to have a damping of 5% of critical.

(d) The design would permit plastic deformations for the strongest ground motion (N = 4) envisaged. This strong ground motion is likely to occur at least once during the life of the structure. Ground motions of lesser intensity are likely to occur more often. The deformation, say, for N=1 will be in the elastic range.

DYNAMIC BEHAVIOUR OF WATER TOWERS.

Water Towers consist of a heavy mass (tank proper and water) supported on top of columns, which act as springs. If the columns are braced only horizontally, the column and brace system will act as a Vierendeel girder to resist the horizontal forces. In this system, the lat ral force is resisted by moments and shears. If diagonal bracing is provided and if sufficient rotation is permitted at the joints, the system would act as a hinged truss system where the forces in the members would be axial.

A water tower resting on columns could be represented as a single degree of freedom system (fig. 1).

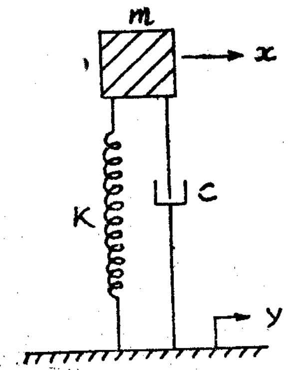

Fig. 1.

The equation of motion 'x' of such a system subjected to a ground motion represented by 'y' is given by

$$m\frac{d^2x}{dt^2} + C\left(\frac{dx}{dt} - \frac{dy}{dt}\right) + k(x-y) = 0 \qquad (1)$$

where 'm' is the mass, 'c' is the coefficient of damping and 'k' the spring constant.

The solution of equation is given by

$$x - y = \frac{1}{p} S_v \qquad \dots (2)$$

where 'p' is the natural frequency of the system in radians/sec. and equal to $\sqrt{k/m}$.

$$\text{and} \quad S_v = \int_0^t \ddot{y}(\tau) e^{-p(C/C_c)(t-\tau)} \sin p(t-\tau) d\tau \qquad (3)$$

and known as the "Velocity Response Spectrum" of the ground motion. An average velocity spectrum curve is shown in Fig. 7 (IS : 1893–1962). Calculating the period of the sturcture and assuming a suitable value of N, it is possible to evaluate the relative displacement (x–y) which the system would undergo during a corresponding ground motion.

The behaviour in the elastic range could be reasonably predicted as above. For a severe earthquake a design would be possible only if plastic deformations are permitted. The energy input to the structure due to the earthquake would be stored as plastic strain energy in the diagonal braces. The forces in the members may be evaluated on the basis that diagonal braces are subjected to yield stresses. This method of design will localise the failure, if any.

ANALYSIS OF FORCES.

(For details of analysis and design refer Krishna and Chandrasekaran 1963).

(A) Estimation of Weight of the System.

A part of the weight of the column and braces is assumed to be added to the weight at top, to take care of the effect of the weight of column and braces on the vibration characteristics of the system on the basis of Rayleigh's method.

i.	(a) Weight of tank	=	301,500
	(b) Weight of water	=	500,000
			801,500
ii.	(a) Columns	=	130,100 lbs.
	(b) Horizontal Braces	=	61,150 lbs.
	(c) Diagonal Braces	=	2,750 lbs.
			194,000 lbs.
iii.	Weight at top	=	801,500 lbs.
	$\frac{1}{3}$ weight of columns and braces	=	64,700 lbs.
	Total equivalent weight		866,200 lbs.
			Say 870 kips.

Without water, equivalent weight = 370 kips.

(B) Estimation of the Stiffness of the Structure.

In the elastic stage, bulk of the force would be resisted by the columns which are considerably more stiff than the tie rods.

The supporting system could be assumed to be made of springs in series. The stiffness of the spring (one bay) is made of stiffness of columns acting as parallel systems with diagonal braces.

(a) Stiffness of one bay:

i. Stiffness of column k_c is given by

$$k_c = \frac{12EI}{L^3} = 70.0 \text{ kips/in.}$$

Stiffness for 8 columns acting together will be = 560.0 kips/in

ii. Stiffness of the Diagonals.

$$k_b = F/\Delta = \frac{AE}{L}$$

where k_b = stiffness of one diagonal (refer fig. 2)

F = horizontal force

A = Area of Cross-section of member

Δ = horizontal deflection due to F

E = modulus of elasticity of member

L = equivalent length of member

k_{b_1} = 30.0 kips/in

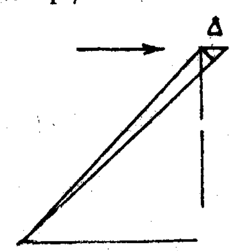

Fig. 2.

Similarly k_{b_2} for the adjoining one is given by

k_{b_2} = 44.0 kips/in

for all braces

$$k_b = 4\,k_{b_1} + 2k_{b_2}$$
$$= 208 \text{ kips/in}$$

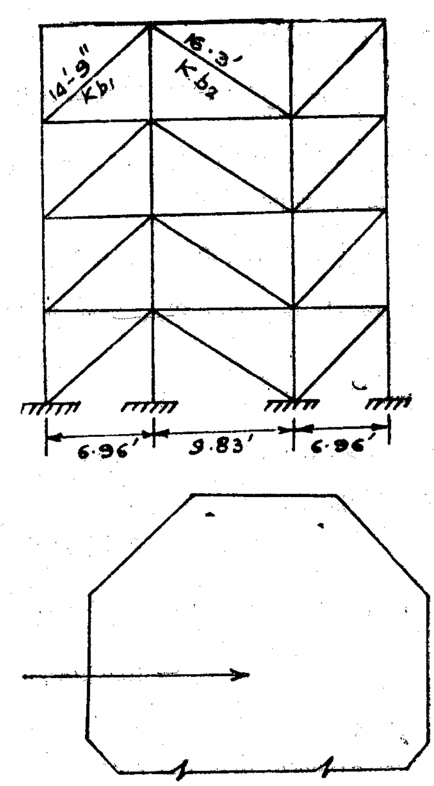

Fig. 3.

iii. Total 'K' per bay
$= 560 + 208$
$= 768$ kips/in

(b) Stiffness for entire structure
$= \frac{1}{4}$ of K
$= 192$ kips/in,

Period of the Structure

$$T = \frac{2\pi}{\sqrt{g}}\sqrt{w/k}$$

$$=0.32\sqrt{870/192} = 0.666 \text{ sec,}$$

(C) Lateral Force due to Earthquake,

i. Elastic Range

Let us assume that $N = 1$

and the damping factor $C/C_c = 5\%$ of the critical for deformations in the elastic range,

From the average velocity spectrum curves (Fig. 7, IS : 1893—1962) for $T = 0.666$ sec, $C/C_c = 0.05$ and $N=1$, we have

$$S_V = 0.46 \text{ ft/sec:}$$

$$(x-y) = \frac{S_V}{p} = \frac{0.666}{2\pi} \times (0.46 \times 12) = 0.58 \text{ inch}$$

This is the total horizontal displacement, and therefore, total shear 'V' is given by

$$V = K(x-y) = 200 \times 0.58 = 116 \text{ kips}$$

The horizontal seismic coefficient α_h is thus given by

$$= \frac{2000 \times 0.58}{870} = 0.133$$

Displacement per bay $\triangle = \frac{1}{4}$ of total $= 0.145''$

(a) Moment in the columns $= k_c \times \triangle \times \frac{1}{2}$
$= 55.2$ kips/ft.

(b) Axial forces in diagonals$= K_{b2} \times \triangle \times \frac{16.3}{9.83}$
$= 10.6$ kips.

ii. Plastic Range

It will be assumed that the energy input to the structure would correspond to a spectral velocity for $N = 4$ and $c/c_c = 5\%$ and S_V calculated on the basis of elastic behaviour. The spectral velocity in the plastic range would be $4 \times 0.45 = 1.84$ ft/sec.

The lateral force coefficient would be calculated on the basis that diagonal braces are subjected to yield stresses. On this basis the lateral force coefficient works out to be 0.20.

DESIGN DETAILS

(A) Columns

Elastic Range

The columns are subjected to direct load, upward and downward forces due to moment of lateral force and bending moment due to shear in the columns.

(i) Direct load on the columns $=125.0$ kips. (adding together the tank and water load as well as self weight of columns and braces we get 1000 kips. approximately, Each column will thus have 125 kips.)

(ii) Maximum Moment in the columns (at the junction of the column and brace)
$= 55.2$ kips ft.

(iii) Upward and downward forces in columns due to moment of lateral force.

(a) Bottom of columns between foundation and lowermost (third) brace $= \pm 135.0$ kips.

(b) Bottom of columns between second and third brace $= \pm 106.0$ kips.

(c) Bottom of columns between first and second brace $= \pm 76.6$ kips.

(d) Bottom of columns between first brace and tank $= \pm 47.2$ kips.

Net forces in columns

(i) Maximum moment $= 55.2$ kips. ft.

(ii) Direct load in kips

(a) above foundation $= +260$ or -10.0
(b) above third brace $= +231$ or $+19.0$
(c) above second brace $= +201.6$ or $+48\ 4$
(d) above first brace $= +172.2$ or $+77\ 2$

Section Assumed

18″ × 18″ square

Longitudinal reinforcement: 8 bars 7/8″ ϕ + 4 bars 3/8″ ϕ

Laterals

(i) Ties ⅜″ ϕ at 8″ c/c

(ii) Spiral ⅜″ ϕ at 4″ pitch for a length of 3′ 0″ on either side of brace.

Minimum ultimate cube strength of concrete 2250psi.

m = 15

Area provided = 397.4 sq. in.

I = 12,210 in^4.

Stresses

Just above foundation

(a−1) when direct load = 260 k
and Moment = 55.2 k ft.
Stresses = 1144 psi (Comp) or 166 psi (Comp)

(a−2) when direct load = − 10.0 kips
and Moment = 55.2 k. ft.

Stresses are

Concrete = 641.5 psi

Steel = 26,830 psi

These high stresses in concrete (for a−1 condition) and in steel (for a−2 condition) is permitted because these include stresses due to earthquake the force due to which has been taken to be much higher than that recommended in the code. The increase of working stresses by 50% under the circumstances is reasonable. At all other levels, stresses are lower than the above condition. In actual practice, however, there will be a redistribution of stresses due to semi-plastic rotations, and will lower the stresses below the values obtained above.

Plastic range

Loads

(i) Downward load due to self weight = 125 kips.

(ii) Upward and downward forces due to moment of lateral force of 0.2 w = ± 202.5 kips.

Net forces

(a) Downward = 327.5 K

(b) Upward = 77.5 K

Stresses

824 psi in concrete (comp)

14,800 psi in steel (tension)

(B) Diagonal Braces

Elastic Range

1″ ϕ m.s. rods are used as diagonals

Area of steel = 0.783 sq. in

Stress = 13,550 psi (tensile)

If the earthquake is very severe, the joints would become plastic and permit large deformations of diagonals. In that case, the diagonals would be fully stressed.

Plastic Range

Energy to be dissipated

$= \frac{1}{2}$ W/g. S_v^2

= 45,600 ft. lbs.

Considering half the diagonal braces acting in tension, average plastic strain = 0.0068

In extreme cases actual values of strain may be double that of the average strain and this would correspond to a maximum displacement of 6″ at the tank level which is considered as an upper limit.

(C) Horizontal Braces

Elastic Range

Maximum moment = 156.4 kft.

Maximum shear = 22,400 lbs.

Assume 12″ × 21″ section

4 bars 1½″ ϕ at top and bottom

⅜″ ϕ stirrups at 6″ c/c throughout. Minimum ultimate cube strength of concrete = 2250 psi.

Stresses.

Stress in tensile Steel
= 27,700 psi

Stress in concrete
= 1,180 psi

Shear stress
= 110.5 psi

This is permissible, but as a precautionery measure against small torsion that may develop $\frac{3}{4}''$ dia. stirrups at 6″ are provided.

Plastic Range

Horizontal Braces act as structs.

Axial force = 15,600 lbs.

considering steel alone

stresses in steel = 15,600/8 = 1,950 psi.

(D) Design of Foundation.

Design coefficient α_h	=	0.20
Depth of foundation	=	8′ 0″
Inner Radius of annular raft	=	6′ 0″
Outer Radius —do—	=	24′ 0″
Area of the footings A	=	177, 500 in^4
Downward load P	=	2342 kips.
Overturning moment M	=	12,200 kips ft.
Direct Stress	=	1536 psf.
Due to Moment	=	±1650 psf.
Total pressure	=	3125 psf. or—114 psf.

For the extreme case of a severe earthquake these values of stresses in soil may be permitted.

Width of footing = 10.5′

Max. Moment = 1394,000 lbs. ft.

Depth regd. = 32″

Overall depth provided is 42″ with clear cover to bars 3″

Steel required = 2.24 sq. in.

Provide 1″ ϕ at 4″ c/c.

At top provide $\frac{3}{8}''\phi$ at 4″ c/c to take care of negative moment.

Check for Torsion and shear.

Taking moment of the net forces.

M = 758,000 lbs. ft.

Torsion = 379,000 lbs. ft.

Torsional Resistance.

Consider a rectangular beam 30″ deep and 102″ wide

$$f_s = \frac{T}{q\, b^2 d} = 54 \text{ psi where } q = 0.27$$

Toal Load = 230 kips

f_s = 42 psi

Direct Shear

Total Max. Shear = 54 + 42 = 96 psi

Beam—30″ deep and 102″ wide.

Moment = 3190 kips in.

Depth required = 15.2″ wide beam

A_s=5.04 sq. in

HYDRODYNAMIC PRESSURES IN THE TANK

Suppose the tank to be given a maximum acceleration of 0.20 g and subjected to a maximum displacement of 6″.

For the purpose of analysis let us assume that the tank is a cylinder of radius 13.5 ft. and height 14.0 ft.

(Formulae from reference 1).

Impulsive Pressures on the Tank.

$$P_i = W.\alpha_h.\frac{\tanh \sqrt{3}\, R/h}{\sqrt{3}\, R/h}$$

$$= 0.11\ W$$

acting at a height h_o from bottom

$$\text{where } h_o = \tfrac{3}{8} h \left\{ 1+(4/3\frac{\sqrt{3}\, R/h}{\tanh \sqrt{3}\, R/h} - 1) \right\}$$

$$= 0.77\ h$$

Convective Pressures on the tank.

$$P_c = 1.2\ M_1\ g\ \theta_h$$

$$M_1\ g = W\times\tfrac{1}{4}\times\tfrac{25}{34}\times\sqrt{\frac{27}{8}}\frac{R}{h}\tanh\sqrt{\frac{27}{8}}\ h/R$$

$$\theta_h = A\sqrt{\frac{27}{8}}\frac{1}{R}\tanh\sqrt{\frac{27}{8}}\frac{h}{R}\times 5/6$$

$$P_c = 0.019\ W$$

Acting at a height of h_o from bottom

$$\text{where } h_o = h\left(1-\frac{\cosh\sqrt{\frac{27}{8}}\cdot\frac{h}{R}-2.0}{\sqrt{\frac{27}{8}}\frac{h}{R}\sinh\sqrt{\frac{27}{8}}\frac{h}{R}}\right)$$

$$= 0.77\ h$$

Hydro - static pressure

$$P_h = \frac{wh}{2}\times\frac{\pi\times 2R\times h}{w\times\pi R^2 h}W$$

$$= Wh/R = 1.07\ W$$

Acting at a height of 0.33 h from bottom

The sum of impulsive and convective pressures, if assumed to attain maximum values simultaneously will be equal to 0.129 W which is equal to 12% of that of hydrostatic pressure. The sum of the moment of impulsive and convective pressures will be equal to 0.099 Wh which is 27.8% of that of hydrostatic pressure.

The above increase in pressure and moment due to hydrodynamic effect is accounted for by the normal increase in the working stresses. However, care should be exercisd ethat all reinforcements are anchored properly.

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FREE VIBRATIONS OF BEAMS AND CANTILEVERS WITH ELASTIC RESTRAINTS

M. B. Kanchi*

SYNOPSIS

Free undamped flexural vibrations of uniform beams and cantilevers with elastic restraints are studied using Ritz's minimizing method. An approximate mode shape is assumed in terms of suitably selected parameters and by adjusting the relative ratios of these parameters, solutions corresponding to arbitrary elastic restraints applied at the ends of the members are presented. Frequency equations, which otherwise would be in the form of transcendental equations involving trignometric as well as hyperbolic functions, are derived in the form of quadratic or cubic equations. Suitable charts are appended to aid analysis and design.

INTRODUCTION

The governing differential equation for free undamped, flexural vibrations of uniform member of mass m per unit length and uniform flexural rigidity EI, is given by

$$\frac{d^4X}{dx^4} - \lambda^4 X = 0 \qquad \dots (1)$$

in which,

$$\lambda^4 = \frac{mp^2}{EI} \qquad \dots (2)$$

p, being the natural frequency in angular measure; X, is the dynamic deflection mode shape and is of the form

$$X = C_1 \operatorname{Sin} \lambda x + C_2 \operatorname{Cos} \lambda x + C_3 \operatorname{Sinh} \lambda x + C_4 \operatorname{Cosh} \lambda x \quad (3)$$

Four boundary conditions, two at each end of the member, determine these four arbitrary constants and the mode shape is determined to within an arbitrary constant, as follows:

$$\overline{X} = A_1 \left(\operatorname{Sin} \lambda x \frac{C_2}{C_1} \operatorname{Cos} \lambda x + \frac{C_3}{C_1} \operatorname{Sinh} \lambda x + \frac{C_4}{C_1} \operatorname{Cosh} \lambda x \right) \quad \dots (4)$$

Only three of the four arbitrary constants are independent and determine the mode shape which is characteristic of the given boundary conditions. The fourth determines its magnitude and has to be determined from the given initial conditions. To calculate the natural frequency only the shape is of concern and is obtained to within an arbitrary constant.

For members with elastic restraints solution of the frequency equation, resulting from the elimination of the four arbitrary constants from the four end conditions of the members, reduces to the solutions of transcendental equations involving trigonometric as well as hyperbolic functions. The solutions will have to be effected graphically, or by trial and error. Since the natural frequency is insensitive to dynamic deflection shape and only depends on the 'overall' or 'average' shape an approximate shape always yields values which are in good agreement with the exact values. Either of the Ritz's methods—the averaging method, or the minimizing one, can be used to formulate the problem. Analysis of beams and cantilevers with elastic restraints at their ends, using these procedures is the main subject matter of the paper. The Ritz minimizing procedure whose usage is more popularly known than that of the averaging method, is used here to derive the expressions.

CANTILEVER WITH ELASTIC MOMENT RESTRAINT AT THE FREE END.

Consider a cantilever of height h, of uniform mass m per unit length, and flexural rigidity EI, as shown in Fig. 1 (a). It is restrained at the top by a moment which is proportional to the rotation thereat. This has been schematically represented in Fig. 1 (a) by a massless spring. Let K be the stiffness of such a restraint. K is the moment per unit rotation, and always opposing the rotation. This may be looked upon as the free body of the column of a portal frame in which the mass of the beam is not considered. Extension of the results obtained for the present case to include the effect of the

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mass will be made. Assume that the dynamic deflection mode shape is of the form

$$X = \frac{a}{2}\left(1-\text{Cos}\ \frac{\pi x}{h}\right)+b\left(1-\text{Cos}\ \frac{\pi x}{2h}\right) \quad (5)$$

This is an approximation to the shape given by Equation (3). The first part of the assumed shape corresponds to the case of infinite restraint as shown in Fig. 1 (c), while as the second part to the case of zero restraint,

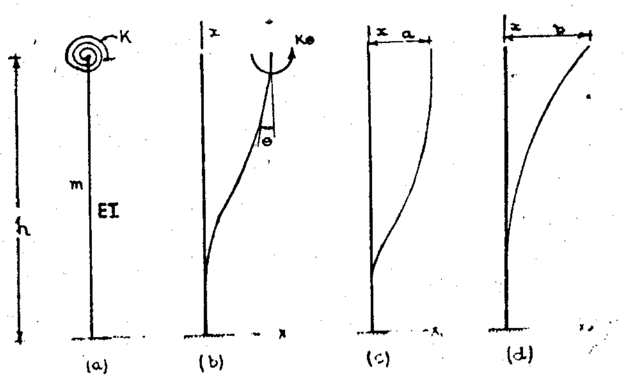

Fig. 1.

or free end, as shown in Fig. 1 (d). Writing Equation (5) in the form analogous to that of Equation (4), namely,

$$\overline{X} = A_1\left[\frac{1}{2}\left(\frac{a}{b}\right)\left(1-\text{Cos}\ \frac{\pi x}{h}\right)+\left(1-\text{Cos}\ \frac{\pi x}{2h}\right)\right] \quad \dots (6)$$

It can be seen that the ratio $\left(\frac{a}{b}\right)$ determines the shape of the mode, and A_1 the size of it. The shape parameter (a/b) will now be determined as a function of the stiffness of the restraint applied, so that for different stiffnesses, ranging from zero to infinity, this parameter can be adjusted.

The maximum strain energy of the system is given by

$$V = \frac{EI}{2}\int_0^h \left(\frac{d^2X}{dx^2}\right)^2 dx + \frac{1}{2} K\,\theta^2 \quad \dots (7)$$

Evaluating the derivative with respect to x, of X from Eq. (5), at x=h, it can be seen that,

$$\theta = \left.\frac{dX}{dx}\right|_{x=h} = \frac{\pi b}{2h} \quad \dots (8)$$

Substituting X and θ from Equation (5) and Equation (8) into Equation (7) and evaluating the integrals, the strain energy expression takes the form

$$V = \frac{EI\pi^4}{16h^3}\left[a^2 + \frac{4}{3\pi}ab + \frac{1+\tau}{4}\ b^2\right] \dots (9)$$

in which τ is the dimensionless ratio of stiffnesses, denoted by

$$\tau = \frac{8}{\pi^2}\cdot\frac{h}{EI}\cdot K \quad \dots (10)$$

This actually happens to be the approximation of the ratio, $K/\left(\frac{EI}{h}\right)$, in which $\left(\frac{EI}{h}\right)$ represents the flexural stiffness of the cantilever as used in structural statics.

