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DYNAMIC CHARACTERISTICS OF A SHEAR WALL-FRAME COMBINATION†

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INTRODUCTION

To find the dynamic response of a building it is essential to know its dynamic characteristics like natural frequency, mode shape and the modal damping coefficient. Various theoretical methods are available for finding natural frequencies and mode shapes. The accuracy of these theoretical methods together with the validity of assumptions can only be assessed by comparing the theoretical predictions with experimental results. It is found on a review of available literature that very few experimental data on dynamic testing is available. This paper deals mainly with the experimentation of natural frequencies of a perspex model of a six-storey building consisting of frames combined with shear walls.

The straight forward approach of finding natural frequencies and mode shapes of multi-storey buildings based on the concept of 'shear structure' yields a shape of elastic line with vertical tangents at floor levels, which is equivalent to the assumption that the rotations at the ends of the vertical member are zero. However, 'such simplification is not always admissible because the actual shape of an elastic curve of the structure at any instant will not be vertical at the floor levels but will be more like a cantilever's deflected curve, that is, the end sections of the elastic joints in each storey are not restrained to fixity of the floors. A more accurate dynamic analysis of the multistorey building as against the "shear structure" hypothesis can be made by assuming that the masses of the multi-storey are lumped at the floor level and are vertically connected by weightless elastic joints having two degrees of freedom (rotation and translation) at each joint.

Clough, Wilson and King[1] gave a computer solution of analysing large multi-storey buildings which also included shear walls. The programme, which was based on stiffness method took into account flexural, shear, and axial deformations of the member. In addition to reducing the solution time significantly, this approach has a major advantage in that the lateral stiffness matrix which is obtained may be used for finding the vibrational characteristics of buildings or in doing a direct dynamic analysis.

METHOD OF ANALYSIS

The method suggested in this paper for finding the complete dynamic response of a structure is similar to that of Clough, Wilson and King[1]. Direct stiffness method is used

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to formulate the stiffness matrix of the structure. If the structure is laid out in a rectangular grid pattern in plan as shown in Fig. 1. then for finding the stiffness of the structure in the considered direction, the structure is separated into two distinct systems[3], shear wall and frame system as shown in Fig. 2. linked together by link beams. This idealised combination of wall and frame system connected by link beams as shown above can be used to find the dynamic characterstics of the complete structure. The various assumptions made in the analysis are:-

(i) The material of the structure is perfectly elastic and only small deformations are considered.

(ii) For beam and column elements only flexural deformations are allowed, but for the shear wall, both flexural and shear deformations are taken into account.

(iii) The floor diaphragms are rigid in their own plane, but have no stiffnesses normal to this plane Thus, each floor level is constrained to translate without rotation and each vertical member is subjected to same displacement in the considered direction at any storey level.

(iv) Each shear wall is treated as a vertical flexural element in which plane horizontal sections remain plane as the wall deflects.

(v) Shear walls and columns continuously extend from base to top and beams from side to side. Minor variation in regularity from omission of members at any storey level can be allowed for by assuming zero stiffness for the omitted members.

(vi) The direction of the ground motion coincides with one of the main geometrical direction of the building ground plan.

(vii) For dynamic anlysis, the masses of the multi-storey are lumped at the floor levels thus yielding a diagonal mass matrix.

(viii) Damping effects are neglected for finding the natural frequencies and mode shapes of the structure.

