Paper No. 75

Bulletin of the Indian Society of Earthquake Technology. Vol. 6, No. 3, pp. 95-108. Sept. 1969

Bulletin of the
Indian Society of Earthquake Technology

Vol. VI Sept. 1969 No. 3

EXPERIMENTAL STUDY OF FRAME MODELS WITH TWIN DIAGONAL BRACES

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Synopsis

In an unbuckled state, the compression diagonal in a panel with a cross type of bracing system, also resists lateral load. An attempt has been made to determine its contribution towards spring constant, which is used for computing the frequency of vibration of a framed structure. For the experiments conducted it was found to have nearly the same contribution as tension diagonal until it buckled. Also, the simplified method of spring constant computation by assuming that braces and frame act independently and form a paralled system of shear springs was found to be approximately equal to the stiffness coefficient method. It was further observed that such analytical methods do not give the correct value of spring constat for all location of braces.

Introduction

In the domain of experimental study of braced frames, attempts have been made to determine the following :

(i) Damping[1,2]‡

(ii) Dynamic characteristics like period and mode shapes[3]

(iii) Restoring force characteristics under lateral load[1,4]

(iv) Contribution of stiffness by braces in a frame under lateral load[2]

Stiffness distribution in a multistorey framed structure influences both the period and mode shapes remarkably and axial forces present in a braced frame modify the dynamic characteristics appreciably[3]. Wakabayashi and Tsuji[4] found the braced frames to have an unstable equilibrium after the buckling of the compression bracing. With an increase in deflection amplitude, the unstable equilibrium disappears and the curve shows a spindle shape. Funahashi, Kinoshita and Saito[1] found a double bilinear hysteretic characteristics for braced frames. Gosain and Chandrasekaran[2] found experimentally that spring constant of braced frame computed by considering the braces and frame acting independently and forming a parallel system of shear springs, is not valid for all location of braces. In the

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‡ Refers to serial number of reference given at the end.

above investigation, compression braces were so designed that they buckled under a nominal load, and all observations were made for one tension diagonal active in a panel. The number of bays were also limited to two.

In order to further testify the assumptions made in computation of spring constant of braced frames, and to investigate the contribution of compression diagonal towards stiffness, another experimental study was taken up. Lateral load and free vibration tests were conducted on perspex models having one bay one storey, two bay two storey and three bay two storey frames. Experiments revealed a contiguity in the stiffness contribution by tension and compression diagonals, unless the compression diagonal buckled. It was also noted that stiffness of a brace increases significantly by increasing its end and lateral restraints.

Spring Constant of Braced Farmes

In calculating the spring constant of braced frames, it has generally been assumed that frame and braces act independently and form a parallel system of shear springs(5).

Spring constant of the columns, taking into account joint rotation, is given by :

$$K_c = \alpha \cdot \frac{12\,EI}{L^3} \cdot n$$

where α = $1/F^3$, a multiplication factor given by Chandrasekaran(6)

E = Young's modulus of elasticity

I = Moment of inertia of column

L = Length of column

n = number of columns

Due to an application of a lateral load, any particular storey in a multistorey frame undergoes deflections due to the following effects :

(a) Brace elongation

(b) Beam shortening

(c) Column elongation and shortening.

Figs. 1(a), (b) and (c) illustrate such deflections under a lateral load F. Deflection due to axial deformation of the brace is given by

$$\triangle H_1 = \frac{F.\,L_D\,\sec^2\phi}{A_D.\,E}$$

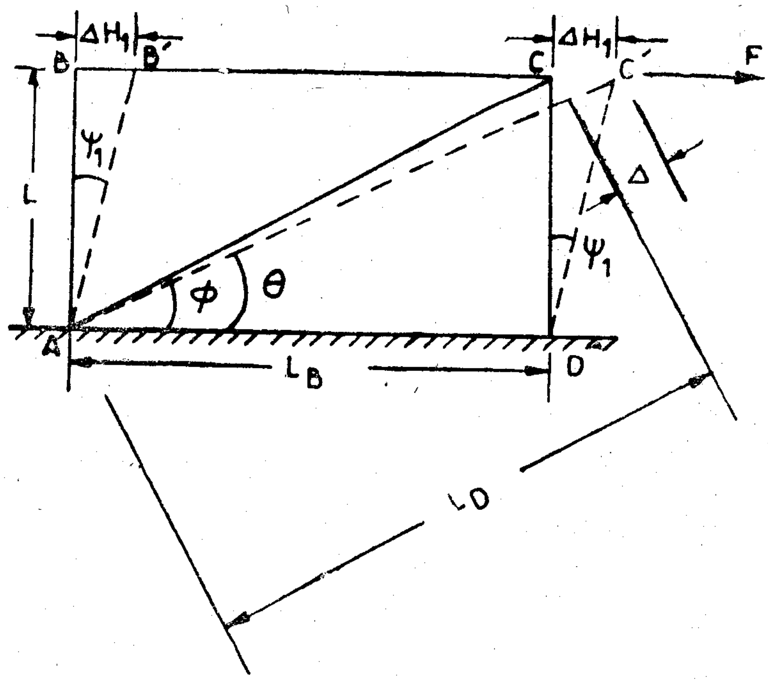
The beam will shorten due to force F by an amount $\triangle H_2$ and thus cause horizontal deflection.