The maximum kinetic energy of the system is given by

$$T = \frac{1}{2}\ mp^2 \int_0^h X^2\, dx \quad \dots (11)$$

On substituting Equation (5) into this, and evaluating the definite integral, finally,

$$T = \frac{mp^2h}{16}\left[3a^2+8\left(1-\frac{4}{3\pi}\right)ab+8\left(\frac{3}{2}-\frac{4}{\pi}\right)b^2\right] \quad \dots (12)$$

Equating expressions given by Equations (9) and (12) and denoting,

$$\overline{\lambda} = \left(\frac{\lambda h}{\pi}\right)^4 = \frac{mp^2}{EI}\frac{h^4}{\pi^4} \quad \dots (13)$$

Rayleigh's quotient takes the form,

$$\overline{\lambda} = \frac{a^2 + \frac{4}{3\pi}ab + \frac{1+\tau}{4}b^2}{3a^2 + 8\left(1-\frac{4}{3\pi}\right)ab+8\left(\frac{3}{2}-\frac{4}{\pi}\right)b^2} \quad (14)$$

Denoting by N and D, the mumerator and denominator, we can write

$$\overline{\lambda} = \frac{N}{D} \quad \dots (15)$$

Equations

$$\frac{\partial\overline{\lambda}}{\partial a} = 0;\ \frac{\partial\overline{\lambda}}{\partial b} = 0 \quad \dots (15)$$

determine the conditions to evaluate the parameters a and b. The first of these equations becomes

$$D\ \frac{\partial N}{\partial a} - N\ \frac{\partial D}{\partial a} = 0$$

or

$$D\left(\frac{\partial N}{\partial a}-\frac{N}{D}\ \frac{\partial D}{\partial a}\right)=0$$

Since D cannot be zero and $\frac{N}{D}=\bar{\lambda}$

Equations (15) can be written as

$$\frac{\partial}{\partial a}(N-\bar{\lambda}D)=0$$

$$\frac{\partial}{\partial b}(N-\bar{\lambda}D)=0 \quad \dots (16)$$

Using for N and D, the numerator and demoninator of Equation (14), these give,

$$\left\{2-6\bar{\lambda}\right\}a+\left\{\frac{4}{3\pi}-8\left(1-\frac{4}{3\pi}\right)\bar{\lambda}\right\}b=0$$

$$\left\{\frac{4}{3\pi}-8\left(1-\frac{4}{3\pi}\right)\bar{\lambda}\right\}a+\left\{\frac{1}{2}(1+\tau)-16\times\left(\frac{3}{2}-\frac{4}{\pi}\right)\bar{\lambda}\right\}b=0 \quad \dots (17)$$

Therefore, the frequency determinant is as follows :

$$\begin{vmatrix} 2-3\bar{\lambda} & \frac{4}{3\pi}-8\left(1-\frac{4}{3\pi}\right)\bar{\lambda} \\ \frac{4}{3\pi}-8\left(1-\frac{4}{3\pi}\right)\bar{\lambda} & \frac{1}{2}(1+\tau)-16\left(\frac{2}{3}-\frac{4}{\pi}\right)\bar{\lambda} \end{vmatrix}=0 \quad (18)$$

Expanding the determinant, frequency equation reads as follows :

$$\bar{\lambda}^2-(11.2354+5.3097)\bar{\lambda}+1.4513+1.7699\tau=0 \quad \dots (19)$$

Frequencies being obtained from this equation, Equations (17) then determine mode shapes. These two equations can be written, in terms of the ratio:

$$\alpha=\left(\frac{a}{b}\right) \quad \dots (20)$$

From Equations (17)

$$\bar{\lambda}=\frac{\alpha+\frac{2}{3\pi}}{3\alpha+4\left(1-\frac{4}{3\pi}\right)}=\frac{\alpha+0.2122}{3\alpha+2.3024} \quad \dots (21)$$

$$\alpha=-\frac{\frac{1}{2}(1+\tau)-16\left(\frac{3}{2}-\frac{4}{\pi}\right)\bar{\lambda}}{\frac{4}{3\pi}-8\left(1-\frac{4}{3\pi}\right)\bar{\lambda}} \quad \dots (22)$$

Between these two, elimination of $\bar{\lambda}$ gives

$$\tau=\frac{6.6632\,\alpha^2+4.2564\,\alpha-0.7627}{3\alpha+2.3024} \quad \dots (23)$$

From Equation (19) it can be verified that for $\tau=0$ $\bar{\lambda}=0.1307$. This case corresponds to the case of a cantilever. This value of $\bar{\lambda}$ as given by Equation (13), corresponds to $p=3.5681\sqrt{\frac{EI}{mh^4}}$ which is only 1.48% higher than the true value, namely, $3.5159\sqrt{\frac{EI}{mh^4}}$.

The value $\bar{\lambda}=0.1307$ in Equation (21) gives $\alpha=0.1459$. This states the ratios in which the two shapes shown in Figs. 1 (c) and 1 (d) are mixed up. Similarly for the case when $\tau=\infty$ from Equation (19) $\bar{\lambda}=0.3333$ and then from Equation (21) $\alpha=\infty$, that is, $b=0$. This means that the shape is purely as shown in Fig. 1 (c). The value of the frequency will be $p=5.6977\sqrt{\frac{EI}{mL^4}}$ which is of the order only 1.8% higher than the true value $p=5.5932\sqrt{\frac{EI}{mL^4}}$ corresponding to $\bar{\lambda}=0.3211$. The exact frequency equation, as obtained by applying the boundary conditions,

$$\left.\begin{aligned} X\Big|_{x=0}&=0; \\ \frac{dX}{dx}\Big|_{x=0}&=0 \\ -EI\frac{d^2X}{dx^2}\Big|_{x=h}&=K\frac{dX}{dx}\Big|_{x=h} \\ EI\frac{d^3X}{dx^3}\Big|_{x=0}&=0 \end{aligned}\right\} \quad \dots (24)$$

is as follows :

$$\frac{\text{Cos}\,\lambda L\ \text{Cosh}\,\lambda L+1}{\text{Cos}\,\lambda L\ \text{Sinh}\,\lambda L+\text{Sin}\,\lambda L\ \text{Cosh}\,\lambda L}=\frac{\pi^2}{8}\left(\frac{\tau}{\lambda L}\right) \quad \dots (25)$$

Equation (19) has taken now the place of Equation (25). The true mode shape governing the frequency equation (25) is quite involved. The mode shape corresponding to equation (25) is as follows :

$$\bar{X}=A_1\{(\text{Sin}\,\lambda x-\text{Sinh}\,\lambda x)+\alpha'(\text{Cos}\,\lambda x-\text{Cosh}\,\lambda x)\} \quad \dots (26)$$

wherein α' is given by

$$\alpha'=\frac{-(\text{Sin}\,\lambda L+\text{Sinh}\,\lambda L)+\frac{\pi^2}{8}\frac{\tau}{\lambda L}(\text{Cos}\lambda L-\text{Cosh}\,\lambda L)}{(\text{Cos}\,\lambda L+\text{Cosh}\,\lambda L)+\frac{\pi^2}{8}\frac{\tau}{\lambda L}\ (\text{Sin}\,\lambda L+\text{Sinh}\,\lambda L)} \quad \dots (27)$$

Equation (6) has here taken the place of Equation (26) and Equation (21) the place of Equation (27). It is seen that Equations (19), (6) and (21) are easier to be handled than their corresponding true expressions—Equations (25), (26) and (27). It is indeed fortunate that frequency is insensitive to these approximations.

Equations (21) and (23) are used to prepare the freqency curve as shown in Fig. 2. To obtain $\bar{\lambda}$ in terms

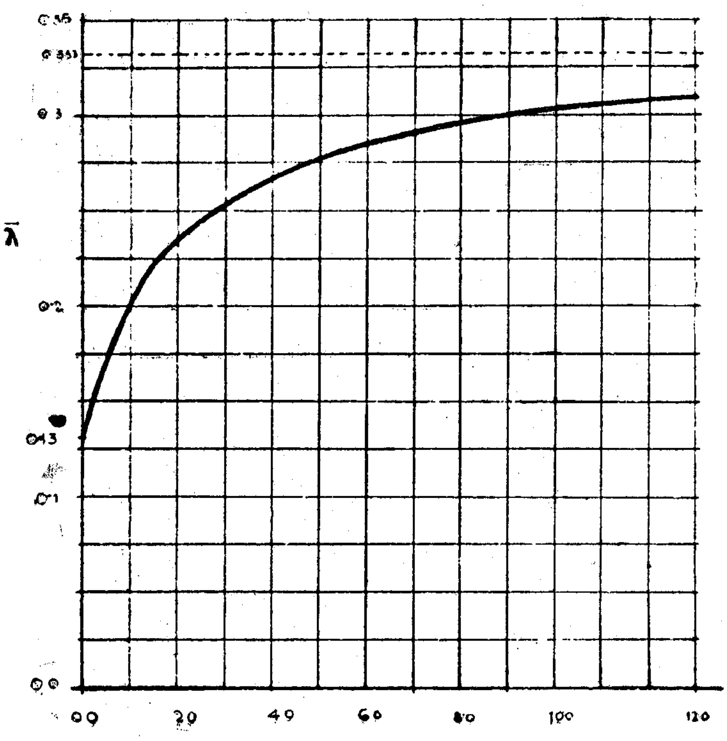

Fig. 2.

of τ, Eq. (19) need be solved. However, Equations (21) and (23) together can yield a set of values to plot $\bar{\lambda}$ Versus τ. Equations (21) and (23) give respectively the plots of $\bar{\lambda}$ Vs α and τ Vs α. From these plots the curve of $\bar{\lambda}$ Vs τ is obtained as shown in Fig. 2. From this figure it is seen the frequency is sensitive to restraints onlyin the beginning upto $\tau = 2$ while as it is insensitive beyond $\tau = 10$.

CANTILEVER WITH ELASTIC RESTRAINT AND A MASS CONCENTRATED AT FREE END

The free body of one of the columns of a symmetrical protal frame, shown in Fig. 3 (a) is as shown in Fig. 3 (b).

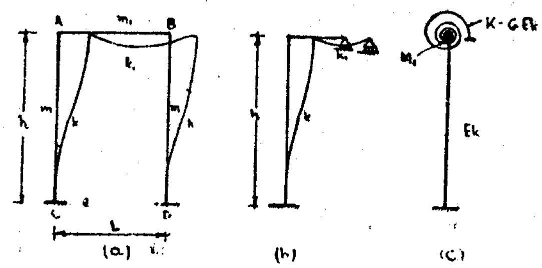

Fig. 3.

The column AB for restraints is equivivalent to a cantilever with moment restraint at top. When sidesway mode of vibrations is considered, one has to consider the kintetic energy as follows :

(a) kinetic energy of the mass of this cantilever assumed uniformly distributed along its length, vibrating horizontally;

(b) kinetic energy, of the mass of the beam BC, vibrating horizontally, displacement of each of the elemental mass of the beam being equal to the horizontal displace-ment at B of the cantilever, (extensions of the beam being neglected), and

(c) kinetic energy of the mass of the beam BC, asssumed as a distributed mass, vibrating vertically.

Of these three contributions the first two are usually considered and to include the last, the mode shape of the beam BC has to be assumed, consistant with the continuity condition at the joint B. If only the first two are taken into account, the analysis of single bay symmetrical portal frame reduces to that of a cantilever with moment restraint of stiffness,

$$K_1 = 6 \, E \, k_1 \qquad \ldots (28)$$

and a mass. M_1 concentrated at free end equal to

$$M_1 = m_1 \frac{L}{2} \qquad \ldots (29)$$

where m_1 is the mass per unit length of beam. This has been schematically represented in Fig. 3 (c).

To proceed to analyse such a cantilever it will be assumed that the mode shape is still given by Eq. (5). Strain energy expression, given by Equation (9) remains the same, while as to the expression of Kinetic energy, given by Equation (12) the following expression, representing the contribution from the mass M_1 vibrating with sinusoidal displacement having amplitude $X\Big|_{x=h}$,

namely, (a+b), has to be added:

$$T_1 = \tfrac{1}{2} M_1 (a+b)^2 \qquad \text{... (30)}$$

Adding this to the right hand side of Eq. (12) and equating it to the right hand side of Equation (9) one obtains

$$\bar{\lambda} = \frac{a^2 + \frac{4}{3\pi} ab + \frac{1+\tau_1}{4} b^2}{\left(3+\frac{\gamma_1}{8}\right)a^2 + \left\{8\left(1-\frac{4}{3\pi}\right)+\frac{\gamma_1}{4}\right\}ab + \left\{8\left(\frac{3}{2}-\frac{4}{\pi}\right)+\frac{\gamma_1}{8}\right\}b^2} \qquad \text{... (31)}$$

in which γ_1 is the ratio of the mass of the beam to that of column,

$$\gamma_1 = \frac{M_1}{mh} = \frac{1}{2}\,\frac{m_1}{m}\cdot\frac{L}{h} \qquad \text{... (32)}$$

and τ_1, the dimensionless ratio of stiffnesses:

$$\tau_1 = \frac{48}{\pi^2}\,\frac{k_1}{k} \qquad \text{... (33)}$$

Equation (31) is the extension of Eq. (14). Proceeding the same way as was adopted in deriving Equations (21) and (22) we obtain instead of them the following equations.

$$\bar{\lambda} = \frac{\alpha + \frac{2}{3\pi}}{\left(3+\frac{\gamma_1}{8}\right)\alpha + 4\left(1-\frac{4}{3\pi}\right)+\frac{\gamma_1}{8}} \qquad \text{... (34)}$$

$$\alpha = -\frac{\frac{1}{2}(1+\tau) - \left\{16\left(\frac{3}{2}-\frac{4}{\pi}\right)+\frac{\gamma_1}{4}\right\}\bar{\lambda}}{\frac{4}{3\pi} - \left\{8\left(1-\frac{4}{3\pi}\right)+\frac{\gamma_1}{4}\right\}\bar{\lambda}} \qquad \text{... (35)}$$

From these two equations $\bar{\lambda}$ and α can be obtained for given values of τ_1 and γ_1 either by trial and error, or by solving the resulting quadratic by elimination of α between these two equations. The quadratic would be the same form as equation (19).

BEAMS WITH ELASTIC RESTRAINTS.

Analysis of the free flexural vibrations will now be carried to the case of beams with uniformly distributed mass and flexural rigidity. The ends of the beam are held against relative displacement, but are elastically restrained against rotations, as shown schematically in Fig. 4 (a). This may be looked upon as the freebody of the vertically vibrating beam AB of a portal frame shown in Fig. 5, in which the columns have different stifnesses, In analysing the beam AB of Fig. 4 (a) the

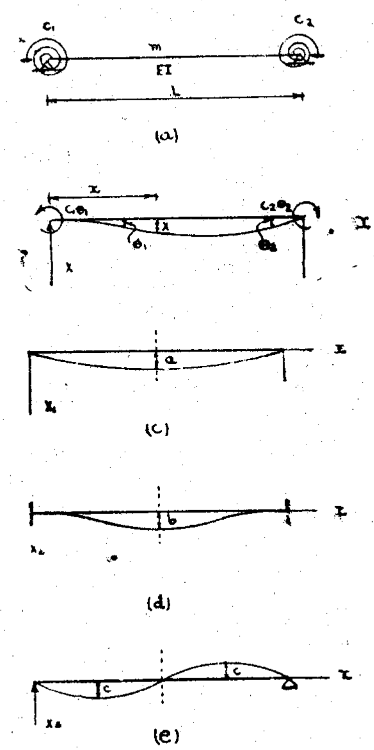

Fig. 4.

springs will be assumed massless. It differs from being the complete dynamic analogue of the beam AB of the portal frame of Fig. 5, in as much as the Kinetic energy

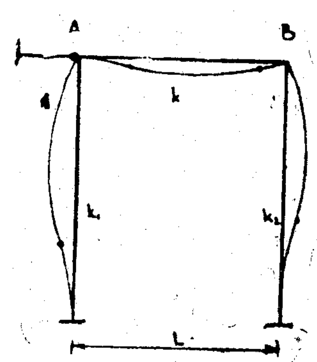

Fig. 5.

of the column masses vibrating horizontally is excluded. The results obtained for the beam AB hold for the beam

AB of the portal frame in which the masses of the columns are either too small or are prevented from vibrating laterally.

For the beam AB of Fig. 4 (a) assume the dynamic deflection mode shape as

$$X = a \sin \frac{\pi x}{L} + \frac{b}{2}\left(1 - \cos \frac{2\pi x}{L}\right) + c \sin \frac{2\pi x}{L} \quad (36)$$

Each of the shapes are shown in Figs. 4 (c), 4 (d) and 4 (e). The shape function will be

$$\bar{X} = A_1\left\{\sin \frac{\pi x}{L} + \frac{\alpha_1}{2}\left(1 - \cos \frac{2\pi x}{2}\right) + \alpha_2 \sin \frac{2\pi x}{L}\right\} \quad \ldots (37)$$

where

$$\alpha_1 = \left(\frac{b}{a}\right);\ \alpha_2 = \left(\frac{c}{a}\right) \quad \ldots (38)$$

and A_1 is the size parameter. As in the previous case, here too α_1 and α_2 will be determined in terms of the stifnesses C_1 and C_2.

Maximum strain-energy will be given by

$$V = \frac{EI}{2}\int_0^L \left(\frac{d^2X}{dx^2}\right)^2 dx + \tfrac{1}{2} C_1\theta_1^2 + \tfrac{1}{2} C_2\theta_2^2 \quad (39)$$

where the second and third terms on the right hand side represent the energy stored in restraints at the supports. Evaluating the drivative with respect to x of X as stated by Equation (36) at x = 0, and x = L, one obtains

$$\theta_1 = \frac{\pi}{L}(a+2c);\ \theta_2 = \frac{\pi}{L}(-a+2c) \quad \ldots (40)$$

Substituting Equation (36) and Equation (40) in Equation (39) and evaluating the definite integrals, one arrives at

$$V = \frac{EI\pi^4}{4L^3}\left[a^2 + 4b^2 + 16c^2 + \frac{16}{3\pi}ab + (a^2+4c^2)(\tau_1+\tau_2) + 4ac(\tau_1 - \tau_2)\right] \quad \ldots (41)$$

in which τ_1 and τ_2 are the relative stiffness ratios

$$\tau_1 = \frac{C_1}{\left(\frac{\pi^2 EI}{2L}\right)};\ \tau_2 = \frac{C_2}{\left(\frac{\pi^2 EI}{2L}\right)} \quad \ldots (42)$$

The factor $\frac{\pi^2 EI}{2L}$ is Fourier approximation of the exact value $\frac{4 EI}{L}$, representing the flexural stiffness of beam without vibrations. Substituting for x, its value given by Equations (36) into the right-hand side of Equation (11) and evaluating the integrals, expression for maximum Kinetic Energy becomes

$$T = \frac{mp^2L}{4}\left\{a^2 + \tfrac{3}{4}b^2 + c^2 + \frac{16}{3\pi}ab\right\} \quad \ldots (43)$$

Equating Equation (41) to Equation (43) and denoting

$$\bar{\lambda} = \left(\frac{\lambda L}{\pi}\right)^4 = \frac{mp^2}{EI}\cdot\frac{L^4}{\pi^4} \quad \ldots (44)$$

one abtains

$$\bar{n} = \frac{a^2 + 4b^2 + 16c^2 + \frac{16}{3\pi}ab + (a^2+4c^2)(\tau_1+\tau_2) + 4ac(\tau_1-\tau_2)}{a^2 + \frac{3}{4}b^2 + c^2 + \frac{16}{3\pi}ab} \quad \ldots (45)$$

Proceeding in the same way as was adopted in deriving Equations (16),

$$\left.\begin{aligned} &\frac{\partial}{\partial a}(N - \bar{\lambda} D) = 0 \\ &\frac{\partial}{\partial b}(N - \bar{\lambda} D) = 0 \\ &\frac{\partial}{\partial c}(N - \bar{\lambda} D) = 0 \end{aligned}\right\} \quad \ldots (46)$$

Where N and D respectively denote the numerator and denominator of Equation (45). These equations give the governing equations of mode shapes. These are as follows :

$$\left.\begin{aligned} &\left\{1 + (\tau_1+\tau_2) - \bar{\lambda}\right\} a + \frac{8}{3\pi}(1-\bar{\lambda}) b + 2(\tau_1-\tau_2) c = 0 \\ &\frac{8}{3\pi}(1 -)\bar{\lambda}\, a + (4 - \tfrac{3}{4}\lambda) b = 0 \\ &2(\tau_1-\tau_2)\, a + \{16 + 4(\tau_1+\tau) - \bar{\lambda}\}\, c = 0 \end{aligned}\right\} \quad (47)$$

The condition that the determinant of these equations should vanish yields the frequency equation. Alternatively by a systematic elimination of a, b and c the same equation can be derived. From second and third equations of (47), ratios (b/a) can be obtained. Using the notation stated by (38)

$$\alpha_1 = \frac{\frac{8}{3\pi}(1-\bar{\lambda})}{\frac{3}{4}\bar{\lambda} - 4} \quad \ldots (48)$$

$$a_2 = \frac{2(\tau_1-\tau_2)}{\bar{\lambda}-4(\tau_1+\tau_2)-16} \quad \dots \quad (49)$$

Dividing the first of Eqs. (47) by a throughout and using therein Eqs. (48) and (49) and simplifying the frequency equation will be obtained as

$$0.0295\bar{\lambda}^3-\bar{\lambda}^2[0.868(\tau_1+\tau_2)+3.7810]$$
$$+\bar{\lambda}[29.2360(\tau_1+\tau_2)+12\tau_1\tau_2+56.2240]$$
$$-64\tau_1\tau_2-77.1180(\tau_1+\tau_2)-52.4720=0 \quad (50)$$

$\bar{\lambda}$ being determined by this equation Equations (48) and Equations (49) will then give the mode shape parameters a_1 and a_2.

It is seen that Equation (50) is symmetrical with respect to τ_1 and τ_2. For the case of fixed beam $\tau_1=\tau_2=\infty$, Equation (50) gives

$\bar{\lambda}=\frac{16}{3}$ .Using Eq.(44), $p=22.7985\sqrt{\frac{EI}{mL^4}}$ which is about 1.5% higher than the true value of $22.36\sqrt{\frac{EI}{mL^4}}$ For this case $a_2=0$ and $a=\infty$ that is, a=0, as given by Eqs. (48) and (49).