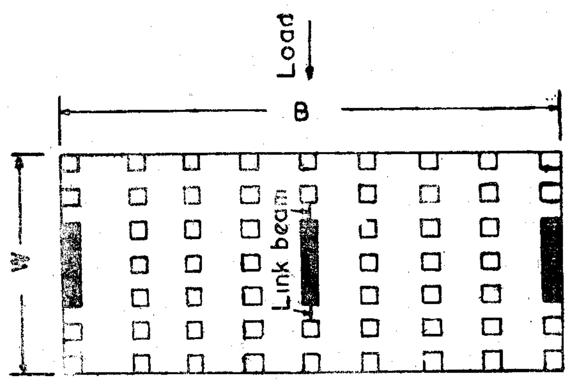


Fig. 1 Floor Plan

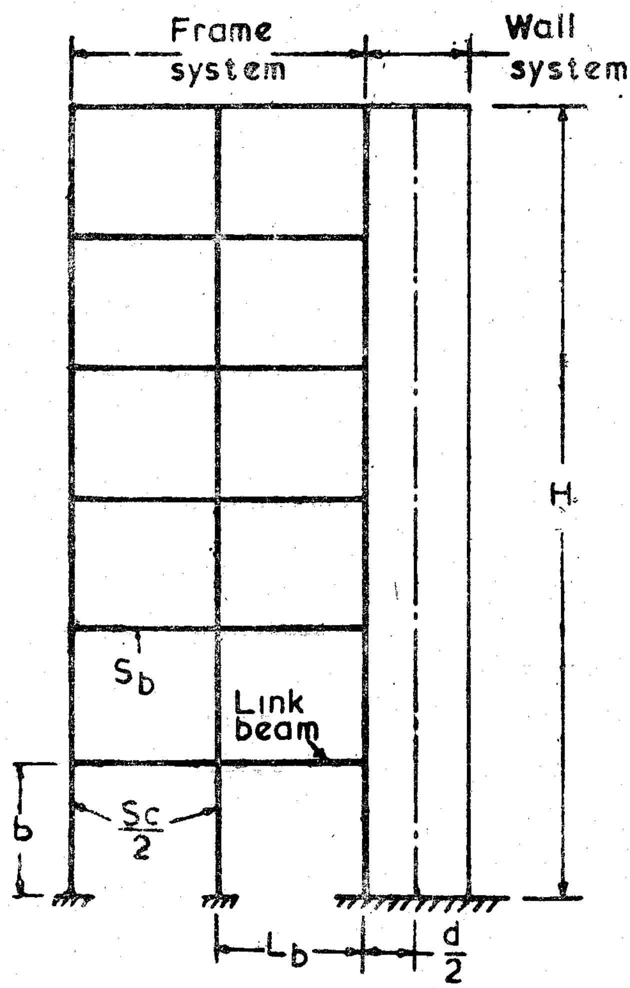


Fig. 2 Idealised Structure

BASIC MEMBER EQUATIONS

The load displacement relationship for a member having two degrees of freedom at each node point as shown in Fig. 3 (a) is given by the matrix,

$$(K_1) = \frac{EI}{L}\begin{bmatrix} 4 & & & \text{symmetrical} \\ -\frac{6}{L} & \frac{12}{L^2} & & \\ 2 & -6/L & 4 & \\ +\frac{6}{L} & -\frac{12}{L^2} & \frac{6}{L} & \frac{12}{L^2} \end{bmatrix}$$

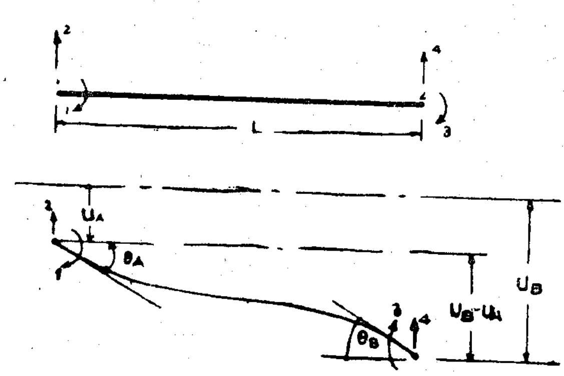


Fig. 3 (a) Beam element subjected to moment and shear at each node point.

when shear deformations are neglected. If the effect of both flexural and shear deformations is included then the modified stiffness matrix $[\bar{K}_1]$ is given by

$$(\bar{K}_1) = \begin{bmatrix} C & & & \text{symmetrical} \\ \frac{-(C+D)}{L} & \frac{2(C+D)}{L^2} & & \\ D & \frac{-(C+D)}{L} & C & \\ \frac{(C+D)}{L} & \frac{-2(C+D)}{L^2} & \frac{(C+D)}{L} & \frac{2(C+D)}{L^2} \end{bmatrix}$$

where

$$C = \frac{2EI}{L}\,\frac{(2+\lambda)}{(1+2\lambda)}$$

$$D = \frac{2EI}{L}\left(\frac{1-\lambda}{1+2\lambda}\right)$$

and $$\lambda = \frac{6EI}{L^2 \bar{A} G}$$

where λ is called the shear flexibility factor and $\bar{A}$ is the effective shear area.