$$\triangle H_2 = \frac{F\,.\,L}{A_B.\,E}$$

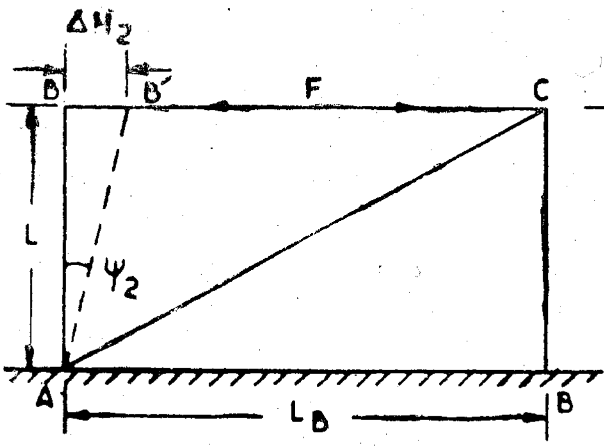
Changes in length due to axial forces in two columns of the braced bay also cause horizontal storey deflection $\triangle H_3$.

$$\triangle H_3 = \frac{2F\,.\,L}{A_C\,.\,E}\,.\,\tan^2\phi$$

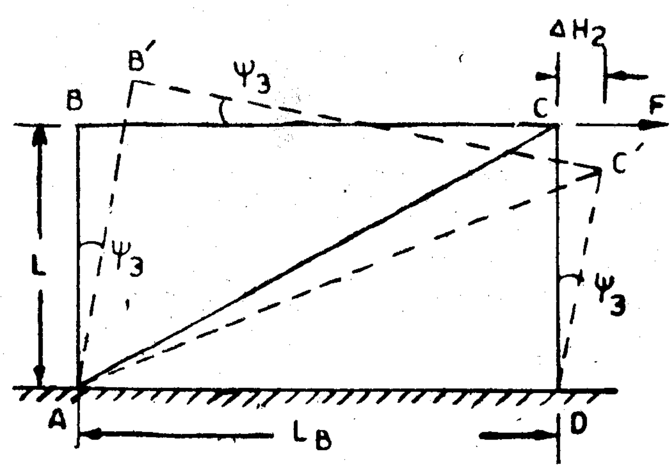
Total deflection, $\triangle H = \triangle H_1 + \triangle H_2 + \triangle H_3$.



(a) Effect of Brace Elongation



(b) Effect of Beam Shortening



(c) Effect of column Deformation

Notations :

AB′C′D = Deflected Shape of Frame ABCD

$\triangle H_1, \triangle H_2, \triangle H_3$ = Horizontal Deflections of the frame at storey level considered in the above mentioned three effects respectively

$\triangle$ = Axial deformation of brace AC

ϕ = Angle made by the brace with the horizontal

θ = Angle made by the brace with the horizontal in the deformed state

$\theta \approx \phi$

A_B, A_C, A_D = Cross sectional areas of beam, column and diagonal brace respectively

L, L_B, L_D = Length of column, beam and diagonal brace respectively

ψ_1, ψ_2, ψ_3 = Angles made by the deformed column with the vertical considered in the three effects respectively.

Fig. 1

Since spring constant is defined as load per unit deflection,

$$K_B = F/\Delta H$$

$$= \frac{F}{\Delta H_1 + \Delta H_2 + \Delta H_3}$$

$$= \frac{1}{\frac{L_D \sec^2 \phi}{A_D.E} + \frac{L}{A_B.E} + \frac{2L}{A_C.E} \cdot \tan^2 \phi}$$

where K_B = spring constant due to axial strains in brace, beam and columns.

Test Model

The bay width and storey height of each panel of the perspex model was kept as 30 cm. Cross section of both beams and columns were kept as 2.5 × 0.635 cm., thereby giving a ratio of moment of inertia per unit length of beam to that of column, i.e. $S_b/S_c = 1$. Beams were cemented to the columns by araldite. The enlarged portion of the column was clamped in a fixing frame, thus providing fixity at the base. For fixing diagonal braces (cross section 0.635 × 0.157 cm) gusset plates in the form of quadrants of circular perspex plates 7 cm dia. and 3 mm thick, were fixed at each joint of the frame by means of araldite. These gusset plates were fixed exactly at mid depth of the beams and columns.

In order to reduce the frequency of vibration of the model to a recordable value, flat iron strips having a weight of 1.46 kgm were fixed rigidly to all beams of the frame.

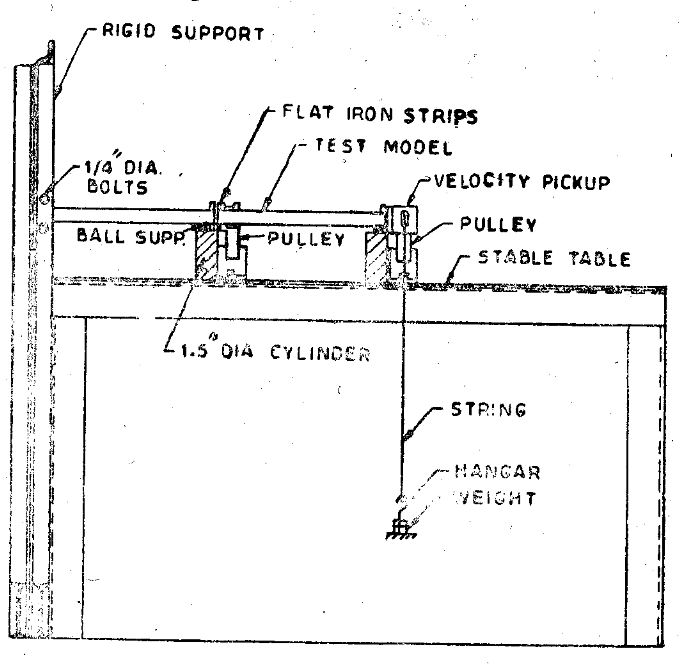


Fig. 2. Experimental Set-up for Static Testing

The model was kept in a horizontal position supported at the storey level by ball supports free to move in all directions. As seen in Fig. 2, the balls were placed on cyclindrical uprights with machined surface for free movement of the balls, and hence the frame. Such an orientation of the frame was adopted to prevent buckling in a direction perpendicular to the direction of application of load. This also eliminates any torsional effect during free vibration test.