Preparation of frequency charts in this case is more

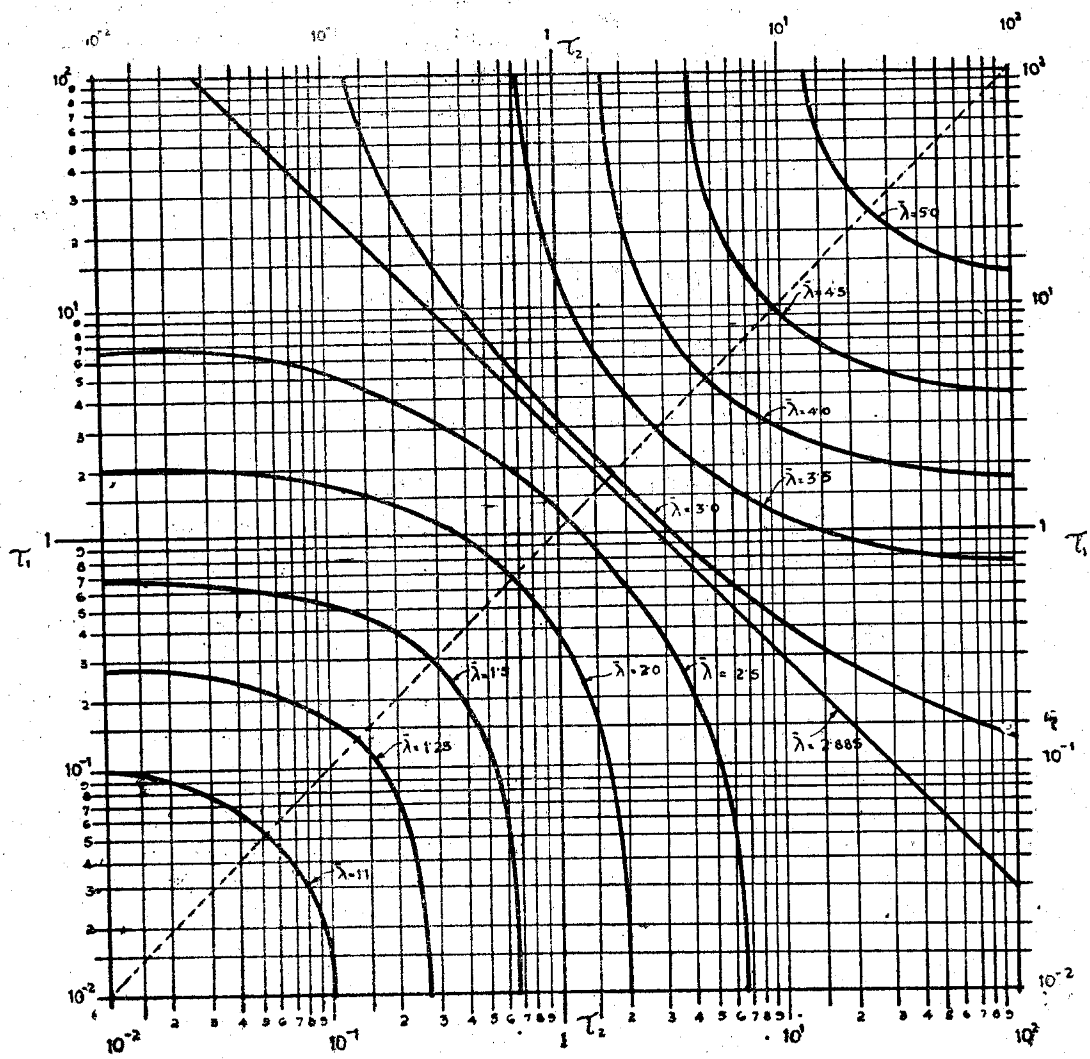

Fig. 6.

involved than in the previous case of cantilever in which only one parameter α was involved. In this case Equation (50) has been solved for τ_2 in terms of τ_1 and $\bar{\lambda}$, and for different constant values of $\bar{\lambda}$ τ_2 has been solved in terms of τ_1. For example for $\bar{\lambda} = 2$,

Equation (50) reduces to

$$\tau_1+\tau_2+1.8080\,\tau_1\,\tau_2-2.0380 = 0 \quad \dots \quad (51)$$

This is symmetrical with respect to the diagonal line $\tau_2=\tau_1$ (Fig. 6). Any set of values for τ_1 and τ_2 that satisfies this equation gives frequency $\bar{\lambda}=2$. If $\tau_1=\tau_2$, that is, if the end restraints are identical, then $\tau_1=\tau_2=0.644$. Also if one of the ends is hinged, then the other should have the restraint of stiffness ratio equal to 2.038, as obtained by putting either of τ_1 or τ_2, equal to zero. Equations similar to Equation (51) have been determined for different assigned values for $\bar{\lambda}$ rangingfrom $\bar{\lambda} = 1$ to $\bar{\lambda} = \frac{16}{3}$, respectively the lower and the upper bounds for the frequency.

Curves given in Fig. (6) can be used to try the various stiffness ratios to produce a desired frequency, or conversely, given the stiffness ratios of the restraints to calculate the frequency. Since the curves are on logarithmic scale interpolation may cause errors. Equations (48), (49) and (50) can always be used to rectify the values picked up from the curves. For example suppose $\tau_1=\tau_2=1$. From the curves $\bar{\lambda} = 2.4$. To correct it for inaccuracies involved in interpolation, Newton's formula can be applied to Eq. (50). It was found that the first correction was -0.02 giving $\bar{\lambda} = 2.38$. A second correction gave $\bar{\lambda} = 2.3802$.

Mode shape can be determined from Equations (48) and Equations (49) when once the value of $\bar{\lambda}$ is known. Suppose $\bar{\lambda}=2.5$. From the frequency chart, $\tau_1=2$, $\tau_2=0.6$ is one of the admissible sets. Eqbations (48) and (49) for these values of $\bar{\lambda}$, τ_1 and τ_2 give $\alpha_1 = 0.5992$ and $\alpha_2 = -0.1171$. Therefore, Eq. (37) becomes

$$\bar{X}=A_1\left\{\text{Sin}\,\frac{\pi x}{L}+0.2996\left(1-\text{Cos}\,\frac{2\pi x}{L}\right)-0.1171\,\text{Sin}\,\frac{2\pi x}{L}\right\} \quad \dots \quad (52)$$

This shape function corresponds to points of contrflexures at $x = 0.22$ L and $x = 0.86$ L.

CONCLUDING REMARKS.

By assuming an approximate shape for the dynamic deflection mode in terms of some suitably selected parameters solution for natural frequencies of beams and cantilevers with elastic restraints can be effected with a greater ease. The errors admitted in the frequencies so obtained are of permissible order. Solution of frequency equations which are transcendental in nature involving trigonometric and hyperbolic functions are replaced by algebraic equations. It is easier to vary the parameters and prepare the frequency charts.

ACKNOWLEDGEMENTS.

Author wishes to acknowledge greatfully the assistance rendered by Sri S. S. Saini for computations involved in this paper.

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A REVIEW OF MACHINE FOUNDATION BEHAVIOUR

Shamsher Prakash* and U. K. Bhatia**

SYNOPSIS

A machine foundation differs from any other type of foundation, because of the dynamic nature of loads. Till about 1930, a machine foundation design was based upon emperical methods. These methods did not take into account the properties of the underlying soil. In the years to follow, attempts have been made to understand the problem scientifically.

Characteristics of the underlying soil strata affect the resonance of the system with the machine. The available literature on the subject is scattered and no systematic investigation, covering the present trend is available. The present investigation of the Behaviour of Machine Foundation is intended to make a systematic study of available literature and is believed to lead to a better understanding of the problems connected. The various methods to determine the resonant frequency of machine foundation are critically reviewed, compared with each other and their limitations discussed.

A simple emperical equation for determination of resonant frequency of the system is proposed. Suggestions for further research have also been made.

INTRODUCTION

General :

The function of a machine foundation, similar to any other foundation, is to transmit the imposed loads safely on to the soil on which it is placed. Its special feature, however, is that in addition to the static load, due to the weight of the machine, and the foundation, vibrating or pulsating forces varying with time have to be considered. Such forces may be of short duration, such as shock or impact forces in forging hammers or may vary periodically as in reciprocating and rotating machines. As a result, waves or steady vibrations are set up in the foundation soil. If the natural frequency of the foundation soil system happens to coincide with or lie close to the frequency of the exciting forces generated by the machines, excessive vibration amplitudes may occur, which lead to structural damage or operational failure of the machine.

Role of soil mechanics:

Problems connected with machine foundations, were not considered to be important till the advent of the thirties, when greater use of heavy industrial plants and consistent failure of machine foundations attracted attention of designers (Tschebotarioff 1951). Before that, the design of machine foundations was purely emperical, the simplest being to provide heavy foundation blocks. It was considered adequate to rest the machine on rigid foundation so as to avoid excessive amplitudes of vibration. It was believed that natural frequency of the rigid foundation would be higher than the operating frequency of the machine. But invariably the rigid foundation has to rest on the ground. The result is that rigid foundation transmits the vibrations to the ground, which is relatively flexible, so that danger of resonance is still there. The importance of this fact was realized when even after the provision of rigid foundations, excessive vibrations due to resonance were caused. With the advent of soil mechanics, the problem of machine foundation has been tackled more rationally and scientifically. The investigations, both theoretical and experimental have led to better understanding of the behaviour of machine foundations resulting in economy. At the same time, the number of failurs of machine foundations which are mostly due to excessive amplitudes of vibrations have been minimized.

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Concept of Resonant Frequency :

Experimental studies on the phenomena of ground vibrations were first systematically conducted by Degebo (Deustche Gesellschaft für Bodenmechanik) organisation in Germany from 1928 to 1939. Their earlier experiments were with a vibrator with fixed static and dynamic forces. The results showed that soil at any site had a natural frequency or self frequancy of vibrations, depending only on the soil type (Lorenz 1934). This was further substantiated by Andrews and Crockett (1945) who independently determined the natural frequency by a study of resonance between heavy industrial plant, and the ground. At about the same time, Vios, the Institute for Engineering Foundation Research performed similar tests in Russia (Barkan 1936).

Later experimental and analytical studies (Lorenz 1953, Reissner 1936, Sung 1953, Quinlan 1953, Richart 1960, Tschebotarioff and Ward 1948) have shown that the natural frequency of soil as such is meaningless. It varies not only with the type of underlying soil but depends also on the shape and extent of the contact area and magnitude of dynamic and static loads. Hence it will be proper to use the term resonant frequency of the machine foundation-soil system, or simply the resonant frequency of the system.

The problem of determining the resonant frequency of the system has been tackled both by theoretical methods and emperical methods based on past data of resonance. The theoretical approaches are based on the assumption of (a) Soil as semi-infinite elastic solid and (b) Soil as a spring, usually elastic.

Semi-Infinite Elastic Solid :

One of the theoretical approaches is concerned with the "*Dynamic Boussineq's Problem*". This approach considers the machine foundations resting on the ground and oscillating on the surface of the semi-infinite elastic, isotropic and homogeneous medium. Prominent contributions based on this concept are those of Reissner, (1936), Quinlan (1953), Sung (1953) for vertical vibrations and those of Arnold, Bycroft and Warburton (1955), Bycroft (1959), and Hesih (1962), for other modes of vibration viz., horizontal and rotational modes.

This analysis is based on the knowledge of the dynamic response of the ground in terms of dynamic soil constants i.e. modulus of elasticity or modulus of rigidity, and Poisson's ratio. Dynamic testing of soils is done by measuring the velocities of propogation in the medium. Rayleigh (1865), Lamb (1904), Leet (1950) and others have analysed the velocity of wave propegation in the semi infinite elastic homogeneous and isotropic medium. Bergstrom and Linderholm (1946), Bernhard and Finelli (1953), Jones (1955, 1958), Bernhard (1956), Vanderpoel (1951), Nijboer and Vanderpoel (1953) Nijboer (1959) Heukelom and Foster, (1960), and many others have given the analysis applicable to soils for determining in-situ dynamic constants.

Mass Spring System :

The other theoretical approach is to assume the ground to be a spring, with or without, damping. In initial studies (Rausch 1936), this spring was assumed to be weightless and linear. But experimental studies by Degebo (Lorenz 1934), Vios (Barkan 1936) and their subsequent analysis have shown that some soil mass also oscillates with the machine foundation. This mass of soil was found to be 4 to 10 times the vibrator mass by Degebo. Vios concluded it to be relatively insignificant and neglected it to obtain the dynamic modulus of subgrade reaction. But indirectly it was accepted that soil mass could not be neglected and that its value must lie between two third and one and a half times that of foundation. The above two statements seem to be contrary. Terzaghi (1943), reccommends the soil mass to be three times the dynamic force transmitted to the ground. Andrews and Crockett (1945), Crockett and Hammond (1948, 1949) suggested that mass of soil which vibrates with the foundation must bear some relation to the bulb of pressure which gives the stress distribution under a uniformly loaded area on an elastic medium. None of them gave any precise relationship. It was, however, regarded that this mass must vary with the area of contact and the dynamic unbalance forces. Balakrishna and Nagraj (1960) and Balakrishna Rao, (1961, 1962) further advanced the concept of pressure bulb and have suggested that the mass oscillating should be taken as the mass of the soil within the pressure bulb of the same intensity (lb./sq. ft.) as the density of the soil (lb./cu. ft.).

The value of the spring constant has been taken as the load required per unit reversible deflection (Barkan 1936, Newcomb 1951). Lorenz (1934) has obtained k' dynamic modulus of subgrade reaction and W_s the soil weight, from two vibrator tests under different loading conditions and with different areas, by assuming both k' and W_s to be constant. But this assumption is not justified, as the tests have shown that both spring constant and soil weight vary with different vibrator sizes and loading conditions even on the same type of soil.

Pauw (1953) by assuming soil as truncated spring, has developed expressions for spring factors and mass factors for different modes of vibration.

Use of Experimental Behaviour :

Another approach uses the past records of resonant frequencies observed. Emperical relations have been developed. Tschebotarioff and Ward (1948) and Tschebotarioff (1951, 1953) have obtained logarithmic relationship between reduced natural frequency and the contact area. Newcomb (1951) observed a linear relationship between the resonant frequency and the static pressures.

Another approach which considers soil as sublinear spring, uses the resonance curves obtained from the test vibrator to plot the sublinear characteristic of soil and has been put forward by Lorenz (1953) and Alpan (1961).

Scope of Study :

The problem of machine foundation has been receiving importance since the thirties of this century. The importance of underlying soil strata has been realized in respect of the resonance phenomena. The available literature is scattered and no systematic investigation covering the present trend is available. It is felt that this investigation of the behaviour and design of machine foundation, based on a systematic study of available literature, will lead to better understanding of the problem connected. The nature of the problem consists of a system to be analyzed (Alpan 1961). This system consists of the machine, the foundation and the soil and involves the following procedure :

(a) Weight and operating frequency of the machine and the magnitude of the dynamic forces is given.

(b) The properties and the dynamic response of the foundation soil are to be determined, or assumed.

(c) The foundation is designed based on soil properties. The general shape and dimensions of which may be assumed for preliminary design.

The type of the foundation considered in this paper is massive block resting directly on the ground. Isolators and shock absorbers are not considered.

Review of the available literature on resonant frequency leads to an interesting observation. Spring constant in all the approaches can be expressed as a simple multiple of G r_0, while the mass factor a simple multiple of $\rho\, r_0^3$,

where G is the modulus of ridiclity of underlying soil,

ρ is the mass density of underlying soil and

r_0 is the radius of the foundation base in contact with soil.

Out of the available approaches to resonant frequency determination, Pauw's (1953) analysis is recommended. Balakrishna Rao's (1962) modification is recommended in connection with vertical vibrations only.

Almost all the experimental investigations reported in literature have been carried out for vertical vibrations. The approaches already existing should be verified for other modes of vibrations as well. Pressure distribution under machine foundation have not been investigated. Information on effect of dynamic load on bearing capacity is not available. Based on these observations, suggestions for further research have been made.

It is felt that the dynamic behaviour of soil be evaluated by observing the resonant frequency of a test vibrator under different loading conditions and contact areas. The information obtained then can be used for determination of resonant frequencies of the foundation soil system.

BEHAVIOUR OF MACHINE FOUNDATIONS

General :

Machine foundations are important substructures. For the safety of operation of every factory, dependable foundations of its machines are essential. If the foundation is not properly designed, not only the machine gets damaged but the adjoining structures may also be damaged. In addition, for proper working conditions in a factory, the vibrations produced by the machine should be such as not to interfer with the worker's comfort. For a proper design of machine foundation it is essential that its behaviour be understood. The discussion to follow has been prepared from evidence reported in literature from time to time. It is essential to know the magnitude of the dynamic forces and its frequency. Based on its behaviour, the requirements of machine foundation are also discussed.

Behaviour of a Machine Foundation :

A machine foundation is different from other foundations, mainly because it is subjected to a dynamic load, which is usually periodic. Under the influence of this load, the foundation starts vibrating. For every system, there is a natural frequency, which is defined as the frequency with which it will vibrate, when subjected to free vibrations. For a body with spring stiffness as k and mass m_0, the natural frequency ω_0 is given by (neglecting damping).

$$\omega_0 = \sqrt{k/m_0} \quad \text{... (1a)}$$

$$= \sqrt{kg/W_0} \quad \text{... (1b)}$$

Under forced vibrations, as in machine foundations the phenomena of resonance occurs, if the operating frequency coincides with this natural frequency. For no damping the amplitude at resonance tends to infinity. If damping is included in the system, the amplitude of vibration is still maximum close to resonance, though of finite value. The ratio of actual amplitude to free amplitude (the static deflection of spring, due to dynamic load) is called magnification factor N_1 (Denhartog 1947). At frequency ratio (the ratio of operating frequency to the natural frequency) of 1.0, the magnification factor is maximum.

The transmissiblity is defined as the ratio of force transmitted to the foundation and dynamic force. For small damping, transmissibility is maximum at frequency ratio of unity. For machines having dynamic loads independent of frequency, it is maximum at frequency ratio of 1.0, for large damping as well. If the dynamic load is proportional to the square of the frequency (which is true for rotating and reciprocating machines) transmissibility may be maximum for higher frequency ratios (Mykelstad 1956). In such cases it is preferable to keep frequency ratio lower than 1.0.

In addition, at resonance, the power required to keep the system oscillating is maximum. This has been observed experimentally by Lorenz (1934), Crockett and Hammond (1948) and analytically by Reissner (1936), Sung (1953) and Quinlan (1953). Thus it is seen that at resonance, the amplitude of vibration, the force transmitted, and the power input requirement of the machine, are maximum. Hence resonance has to be avoided.

In an attempt to avoid resonance, the foundation was made rigid and firm. The natural frequency of such a rigid body like mass concrete foundation is very high. The equivalent spring constant (k) is (Timoshenko 1937):

$$k \propto \frac{E}{1-\nu^2} \quad \text{... (2)}$$

where E is modulus of elasticity and

ν is the Poisson's ratio.

Values of E and ν for the concrete are of the order of 2 to 5×10^6 psi and 0.15 respectively. This would give a very high natural frequency of the foundation, with hardly any chance of resonance with machine's operating frequency. But it has been observed that even these massive foundations start vibrating and sometimes the amplitudes become quite large. The answer lies in the fact, that though the foundation is rigid in itself, it is resting on the ground. The ground is not as rigid. The value of E for soils is of the order of 10 to 15×10^3 psi and Poisson's ratio of the order of 0.3 to 0.4. This means that soil and foundation are in series (two springs in series), with the soil characteristics predominating. That is why phenomenon of resonance can be noticed even after providing a rigid foundation.

Consider a rigid, concrete foundation block, which supports a steam engine with speed of 250 r.p.m. resting on ground. Assume that resonant frequency of the system is 300 r.p.m. This will lead to fairly excessive amplitude of vibration, as the frequency ratio is close to unity. If this foundation is made 'stronger' by adding more concrete mass to the foundation block, the value of m_o (equation 1 a) increases. This will lead to decreased resonant frequency of the system, and the frequency ratio approaches closer to unity leading to still more severe vibration amplitudes. This example shows that the quality of the foundation does not necessarily improve with the mass of the foundation- block.

Forced vibrations are transmitted through the ground. Even at a distance, if some adjoining foundation has a natural frequency close to the frequency of transmitted vibrations, resonance may occur leading to damage. For this reason, the foundation under heavy machines and forging hammers are isolated, and shock absorbers are used.

Classification of Dynamic Loads :

The nature of vibrations and dynamic forces associated with a machine can, in general, be classified as :—

(a) Shock loads occuring at regular intervals e.g. vertical loads as in punching press, forging hammers and horizontal shock loads, as in looms.

(b) Vibratory loads, which repeat after a particular period and are cyclic in nature. These may include the vibrations caused in any of the six degrees of freedom (for a single mass) by three translatory loads and three rotational torques.

Translatory vibrations include vertical, longitudinal and lateral movements along three coordinate axes, x, y, z. Rotary motion about vertical axis (z axis) is called "ya wing" while about longitudinal y-axis is called "rotation" and pitching about lateral axis (x-axis). For symmetrical foundations, vertical vibrations and ya-wing can exist independently, but rocking is associated with longitudinal vibrations.

In most of the machine foundations, the vibrations occur in vertical direction or in rocking.

Requirments of Machine Foundations :

A properly designed foundation for a machine must first of all meet the general requirements for all foundations for the particular load transmitted to the ground. These are as follows (Tschebotarioff 1951):—

1. The loads of the structure should be transferred to soil layers capable of supporting them without a shear failure.
2. The deformation of the soil layers underlying the foundation should be compatible with those which the foundation, the super-structure, as well as adjoining existing structures can safely undergo.

In addition, the machine foundation must meet the following additional requirements, which are characteristics of dynamic loading:—

(a) Vibrational Amplitude:

It is not possible to eliminate the oscillating motion completely from a foundation which is subjected to significant dynamic impulses. The designer can only attempt to reduce the foundation vibration to a magnitude which is tolerable at the operating frequency.

In the absence of the design specifications for limiting vibrations, either of the following recommendations may be used as a guide. One of those is originally suggested by Rausch, (1936), and reported in English by Converse (1962). According to this, the permissible amplitude in inches is given by:

$$\text{Permissible amplitude} = \frac{9.54}{f} \text{ for frequencies less than 1800 r.p.m.} \quad \ldots (3a)$$

Lorenz (1934) and Balakrishna Rao (1961) observed experimentally that the resonant frequency of the system decreases with increase in dynamic load. This may be explained on the assumption that in such a case, the pressure tends to become more intense near the centre of the oscillator resulting in smaller effective radius. However, no quantative explanation is available.

Arnold, Bycroft and Warburton (1957) and Bycroft (1959) have extended the analysis to the other vibratory modes. They have considered circular vibrator with rigid base distribution. As the circular vibrator is the case of axial symmetry, in reality the vibrating mass has four degrees of freedom, i.e., translation horizontally and vertically and rotation about horizontal and vertical axes.

The work done by Reissner, Quinlan, Sung and Arnold Bycroft and Warburton, dealt with one of the six modes of vibration at a time and, therefore, is limited to the case, where the six modes exist independently. It is possible to evaluate the equations of motion directly from the above theories. An interesting transformation suggested by Hseih (1962) makes it possible to find the equations of motion.

But if the machine foundation is symmetrical about contact base, the vertical translation and rotation about vertical axis exist independently while the horizontal translation is coupled with rotation about horizontal axis. If the centroid of contact area and c.g. of the machine foundation coincide all the six degrees of freedom are decoupled. This is a hypothetical case and is not possible in practice. The same conclusions are reached by Pauw (1953) by considering the equations of motion, which are obtained by the soil spring analogy.

This method offers a correlation between the two theoretical approaches viz., soil as elastic solid, and soil as spring.

Ford and Haddow (1960) have obtained the natural frequency of machine foundation based on Rayleigh's principle for rigid foundations. For a conservative system, according to Rayleigh's principle, the maximum strain energy is equal to the maximum kinetic energy. It is based mainly on the following assumptions:

(a) Vertical Vibration:

1. The system may be considered as conservative in order to determine the natural frequency.
2. Dynamic pressure is transmitted through soil contained in a solid formed by the base of foundation and a soil surface.
3. The dynamic stress at depth z is uniformly distributed over a section parallel to the base of the foundation.

The last assumption is inaccurate, but is useful for the development of expressions.