If the shear wall is replaced by uniform line elements having the same stiffness and, placed along its centroidal axis then the effects of finite width of shear wall on the link beam, connecting frame and wall systems, will be equivalent to that produced by an end gusset fixed on the link beam as shown in fig. 3 (b). The length of gusset is equal to the distance from the end of the link beam to the centroidal axis of the wall to which it is attached. The gusset end behaves as a rigid member.

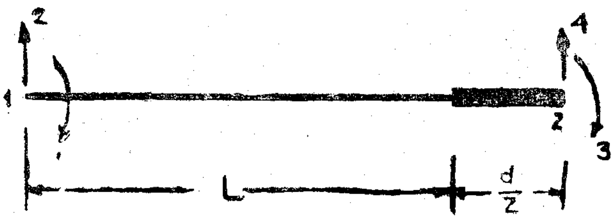


Fig. 3 b

Consider a typical link beam with rigid end gusset as shown in Fig. 3 (b) subjected to two degrees of freedom at each node point.

The stiffness matrix, $[K_2]$ defining the load deflection relationship of the above modified beam element is given below,

$$K_2 = \frac{EI}{L}\begin{bmatrix} 4 & & & \\ -6/L & \frac{12}{L^2} & & \\ \frac{6}{L}(d/2)+2 & -\frac{6}{L}+\frac{12}{L^2}(d/2) & 4+\frac{12}{L}\left(\frac{d}{2}\right)+\frac{12}{L^2}\left(\frac{d^2}{4}\right) & \\ \frac{6}{L} & \frac{12}{L^2} & \frac{6}{L}+\frac{12}{L^2}(d/2) & \frac{12}{L^2} \end{bmatrix}$$

SYNTHESIS OF THE STIFFNESS MATRIX FOR THE IDEALISED FRAME SYSTEM

Consider an idealised frame having m storeys and n-1 bays, subjected at each storey level i, to a lateral point load F_i. These loads will produce lateral displacement $\triangle i$ and rotation θ of each joint as shown Fig. 3 (c).

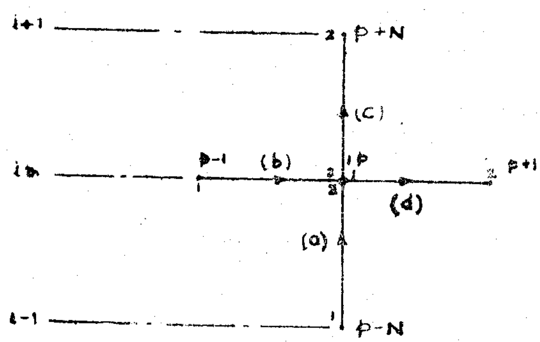


Fig. 3 c

From compatibility of deformations we obtain

$$(\theta_2)_a = (\theta_2)_b = (\theta_1)_c = (\theta_1)_d = (\theta_p)$$
$$(\theta_1)_a = \theta_{p-N} = (\theta_1)_b = \theta_{p-1}$$
$$(\theta_2)_c = \theta_{p+N} \quad , \quad (\theta_2)_d = \theta_{p+1}$$
$$(u_2)_a = (u_1)_c = \triangle_i$$
$$(u_1)_a = \triangle_{i1} \quad , \quad (u_2)_c = \triangle_{i+1} \tag{1}$$