In all, thirty three arrangements of bays, storeys and braces have been studied. Their distribution is as follows :

(a) Three arrangements of braces in single bay, single storey model.

(b) Five arrangements of braces in single bay, two storey model.

(c) Nine arrangements of braces in two bay, two storey model.

(d) Sixteen arrangements of braces in three bay, two storey model.

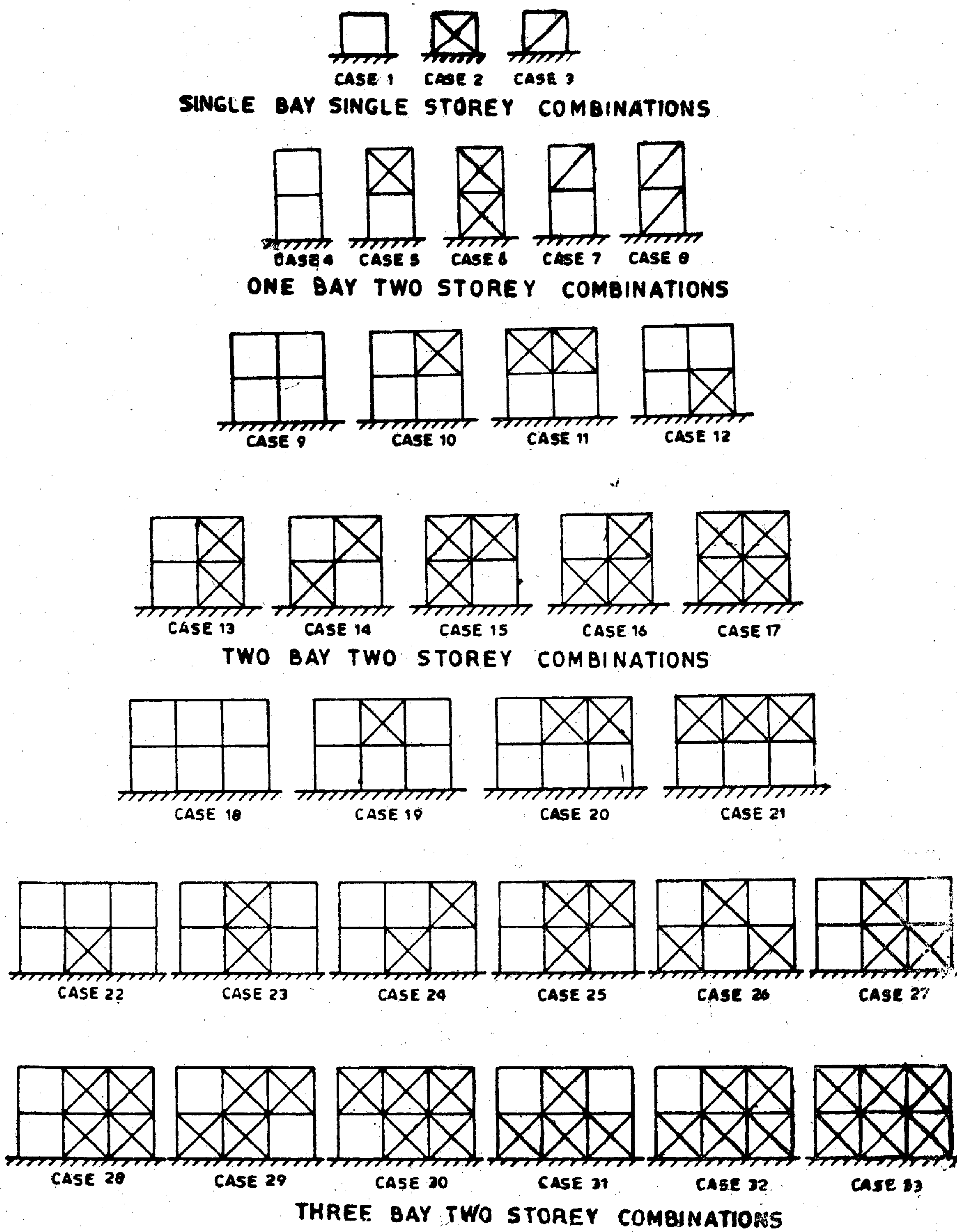

CASE 1
CASE 2
CASE 3
SINGLE BAY SINGLE STOREY COMBINATIONS
CASE 4
CASE 5
CASE 6
CASE 7
CASE 8
ONE BAY TWO STOREY COMBINATIONS
CASE 9
CASE 10
CASE 11
CASE 12
CASE 13
CASE 14
CASE 15
CASE 16
CASE 17
TWO BAY TWO STOREY COMBINATIONS
CASE 18
CASE 19
CASE 20
CASE 21
CASE 22
CASE 23
CASE 24
CASE 25
CASE 26
CASE 27
CASE 28
CASE 29
CASE 30
CASE 31
CASE 32
CASE 33
THREE BAY TWO STOREY COMBINATIONS

Fig. 3

All these cases are shown in Fig. 3. Only single bay single storey, and single bay double storey frames were tested with and without compression diagonals. Rest of the frames were tested with twin diagonals.

Computation of Spring Constant of Test Model

(a) *By Considering Brace and Frame to be Acting in Parallel*

For the single storey model and $S_b/S_c = 1$, $F = 1.126$

$$K_c = \frac{1}{(1.126)^3} \frac{12 \times 30 \times 10^3 \times 0.0533}{(30)^3} \text{ (for one column)}$$

$$= 0.500 \text{ kgm/cm}$$

For the two storey model and $S_b/S_c = 1$, $F = 1\ 248$

$$K_c = \frac{1}{(1.248)^3} \times \frac{12 \times 30 \times 10^3 \times 0.0533}{(30)^3} \text{ (for one column)}$$

$$= 0.326 \text{ kgm/cm.}$$

(From some preliminary experiments it was found that the value of E of perspex varied with the thickness of the material. Therefore, the value used for theoretical computation, i.e. $E = 30 \times 10^3$ kgm/cm², is an average value).