(b) Horizontal Vibration:

The same assumption as for vertical vibrations are made with additional assumption that the dynamic shearing stress is uniformly distributed over a section of the solid parallel to x, y plane.

Equating the kinetic energy of the soil and machine foundation to the maximum strain energy of the soil, the author has obtained the natural frequencies in vertical and horizontal modes of vibration.

Discussion

Reissner's analysis (1936) forms the basis of the subsequent analysis given by Quinlan (1958), Sung (1953), Hseih (1962) Richart (1953, 1960), Bycroft (1959), and Arnold, Bycroft and Warburton (1955). It forms a sound basis as long as soil can be assumed homogeneous, elastic and isotropic solid. Reissner's analysis assumes the distribution under the circular base as uniform, which obviously is not the case. It is evidenced from the experiments that the resonant frequency decreases and maximum amplitude of vibration increases with the increase in the *exciter forces* as indicated by Lorenz (1934, 1953, 1959) and Balakrishna Rao (1961). Reissner's analysis does not give varying frequency of resonance for change in the exciter force and as such deviates from the experimental data.

The modifications by Sung (1953) and Quinlan (1953) for the different load distribution (parabolic, uniform, rigid) show that as the pressures tend to concentrate near the centre of circular base, the resonant frequency decreases and amplitude of vibration increases. The increase in dynamic force, may qualitatively be assumed to be associated with the change in pressure distribution or the decrease in "effective radius" corresponding to Richart's (1953) suggestion. This tries to bridge the gap between theory and experimental evidence. However, no quantitative explanation as to the change in pressure distribution, or the change in effective radius, with the change in dynamic forces is available. Sung (1953) and Quinlan (1953) observed the natural frequency of vertical vibrations on the test vibrators and compared the results with theory and found them in good agreement. Richart (1960) has applied this theory to the design of machine foundations.

Bycroft (1953) Arnold, Bycroft and Warburton (1955) have extended the above theory for other modes of vibrations as well. But they consider only the rigid base distribution. The expressions obtained by them are of the similar nature and form as in case of vertical vibrations. Now this can be applied only if the vibration in a particular mode can exist independently, which is true only for motion in vertical direction when the machine foundation is symmetrical about the contact base. The theoretical analysis of horizontal and rotational modes of oscillation have not been experimentally verified.

The equation of motion for six degrees of freedom cannot be directly obtaind from this theory. A transformation suggested by Hseih (1962) makes it possible to obtain the six second order differential equations of motion which may be solved by means of analogue computors. For the case of machine foundation symmetrical about contact base (which is usually the case) the horizontal translatory vibration is coupled with rotary motion about horizontal, perpendicular to the translatory vibrations. This has been independently confirmed by Pauw (1953) by truncated spring analogy. Only for the case when centre of gravity of machine foundation coincides with the centroid of the contact area, all motions are decoupled (This is not feasible, practically).

Ford and Haddow (1960) obviously have given a basically different approach to the problem by considering the machine foundation soil as a conservatives ystem. But this gives the resonant frequency which is independent of the dynamic load, which certainly is not correct as shown by the experiments of Lorenz (1934, 1953, 1959) and Balakrishna Rao (1961).

For the foundations other than circular ones, which have been considered in the theoretical analyses, following modifications have been suggested by various investigators.

1. For translatory motion, use an equivalent radius which gives the area of circle equal to that of the contact area of the foundation with ground (Sung 1953, Richart 1960, Hseih 1962).
2. For rotational motion use an equivalent radius which gives the moment of inertia of the circle equal to that of the contact area of the foundation about the axis of rotation (Hseih 1962).
3. Bycroft (1959) has suggested that if Z is the amplitude for equivalent circular base and Zr for the rectangular base, then $Zr = m \times Z$, where m is a shape factor and may be taken as for static case given by Timoshenko (1937).
4. As the distribution assumed by Bycroft (1959) is the rigid base distribution, the concept of effective radius is suggested to find the values of frequency and amplitude for other type of distribution.

In all the above theoretical methods based on the homogeneous semi-infinite, isotropic, elastic solid, certain values of G (modulus of rigidity) and ν (Poisson's ratio), have to be estimated. The value of modulus of rigidity G vareis with depth and so does the Poisson's ratio ν. The test on oscillator, will no doubt give certain value of G and ν, but it is valid only for the depth, which may be taken approximately equal to three times the base width of test vibrator. With increase in prototype area, the value of G and ν should be valid for depth upto three times the foundation width, and these values will obviously be different from those in test vibrator.

Another short-coming is the pressure distribution to be assumed. No data is available from field records, regarding the actual distribution and change in distribution with increase in dynamic loads. It has further been found from the survey of available literature that while the ratio of dynamic to static force in the prototype is about 4% to 5%, its value in the model vibrator is any where from 25% to 90%. The distribution in two cases may be different, but no quantative information is available.

Soil as Elastic Spring:

The first known approach to analyse the foundation vibrations considered the vibrating system to behave as a single mass supported by a weightless spring and subjected to viscous damping (Lorenz 1934, and Barkan 1936).

For undamped forced vibrations, resonance occurs at $f/f_0 = 1.0$, where f is frequncy of forced vibrations and f_0 is natural frequency of the system. For small values of damping the peak amplitude occurs at frequency ratio so close to unity that the difference is usually negligible.

Another possible variable with frequency ratio is the work per unit of time required to operate the vibrator. This work consists of two parts (Terzaghi*1943). One part is used up in overcoming the friction in bearings and other resistances within the mechanism. It has been found that this part increases approximately in direct proportion to the square of frequency. The second part is consumed by the viscous resistance of soil against periodic deformation. The rate of work for operation is maximum at resonance frequency.

From 1930 onwards, the work was carried by Degebo (Lorenz 1934). Their standard experimental set up consisted of a vibrator, weighing 2700 kgm with base area 1 sq. meter.

Experiments were conducted on different sites using the above vibrator. Amplitude of vibration, the phase angle between the exciting force and the resulting vibrations and the power requirement of the vibrator were determined at various frequencies. The reasonant frequency is determined where maximum amplitude occurs and checked with the frequency where maximum power is required and also where phase difference ψ is $\pi/2$. Comparison of the experimental plots with the theoretical plots gives the value for damping factor.

The frequencies which correspond to individual soil type and hence to bearing capacity according to Lorenz (1934) indicate that the higher the natural frequency, the higher the safe soil pressure. λ the damping factor was found to have the following significance. A value in excess of about 3 to 4 sec^{-1} combined with an important settlement of the base was considered an indication of high compressibility and sensitivity to vibrations.

During these experiments it was found that W in equation 1b is not the weight of vibrator alone, but also includes the weight of the soil vibrating with it. Expression for natural frequency then becomes;

$$\omega_0 = \sqrt{\frac{K.g}{W_0+W_s}} \quad \ldots (3)$$

where W_s *is equivalent soil weight which is assumed to be concentrated at the c.g. of foundation mass.*

In order to determine W_s, the weight of vibrator was increased by means of surcharge, and the test repeated. The natural circular frequency of system decreases from ω_0 to ω_0'. Assuming, for the sake of simiplification, that the increase in weight of the vibrator has no effect on W_s, two equations are obtained which make it possible to determine W_s.

Another suggested method is to increase the area of the vibrator base, keeping weight of the vibrator same. Replacing k by k'.A in equation (3) ω_0 is obtained as;

$$\omega_0 = \sqrt{\frac{k'.\,A.g.}{W_0+W_s}} \quad \ldots (4)$$

where A is the area of base plate, and
k′ is the modulus of dynamic subgrade reaction.

Assuming value of k′ to be same from one test to another, value of W_s can be determined.

By increasing the weight of vibrator from 1.8 to 3.4 metric tons, the value W_s was found to be 12.5 tons (Lorenz 1934). Similarly keeping the vibrator weight at 2700 kgm (2.7 metric tons), and changing the area from 1/4 sq. meter to 1 sq. meter, the value of W_s for the same site was found again to be 12.5 tons.

In another series of the tests, W_s was equal to 1 metric ton, when the weight of vibrator was increased from 2060 kgm to 2700 kgm. These results indicate that value of W_s is likely to vary between wide limits. For change in eccentricity (increase in dynamic loads), the natural frequency was found to decrease.

At about the same time, the independent tests were carried out in Russia which are reported by Barkan (1936, 1963). For the vabrations so produced as to give both gyration (rotation) and translatory d splacement the system has two degrees of freedom, the resonant frequencies are coupled, and two resonant frequencies were noted.

The experimental foundations weighed upto 30 tons and had an area at bottom upto 8 sq.m.. Value of $k=k'.A$ was determined by the statical tests (reversible displacement x k′ = normal stress). k′ was determined for areas 2, 4 and 8 sq.m. From the determined values of k′, the frequencies of the vertical vibrations only were calculated. The foundation was subjected to forced vertical vibration with the aid of vibrating machine and resonance recorded. In nearly all the cases, frequencies differed but little from the theoretically calculated ones. In the analysis the weight of the soil participating has been neglected. Barkan (1936) also observed the resonant frequency of 11 cps for the eccentricities of 22.5 mm, 17.5 mm, 6.5 mm. This seems erronous as the resonant frequency has been found to decrease with increase in eccentricity or the dynamic loading (Lorenz 1934, Crockett and Hammond 1948, 1949, Lorenz 1953).

Later experiments in Sweden (Bergstrom and Linderholm 1946) have shown that for large base plates (of the order of 3 m. radius), the value of subgrade reaction k′ corresponds to the values obtained from wave velocity measurements.

Andrews and Crockett (1945), and Crockett and Hammond (1948. 1949, 1958) also measured natural frequencies using a vibrograph to pick up the oscillations in the vicinity of large hammers. These frequencies are roughly the same as those reported by Degebo (Lorenz 1934). Crockett and Hammond (1948) also obtained the same natural frequency irrespective of the size of the foundation for any particular type of ground. The largest foundation tested had an area of 2500 sq. ft.

However, they suggested that mass of soil which vibrates with the foundation must bear some relation to bulb of stress, which gives the stress distribution under a uniformally loaded area on an elastic medium. The active ground weight is assumed to be within a certain bulb of pressure, but no relationship had been indicated.

Pauw (1953) has given an analytical procedure whereby the dynamic soil constants required for the prediction of natural frequencies of a foundation soil system may be determined. The foundation soil system is treated by considering the foundation to be supported by a truncated pyramid of "soil springs".

Based on the concept that the modulus of elasticity is approximately proportional to the shearing strength, Pauw made the following assumptions :—

1. For Cohesionless soils the modulus of elasticity is proportional to the effective depth which equals the actual depth plus equivalent surcharge.
2. For cohesive soils the modulus of elasticity is constant. Intermediate soil conditions may be interpolated on the basis of Coloumb's law.
3. The distribution of stress takes place within a truncated pyramid.
4. The soil pressure below the foundation and also at any depth is uniform.

Spring factor is defined as the force or moment exerted on a system when it is displaced a unit distance or rotated through a unit angle, from the equilibrium position. For a foundation with six degrees of freedom, six spring constants are required for each surface in contact with soil. Apparent mass of soil vibrating with the foundation is estimated by equating the kinetic energy of an equivalent concentrated mass at the surface to the total kinetic energy in the effective zone. He has given these factors for horizontal and vertical surface.

The integral for mass factor in case of translatory vibrations for cohesive soils does not yeild to a converging solution. He has also considered the equations of motion for a symmetrical foundation (c.g. of machine foundation directly above centroid of the contact surface) and found that only vertical vibrations and rotation about vertical axis exist independently. The horizontal translatory motion is coupled with rotation about horizontal axis. Balakrishna Rao and Nagraj (1960) and Balakrishna Rao (1961, 1962) have developed further the concept of oscillation of Bulb of pressure as advanced by Crockett and Hammond (1948, 1949). This has been modified to the density pressure bulb concept.

The weight of the soil mass participating in vibration (equation 3) is estimated by taking the weight of the soil contained in a definite pressure bulb. This pressure bulb is obtained by considering the sum of static and maximum positive dynamic load of the machine and the foundation block to act as concentrated load at the mass centre of the foundation block. The reason advanced for adding the dynamic load is that the additional static stresses are developed by dynamic load (Nagraj and Balakrishna Rao 1959). The boundary of this pressure bulb is supposed to be given by pressure intensity of $|\gamma|$ lbs per sq. ft. where γ is the density of the soil mass in lbs per cft.

The above authors have suggested to take the value of K or spring constant according to that given by Pauw, or for that matter from the dynamic soil tests. Knowing the value of K and W_s, the natural frequency can be determined.

Balakrishna Rao has checked the resonant frequency as calculated by pressure bulb concept with the published results of Converse (1953) and Eastwood (1953).

Barkan (1963) has developed an analysis on the analogy that soil acts as spring. For horizontal forces, the foundation soil system has been considered to be a two degree of freedom system.

Discussion

The approach is based on the assumption that soil acts as a linear spring, and for the purpose of resonant frequency it is sufficiently accurate to state that resonance occurs at the frequency ratio of unity. Barkan (1936) has verified this for the assumption that no soil mass is participating in the vibration, but Lorenz (1934, 1953 b), Crockett and Hammond (1948), Eastwood (1953), have shown that this soil mass cannot be neglected, though it is an uncertain factor. Hence it seems that neglecting the soil mass in denominator may have been compensated by the fact that Barkan has taken the value of spring constant from the static deflection test. (He has defined the spring constant as the load required to produce the unit reversible settlement).

It seems only proper that while giving W_s a certain value, the spring constant also must be considered. It is both W_s and K that vary rather than any single factor. Hence to suggest that Degebo obtained W_s as 4 to 10 times the static weight and Vios got W_s as 2/3 to 1.5 times the static weight is an indefinite statement.

Observations of Crocket and Hammond (1948) that natural frequency of foundation soil system is independent of the size of the contact area is inconsistent with recent tests of Eastwood (1953).

Pauw (1953) has given the spring factors, and mass factors for different modes of vibration based on theoretical derivation which assumes soil as truncated pyramid spring. But these spring and mass factors seem to be alright for the soils where Young's modulus increases linearly, with depth. If the Young's modulus is a constant, the approach does not give definite results, and a modification in surcharge as suggested by Converse (1962) may be used.

Andrews and Crockett (1945) Crockett and Hammond (1948, 1949) have suggested that the mass of soil oscillating may be taken as that contained within a pressure bulb, which they have not specified. Also as stated earlier, specifying W_s is not significant unless the spring constant is also defined.

Balakrishna Rao (1960. 1961, 1962) considers the pressure bulb for the combined static plus dynamic load, and has given a specific value to this pressure bulb. This explains the phenomenon of decreased resonant frequency for higher dynamic load as it assumes that the soil mass will increase. The pressure bulb for distributed load based on equivalent sphere did not give appreciable difference. He recommended the use of spring constants according to Pauw's approach. Thus W_s is calculated on the assumption of soil as uniform, homogeneous, elastic medium (having same Young's modulus at various depths), the spring constant K is calculated on the assumption that Young's modulus increases with depth (Pauw's approach). That is, the value of W_s and K are calculated on the basis of contrary assumptions. But it offers a good emperical means to evaluate the effect of changed dynamic force.

It is important therefore, that unless and untill both values of K and W_s are specified, the approach is not bound to be rational. In latter experiments with road subgrades Heukelom (1959) and Heukelom and Foster (1960) have kept the spring constant same, and embodied the deviation solely in the mass of vibrating soil.

Miscellaneous Methods :

Emperical approaches to the problem of determining resonant frequency will be discussed under this section :—

Tschebotarioff and Ward (1948) and Tschebotarioff (1951, 1953) have suggested a logarithmic relation between the area of foundation and the reduced natural frequency. The resonant frequency is given by ;

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k'.A.g}{W_o+W_s}} \quad \ldots \quad (5a)$$

$$= \frac{1}{2\pi}\sqrt{\frac{k'.g}{1+W_s/W_o}}\cdot\sqrt{\frac{A}{W_o}} \quad \ldots \quad (5b)$$

W_o/A is equal to the static load on foundation per unit area (σ st). Thus at unit static pressure, the frequency would be ;

$$f_{nr} = \frac{1}{2\pi}\sqrt{\frac{k'.g}{1+W_s/W_o}}\cdot\sqrt{1} \quad \ldots \quad (5e)$$

This is termed as the reduced natural frequency.

$$\text{or} \quad f_{nr} = f_o\sqrt{\sigma\,st} \quad \ldots \quad (6)$$

Tschebotarioff worked out values of reduced natural frequency from the published data of Degebo, Vios, Newcomb and other records and also the result of some tests which he simself carried out in a C.B.R. test mould and found that a straight line relationship existed between area of contact and reduced natural frequency on a log log plot for one soil type. For different soils, the straight lines were parallel and can be represented by an equation,

$$A = j_1\,(f_{nr})^t \quad \ldots \quad (7a)$$

where A is the contact area,

j_1 and t are constants for the plot.

Substitute for f_{nr} in terms of f_o. we get ;

$$A = j_1\left(\sqrt{\frac{W_o}{A}}\cdot f_o\right)^t \quad \ldots \quad (7b)$$

$$\text{whence } f_o = j_1^{-1/t}.\ \frac{A^{\left(\frac{1}{t}+\frac{1}{2}\right)}}{\sqrt{W_o}} \quad \ldots \quad (7c)$$

Goyal and Alam Singh (1960) have expressed the emperical equation of the straight line of above plot for sands and clays. Transferring their value to the form of equation (7c), the exponent of A for sands is 0.242 and for clays is 0.248.

Alpan (1961) found the value of t in the above equation as – 3.98 and hence, the exponent of A is 0.248 for all type of soils. Equation (7c) can then be written is ;

$$f_o = \frac{J'}{\sqrt{W_o}}A^{0.248} \quad \ldots \quad (8)$$

where J' is a constant.

The contact area in Tschebotarioff's diagram ranges from 1 to 1000 sq. meters (10 to 10000 sq ft). For smaller areas, results of laboratory experiments with model footings are available (Eastwood 1953). These tests were made to investigate the factors influencing the resonant frequency on dry and inundated sand. The oscillations were generated by impact. The sand employed in his tests had a dry density of 1.74 g/c.c. and a void ratio of 0.525.

Alpan (1961) plotted the results of Eastwood in terms of reduced natural frequency f_{nr} versus area A on log-loge scales and compared it with extrapolated values as obtained by Tschebotarioff's plot for sands. The lines obtained by Alpan are quite different from those of Tschebotarioff. Actually the points are anything upto 100% high or 50% low, the errors being masked by the log-log scale (Eastwood's comments 1953). From the data which Tschebotarioff has used for his plot (Tschebotarioff 1953) it will be seen that the resonant frequency was obtained by forced vibration test and shock or impact and exciting force was either vertical or horizontal and vertical or, only horizontal. But it is a known fact that the nature of the vibrations and the method by which they are induced materially affect the frequency response of the system. Hence Tschebotarrioff's plot is not the true picture of frequencies.

Eastwood's (1953) tests show that for the same applied load per unit area, the natural frequency of a 12" × 3" model footing is the same as that for a 24" × 3" model footing. Thus they will also have the same reduced natural frequency even though area of one is twice that of the other. He has suggested a possible relation between reduced natural frequency and the least dimension of footing.

To obtain same values of f_{nr} (for same area), whatever be the applied load expression $\sqrt{k'.g/1 + W_s/W_o}$ in quation (5b) has to be constant for different areas. This means that either W_s must increase at exactly the same ratio as W_o or alternatively W_s is always negligible compared to W_o. The latter is impossible and the former extremely unlikely (Eastwood 1953).

Alpan (1961) has made an attempt to analyze from first principles, the relation between frequency and area and obtained an expression for natural frequency as;

$$f_o = \frac{\text{constant}}{\sqrt{m}} \cdot \frac{A^{0.25}}{\sqrt{W_o}} \qquad \dots (9)$$

where m is shape factor. Equation (9) differs from equation (8) in only that a shape factor m is involved, and that exponential power of A is 0.25 instead of 0.248.

Now shape factor is not only dependent upon the length width ratio, but may depend also on the type of the load distribution. This probably may be able to remove the discrepancies in the plot of Tschebotarioff. For example, Eastwood (1953) obtained the same natural frequency f_o, for 24"×3" and 12"×3" foundation models, for the same static load intensity though the area is twice. This may be explained by introduction of shape factor. Further work has to be done along these lines.

From the results of the field testing programme mainly conducted to note the effect of various parameters, on the compaction of sand by vibration in a test pit 6 feet deep, 10 feet square, an empirical equation (Converse 1953) for resonant frequency of a vibrator sand mass was developed as follows:

$$f_o = \frac{\sqrt{g}}{2\pi} \sqrt{840 \frac{\gamma}{G} (1.64 - F_o/W_o) + 0.55\, G r_o/W_o} \qquad \dots (10)$$

$$o < \frac{F_o}{W_o} < 1.$$

where f_o is frequency in cps
G is modulus of rigidity of soil, psi.
F_o is maximum dynamic force, lbs.
W_o is static weight of vibrator lbs. and
r_o is radius of vibrator, in. and
γ is unit weight of soil, lb. per cuft.

Converse verified the resonant frequency based on the above formula with that of the field test results obtained with base plates 15.7, 19.2, 24.0 and 45.0 inch in diameter.

The development of the emperical equation is significant, as it involves not only the soil constants, but also the vibrator dimensions (r_o), weight (W_o) and dynamic force (F_o) still, the equation is developed only for one type of soil and it is only reasonable to expect that this will vary with the type of soil and such an equation is not universal in nature.

Blessing has presented curves for determination of resonant frequency and amplitudes of compressor or engine fondations. These are based partly on elastic analysis and on the experience gained from their installations. These curves have been adopted in the Indian Standard on Design of Machine Foundations.

BASIC SIMILARITY OF VARIOUS APPROACHES.

In all the methods for predicting resonant frequency which assume soil to be homogeneous, an interesting similarity is pointed out. Based on this an empirical formula for resonant freqnency is suggested.

For static loads, the value of spring constant for various contact pressure distributions are reported by Jones (1958) and Heuklom (1959).

$$k = 4\ Gro/\ (1 - \nu) \qquad \ldots \quad (11a)$$

For rigid plate condition. (Sneddon 1951)

$$k = \pi\ Gro/\ 1 - \nu \qquad \ldots \quad (11b)$$

for uniform stress distribution, (Boussinesq 1885).

$$k = \frac{3\pi}{4}\ \frac{Gro}{1-\nu} \qquad \ldots \quad (11c)$$

for parabolic stress distribution. (Frohlich 1934).