The moment equilibrium applied at the joint, p, yields

$$(M_2)_a + (M_2)_b + (M_1)_c + (M_1)_d = 0 \tag{2}$$

Also for horizontal equilibrium

$$\Sigma(\,(S_1)_a - (S_2)_c\,) = F_i \tag{3}$$

Substituting the values of the moments and shears in equations (2) and (3) for moments and shear equilibrium at each joint, we will get a set of equations which may be expressed in matrix form as [6]

$$\left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{21} & K_{22} \end{array}\right] \left\{\begin{array}{c} \theta \\ \hline \triangle \end{array}\right\} = \left\{\begin{array}{c} \theta \\ \hline F \end{array}\right\} \tag{4}$$

where K_{21} is equal to K_{12}^T

The above equation can be written as

$$[K]\{\triangle\} = \{P\}$$

or

$$\{\Delta\} = [K]^{-1}\ \{P\} = [F]\ \{P\}$$

or

$$\left\{\frac{\theta}{\Delta}\right\} = \left[\begin{array}{c|c} F_{11} & F_{12} \\ \hline F_{21} & F_{22} \end{array}\right] \left\{\frac{O}{F}\right\} \tag{5}$$

The submatrices of (E) being of the same order as the corresponding submatrices of [K] matrix in equation (4).
Equation (5) yields

$$\{\Delta\} = [F_{22}]\ \{F\}$$

or

$$\{F\} = [F_{22}]^{-1}\ \{\Delta\} \tag{6}$$

The equation (6) relates the lateral loads applied at each storey level with the storey level displacements. Thus $[F_{22}^{-1}] = [\overline{K}]$ defines the stiffness matrix of the size $n \times n$: where n is the number of storeys.

It can be shown that $(K) = (K_{22}) - (K_{21})\,(K_{11}^{-1})\,(K_{12})$.

A computer programme was written which finally gave the value of (K) matrix for direct use in the dynamic analysis. For the six-storey problem discussed in this paper, the inverse matrix, $[K_{11}]$, was determined directly. Since in tall buildings this matrix $[K_{11}]$ is generally banded and is of high order, it is more efficient to find the product $[K_{11}]^{-1}\,[K_{12}] = [X]$ by the solution of equations $[K_{11}]\,[X] = [K_{12}]$ using some elimination procedure

DETERMINATION OF NATURAL FREQUENCIES AND MODE SHAPES OF VIBRATION.

The equation of motion of a freely vibrating, undamped system can be written in matrix form as

$$[m]\,\{\ddot{q}\} + [K]\,\{q\} = 0 \tag{7}$$

If the system is vibrating in one of its normal modes then we can write

$$\{\ddot{q}\} = -\,p^2\ \{q\} \tag{8}$$

where p is the angular natural frequency of vibration of the structural system vibrating in one of its normal modes. [m] is the diagonal mass matrix because masses are assumed to be lumped at each storey level. Substituting from equation (8) into (7), one gets

$$[K]\,\{q\} = p^2\,[m]\,\{q\} \tag{9}$$

If we premultiply each side of equation (9) by $[m^{-1}]$, we obtain

$$[m]^{-1}\,[K]\,\{q\} = p^2\,\{q\}$$

or

$$[L]\,\{q\} = p^2\,\{q\}$$

where

$$[L] = [m^{-1}]\,[K] \tag{10}$$

Equation (10) represents the eigenvalue problem in which the quantities p^2 are the eigenvalues and the vector {q} defines the eigen vectors or the mode shapes.

The eigenvalue problem has been solved by using Matrix Iteration procedure. As stiffness matrix is used in formulating the eigenvalue problem, the iteration converges to the highest value of p^2 or to the highest mode first. The matrix iteration procedure was programmed in Fortran 1900 using an accuracy of 0.000001 for convergence.

For the six-storey perspex model (see fig. 5), the various cross-sectional properties are given below.

Moment of inertia of all girders $= 0.00455\ \text{in}^4$
Moment of inertia of all columns $= 0.00455\ \text{in}^4$
Moment of inertia of shear wall $= 0.96\ \text{in}^4$
E for perspex $= 4.33 \times 10^5$ psi.