Also

$$K_B = \frac{E}{\frac{L_D \sec^2\phi}{A_D} + \frac{L}{A_B} + \frac{2L}{A_C} \tan^2 \phi}$$

$$= \frac{30 \times 10^3}{\frac{42.42 \times 2}{0\ 063} + \frac{30}{1\ 588} + \frac{2 \times 30}{1\ 588} \times 1}$$

$$= 21.5 \text{ kgm/cm}$$

If the effect of beam and column strain is reglected,

$$K_B = \frac{A_D\, E}{L_D \sec^2 \phi} = \frac{0.063 \times 30 \times 10^3}{42.42 \times 2}$$

$$= 22.25 \text{ kgm/cm}$$

Thus the beam and column strains reduce the stiffness by 3.38% in the test model.

(b) *Spring Constant by Stiffness Matrix Method*

Once the area of cross section and moment of inertias of beams, column and braces are known, the spring constant can be obtained by the stiffness Matrix method. A digital computer program(7) was used to solve a few typical cases. The analysis was carried out by considering the compression diagonal also to be effective. Stiffness matrices of cases 1 to 4, 13, 27 and 33 obtained by the above program, are given in Table 1, along with the stiffness matrices obtained by considering the brace and frame to be acting in parallel. There is a fairly good correspondence in the values obtained by the two methods.

Experimental Determination of Spring Constant

In order to get the influence coefficient matrix for the test model, load was applied at the top storey level (1) to give g_{11} and g_{12}, and at the bottom storey level (2) to give g_{22} and g_{21}. Load was applied directly by weights and deflections were measured both during loading and unloading by dial gauges having a least count of 0.01 mm.

Fig. 4 shows representative load—deflection curves. These curves are approximately bilinear. Upto a certain load, the compression diagonal also participates actively in taking the shear, but after it buckles, only the tension diagonal remains effective. Such an observation was also made by Funahashi, Kinoshita and Saito[1] Influence coefficients have been determined by using the slope of the line joining the origin to the peak of the curve. These are termed as secant slope values. These influence coefficients are then inverted to get the stiffness matrix.

The spring constant of the brace, K_B is obtained by deducting the experimental spring constant of the skeleton frame K_c from the experimental value K_T for different cases[2], i.e.

$$[K_B] = [K^T] - [K_c]$$

Tables 2, 3, 4 and 5 give a comparison between experimental and theoretical spring constant values for single bay single storey frames, one bay two storey frames, two bay two storey frames and three bay two storey frames respectively.

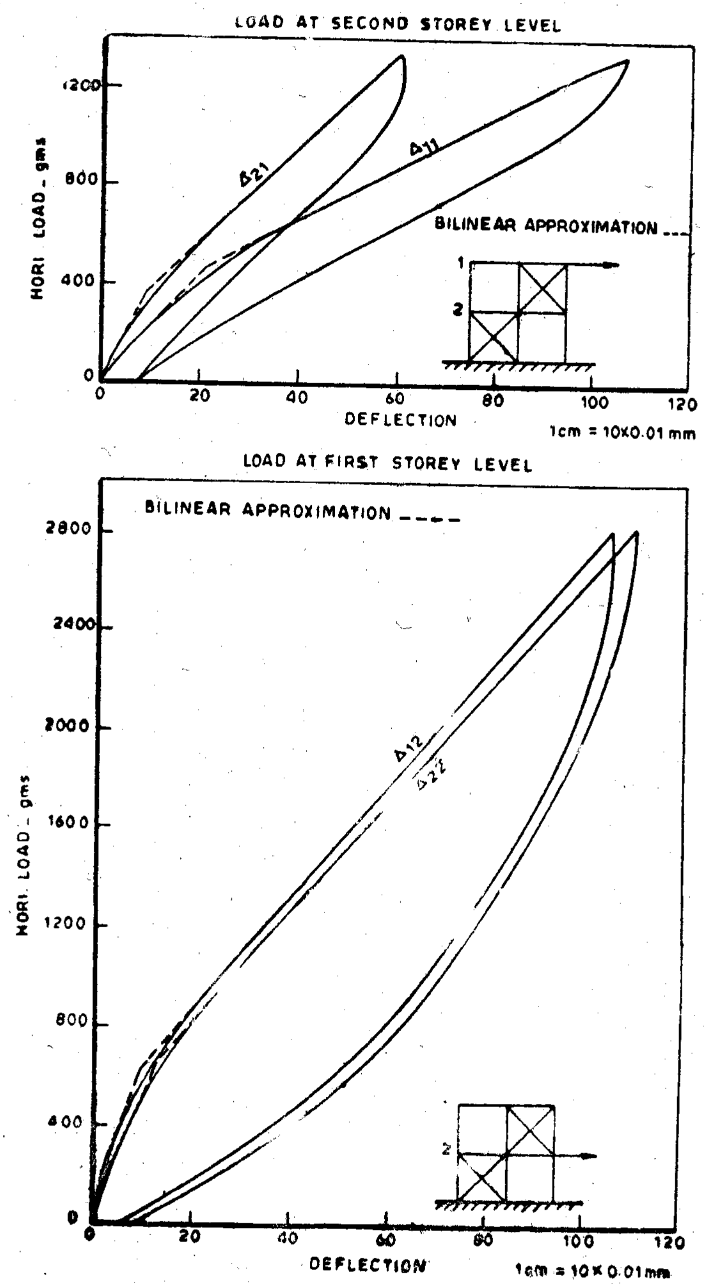


Fig. 4. Load deflection curves for case 14

Free Vibration Test

For determining the natural frequencies of vibration of the models, a self generating type velocity pick-up was fixed at the upper storey level as shown in Fig, 2. The signals were amplified by a high gain D. C. Amplifier and these were recorded by an Ink Writing Oscillograph.