Equations (11) show that spring constant is multiple of Gro. Now consider the theory of vibration of elastic homogeneous, semi-infinite, isotropic medium (soil). In this analysis, mass ratio 'b' and dimensionless frequency term 'a' are used, where

$$b = \frac{m_o}{\rho ro^3} \text{ and } a_o = \omega_o r_o \sqrt{\rho/G} \qquad \ldots \quad (12a)$$

$$\therefore ba_o^2 = \frac{m_o}{\rho ro^3}\ \omega_o^2\ r_o^2\ \frac{\rho}{G} \qquad \ldots \quad (12b)$$

which after re-arranging gives,

$$\omega_o^2 = \frac{Gro}{m_o}\ (ba_o^2) \qquad \ldots \quad (12c)$$

Value of equivalent spring constant is, therefore, (assuming $\omega_o^2 = k/m_o$).

$$k = Gro.\ (ba_o^2) \qquad \ldots \quad (12d)$$

(ba_o^2) is, a function of load distribution Poisson's ratio and 'a'.

Hseih's transformation (1962) gives spring constant as

$$k = Gro\ F_1 \qquad \ldots \quad (13)$$

where F_1 has been evaluated in terms of 'a' for rigid distribution and different Poisson's ratio

$$\therefore \omega_o^2 = \frac{Gro\ F_1}{m_o} \qquad \ldots \quad (14a)$$

Putting value of F_1 as given by him, we get (for $\nu = 1/2$),

$$\omega_o^2 = \frac{Gro\ (8-2.0\ a_o^2)}{m_o} \qquad \ldots \quad (14b)$$

Substituting value of 'a_o' $=\omega_o ro\ \sqrt{\rho/G}$ and rearranging we obtain,

$$\omega_o^2 = \frac{8\ Gro}{m_o + 2\ \rho\ ro^3} \qquad \ldots \quad (15a)$$

Similarly for $\nu = 1/4$,

$$\omega_o^2 = \frac{5.3\ Gro}{m_o + \rho ro^3} \qquad \ldots \quad (15b)$$

For $\nu = 0$,

$$\omega_o^2 = \frac{4.0\ Gro}{m_o + 0.5\ \rho ro^3} \qquad \ldots \quad (15c)$$

Values of spring constant from equation (11a) for rigid base distribution is 4 Gro/1 — ν, which for ν = 1/2, 1/4,0 is 8 Gro, 5.33 Gro and 4.0 Gro respectively. This tallies with spring constants in equations (15).

In Ford and Haddow's analysis (1960) it can be shown (Bhatia 1963) that the equivalent spring constant is a multiple of Gro, and the mass of soil participating in vibration is a multiple of $\rho\ ro^3$.

Now consider the emperical plot of Tschebotarioff (1948, 1951, 1953). The equation of the plot is

$$f_o = \frac{J'}{\sqrt{W_o}}\ A^{0.248} \qquad \ldots \quad (8)$$

Substituting $A = \pi\ ro^2$ in equation (8), we get

$$f_o = \text{const} \times \frac{r_o^{0.496}}{\sqrt{W_o}} \qquad \ldots \quad (16a)$$

$$= \text{constant} \times \sqrt{\frac{r_o^{0.992}}{W_o}} \qquad \ldots \quad (16b)$$

where the constant depends upon soil type (and hence on value of G)

Therefore, spring constant may be taken as a multiple of Gro. Note very small power difference between 1.0 and 0.992.

Converse (1953) has given an emperical equation for resonant frequency of sand vibrator system (Eq. 10).

For large values of G, first term under the root is small and may be neglected and this reduces to,

$$f_0 = \sqrt{\frac{Gro}{W_o}} \times \text{a constant} \quad \dots (17)$$

Pauw (1953) has evaluated the spring constants for the cohesive soils for which values of E or G can be assumed to be constant. Though it was not possible to evaluate the mass factor, it will be of interest to see nature of spring constant by given him. Expression for vertical vibration is ;

$$k = E\, a\, B\, \gamma_z \quad \dots (18a)$$

where $\tan^{-1} a/2$ denotes the angle of pressure distribution and γ_z is a factor depending upon L/B ratio.

For circular vibrator $B = 2$ ro, and $\gamma_z = 1.0$, (Pauw 1953).

$$\therefore \quad k = E.a\,(2\,ro)\,.1 = 2\,G\,(1 + \nu).\ (2ro).$$
$$= Gro\,\{4\,(1 + \nu)\,\} \quad \dots (18b)$$

i.e. the spring constant is a multiple of Gro.

Experiments by Nijboer (1953, 1959) Vander Poel (1951, 1953) Heukelom (1959) Heukelom and Foster (1960), obtained the value of spring constant as approximately 7.6 to 7.7 Gro, in their dynamic tests.

i.e. $k = 7.6$ Gro ... (19)

From consideration of equations (11), (12), (14a), (15), (16b), (17), (18) and (19) a simplified form of the natural frequency expression is suggested viz.,

$$\omega_0^2 = \frac{\lambda_1 \times Gro}{m_o + \lambda_2\, \rho ro^3} \quad \dots (20)$$

Introducing the shape factor 'm' in equation (20) we obtain,

$$\omega_0^2 = (\lambda_1/m).\frac{Gro}{m_o + \lambda_2\, \rho\, ro^3} \quad \dots (21)$$

where λ_1, λ_2 are constants for the system and
m is the shape factor depending upon L/B ratio

The effect of change in dynamic load upon λ_1 and λ_2 will have to be investigated experimentally.

DESIGN OF A MACHINE FOUNDATION.

The general method of designing a foundation is to determine the natural frequency of the system based on analytical mothods discussed in this report and ascertain the amplitude of oscillation. The various steps in these methods are :

1 The dynamic unbalanced forces, and their frequencies of operation are calculated, or these may be supplied by the manufacturers.

2 The dimensions of the foundation block are assumed, taking care that allowable soil pressures (which are less than in case of static loads only) are not exceeded.

3 The soil type is analysed by borings, and sampling. To analyse behaviour of the soil, in-stitu vibrator tests, should be conducted. In these tests, either the resonant frequency and amplitude of vibration are determined for different combination of static and dynamic weights, or the velocity of wave propogation is determined. The value of β (rate of increase of Young's modulus with depth) can also be calculated with the help of Pauw's analysis. The values of decay factor or B_1, a constant in Ford and Haddow's analysis can be calculated. It is recommended that tests with at least 3 different areas of vibrator be performed. Balakrishna Rao (1962) has suggested a linear variation of β with area A, based on analysis of data of Ford and Haddow. Pauw (1953) has however, assumed β to be constant.

4 Knowing the above soil properties, the resonant frequency of the actual machine foundation soil system can be calculated by any one of the methods described previously.

5 Resonant frequency is then checked with operational frequency of the machine, and if the frequency ratio is within safe limits (less then 0.5, or more then 2.0), the design may be checked for amplitude of vibrations.

6 Usually the amplitude of vibrations, can be determined with sufficient accuracy by assumption of a simple spring, in which the damping value may be neglected or a reasonable value usually 0.25 may be assumed.

7 If this is found to be within permissible limits, which can be tolerated by the machine and the structure, then the design is safe.

8 If not, assume another preliminary design and repeat the above steps.

Usually vertical vibrations exist independently. If the vibrations occur in more then one direction the frequencies will be coupled as shown by Pauw (1953), and, Hseih (1962). In that case Pauw's method is recommended: Pauw's method is applicable only for cohesionless soils.

For fairly homogeneous soils, either modification in surcharge as suggested by Converse (1962) may be used, or Barkan's (1963) method may be used.

SUGGESTIONS FOR FOURTHER RESEARCH.

In view of the study made, the following suggestions for further research are made :

1 A simple equation for the natural frequency of the system is evolved (Equation 21). The factors affecting λ_1 and λ_2 need be studied systematically.

2 Shape factors (m) i.e. the effect of shape of base area (characterized by L/B ratio) needs to be investigated experimentally.

3 Balakrishna Rao (1960, 61, 62) has evolved density pressure bulb concept. It accounts for the change in the natural frequency of system with change in dynamic loads. This approach has given good results for vertical vibrations when Pauw's spring factor for cohesionless soils (E increasing with depth) is used. The procedure needs experimental verification for cohesive soils.

4 Experimntal investigations of bearing capacity of soil under dynamic loads are being undertaken (A.S. T.M. 1961). A systematic study of the problem will prove useful to the designer of machine foundations.

5 The problem of machine foundations on piles has not been tackled at all. Experimental and analytical investigations of this problem need be made.

6 No data from the actual machine foundations in India is available. It is suggested that a questionaire be prepared and sent to various industries, designers, and research workers dealing with machine foundations. The purpose will be to undertake a systematic analysis of field data regarding actual behaviour of machine foundations. This will help in co-ordinating the efforts of various workers and will consequently lead to a standard practice for the design of machine foundations.

COCNLUSIONS

From the study of the available literature the the following conclusions could be drawn :—

1. Resonance phenomena cannot be ignored in machine foundations and design should account for it.
2. For soils in which the value of E increases linearly with depth (for sand, and normally loaded clays), Pauw's method is recommended. For vertical vibrations only, Balakrishna Rao's (1962) method to calculate apparent mass factor should be used.
3. For soils in which value of E, can be assumed to be uniform, Pauw's method cannot be applied as such. Modification in surcharge as suggested by Converse (1962) is recommended.
4. For other modes of vibration, Pauw's (1953) analysis can be used.
5. In most cases the amplitude of vibration can be determined with sufficient accuracy by simple mass spring analogy.
6. A simple equation, based on the general similarity of analytical approach is developed, for determination of the natural frequency of system.

ACKNOWLEDGEMENT.

The junior author was provided with a Scholarship by the Ministry of Scientific Research and Cultural Affairs under the Technical Teachers Training Programme. The investigation was carried out in the School of Research and Training in Earthquake Engineering University of Roorkee, Roorkee (U.P.).

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COMPUTING PERIODS OF VIBRATION OF MULTISTOREYED BUILDING FRAMES

A. S. Arya*

SYNOPSIS

It is emphasised in this paper that in the dynamic analysis of multistoreyed buildings, the flexibility of girders must be considered along with the stiffness columns because assuming the girders rigid may lead to large errors in the calculated periods of vibration. A simplified method is suggested for computing the natural periods and the corresponding mode shapes taking the girder flexibility into account.

INTRODUCTION

Dynamic analysis of any structure subjected to impulsive or vibrating forces requires the knowledge of its stiffness and the damping characteristics. Stiffness determines the natural periods of vibration and the corresponding mode shapes almost exclusively since the effect of damping on natural periods is small and negligible. Damping has the influence of reducing the amplification factors under forced vibration. The effect is more pronounced when imposed period of vibration is close to the natural period, otherwise when forcing frequency is considerably less or more than the natural frequency, the effect of damping is small and negligible. The amplification of displacement and consequently of forces produced in the structure depends upon the ratios of forcing periods to the natural periods and the corresponding mode shapes. Therefore, determination of natural periods of vibration of a structure constitutes the most important step in the process of its dynamic analysis.

Multistoreyed buildings are very complex structures from the point of view of structural analysis especially under horizontal loads causing sidesway. This fact alone has resulted in the introduction of a large number of approximate methods of analysis for sidesway problems under static loads. Under dynamic loads the problem is all the more complicated because the natural periods of the building are found to be influenced to a great extent by the method of construction, floor slabs, wall panels, partitions, etc., which are otherwise assumed not to contribute to the stiffness of the building. How far they influence the stiffness and whether such contribution is of a permanent nature or may be destroyed in the very first shock given to the building is not known. It has been found (1)† that for existing multistoreyed buildings with steel frames, the measured fundamental period comes close to that computed by assuming building floors to be rigid, that is, having infinite moment of inertia as compared with the columns But for a bare frame or if the contribution to stiffness of filling material (which is not designed to carry forces due to shocks) is destroyed the stiffness is bound to decrease and actual girder stiffness must be considered for computing natural periods. Whereas assumption of rigid girders greatly simplifies the problem of analysis and much literature exists on this procedure, consideration of flexibility of girders complicates the problem because of consequent joint rotations. But this must be taken into account since the influence of girder flexibility on the natural periods is considerable. For example, for a particular 19-storey steel-frame building, the fundamental period was 1.3 sec. when girders were assumed rigid but 3.4 sec. when girder flexibility was considered(2). Figure (1) further shows the influence of girder flexibility on the fundamental periods of single bay multistoreyed frames (1). In the case of actual buildings, where beams are stiffened by the floor slabs, the margin between the two assumptions decreases considerably.

From the above discussion it follows that in dynamic computations stiffness properties of beams and

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† Numbers in parentheses refer to the list of References at the end of the paper.

columns must be taken into account. Where a digital computer is available, such calculations may be done easily. But where such a facility is not present, this computation involves too much work. It is the aim of this paper to present a simplified method for calculating the fundamental period by Rayleigh's procedure, or all the periods and mode shapes if desired by the flexibility equations.

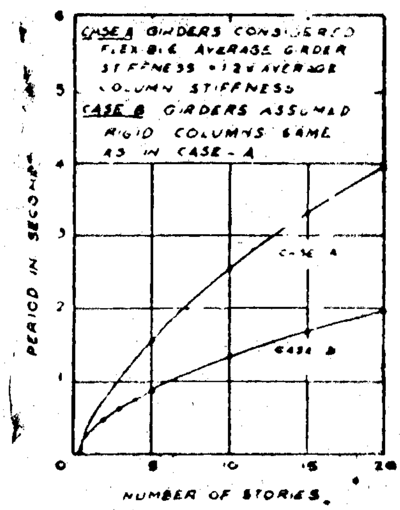


Fig. 1

METHOD OF ANALYSIS

It has been shown in (3), (4) and (5) that any general multistoreyed frame can be represented by a modified frame with single column. The stiffnesses in the single-column frame are modified in such a way that for the horizontal loads producing sideway of the original frame, the modified frame could be analysed as a non-sway frame. The sidesway deflections and moments in the modified frame are thus determined in a direct manner. From these, the moments in the original frame are then obtained by moment distribution done on the original frame. For determining the time-periods and corresponding mode shapes, the horizontal displacements are the only quantities required which can be obtained from the modified frame itself. Therefore no further reference to the original frame is necessary and hence the simplification. The modified frames for a few cases are shown in Fig. 2. In case (a) a two—column symmetrical frame is shown. The modified frame has its column twice as stiff as the individual columns of the original frame and the beams are twelve times as stiff as the corresponding beams. In this case the equivalence of the modified frame is "exact". The resulting deformations—slopes and displacements—of the modified frame will be exactly equal to those of the original frame without any approximation. In case (b) a frame obtained on the principle of multiples is shown together with the component one-bay frames which may be assumed to be making it. If the applied horizontal load is proportioned in the two component frames in the ratio 1:m, it is easily seen that the rotations and displacements of joints in the two component one-bay frames will be the same. Thus if the two adjacent columns of the one-bay frames are combined in one, neither compatibility nor equilibrium is disturbed. Therefore, for this frame also the column stiffness of modified frame is the sum of corresponding column stiffnesses of the original frame and the beam stiffness of the single column frame is 12 times the sum of all corresponding beam stiffnesses. The equivalence is again exact since the rotations of all the joints at any floor of such a frame are exactly equal. In case (c) a general frame is shown with irregular stiffness and nonuniform heights in different bays. It is well known that the joint rotations at any floor of this frame will in general be unequal. If the average value of all such rotations at any floor is considered, the general frame can be represented by a single column frame shown in Fig. 2 (c). In this case also the single column will have an aggregated stiffness of all columns in any storey but the factor with beam stiffnesses will be different from 12. A study of large number of frames (3) indicates that a factor 'A' may be used for the floors and 'A'' for the roof. Factors A and A' are in general different from 12, but near to it. For certain frames their values may be taken from Table 1.

If the bases of the columns are hinged, the stiffness of single column of the modified frame becomes zero in the bottom storey as shown in Fig. 2 (d). The modified frame in cases (c) and (d) is not exactly equivalent to the original frame as its joint rotations are approximately equal to the average of joint rotations at any floor

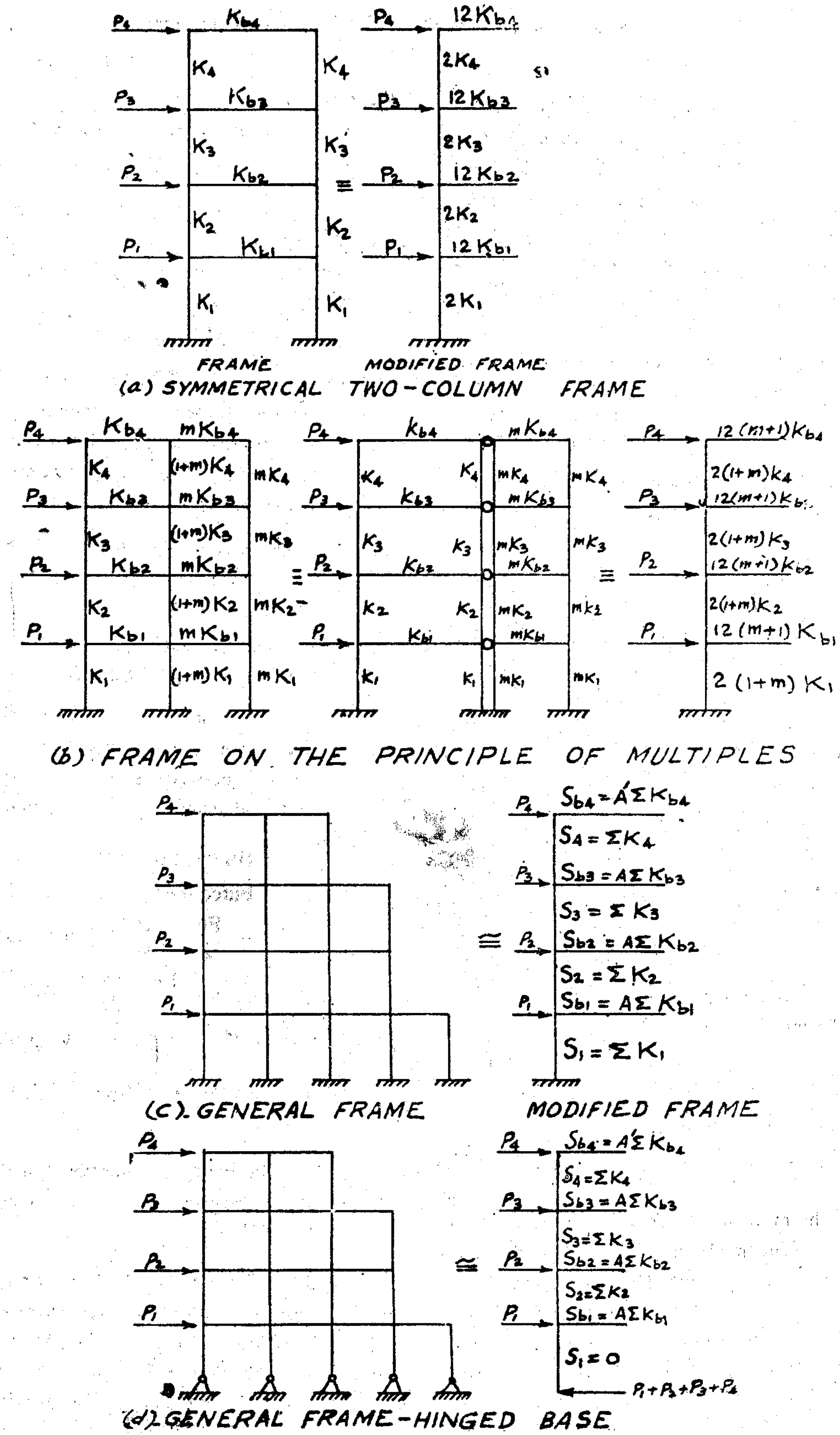


Fig. 2.—Modified Frames

of the original frame. Fortunately the effect of this approximation on the horizontal displacements of the frame is small. Therefore, the storey displacements and hence the periods and mode shapes may be computed with sufficient accuracy. If values of A and A′ are assumed according to Table 1, the results are expected to lie within ± 2% of the exact values.

TABLE 1.

Type of Frame	Factor A for Floors	Factor A′ for Roof
1. Symmetrical two column frames or frames on principle of Multiples	12.0	12.0
2. Frames approximately as in 1.	11.8	11.8
3. Irregular two column frame, column ratios on any floor not to exceed 4 to 1.	11.8	10.8
4. Irregular two column frame with higher ratios say 10 to 1 of column stiffnesses on any floor.	9.5	9.5
5. (a) Three column frame with same beam stiffnesses	11.9	11.4
(b) Many column frame with same beam stiffnesses	12.0	11.6
6. Four column frame with same beam stiffness in outer span and uniform in the interior spans of all storeys.	11.4	10.5
7. General Frame with irregular beam and column stiffnesses within ordinary limits.	11.5	11.5

For finding the moments in the single-column frame, the method of moment distribution is used with the modification that the carry over factor becomes '−1' instead of + ½. Then from the end moments of the column, the storey displacements, are determined immediately. The following example will illustrate the procedure.

Example 1.

To determine the horizontal displacements of frame shown in Fig. 3 (a)

Fig. 3 (b) shows the modified frame obtained by taking A and A′ equal to 11.5 as per case 7 of Table-1. The various steps in the calculation are as follows (Refer Fig. 3 a).

1. Distribution factors are determined as usual. Factors for column ends only need be written.
2. Initial moments are obtained by multiplying the storey shears by half of the storey heights that is, Fh/2. Here anticlockwise moments are taken negative.
3. Unbalanced joint moments are distributed and carried over as usual excepting that the carry-over factor is '−1'.
4. Column moments are found by summing the initial, the distributed and carry-over moments.
5. To obtain the relative storey displacements it was shown in Ref. (5) that the relative displacement △ between the two ends of a storey may be found from the following relation:—

$$\frac{6ES\triangle}{h} = \text{(Initial end moment)} - (3\times \text{ sum of distributed moments at the ends of the storey)}$$

$$= -\frac{Fh}{2} - \sum (3\times \text{Dist. moments})$$

$$\triangle = \frac{h}{6ES}\left[-\frac{Fh}{2} - \sum(3\times \text{Dist. moments})\right] \quad (1)$$

where S is the column stiffness in the story.

This explains the calculations of △'s through steps (5) to (8).

9. The floorwise displacements are obtained by summing up the relative storey displacements starting from the foundations.