The $(\bar{K})$ matrix, relating lateral forces at each storey level with the storey level displacement, Δ i, for the model as obtained from computer analysis is given below.

$$(\bar{K}) = 10^4 \begin{bmatrix} 8.65718 & & & & & \\ -6.65222 & 9.28221 & & \text{symmetrical} & & \\ 2.76680 & -6.92088 & 9.37485 & & & \\ -0.721082 & 2.82513 & -6.90120 & 9.20256 & & \\ 0.177104 & -0.692433 & 2.65284 & -6.21984 & 6.60023 & \\ -0.0292698 & 0.114349 & -0.437438 & 1.67468 & -2.48354 & 1.15533 \end{bmatrix} \text{lbs./in.}$$

The diagonal mass matrix is given by

$$(m) = 10^{-4} \begin{bmatrix} 11.22 & & \text{symmetrical} & & & \\ & 9.35 & & & & \\ & & 9.35 & & & \\ & & & 9.35 & & \\ & & & & 9.35 & \\ & & & & & 6.26 \end{bmatrix} \text{lbs./sec}^2.$$

The theoretical natural frequencies and mode shapes as obtained from Matrix iteration procedure from the computer are given below.

No.	1	2	3	4	5	6
p^2	0.4744x10⁶	2.563x13⁶	15.17x10⁶	52.66x10⁶	134.7x10⁶	259.0x10⁶
mode shape	1.000	1.000	1.000	1.000	1.000	1.000
	2.644	2.049	1.121	0.00898	−1.077	−1.901
	4.445	2.231	−0.0507	−1.001	0.0776	2.212
	6.061	1.193	−1.080	0.1366	0.994	−1.967
	7.377	−0.7931	−0.576	0.894	−1.080	1.182
	8.480	−3.15	1.208	−0.7393	0.5715	−0.494

The mode shapes are plotted in Fig. 8.

EXPERIMENTAL TESTS

The validity of assumptions can only be assessed by comparing the theoretical predictions with the experimental results. Very little experimental data in published literature is available on resonance testing of multi-storey frames. The authors have carried out resonance tests on a perspex model of a six-storey building consisting of frames combined with a shear wall as shown in fig. 4 in order to determine—

(i) the natural frequency and

(ii) the mode shape.

In this method, the system is excited harmonically and the amplitude and the phase angle of various points on the structure are measured. Depending upon the physical quantities which are measured, and the way in which the experimental data is plotted, there are various techniques available by means of which the plots may be analysed. The peak amplitude method is one of the most widely used techniques for lightly damped systems but has a serious drawback in that no account is taken of changes in the phase lag of the displacement behind the exciting force and this is most predominant when the system is passing through resonance. It is also worthwhile to point out that the peak amplitude can be affected by the presence of vibrations of the supporting structures components. This does not always give the true natural frequency as discussed in our earlier paper[4]. The natural frequency can also be obtained from the analysis of a phase angle plot. The phase angle plot yields as much information regarding the natural frequency and damping as the peak amplitude but is not influenced by the presence of vibrations of the supporting structure's components. Kennedy and Pancu[2] proposed a method in which they simultaneously account for the variation of the amplitude and phase angle with the frequency. Instead of plotting two separate graphs, one for the peak amplitude and the other for the phase angle, the amplitude and the phase angle are plotted simultaneously in an Argand plane. The principal author of this paper has discussed the merits and demerits of different techniques available for the determination of natural frequencies in his earlier paper [5].