Vibrations were imparted by pulling the model by a certain extent and then letting it go. The experimental values of frequencies are tabulated in Table 6 along with the

Table 1
Comparison of Spring Constants by Stiffness Matrix and Parallel Spring Methods

Case No.	Spring Constant by Stiffness Matrix $[K_T]$ kg/cm	Spring Constant $[K_T]$ by consideration of parallel springs kg/cm
1	1.0	1.0
2	44.6	44.0
3	22.8	22.5
4	$\begin{bmatrix} 0.8 & -1.05 \\ -1.05 & 2.42 \end{bmatrix}$	$\begin{bmatrix} 0.73 & -0.73 \\ -0.73 & 1.46 \end{bmatrix}$
13	$\begin{bmatrix} 43.2 & -44.6 \\ -44.6 & 91.5 \end{bmatrix}$	$\begin{bmatrix} 44.1 & -44.1 \\ -44.1 & 88.2 \end{bmatrix}$
27	$\begin{bmatrix} 40.8 & -38.2 \\ -38.2 & 122.0 \end{bmatrix}$	$\begin{bmatrix} 44.5 & -44.5 \\ -44.5 & 130.5 \end{bmatrix}$
33	$\begin{bmatrix} 115.0 & -108.0 \\ -108.0 & 226.0 \end{bmatrix}$	$\begin{bmatrix} 130.5 & -130.5 \\ -130.5 & 261.0 \end{bmatrix}$

Table 2
Comparison Between Theoretical and Experimental Spring Constant of Braces
Single Bay Single Storey Frame

Case No.	Position of braces*	Stiffness (K_T) kg/cm	Stiffness contribution by brace (K_B) kg/cm	Difference between theoretical and exp. values of (K_B)%
1	—	0.81	—	—
2	X	20.60	19.79	8.0
3	/	16.65	15.84	26.4

* *Note* – Locations of braces have been given in the second columns of these tables. The signs denote the following

— denotes no brace,

X denotes a two diagonal brace

/ denotes a single diagonal brace

For example, -X (Fig 3, case 14) denotes a two bay two storey frame having no brace in
X-
left panel of upper storey and right panel of lower storey and has twin diagonal brace in right panel of upper storey and left panel of lower storey. Similarly -XX (case 32) denotes
XXX
a three bay two storey frame in which all panels are provided with diagonal braces except the extreme left panel in the upper storey.

Table 3

Comparison Between Theoretical and Experimental Spring Constants of Braces
One Bay Two Storey Frame

Case No.	Location of brace	Stiffness matrix of braces only [K_B] kg/cm	Stiffness contributed by		Difference between theoretical and experimental stiffness values of braces	
			each upper storey brace kg/cm	each lower storey brace kg/cm	Upper storey brace kg/cm	Lower storey brace kg/cm
5	X —	$\begin{bmatrix} 17.61 & -17.89 \\ -18.77 & 19.00 \end{bmatrix}$	18.32	—	−6.68	—
6	X X	$\begin{bmatrix} 22.84 & -23.02 \\ -23.47 & 51.78 \end{bmatrix}$	23.11	28.67	−1.89	3.67
7	/ —	$\begin{bmatrix} 16.61 & -16.79 \\ -16.97 & 17.16 \end{bmatrix}$	16.88	—	−8.12	—
8	/ /	$\begin{bmatrix} 22.31 & -22.49 \\ -22.67 & 43.88 \end{bmatrix}$	22.49	21.39	−2.51	−3.61

Note : The negative sign indicates that the theoretical value of the stiffness of a brace [K_B] is greater than its experimental value.

frequencies calculated from influence coefficients. In general, it may be noted that the measured frequencies are higher than the computed frequencies. This may be due to the initial slope of the load deflection curves governing the free vibrations[8].

Conclusions

1. Spring constant computation by stiffness matrix method and by assuming the frame and brace to be acting in parallel give a fairly good correspondence.

2. The assumption that only the tension diagonal brace resists the lateral load is limited to cases in which the compression diagonal buckles completely. The stiffness contributed by an unbuckled compression diagonal brace may be as much as that by a tension diagonal brace.

3. With the increase in end restraint of the braces, which may be due to the addition of another bay, storey or an adjoining braced panel, the stiffness contributed by each brace increases significantly.

4. A change in relative location of an isolated brace in a three bay two storey frame model, does not have a significant effect on the spring constant of the brace.

5. Measured frequencies are higher than the frequencies computed by using the influence coefficients obtained from the secant slope of the load-deflection curves. This is so because theoretical studies have indicated that the frequencies are more or less dependent on the initial slope of the bi-linear load deflection curves.

Table 4

Comparison between Theoretical and Experimental Spring Constant of Braces Two Bay Two Storey Frame