Fig. 3 (d) shows an alternative tabulation in case only horizontal displacements are required as in t me-period calculation. Here the distributed mements are ommitted from writing. The carry over moments are written directly since these may be obtain d from the

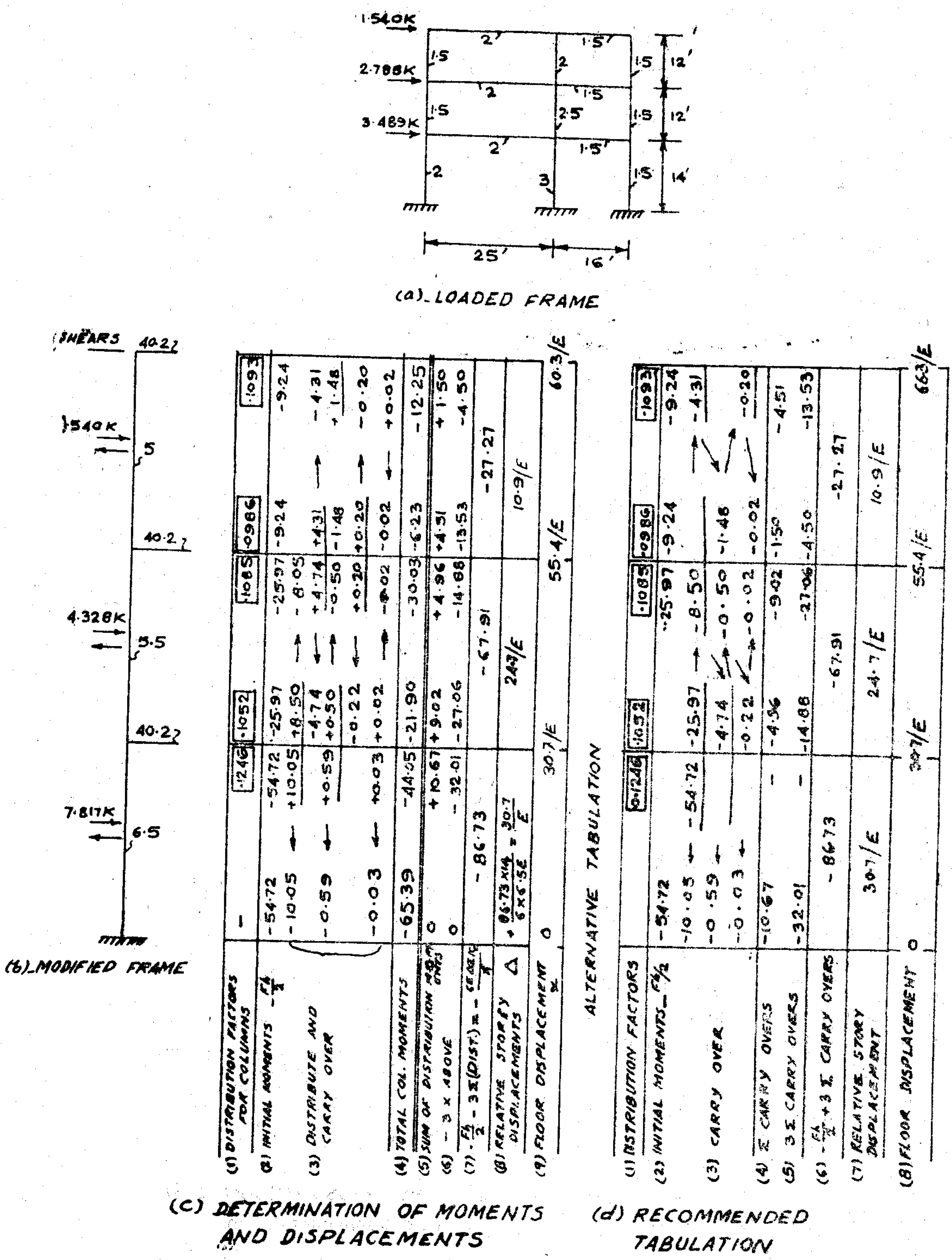

1·540K
2·788K
3·489K
25′
16′
12′
12′
14′
(a) LOADED FRAME
(b) MODIFIED FRAME
ALTERNATIVE TABULATION
(c) DETERMINATION OF MOMENTS AND DISPLACEMENTS
(d) RECOMMENDED TABULATION

Fig. 3

unbalanced moments in proportion of the distribution factors and of the same sign as the unbalanced joint moments. The relative storey displacements may now be obtained from the following equation:—

$$\Delta = \frac{h}{6ES}\left[-\frac{Fh}{2}+(\Sigma\, 3\times \text{Carryover moments})\right] \quad (2)$$

FUNDAMENTAL PERIOD

The fundamental time period of a structure may approximately be determined most easily by Rayleigh's method. In this method it is assumed that the mode shape is proportional to the deflection curve produced by a static load at each floor equal to mass times the acceleration due to gravity. When the building vibrates freely in the first mode, the horizontal displacement x_n at any floor 'n' at any time 't' is given by

$$x_n = CX_n \sin pt$$

where X_n is the mode shape at floor 'n'

C is a constant depending upon stiffnesses.

p is the fundamental natural frequency of free vibrations.

The mode shape is chosen to have the displacement at the top level X_h equal to 1. The maximum kinetic energy of mass m_n is given by

$$\tfrac{1}{2}\, m_n\, (CX_n p)^2$$

and the maximum potential energy is given by

$$\tfrac{1}{2} m_n\, g\, CX_n$$

Summing up the kinetic and potential energies separately and equating we get

$$\sum m_n\,(C_g X_n\, p)^2 = \sum m_n\, g\, C_g\, X_n$$

where C_g is the value of the constant obtained when horizontal load of 1g is considered in calculating static deflections. Putting $p = 2\,\pi/T$ (where T is the fundamental period) and solving

$$T = 2\pi\left[\frac{Cg}{g}\,\frac{\Sigma m_n X_n^2}{\Sigma m_n X_n}\right]^{\frac{1}{2}} \quad (3)$$

Example 2 given below illustrates the procedure of calculating fundamental period of a building by Rayleigh's method.

Example 2.

The four storeyed frame of Fig. 4 has the relative stiffnesses as shown. The actual stiffness of beams is K_o. The floor weights including weights of walls are $4w_o$, $4w_o$, $3w_o$ and $2w_o$ respectively. It is required to determine its fundamental period of vibration.

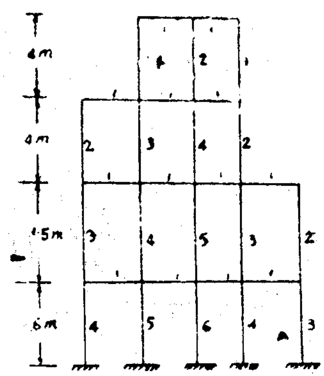


Fig. 4.

Using Rayleigh's method let us give a horizontal acceleration of g to the floor masses. Then the horizontal load applied to the frame at first floor level is

$$\frac{4w_o}{g}\times g = 4w_o$$

Similarly for the other floors the loads will be $4w_o$, $3w_o$ and $2w_o$. The deflections of floors are computed in Fig. 5. The modifying factors A and A′ are assumed 12 and 11.6 respectively.

Using the deflections camputed in Fig. 5, we get the approximate mode shape given in the last line adjusted in such a way that the displacement at the top is 1. This gives $C_g = 13.33w_o/EK_o$

Using Equation 3 we get the time period

$$T=2\pi\left[\frac{13.33w_o}{gEK_o}\cdot\frac{4\times.315^2+4\times.628^2+3\times.855^2+2\times1}{4\times.315+4\times.628+3\times.855+2\times1}\right]^{\frac{1}{2}}$$

$$= 6.3\sqrt{\frac{w_o}{EK_o}}\ \text{seconds.}$$

TIME PERIODS AND MODE SHAPES

To determine all the time periods and mode shapes, the flexibility or the stiffness equations are used. In the present scheme, the flexibility equations are easily derived by obtaining influence lines of horizontal displacements. Let X_{nr} denote the displacement of r^{th} floor produced by a unit static load applied at $.^{th}$ floor level.

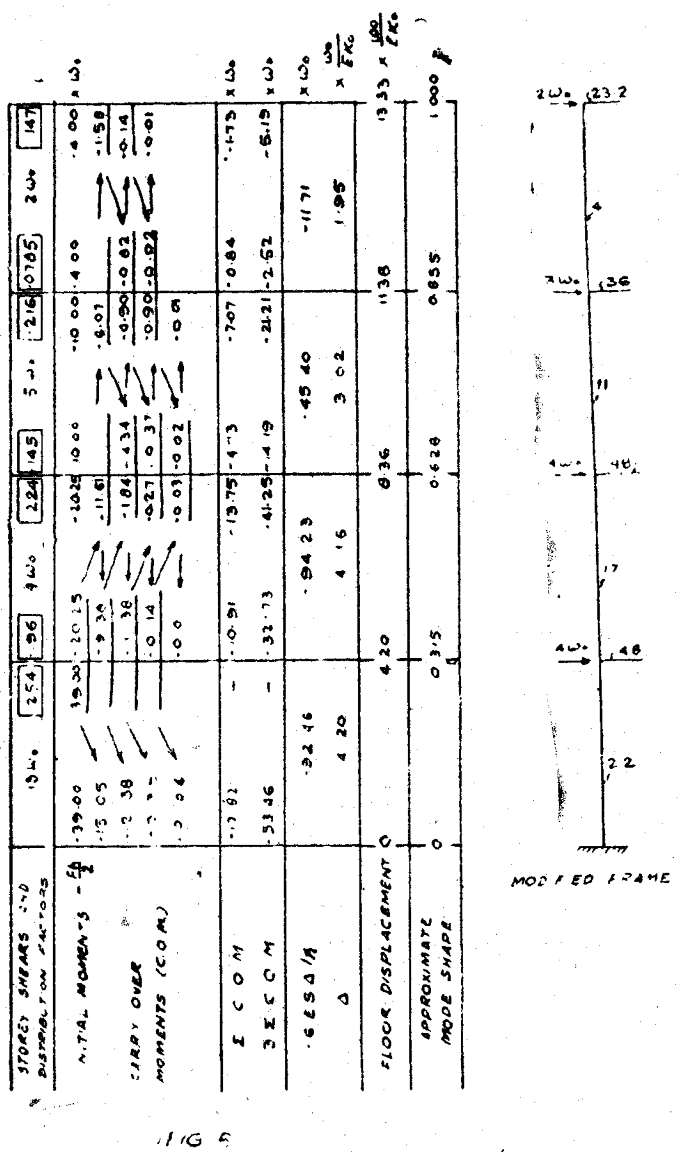


Fig. 5-

If the building is vibrating with a freuqency p, the inertia force applied at the r^{th} floor is $m_r\ p^2 x_r$ where x_r denotes the dynamic displacement. Therefore, the flexibility equation for the n^{th} floor becomes,

$$x_n = p^2 \sum m_r\, x_r\, X_{nr}$$

By transposing

$$(1-p^2 m_n\ X_{nr})\, x_n - p^2 \sum_{n \neq r} m_r x_r\, X_{nr} = 0 \qquad (4)$$

Such equations written for all the floors of a building give a set of homogeneous simultaneous algebraic equations in x_1, x_2, x_3 etc. For a possible non-zero solution, the determinant of the coefficients of x_1, x_2 etc. must vanish. This condition gives the characteristic equation for obtaining values of p and hance the mode shapes. The following example illustrates the prodedure.

Example 3.

All the periods and mode shapes for the frame of Fig. 4 will now be determined.

To obtain the floor displacements due to unit loads, we shall use the modified frame method. The calculations are shown in Fig. 6. Part (a) shows the modified frame assuming A = 12 and A′ = 11.6. The calculations for a unit load at first floor level are shown at (b). The initial moments for this case are

$-1x\ 6/2 = -3.00$ tm.

The calculation of deflections follows the pattern of Example 1, Fig. 3 (d). The final numerical values of x's are to be multiplied by $1/EK_o$ and the results will be in tonne metre units.

The computations of deflections for unit loads applied at other floor levels are shown in Figs. 6 (c), (d) and (e). Since deflections obey the Reciprocal Law, it must be verified that the computed deflections are reciprocal. Results in (b), (c), (d) and (e) satisfy this condition. Now if p is the frequency of vibration of the building, we get the loads.

$$\frac{4w_o}{g}\ p^2x_1,\ \frac{4w_o}{g}\ p^2x_2,\ \frac{3w_o}{g}\ p^2x_3 \text{ and } \frac{2w_o}{g}\ p^2x_4$$

acting at the first, second, third floors and the roof respectively. Hence the compatibility equation at first floor may be written as follows:

$$x_1 = \frac{4w_o}{g}\ p^2x_1\ \frac{0.2453}{EK_o} + \frac{4w_o}{g}\ p^2x_2\ \frac{0.346}{EK_o} + \frac{3w_o}{g}\ p^2x_3\ \frac{0.366}{EK_o} + \frac{2w_o}{g}\ p^2x_4\ \frac{0.37}{EK_o}$$

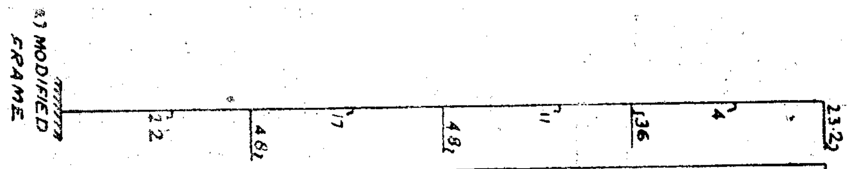
Let $\lambda = \frac{w_o p^2}{gEK}$. Therefore, we get for all the floors and roof the following equations ;

$(1-0.981\lambda)\ x_1-1.384\lambda\ x_2-1.098\lambda\ x_3-0.74\lambda\ x_4=0$ (i)

$-1.384\lambda\ x_1+(1-2.828\lambda)\ x_2-2.463\lambda x_3-1.688\lambda\ x_4=0$ ... (ii)

$-1.464\lambda x_1-3.284\lambda x_2+(1-3.66\lambda)x_3-2.706\lambda x_4=0$ (iii)

$-1.480\lambda x_1-3.376\lambda x_2-4.059\lambda x_3+(1-4.146\lambda)\ x_4=0$ ... (iv)



(b) UNIT LOAD AT FIRST FLOOR

DISTRIBUTION FACTORS		.254	.196	.224	.145	.216	.0765	.147
INITIAL MOMENTS	-3.000	-3.000	-	-	-	-	-	-
Σ C.O.M	-0.799	0	-0.142	-0.616	-0.019	-0.019	-0.001	-0.007
Δ	0.2453		0.1003		0.020		0.004	$\times\frac{1}{EK_0}$
x	0	0.2453		0.3456		0.3656		$0.3696\times\frac{1}{EK_0}$

(c) UNIT LOAD AT II FLOOR

INITIAL MOMENTS	-3.000	-3.000	-2.250	-2.250	-	-	-	-
Σ C.O.M	-1.533	-	-0.783	-1.185	-0.113	-0.514	-0.005	-0.040
Δ	0.345		0.361		0.114		0.0222	$\times\frac{1}{EK_0}$
x	0	0.346		0.707		0.821		$0.843\times\frac{1}{EK_0}$

(d) UNIT LOAD AT III FLOOR

INITIAL MOMENTS	-3.000	-3.000	-2.250	-2.250	-2.000	-2.000	-	-
Σ C.O.M	-1.683	-	-1.384	-1.300	-0.634	-0.898	-0.034	-0.231
Δ	0.366		0.455		0.400		0.132	$\times\frac{1}{EK_0}$
x	0	0.366		0.821		1.221		$1.353\times\frac{1}{EK_0}$

(e) UNIT LOAD AT ROOF

INITIAL MOMENTS	-3.000	-3.000	-2.250	-2.250	-2.000	-2.000	-2.000	-2.000
Σ C.O.M	-1.715	-	-1.506	-1.325	-1.152	-0.977	-0.355	$-0.419\times\frac{1}{EK_0}$
Δ	0.370		0.474		0.509		0.720	$\times\frac{1}{EK_0}$
x	0	0.370		0.844		1.353		$2.073\times\frac{1}{EK_0}$

Fig. 6.

Multiplying equation (iii) by $m_3/m_1=3/4$ and equation (iv) by $m_4/m_1=1/2$, we shall see that the equations become symmetrical about the main diagonal. Thus rearrainging the equations we get

$$\left.\begin{aligned}-(1-0.981\lambda)x+1.384\lambda x_2+1.098\lambda x_3+0.74\lambda x_4=0\\ 1.384\lambda x_1-(1-2.828\lambda)\,x_2+2.463\lambda x_3+1.688\lambda x_4=0\\ 1.098\lambda x_1+2.463\lambda x_2-(0.75-2.75\lambda)x_3+2.03\lambda x_4=0\\ 0.74\lambda x_1+1.688\lambda x_2+2.03\lambda x_3-(0.5-2.073\lambda)\,x_4=0\end{aligned}\right\}\quad(5)$$

The above equations are now in the standard form for determining the eigen values and the eigen modes. The equations are homogeneous, simultaneous algebraic equations and for a possible non-zero solution, the determinant of the coefficients of x's must be zero. This gives a fourth degree characteristic equation in λ the four roots of which are the eigen values. Knowing the eigen values the corresponding modes may the determined by substituting for λ in the obove equations and solving for x_2, x_3 and x_4 say by taking x_1 equal to 1. This procedure becomes involved as number of degrees of freedom increases. In that case, some numerical method must be used. Many such methods, such as Rayleigh quotient method, Holzer's procedure, ploting value of determinant against assumed λ values, method of intensification or successive approximations, etc. are available. In any case, the procedure for obtaining periods and modes after obtaining the compatibility equations is the same as used when the floors are assumed rigid and does not involve any new principles. As such it is left out of consideration here. Just for comparison, the following table gives the time periods and modes for the frame under consideration as computed from above wherein actual girder flexiblility has been considered and as calculated on the assumption of rigid floors.

TABLE

		Rayleighs' method	Actual Stiffnesses				Girders taken Rigid			
		Ist mode	Ist mode	2nd mode	3rd mode	4th mode	Ist mode	2nd mode	3rd mode	4th mode
Period $\times \frac{W_0}{EK_0}$ sec.		6.3	6.31	2.22	1.23	0.687	3.44	1.56	0.96	0.715
Mode	x_4	3.18	3.73	−3.06	0.90	−0 19	2.67	−2.44	0.66	−0.23
Shape	x_3	2.72	2.96	0 33	0.72	0.10	2.06	0.24	−1.25	+0.99
	x_2	2.00	2.12	1.82	−1.90	−0.39	1.59	1.07	0.07	−1.39
	x_1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

CONCLUSION

Time periods and mode shapes of multistoreyed buildings are appreciably different when flexibility of flour is neglected by assuming them rigid as compared with columns and when it is taken into account. Hence actual stiffness of members should be considered for dynamic calcutations. Such computations may be carried out with sufficient accuracy without spending too much labour and time by using the method suggested in the paper.

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SEISMOLOGICAL NOTES

A. N. Tandon*

Earthquakes felt in and near about India during July - September, 1963.

Date 1963	Origin Time (G.M.T.) h. m. s.	Epicentre Lat. (°N)	Epicentre Long. (°E)	Region	Approx. depth (Km)	Magnitude	Remarks
(1)	(2)	(3)		(4)	(5)	(6)	(7)
5 Jul.	07 19 15.8 (U.S.C.G.S.)	27.7	92.1	Assam	33	4.2	—
10 Jul.	02 11 56.3 (U.S.C.G.S.)	36.5	71.8	Hindukush	33	4.9	Recorded at a number of Indian observatories.
13 Jul.	19 08 40 (SHILLONG)	24.5	71	Sind, (West Pakistan)	—	5.6 (New Delhi)	Recorded at almost all Indian observatories.
	19 08 39.1 (U.S.C.G.S.)	24.9	70.3	Pakistan	33	—	—
14 Jul.	10 51 40 (SHILLONG)	36	72	Hindukush	—	5.9 (New Delhi)	Recorded at a number of Indian observatories.
	10 51 42.7 (U.S.C.G.S.)	36.1	70.6	Hindukush	120	5.1	—
14 Jul.	14 48 28.4 (U.S.C.G.S.)	30.3	78.5	Northern India	33	4.8	Recorded at a number of Indian observatories.
1 Aug.	15 32 15	Near Delhi		Delhi - Punjab border	—	—	Epi: about 42 Km to the NW of Delhi, felt at Sonepat (Punjab).
1 Aug.	22 51 15	Near Delhi		Delhi - Punjab border	—	—	Epi: about 41 Km. to the NW of Delhi, felt at Sonepat (Punjab).
3 Aug.	14 48 31	Near Delhi		Delhi - Punjab border	—	—	Epi: about 42 Km. to the NW of Delhi, felt at Sonepat (Punjab).
3 Aug.	19 42 18	Near Delhi		Delhi - Punjab border	—	—	Epi: about 41 Km. to the NW of Delhi, felt at Sonepat (Punjab).
11 Aug.	19 40 52.2 (U.S.C.G.S.)	36.2	71.2	Hindukush	148	—	Recorded at Delhi.

Date 1963	Origin Time (G.M.T.) h. m. s.	Epicentre Lat. (°N)	Epicentre Long. (°E)	Region	Approx. depth (Km)	Magnitude	Remarks
(1)	(2)	(3)		(4)	(5)	(6)	(7)
12 Aug.	18 29 38.8 (U.S.C.G.S.)	25.3	62.7	Near coast of west Pakistan	33	5.2	Recorded at a few Indian observatories.
13 Aug.	07 03 49.6 (U.S.C.G.S.)	36.6	70.9	Hindukush	244	4.7	Recorded at a few Indian observatories
22 Aug.	03 05 08	Near Delhi		Delhi - Punjab border	—	—	Epc. about 40 Km. to the NW of Delhi, felt at Sonipat (Punjab).
2 Sept.	01 34 30 (New Delhi)	34	75	Near Srinagar (Kashmir)	—	5.5	Major property of damage in Kashmir Valley.
	01 34 31.6 (U.S.C.G.S.)	33.9	74.7	Northern India	44	5.1	79 people killed and 400 hundred injured at Badgam near Srinagar.
2 Sept.	22 25 52 (U.S.C.G.S.)	26.2	90.0	Assam - India	220	—	Recorded at some Indian observatories.
6 Sept.	01 58 19	Near Delhi		Punjab - Delhi border	—	3.2	Epc. about 60 Km. North West of Delhi.
9 Sept.	21 41 43.6 (U.S.C.G.S.)	31.3	72.1	West Pakistan	33	4.7	Recorded at many Indian observatories.
12 Sept.	10 35 11 (P-time Delhi)	Near Delhi		Punjab - Delhi border	—	—	Epc. about 40 Km. NW of Delhi, felt at Sonipat (Punjab).
19 Sept.	16 31 15 (U.S.C.G.S.)	31.0	66.8	Afghanistan - Pakistan border	37	4.2	Recorded at some Indian observatories.
20 Sept.	15 01 44	—	—	North West Uttar Pradesh	—	—	Felt at Garbying.
24 Sept.	13 27 06	Near Delhi		Delhi - Punjab border	—	—	Epc. about 39 Km. to the NW of Delhi.
27 Sept.	21 57 40	Near Delhi		—	—	—	90 Km. away from Delhi.

* Director (seismology), Meteorological Observatory, New Delhi.