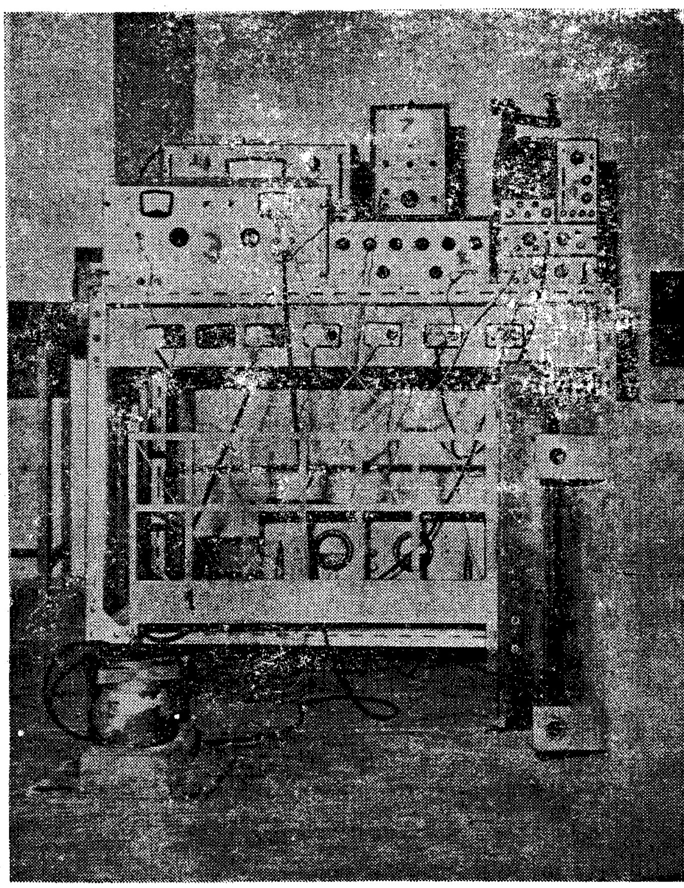


Fig 4. Forced Vibration Set–up
1. Model of the multistorey building. 2. Muirhead decade oscillator 3. Power supply unit to oscillator 4. Phasemeter 5. Oscillator 6. Oscilloscope (double beam) 7. Voltmeter. 8. Heavy concrete block.

EXPERIMENTAL SET-UP

A rig was designed to hold the frame which was fixed at the base. The supporting structure consisted of a heavy concrete block with an I section embeded in it as shown in fig 4. The model was sand witched between to

$4'' \times 4'' \times 3/8''$ angles fixed to the flat surface of the I section through high tensile bolts.

The multi-storey frame as excited harmonically by a Goodman electrodynamic vibrator model No. V50 driven through a Power amplifier by a Muir head decade oscillator K-126-A. It was assumed that the force exerted by the vibrator was proportional to the input voltage, the value of this was maintained constant throughout the single observation for a mode.

The response signal at the top point was picked up by an accelerometer. The output signal from the oscillator was also fed to oscilloscope. The estimate of natural frequencies was first obtained by feeding the response and, the reference signal from the exciter on to the oscilloscope until a 90° phase difference was obtained. The frequencies corresponding to 90° phase angle were defined as the natural frequencies of the frame. After the natural frequencies of the frame had been estimated, the test was repeated, but this time by directly feeding the response and the reference signal to the phasemeter (Airmec type 206) through voltmeter. The amplitude and the phase angle were measured for a range of frequencies of excitation.

RESULTS AND CONCLUSIONS

Initially the readings were taken at an interval ranging from 10 to 5 cps. but this interval was reduced to 1 c/s in the vicinity of the natural frequency. Because in the vicinity of the natural frequency the phase angle and the amplitude variation become very sensitive to frequency. Fig. 5 shows the vector plot around the 2nd, 3rd, 4th and 6th natural frequencies of the mode. The exciting force was applied at A and the response was measured at B as shown in fig. 5. The natural frequency is located by finding the point having maximum frequency spacing around the curve. As mentioned earlier [3] the vector plot method, provides a simple technique of finding the true peak amplitude with good accuracy, free from off resonant contributions, in the resonant mode at a natural frequency. This is done quite easily by fitting a best circle to the are. placing particular emphasis on the part of it in the immediate vicinity of the natural frequeny as shown in fig. 5. The point, 0, denotes the new origin for the mode. Strictly speaking, the maximum spacing can only be determined by measuring the lengths corresponding to small frequency interval around natural frequency. For the present case a frequency interval of 1c/s was chosen and arc length corresponding to this interval measured.

Figs. 6 and 7 represent the Peak Amplitude and phase angle plot around the second natural frequency of the model. Table 1 shows the comparison of theoretically computed natural frequencies with the experimental results for the six-storey model. The results agree closely. It can be seen from the vector plot (fig. 5) that there was a resonant frequency of around 665 cps also. Later this was found to be that of the supporting block. If only the peak amplitude method of finding the natural frequency had been used, it would not have been an easy task to separate the model and support frequencies.