Case No.	Location of braces	Stiffness Matrix of braces only [K_B] kg/cm	Stiffness contributed by		Difference between theoretical and experimental stiffness values of braces	
			each upper storey brace kg/cm	each lower storey brace kg/cm	Upper storey brace kg/cm	Lower storey brace kg/cm
10	— X — —	$\begin{bmatrix} 20.53 & -20.46 \\ -23.70 & 23.67 \end{bmatrix}$	22.1	—	−2.9	—
11	X X — —	$\begin{bmatrix} 42.08 & -42.01 \\ -44.50 & 63.92 \end{bmatrix}$	24.06	—	−0.94	—
12	— — — X	$\begin{bmatrix} 0 & 0 \\ 0 & 22.24 \end{bmatrix}$	—	22.24	—	−2.76
13	— X — X	$\begin{bmatrix} 22.33 & -22.26 \\ -28.25 & 53.82 \end{bmatrix}$	24.28	29.54	−0.72	4.54
14	— X X —	$\begin{bmatrix} 22.53 & -22.46 \\ -27.40 & 52.12 \end{bmatrix}$	24.13	27.99	−0.87	2.99
15	X X — X	$\begin{bmatrix} 53.77 & -53.70 \\ -55.20 & 82.50 \end{bmatrix}$	27.11	28.29	2.11	3.29
16	— X X X	$\begin{bmatrix} 26.83 & -24.91 \\ -28.85 & 86.12 \end{bmatrix}$	26.86	29.63	1.86	4.63
17	X X X X	$\begin{bmatrix} 53.38 & -53.31 \\ -57.70 & 114.42 \end{bmatrix}$	27.4	29.81	2.40	4.81

Table 5

Comparison between Theoretical and Experimental Spring Constant of Braces
Three Bay Two Storey Frame

Case No.	Location of braces	Stiffness Matrix of braces only [K_B] kg/cm	Stiffness contributed by		Difference between theoretical and experimental stiffness values of braces	
			each upper storey brace kg/cm	each lower storey brace kg/cm	Upper storey brace kg/cm	Lower storey brace kg/cm
19	—X— ———	$\begin{bmatrix} 28.68 & -28\ 38 \\ -28.79 & 28.70 \end{bmatrix}$	28.64	—	3.64	—
20	—X X ———	$\begin{bmatrix} 55.28 & -54.98 \\ -55.49 & 53.20 \end{bmatrix}$	27.37	—	2.37	—
21	X X X ———	$\begin{bmatrix} 95.98 & -95.68 \\ -97.19 & 96.80 \end{bmatrix}$	32.13	—	7.13	—
22	——— —X—	$\begin{bmatrix} 0 & -0 \\ 0 & 33.25 \end{bmatrix}$	—	33.25	—	8.25
23	—X— —X—	$\begin{bmatrix} 40.98 & -40.68 \\ -42.69 & 79.40 \end{bmatrix}$	41.78	37.62	16.78	12.62
24	——X —X—	$\begin{bmatrix} 40.38 & -37.08 \\ -39.29 & 70.30 \end{bmatrix}$	38.92	31.38	13.92	6.38
25	—X X —X—	$\begin{bmatrix} 80.88 & -80.58 \\ -82.09 & 115.60 \end{bmatrix}$	40.59	34.42	15.59	9.42
26	—X— X—X	$\begin{bmatrix} 35.28 & -35.82 \\ -37.29 & 111.60 \end{bmatrix}$	36.13	37.73	11.13	12.73
27	—X— —X X	$\begin{bmatrix} 35.78 & -35.20 \\ -37.39 & 100.30 \end{bmatrix}$	36.12	32.09	11.12	7.09
28	—X X —X X	$\begin{bmatrix} 78.48 & -77.28 \\ -84.19 & 147.00 \end{bmatrix}$	39.99	33.51	14.99	8.51
29	—X X X X—	$\begin{bmatrix} 79.38 & -77.78 \\ -85.29 & 147.80 \end{bmatrix}$	40.78	33.51	15.78	8.51
30	X X X —X X	$\begin{bmatrix} 151.58 & -150.38 \\ -153.39 & 223.50 \end{bmatrix}$	50.59	35.86	25.59	10.86
31	—X— X X X	$\begin{bmatrix} 36.08 & -34.23 \\ -36.99 & 141.70 \end{bmatrix}$	35.77	35.31	10.77	10.31
32	—X X X X X	$\begin{bmatrix} 39.03 & -84.78 \\ -99\ 39 & 219.30 \end{bmatrix}$	43.86	43.86	18.86	18.86
33	X X X X X X	$\begin{bmatrix} 169.98 & -124.18 \\ -186.29 & 334.80 \end{bmatrix}$	57.83	53.77	32.83	28.77

Table 6

Frequencies

Case No.	Location of braces	Measured frequency c/s	Computed frequency based on experimental influence coefficients c/s	Percentage difference between measured and computed frequency
1	2	3	4	5
1	—	5.44	3.56	34.60
2	X	17.85	17.95	—0.56
3	/	16.00	16.15	—0.93
4	— —	3.10	3.497	—12.80
5	X —	3.76	3.132	16.70
6	X X	19.22	12.727	33.75
7	/ —	3.65	3.103	15.00
8	//	11.47	11.527	—0.49
9	—— ——	2.55	1.988	22.00
10	—X ——	3.39	2.495	26.40
11	XX ——	3.48	2.650	23.85
12	—— —X	6.57	3.096	52.90
13	—X —X	15.25	8.364	45.10
14	—X X—	16.00	8.611	46.10
15	XX –X	16.45	9.346	43.20
16	—X XX	18.65	11.564	38.00

Paper No. 76

Bulletin of the Indian Society of Earthquake Technology. Vol. 6, No. 3, pp. 109–130. Sept. 1969

DESIGN OF A TYPICAL MACHINE FOUNDATION BY DIFFERENT METHODS

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Synopsis

Procedures for evalution of soil constants from a resonance test, for the design of machine foundations have been illustrated by Barkan, Pauw and Richarts' methods. Based upon these soil constants, two typical foundations, resting on two different types of soils have been checked, by the three methods to illustrate the application of different design methods in practice.

Introduction

For evalution of dynamic soil constants a resonace test is recommended (Prakash and Gupta 1967). However there are several methods by which the results of this test can be interpretted (Prakash 1965). Resonance test data reported on two different sites (Gupta 1965 and Kondner 1964) has been interpretted by Barkan, Pauw and Richart's methods of analysis. Two typical foundations resting on two different soils have then been checked based on the three methods and comparison of the natural frequencies and amplitudes of motion has been made.