LIST OF SEISMOLOGICAL OBSERVATORIES IN INDIA WITH THEIR INSTRUMENTAL EQUIPMENT

A. N. Tandon *

Station	Instruments in operation
1. Bokaro (23° 50′ N, 85° 48′ E)	2 Milne-Shaw seismographs 2 Wood-Anderson type torsion seismographs 1 Sprengnether microseismograph
2. Bombay (Colaba) (18° 54′N, 72° 49′ E)	2 Milne-Shaw Seismographs 1 Sprengnether microseismograph 1 Benioff vertical seismograph (short-period)
3. Calcutta (22° 32′ N, 88° 22′ E)	1 Milne-Shaw seismograph 1 Wood-Anderson type torsion seismograph 2 Omori-Ewing seismographs
4. Chatra (Nepal) (26° 50′ N, 87° 10′ E)	1 Benioff seismograph (Short-period vertical) 2 Milne-Shaw seismographs 2 Wood-Anderson type torsion seismographs 1 Wenner three-component accelerograph
5. Dehra Dun (30° 19′ N, 78° 02′ E)	2 Wood-Anderson type torsion seismographs 2 Milne-Shaw seismographs 1 Short-period Wilson-Lamison type vertical seismograph
6. Delhi (28° 41′ N, 77° 12′ E)	1 Short-period sprengnether microseismograph 2 Short-period Wood-Anderson horizontal seismograph 1 Three component short period Benioff seismograph 1 Three component long period Press-Ewing Seismograph.
7. Howrah (Govt. of West Bengal) (22° 33′ N, 88° 19′ E)	1 Three component Benioff seismograph (short period)
8. Hyderadad (Andhra Govt.) (17° 26′ N, 72° 27′ E)	2 Milne-Shaw seismographs
9. Kodaikanal (10° 14′ N, 77° 28′ E)	1 Milne-Shaw seismograph

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10. Madras (13° 00′ N, 80° 11′ E)	1 Sprengnether microseismograph
11. Poona (18° 32′ N, 73° 51′ E)	1 Sprengnether vertical seismograph (short period) 1 Wood-Anderson type torsion seismograph 1 Milne-Shaw seismograph
12. Port Blair (11° 40′ N, 92° 43′ E)	1 Milne-Shaw seismograph 2 Wood-Anderson type torsion seismographs 1 Sprengnether microseismograph 1 Benioff short-period vertical seismograph (Installed in January, 1963).
13. Sehore (23° 10′ N, 77° 05′ E)	2 Wood-Anderson type torsion seismographs
14. Shillong (25° 34′ N, 91° 53′ E)	1 Three component short-period Benioff Seismographs. 1 Three component long period Press-Ewing Seismograph.
15. Tocklai (Indian Tea Association) (26° 45′ N, 94° 46′ E)	1 Wood-Anderson type torsion seismograph,
16. Visakhapatnam (Andhra University) (17° 53′ N, 83° 18′ E)	1 Sprengnether horizontal microseismograph 2 Wood-Anderson type torsion seismographs.

BADGAM EARTHQUAKE OF SEPTEMBER 2, 1963

L. S. Srivastava*, S. E. Hasan*, R. S. Mithal*,
A. R. Chandrasekaran† and Jai Krishna†

INTRODUCTION

The valley of Kashmir was vigorously shaken by an earthquake which occured at about 7.10 hrs. (01 : 34 : 31 G.M.T.) on September 2, 1963. The shock had a magnitude of 5.5 and was felt over an area of approximately 100,000 square kilometres including practically the entire Kashmir valley.

About sixty-five persons were killed and many others were injured as a result of this earthquake. More than 2000 houses mostly of mud construction, collapsed and had to be completely demolished while about 5000 houses suffered partial damage. The villages worst affected by this earthquake are situated in Badgam Tehsil (Fig. 1) about 15 to 30 kilometres from Srinagar, the capital of Jammu and Kashmir State of India.

Considering the size of the shock and the nature of construction in the area, the damage would have been even more extensive, if the shock had occurred during night and had its focus somewhat shallow. The depth of focus for this shock was about 44 kilometres and hence the shaking was less intense but was felt over quite a large area causing general fright to the people.

HISTORY OF THE REGION

During the past century this region has been subjected to considerable seismic activity. Appendix I (Gutenberg and Richter, 1954) lists most of the large and important earthquakes that have occurred after 1904 and have caused considerable damage in Kashmir during the first half of this century. A big shock which has been well documented occurred on the 30th May, 1885 with its epicentre 20 kilometres west of Srinagar not far from the present shock. About 3000 persons lost their lives in this earthquake. This is a large number considering that the valley is thinly populated. Fig. 4 shows the meisoseismal area and first and second isoseismals of Jones (1885), which approximately correspond to intensities IX and X on the MM scale.

SEISMOLOGICAL DATA

The seismological data for this earthquake as recorded by the India Meteorology Department at their observatories is given below :—

Station	Approximate distance from epicentre in kilometres	Time of arrival of 'P' phase in G.M.T.			Time of arrival of 'S' phase in G.M.T.		
		h	m	s	h	m	s
Dehra Dun	510	01	35	41	01	36	33
New Delhi	625	01	35	55	01	36	59
Bombay	1650	01	38	07	01	41	07
Poona	1680	01	38	10	01	41	11
Madras	2325	01	39	16	01	43	11

Origin time : 01 h. 34 m. 31.6 s. G.M.T. of September 2, 1963

Epicentre : Lat. 33.8° North
Long. 74.7° East

Magnitude : 5.5 (Delhi)

Depth : about 44 kms.

GEOLOGICAL SETTING

The 'Vale of Kashmir' a saucer-shaped valley with

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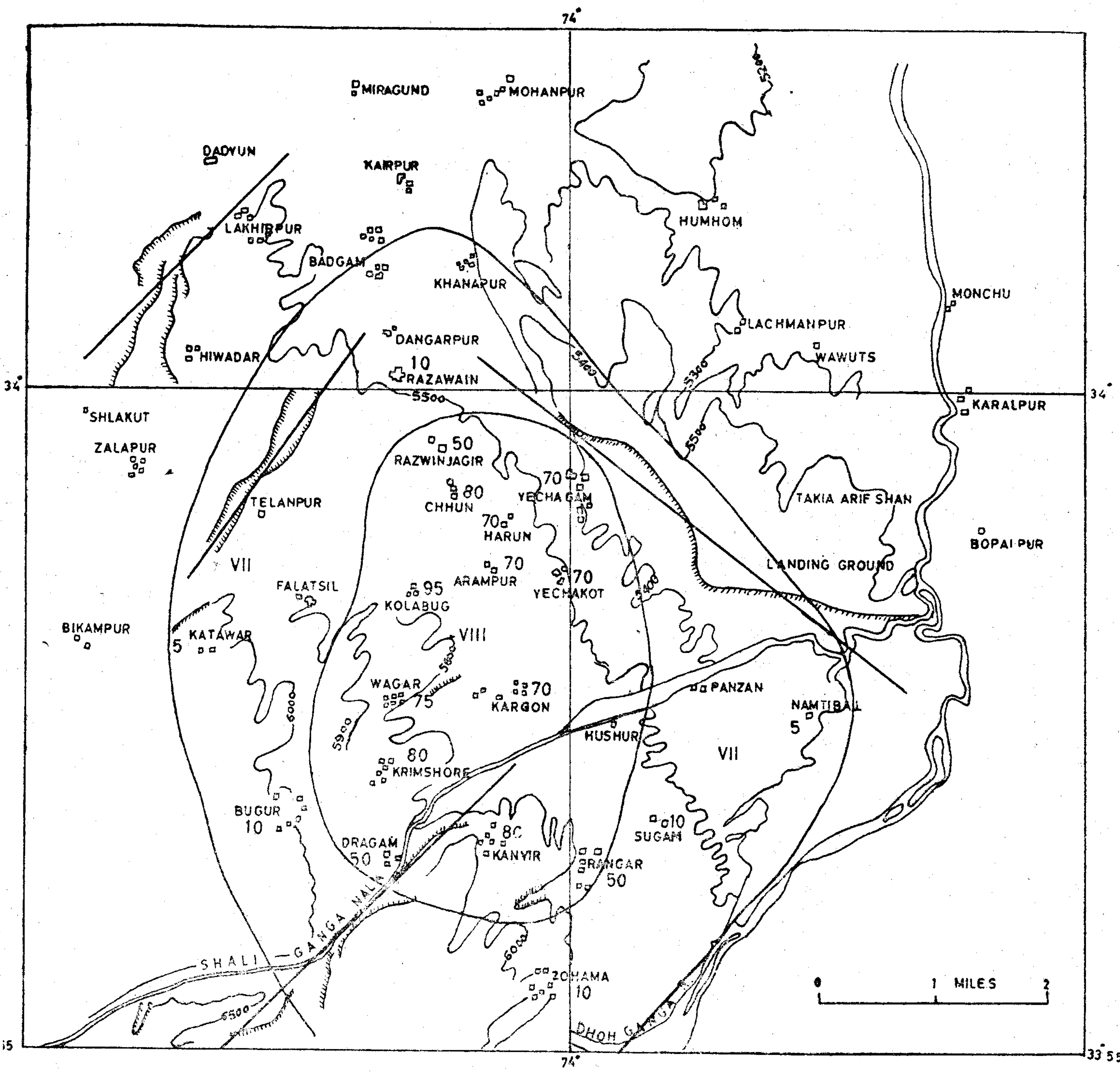
MAP SHOWING DAMAGE INTENSITY AND RELATIONSHIP WITH THE TOPOGRAPHY



THE FIGURES IN ARABIC NUMERALS INDICATE PERCENTAGE OF HOUSES COLLAPSED OR BADLY DAMAGED

THE ROMAN NUMERALS INDICATE INTENSITY ON M/M SCALE

Figure 1.—Map Showing Intensity of the Earthquake and its Relationship with the Topography. Straight lines show trend of escarpments.

a length of about 135 kilometres, a breadth of about 40 kilometres, situated at a mean height of 1500 metres above sea level, lies snugly between the Pir Panjal and the Great Himalayan ranges. Table 1, shows the general geological succession of rocks in Kashmir.

The Panjal range consists of high mountains (3600-4500 metres) cut into by deep ravines and precipitous defiles. In the Pir Panjal, a singularly well defined range of mountains extending from the Kaghan Valley to beyond the Ravi Valley, the mountains are composed of highly compressed and altered rocks of various ages from the Purana and Carboniferous to Eocene, and the axial zone consists of Permo-Carboniferous formations (Fig. 2). The range forming the north-eastern border of the valley consists of peaks, from 4,500 to 6,000 metres in height. These Great Himalayan ranges are composed of formations similar to those of the Pir Panjal (Fig. 2).

TABLE 1.

The table below gives the general succession of rocks in Kashmir (Modified after Wadia, 1957)

Age	Formations	Typical localities
Recent & Pleistocene	River terraces, moraines, Karewas (Upper)	River valleys, alluvia
Pliocene	Lower Karewas, Siwaliks	Foot hills
Miocene	Murree series	Murree hills
Eocene	Nummulites, Laki, Ranikot series	Pir Panjal
Cretaceous	Chikkim series, Spiti shales	Hazara and Ladakh
Jurassic	Spiti shales, Kioto limestone	Banihal, Amarnath
Triassic	Productus beds, Triassic shales with limestone and dolomites, Panjal trap	Lidar valley, Pir Panjal
Permian	Zewan beds, Productus shales	Pir Panjal and Lider valley
Carboniferous	Panjal lavas, slates and agglomerates, Lower Gondwanas - Gangamopteris beds	Pir Panjal Range, Zanskar Range Banihal
Devonian	Muth Quartzite	Lidar anticline
Silurian	Sandy shales, shaly sandstones and impure yellow limestone	Poonch
Ordovician	Quartzites, Limestones	Hemdawar
Cambrian	Cambrian of Hemdawar	Hemdawar
Archean	Dogra slates, Salkhala series, Gneisses and granulites	Cores of Dhauladhar, Zanskar

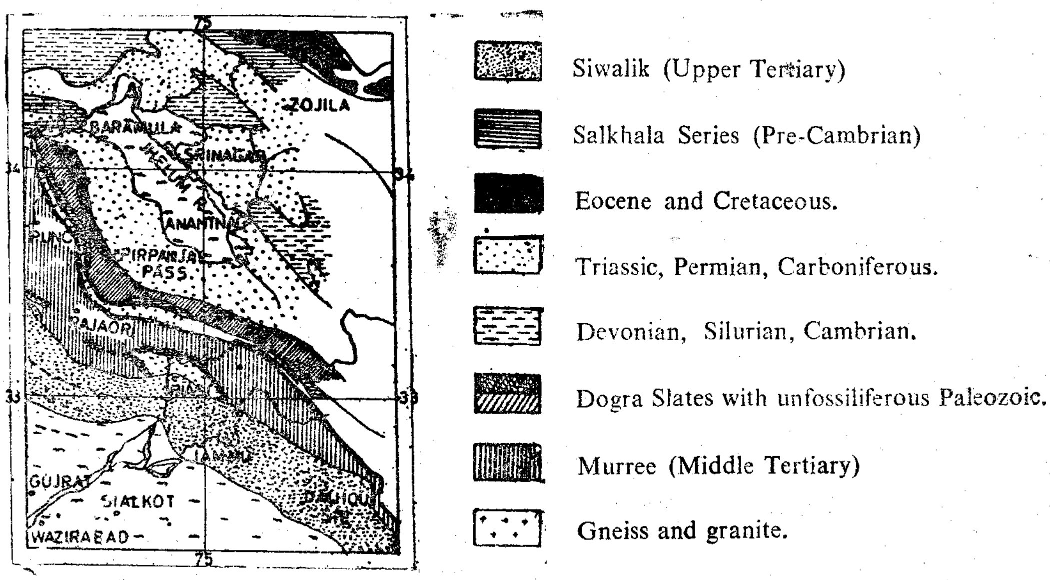


Figure 2.—Geological Sketch map of the Kashmir Valley (After D.N. Wadia).

The intervening 'Vale of Kashmir,' between the above ranges is filled up with the Karewa formations of Pliestocene to Recent age, which were formed in a sinking lake between two slowly rising mountains on either side. The Karewa beds, 1500 metres in thickness, consist of fine grained sand, loam, sandy-clay and gravelly conglomerate, with few lignite horizons. The Karewas, forming flat topped hillocks, plateaus, and terraces, are mostly horizontal or occur with low dips of 2° to 5°. On the flanks of Pir Panjal, where they are found at an elevation of about 3450 metres, these rocks are tilted and their dips vary from 2° to 20° away from the mountains. Dips of over 40° with some folding have also been reported recently.

As indicated above the 'Vale of Kashmir' has developed from a sinking lake between two slowly rising mountains. The big lake, in which the Karewa beds were laid is considered to have been formed by a barrier which was a spur of the mountains between Baramula and Rampur. This divide represents a fault scarp on the Kashmir side and the ancestral Jhelum cut back through this ridge and drained the lake and finally captured what now forms the upper Jhelum, which was originally part of the river Chenab (Krishnan, 1956). The presence of the river deposits at heights of 360 to 420 metres from the present bed level of Jhelum on this spur, and the Karewa beds on the flanks of Pir Panjal at an elevation of 3450 metres indicate that the valley has been subjected to vertical displacements from the time of uplift of the Pir Panjal upto the Present time. The Kashmir Vale thus appears to be a tectonic depression similar to the Gango - Brahamputra region (Mithal and Srivastava, 1959). The present drainage over the Karewa formations and the trend of the escarpments in the vicinity of Badgam (Fig. 1) both indicate a possible existence of criss cross faults in the area. Fig. 3 shows the rose diagram of the trend of the escarpments and

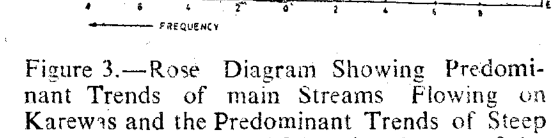


Figure 3.—Rose Diagram Showing Predominant Trends of main Streams Flowing on Karewas and the Predominant Trends of Steep Escarpments in the Meisoseismel area of the 2nd Sept. 1963, Badgam Earthquake

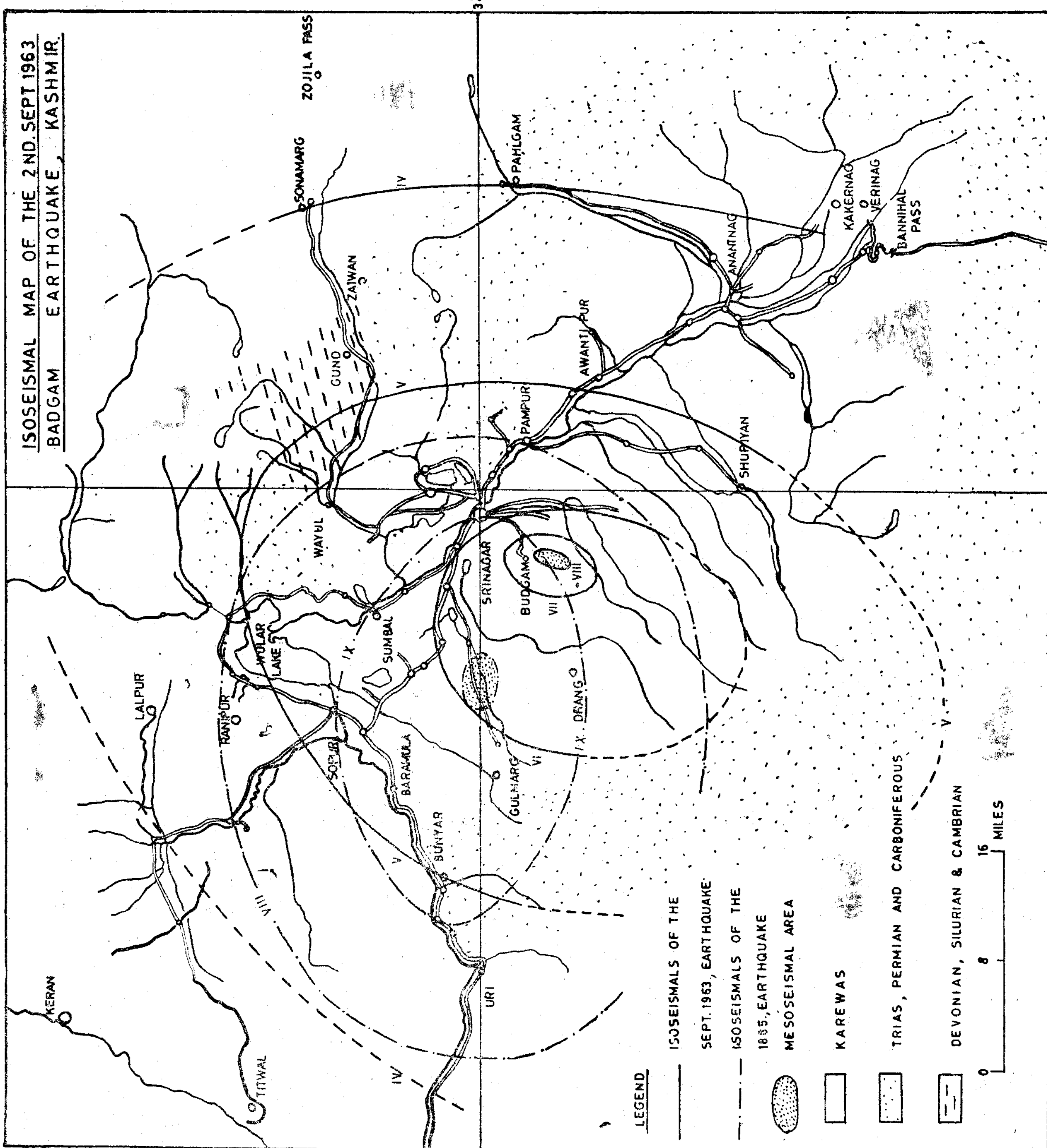


Figure 4.—Isoseismal Map of the 2nd September, 1963, Badgam Earthquake, Kashmir. The Intensities are on M.M. Scale (Modified Version, Richter, 1958).

the main streams flowing over the Karewa formations. As the drainage pattern of a continental region is controlled to a large extent, by the tectonic alignments of the formations of the area, the predominant trends N.N.W. and N.E. of the rose diagram indicate the existence of certain tectonic lineaments in these directions, which had probably controlled the angular drainage pattern of the area.

The Kashmir valley, thus, appears to be a subsided block with criss cross of faults along N.E. and N.N.W. directions. These faults appear to be still active and are responsible for the occasional earthquake shocks in the area. This is a tentative view of the authors and a detailed study of the drainage pattern from ærial photographs, and geological mapping of the area, supported by geophysical investigations will reveal the tectonics of the region.

INTENSITIES

Numerical ratings of intensities during the Badgam earthquake as given in fig. 4, are based on the modified Mercalli Intensity Scale (modified version, Richter, 1958) as applied to observed effects. Data on effects were obtained from reports, field survey and questionnaire card (Appendix II) coverage. The results of the observed effects are summarised in figure 4, where the Kashmir valley has been divided into zones by isoseismal lines. Maximum intenesity based on vibration effects during the earthquake did not exceed a weak eight (VIII). However, the intensities as indicated by some secondary effects of landslides at few places are of higher ratings upto IX. But such ratings are very local and even a few feet away the intensities are much lower and, therefore, these have not been taken into consideration in arriving at the different ratings of the isoseismals.

DAMAGE TO BUILDINGS

No major structures existed in the epicentral area. Most of them are of residential type—single, double or three storeyed buildings. They are predominantly of mud or lime masonry construction. The material used in the mud houses are blocks of dried mud moulded in-situ, each block measuring in length a metre or more and about half a metre or more thick. Pillars made of sun-dried or burnt bricks in mud mortar support the roof, and such blocks are used as filler walls. Almost in all houses, a gable ended roof, generally thatched, rests on pillars. The mud houses in the epicentral area completely collapsed. Figures 5a and 5b show such examples. Figure 6 shows damage to a mosque. This is a double storeyed structure with timber frames and brick-in-mud filler walls. There is a relative movement of the top storey with respect to the bottom storey and the filler walls have given way. Figure 7 indicates a structure made almost fully of timber. It did not collapse, although it got out of plumb and distorted.

The brick masonry houses existing in the area, suffered considerably less damage than mud houses. The walls of such houses consist of two 12 cm. thick brick shutterings with rubble, broken bricks and mud filled in between them, with total thickness of about half a metre. In such buildings the roof thus rests on 12 cm. thick shuttering walls. Such walls during the earthquake failed by buckling and warping (Fig. 8), which caused collapse of the upper portions and roofs of the houses.

In the town of Badgam, in addition to houses, there are a court building, a dispensary, a school and other community buildings. Most of these community buildings have been recently built. They were essentially made of random rubble masonry of brick construction in mud or lime mortar. The jambs and corners were supposed to have been built in lime mortar. This mortar, however, was of very poor lime. Figures 9a and 9b show damage to the court building. The vertical crack at the junction of the side wall has probably resulted from the horizontal bending of the side due to the earthquake jerk helped by small settlement of the corner. Other cracks were typically shear cracks. Figure 10 shows cracking on the floor due to vertical acceleration. This should be expected at a site so close to the epicentre. Figure 11 shows shear movement at the lintel level due to imperfect bond between the lintel and the wall. Figure 12 shows a collape of an interior wall which appears to have been loosely packed in rubble. Figure 13 shows cracking of chimneys even though they were built in good lime mortar, and emphasises the

Fig. 5 (a) —Collapse of mud houses in Kolabug village.