The fifth frequency was missed during the experiment. The experiments had been conducted before the theoretical results were computed.

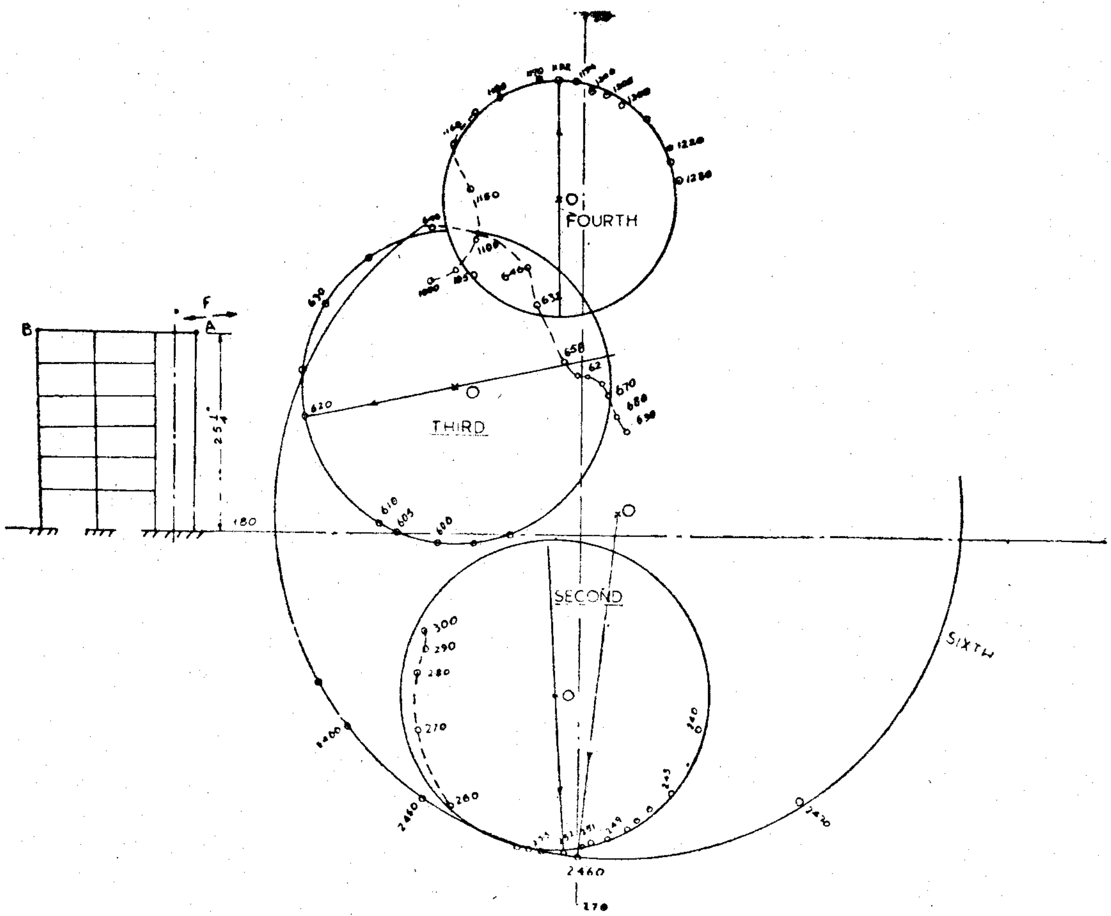


Fig. 5. Vector Plot for Second, Third, Fourth & Sixth Natural Frequencies.