Notation

Symbol		*Units*
a	Length of the foundation	m
a_x	Dimensionless frequency Factor for sliding vibrations	
a_ϕ	Dimensionless frequency factor for rocking vibrations	
a_z	Dimensionless frequency factor for vertical vibrations	
A	Area of the foundation in contact with soil m^2	
A_s	Amplitude factor for sliding	
A_r	Amplitude Factor for rocking	
A_x	Amplitude in sliding	mm
A_ϕ	Amplitude in rocking	Radian
$A_{x\phi}$	Amplitude in combined rocking and sliding	mm
b	Width of foundation	m
C_u	Coefficient of elastic uniform compression of soil	kg/cm^3, t/m^3
C_ϕ	Coefficient of elastic non-uniform compression of soil	kg/cm^3
C_T	Coefficient of elastic uniform shear of soil	kg/cm^3, t/m^3

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E	Modulus of elasticity of soil	kg/cm^2, t/m^2
f_{nx}	Natural frequency in sliding	c.p.s.
$f_{n\phi}$	Natural frequency in rocking	c.p.s.
f_{nz}	Natural frequency in vertical vibration	c.p s.
$f_{nx\phi}$	Natural frequency in combined rocking and sliding	c.p.s.
g	Acceleration due to gravity	$m\text{-}sec^{-2}$
G	Shear modulus	kg/cm^2, t/m^2
h	Equivalent height of surcharge Height of foundation block.	m
I	Moment of inertia of the foundion contact area about an axis passing through the C.G. of the base perpendicular to plane of vibration	m^4
I_m	Mass moment of inertia of foundation and accessories about an axis through the C.G. of the system	$t\text{-}m\text{-}sec^2$
I_{mo}	Mass moment of inertia of foundation and machine about an axis through the C.G. of the base of the foundation	$t\text{-}m\text{-}sec^2$
I_{ms}	Mass moment of inertia of foundation and machine and soil mass about an axis through combined C.G.	$t\text{-}m\text{-}sec^2$
k_x	Spring constant for sliding vibration	t/m
k_z	Spring constant for vertical vibration	t/m
k_{xz}	Spring constant for rocking vibration	t/m
L	Height of C.G. of foundation and machine above the base of the foundation	m
m	Mass of foundation and machine	$t\text{-}m^{-1}\text{-}sec^2$
m_s	Apparent soil mass	$t\text{-}m^{-1}\text{-}sec^2$
M	Exciting moment	t-m
P	Exciting force	t
r_x	Equivalent radius for sliding vibration	m
r_ϕ	Equivalent radius for rocking vibration	m
r_z	Equivalent radius for vertical vibration	m
w_{nx}	Natural circular frequency in sliding	sec^{-1}
$w_{n\phi}$	Natural circular frequency in rocking	sec^{-1}
$w_{nx\phi}$	Natural circular frequency in combined rocking and sliding	sec^{-1}
w_{nz}	Natural circular frequency in vertical mode of vibration	sec^{-1}
ξ	Damping factor	
ρ	Density of soil	t/m^3
ε	Eccentricity factor	
a_x	Length of the element	m
a_y	Width of element	m
a_z	Height of the element	m
γ	Ratio I_m/I_{mo}	
q	Static soil pressure	kg/cm^2

Particulars of Available Test Data :

D. C. Gupta[1] (1965) performed resonance tests on four foundation blocks of one metre height resting on the surface of sandy soil and subjected to sinusoidally varying horizontal unbalance force. The tests were performd by mounting Lazan Oscillator on the top surface of the block and recording amplitudes of motion of the block at different frequencies. The results on a block of 1 m × 1 m × 1 m have been taken up for analysis. The particulars are given below and record of observations in Table I.

Weight of fonndation block	=	2.21 tonnes
Weight of oscillator assembly	=	0.062 tonnes
Density of soil	=	1.8 t/m^3
Base area of the block	=	1 m^2

Table I—Test Data Reported by D. C. Gupta

Eccentricity Factor $\varepsilon \times 10^{-5}$ cm	Observed Natural Frequeney $f_{nx\phi}$ c. p. s.	Peak Amplitude A_{max} mm	Unbalance Force at Resonance F kg
1.45	16.0	0.30	28.0
3.01	15.0	0.43	58.0
5.43	13.0	0.685	90.0
9.28	12.0	1.00	120.0

Konder (1964) reported data on circular footing resting on Silty Clay, tested under vertical vibrations. The particulars of thc test data are as given below :

Diameter of the footing	=	1.57 m
Weight of the footing including vibrator and ballast	=	14.02 tonnes
Unit weight of the soil	=	1.91 t/m^3
Compression modulus of soil at surface E_0	=	740 kg/cm^2
Compression modulus of soil at 8.85 m below surface E	=	1600 kg/cm^2
Shear modulus of soil at surface	=	G = 326 kg/cm^2
Shear modulus at 8.85 m below surface	=	G = 693 kg/cm^2

The compression modulus and shear modulus were determined by seismic methods.