Fig. 6 —Collapsed mosque at Kolabug Village

Fig. 5 (b) –Collapse of mud wall and overtopping of mud block in Kolabug Village.

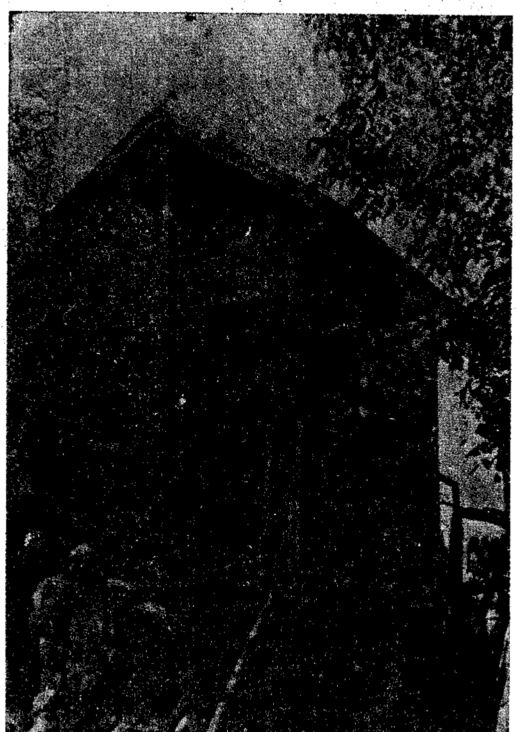


Fig. 7—New, centre of timber construction at Kolabug which suffered very little demage.

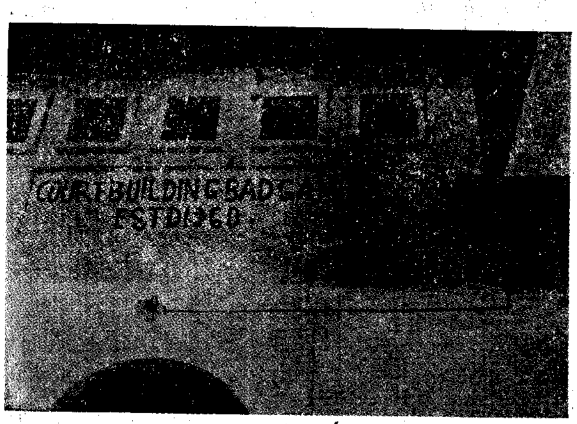


Fig 9 (a)—Court Building, at Badgam with crack in Pont wall.

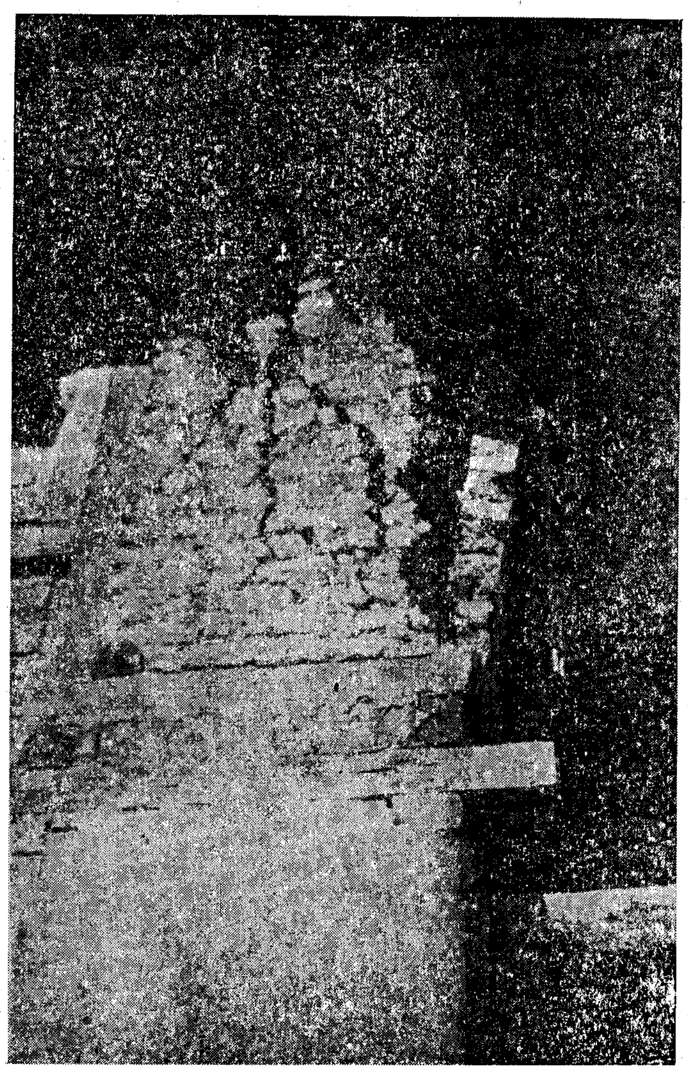


Fig. 8—Buckling of "brick shuttering" at first floor of a brick masonry house at Kolabug.

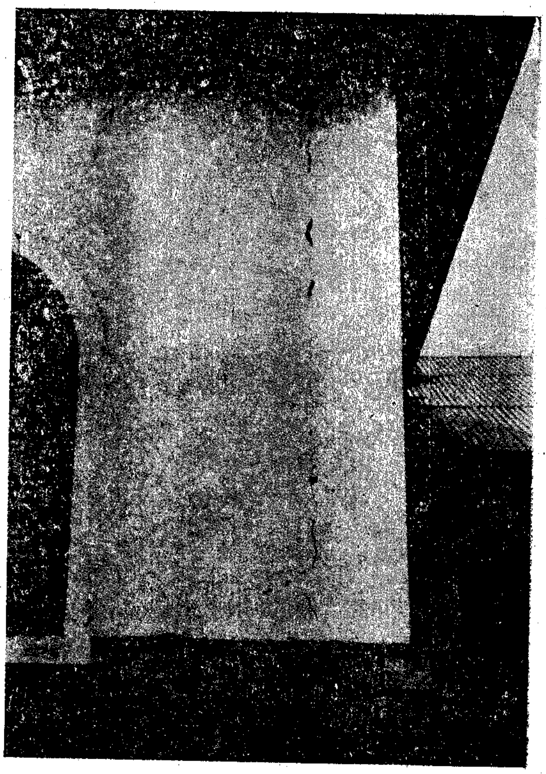


Fig. 9 (b)—Cracks along the junction of the front and side wall of the Court Building at Badgam.

Fig, 10—Cracking and Settlement of the corut at Building, Badgam.

Fig. 11--Crack at lintel in Dispensary at Badgam.

Fig. 12—Collapse of interior wall of the office of the Naib Tehsildar, Badgam.

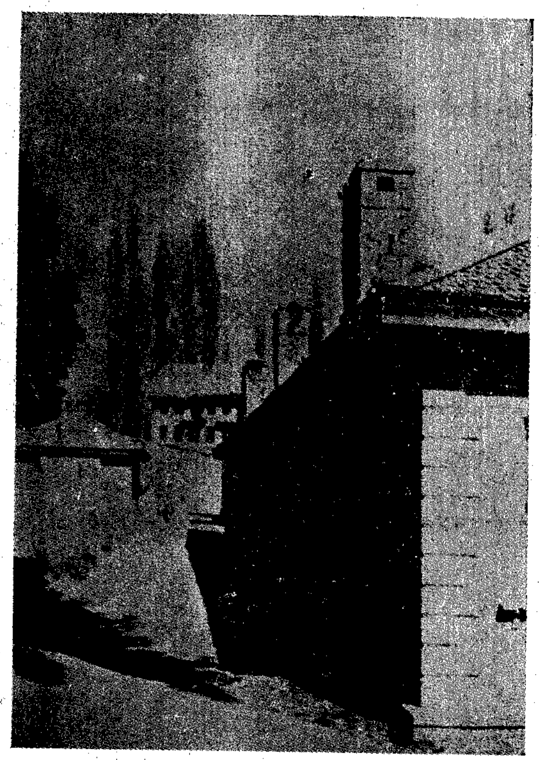


Fig. 13—Cracking of chimneys of the Dispen. sary at Badgam.

Fig. 14—A well constructed house near court Building, Badgam which suffered no damage.

importance of either omitting the cantilever projections or anchoring them suitably into the roofs with adequate reinforcement to take the horizontal forces. Figure 14 shows a house made of diagonally braced timber frames with masonry fillers which had absolutely no damage. This structure is located very close to the structures shown in Figures 9 to 13. This used to be the normal construction in the past.

Damage to structures in the city of Srinagar has not been severe. Some buildings had cracks in filler walls, some floors sunk, and in some others only the plaster had fallen off at places. In the city, the construction is generally of better quality than that described earlier. Poor construction would have been more severely damaged. No building constructed in cement mortar was affected. One of the other reasons of less damage was the depth of focus (44 kms.) of the shock. The waves passing through the soft alluvium lost bulk of their energy.

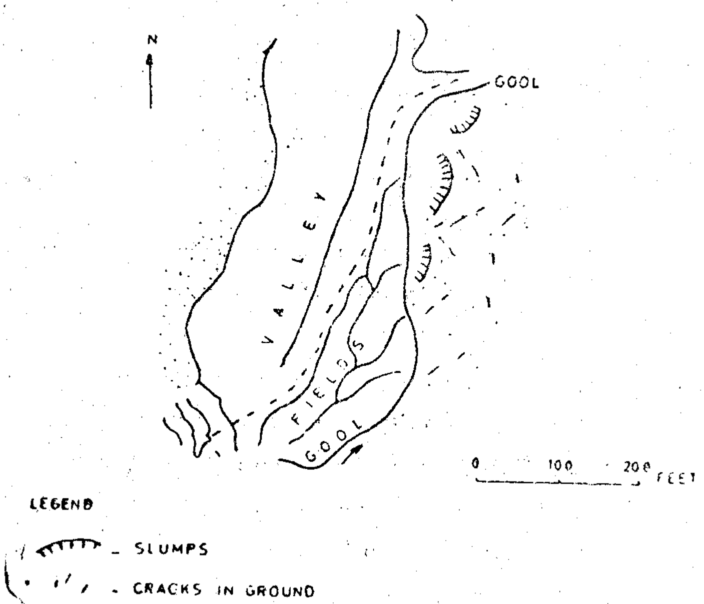


Figure 15.—Slump in Valley East of Chun Village. There is a "gool" (2 feet open unlined water channel) at the base of the slope.

Land Slides and Fissuring :

No large scale fissuring of the ground or landslides occurred during the earthquake. However, isolated failures of potentially unstable slopes were noted by the authors in the area. The landslides in the valley east of Chun village occured in greenish buff silty sands with development of cracks (Fig. 15) upto two inches wide. A slide was also noted in a small area of 10 metres by 50 metres in fully saturated buff coloured silty sand northeast of Arampura (Fig. 16). A retaining wall (Fig. 17) made of rubble stone in lime-sand mortar collapsed in Dragam. All these failures indicate that even a small earthquake shock can be disastrous for the stability of poorly designed retaining walls and steep slopes.

BUILDING CONSTRUCTION IN THE REGION

The older structures constituted of compacted earth or brick construction duly reinforced by timber framing. Provision of the horizontal timber members at different levels and also diagonal members in walls was considered as typical Kashmir construction in the old days. The earthquakes in the past have shaken such houses but were not able to cause collapse because of the timber framing. Some of the houses have existed for more than a hundred years with some of the walls out of plumb but still intact due to the reinforcement thus provided. This practice has, somehow, gone out of use recently because of the fact that timber prices have gone up considerably and it is expensive to provide timber framing. Further if, to compensate the expense on timber framings, the wall thickness is small, the severe winter that this valley experiences for nearly six months does not leave such houses comfortable. Thus, to balance the cost, thicker walls without timber framing have become more common. Consequently the damage to the new buildings is heavy compared with the old buildings in the same locality. This is a problem which is governed by economic considrations but it is generally understood in the area that timber framing or reinforcing the walls with steel would be desirable from the point of earthquake resistance of buildings.

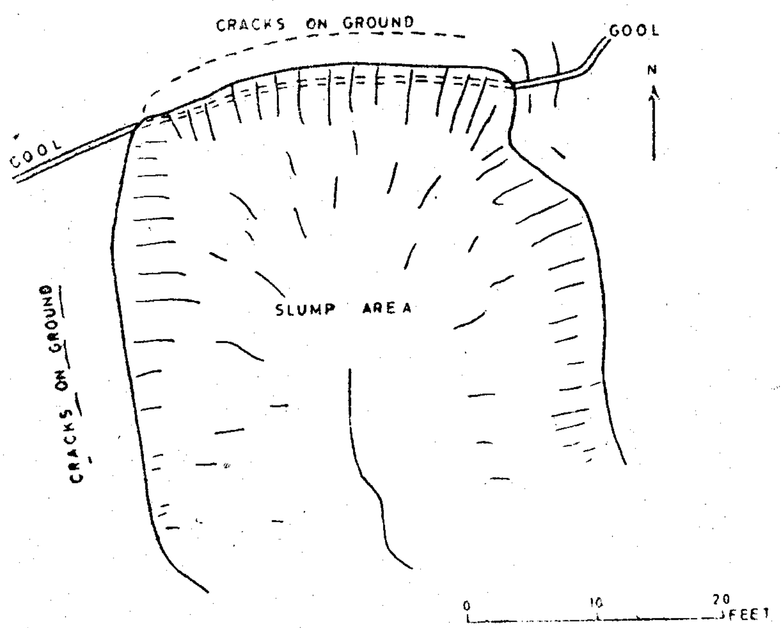


Figure 16.—Slide North East of Arampura. The Soil Was fully Saturated due to Seapage from the " Gool."

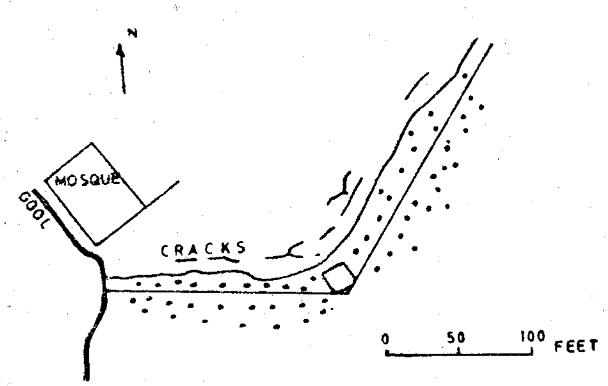


Figure 17.—Failure of Rubble Stone Retaining Wall at Dragam. The Back Fill was fully Saturated due to Seepage from the 'Gool"

RECOMMENDATIONS FOR FUTURE CONSTRUCTION

It is possible to strengthen each of the categories of construction so that they could withstand the lateral forces better than at present. It is, of course, not possible to have the same degree of lateral resistance in all the types of construction for a similar percentage increase in costs. The recommendations contained here aim at providing the best return in terms of additional resistance against earthquake force for more or less the same precentage increase of cost.

In mud construction, it is recommended that split bamboo jafris or timber be use d as reinforcement. The use of compacted earth at optimum moisture content will improve its resistance. Considerable improvement is possible if a bituminous material is used to cover the plinth course so that water does not seep into the wall. A water soaked mud wall has no strength at all.

A composite construction with corners and jambs made of brickwork in good lime mortar or 1 : 6 cement mortar will greatly strengthen the mud houses and prevent their collapse in a moderate size shock. The bamboo jafri or timber frame should, of course, be retained even if the corners and jambs are made in brickwork. Each of these steps will increase the cost to a certain extent, but much more than that is the question of this information being available to the villagers. If these devices are known, they will gradually be used, cutting down losses in future shocks.

If brick costruction in cement mortar is adopted, it is recommended that at least 1 : 6 cement mortar be used. Mortars poorer than 1 : 6 cement mortar have their strength reduced rapidly with the decrease of cement content. In addition, steel reinforcement in jambs and corners in the vertical direction adds to the resistance greatly. If the lintels over doors and windows are extended round the whole structure as a band with the same reinforcement as in the lintel throughout even without extending concrete and embedding steel in brickwork only, the building gets framed with the vertical steel at corners and jambs. Such construction will have resistance very similar to that of a reinforced concrete frame and will stand even strong earthquakes. In areas where earthquakes are frequent this would suggest a very useful form of construction short of a steel frame construction. This type of reinforced brick construction can be achieved within 10 to 12% of the cost of a brick building.

If a structure has more than one storey, then the reinforcement in the corner of the lower storey should be extended through the upper storeys also.

Wherever possible, the old Kashmir type construction may not be discarded. If a structure is made of Reinforced concrete or steel frame construction, a proper allowance could be made for lateral forces. This would, however, be very expensive compared with reinforced brick construction.

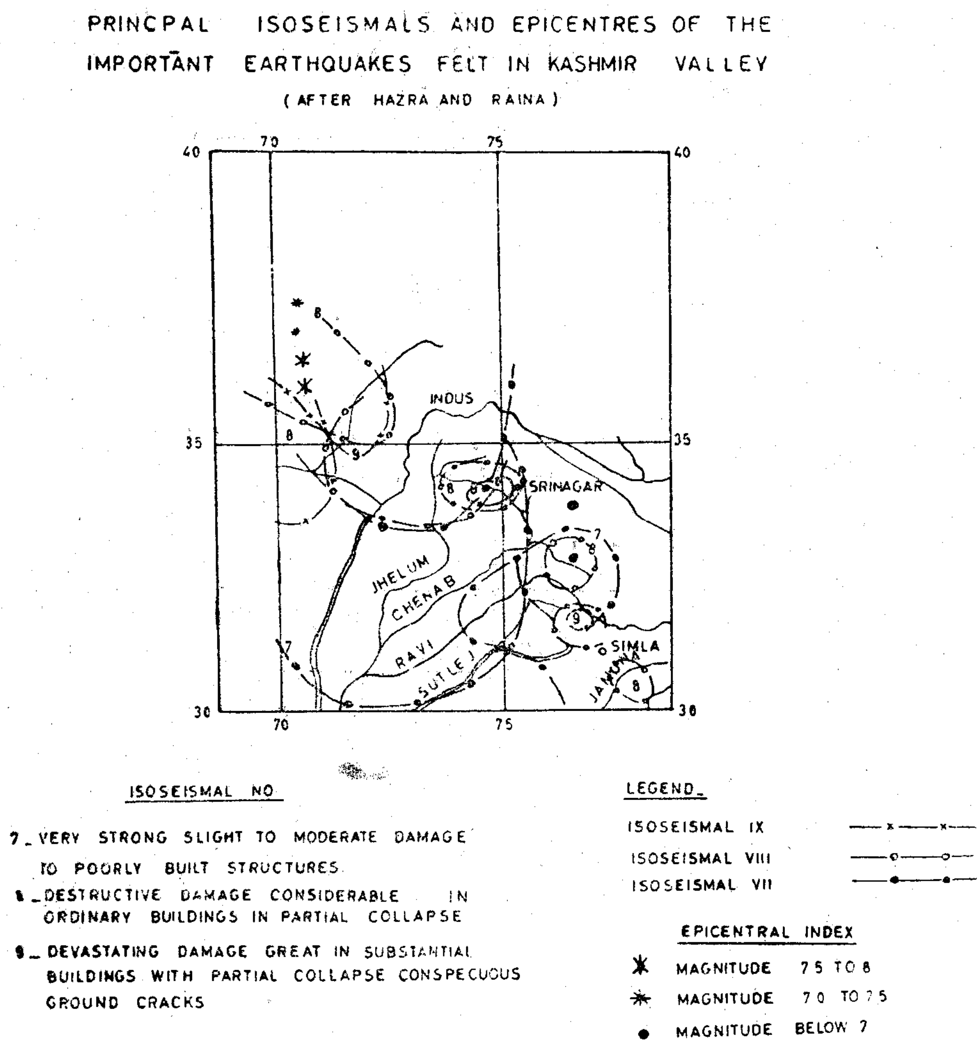


Figure 18.—Principal Isoseismals and Epicentres of the important Earthquakes felt in Kashmir Valley.

Seismicity of the Area :

Kashmir valley is one of the active seismic zones of India. Fig. 18 shows the intensities of the important large earthquakes that have affected the valley. The maximum intensity of X on the M.M. scale have been noted in the past and considerable loss of life and property has taken place. Tectonic movements are still active in the area, and care should be exercised in the design of structures. If the recommendations included in this paper are adopted, damage in future shocks will be cut down considerably.

ACKNOWLEDGEMENTS

The Meteorological Department of the Goverement of India supplied the seismological data. The Deputy Commissioners of Srinagar, Baramula and Anantnag assisted in the collection of information through questionnaire. The officials of the Public Works Department and the Directorate of Geology & Mining of the J & K State fully assisted and gave all necessary informations.

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APPENDIX I

	Date	Time (G.M.T.)	Approximate Epicentre	Richter Magnitude	Depth of focus	Max. Intensity M/M Scale
1.	April 4, 1905	00 : 50 : 00	33N, 76 E	8.00	Shallow shock	X
2.	April 13, 1907	17 : 57 : 03	36 1/2N, 70 1/2E	7±	Large intermediate shock	VIII
3.	Oct. 21 1907	04 : 23 : 06	38 N, 69E	8.00	Shallow shock	X
4.	Oct. 23,1908	20 : 14 . 01	36 1/2N 70 1/2E	7.00	Large intermediate shock	VIII
5.	Oct. 24, 1908	21 : 16 : 06	36 1/2N 70 1/2E	7.00	,,	VIII
6.	July 7, 1909	21 : 37 : 50	36 1/2N 70 1/2E	7.75	,,	VIII
7.	Feb. 18, 1911	18 : 41 : 03	40N, 73E	7.75	Shallow shock	IX
8.	July 4, 1911	13 : 33 : 26	36N, 70 1/2E	7.60	Large intermediate shock	IX
9.	April 21, 1917	00 : 49 : 49	37N, 70 1/2E	7.00	,,	VIII
10.	Nov. 15, 1921	20 : 36 : 38	36 1/2N, 70 1/2E	7.75	,,	VIII
11.	Dec. 6, 1922	13 : 55 : 36	36 1/2N, 70 1/2E	7.5±	,,	VIII
12.	Oct. 13, 1924	16 : 17 : 45	36 N, 70 1/2E	7.30	,,	VIII
13.	Feb. 1, 1929	17 : 14 : 26	36 1/2N, 70 1/2E	7.10	,,	VIII
14.	Nov. 14, 1937	10 : 58 : 12	36 1/2 N, 70 1/2E	7.20	,,	VIII
15.	Feb. 28, 1943	12 : 54 : 33	36 1/2 N, 70 1/2E	7.00	,,	VIII
16.	Sept. 27, 1944	16 : 25 : 02	39N, 73 1/2 E	7.00	Shallow shock	VIII
17.	Nov. 2, 1946	18 : 28 : 25	41 1/2N, 72 1/2E	7.60	Large intermediate shock	VIII
18.	March 4, 1949	10 : 19 : 25	36N, 70 1/2E	7.50	,,	VIII
19.	July 10, 1949	03 : 53 : 36	39N, 70 1/2E	7.60	,,	VIII