Table 1

Comparison of Theoretical and Experimental Values of Natural Frequencies of a Shear Wall-Frame Combination

Mode	Theory cps.	Experiment cps.
1	110	107
2	255	252
3	630	620
4	1160	1192
5	1840	missed
6	2560	2460

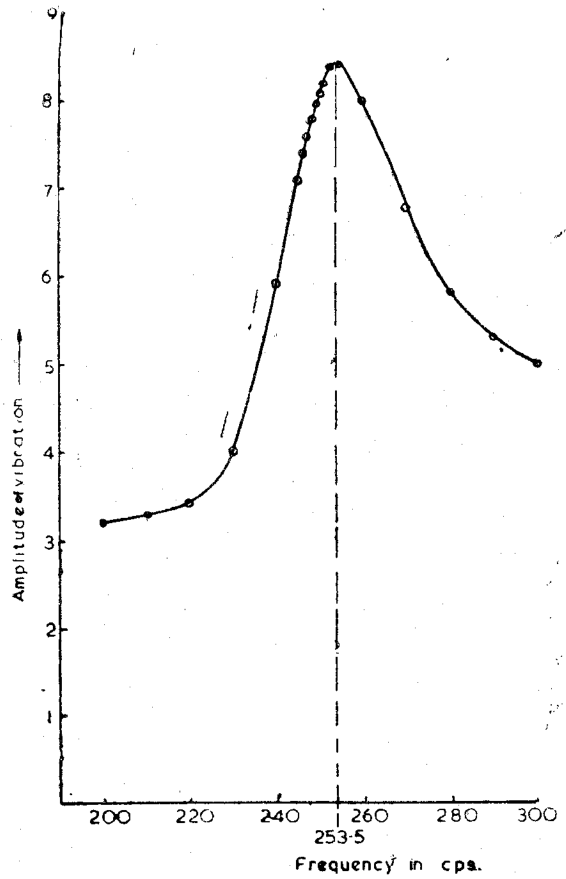


Fig. 6. Amplitude Plot Around Second Frequency.

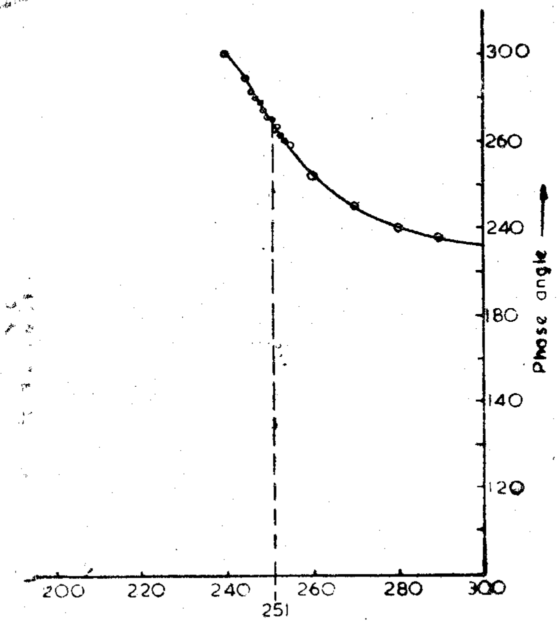
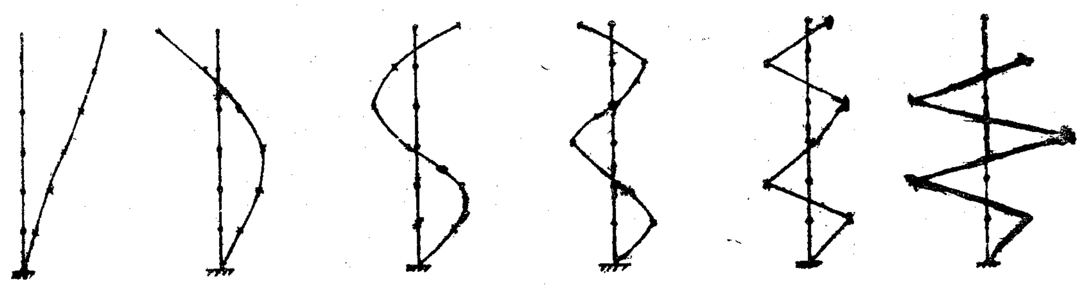


Fig. 7. Phase Angle Plot.



First Second Third Fourth Fifth Sixth

Fig. 8. Mode Shapes of Vibration

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