Table II—Test Data Reported by Kondner

Eccentricity Factor $\varepsilon \times 10^{-3}$ cm	Natural Frequency f_{nz} c.p.s.	Force at Resonant Frequency F kg
1.77	15.2	23.0
3.60	13.6	38.5
5.50	12.8	50.0
7.20	12.0	58.5

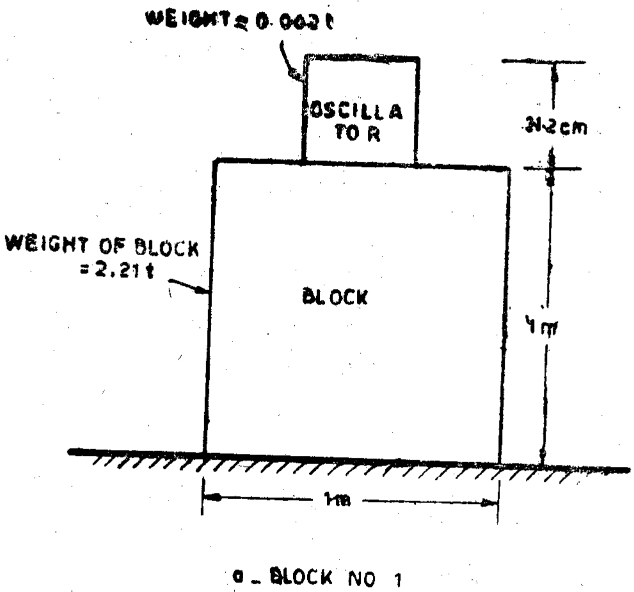

Fig. 1. Section of block

The procedure for analysis of test data will now be illustrated.

Analysis of Test Data

The test data on sandy soil will be analysed first.

Barkans Method

Fig. 1 shows a section of the block 1 m × 1 m × 1 m high. The axis of rotation of the block is perpendicular to the plane of the figure.

1. Moments of Inertia

(a) Base Area.

$$I^* = \frac{1 \times 1^3}{12} = 0.0834 \text{ m}^4$$

(b) Mass of oscillator and block.

For oscillator, $$I_{m_1} = \frac{0\ 062}{9.81} (0.656)^2 = 0.00272 \text{ t-m-sec}^2$$

For foundation block $$I_{m_2} = \frac{m}{12} (a_x^2 + a_y^2)$$

$$= \frac{2.21}{9.81 \times 12} (1+1) = 0.0376 \text{ t-m-sec}^2$$

$$I_m = I_{m_1} + I_{m_2} = 0.00272 + 0.0376 = 0.04032 \text{ t-m-sec}^2$$

$$I_{mo} = 0.0376 + \frac{2.21}{9.81} (0.5)^2 + \frac{0.062}{9.81} (1.156)^2 = 0.10237 \text{ t-m-sec}^2$$

$$\gamma = \frac{I_m}{I_{mo}} = 0.394$$

* All the symbols have been defined in the notation.

2. Determination of C_T

Frequency equation for combined rocking and sliding (Barkan, 1962) is

$$w^4{nx\phi} - \frac{w^2_{nx} + w^2_{n\phi}}{\gamma} w^2_{nx\phi} + \frac{w^2_{nx} \cdot w^2_{n\phi}}{\gamma} = 0$$

$$w^2_{nx} = \frac{C_T . A}{m} = \frac{C_T \times 1}{2.272/9.81} = 4.32\ C_T$$

$$w^2_{n\phi} = \frac{C_\phi . I}{I_{mo}} = \frac{C_\phi \times 0.0833}{0.10237} = \frac{3.74\ C_T \times 0.0833}{0.10237} = 3.02\ C_T$$

assuming $C_\phi = 3.74 C_T$

Substituting in the frequency equation

$$w^4{nx\phi} - \frac{4.32\ C_T + 3.02\ C_T}{0\ 394} w^2_{nx\phi} + \frac{4.32 \times 3.02}{0.394} . C_T^2 = 0$$

$$33.1\ C_T^2 - 18.6\ C_T . w^2_{nx\phi} + w^4_{nx\phi} = 0$$

$$C_T = 0.5\ w^2_{nx\phi}\ ;\ C_T = 0.06\ w^2_{nx\phi}$$

$$w_{nx\phi} = 2\ \pi\ f_{nx\phi}$$

For observed natural frequency, $f_{nx\phi} = 16$ c.p.s. the values of C_T come out to be 5.05 kg/cm³ and 0.595 kg/cm³. Substituting $C_T = 5.05$ Kg/cm³ in the frequency equation, we get $f_{nx\phi 1} = 16.2$ c.p.s.

$f_{nx\phi 2} = 46.0$ c.p.s.

when $C_T = 0.595$ is substituted we get

$f_{nx\phi 1} = 5.45$ c.p.s.

$f_{nx\phi 2} = 15.8$ c.p.s.

The value of C_T selected should be such that it satisfies the condition for two natural frequencies. The observed natural frequency is the lower natural frequency in the combined mode. The second natural frequency will have a higher value which is given only by $C_T = 5.05$ kg/w².

So out of the two values of C_T so obtained, only the higher value will satisfy the condition for two natural frequencies.

The values of C_T have been shown in col. 3, Table III.

Richarts' Method

1. Equivalent Radii

$$r_x = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{1}{\pi}} = 0.564\ m$$

$$\frac{W}{g} \left(\frac{r_\phi^2}{4} + \frac{h^2}{3}\right) = I_{mo}$$

$$\frac{2.272}{9.81} \left(\frac{r_\phi^2}{4} + \frac{1^2}{3}\right) = 0.10237 \therefore r_\phi = 0.64\ m$$

2. Mass ratio

$$bx = \frac{W}{\rho\, r_x^3} = \frac{2.272}{1.8 \times (0.564)^3} = 7.0$$

3. Inertia Ratio

$$b\phi = \frac{I_{mo}\, g}{\rho\, r\phi^5} = \frac{0.10237 \times 9.81}{(1.8) \times (0.64)^5} = 5.07$$

4. **Dimensionless Frequency Factors**

From Richarts charts, Fig. 2b and 2c.

$a_x = 0.85$; $a_\phi = 0.67$

5. **Determination of G**

$$w^4_{nx\phi} - \left[(w^2_{nx} + w^2_{n\phi}) + \frac{m}{I_{mo}} Z^2\, w^2_{nx} \right] w^2_{nx\phi} + w^2 n\phi . w^2_{nx} = 0$$

$Z = 0.516$ m., where Z is the height of combined C.G. above the basis.

$$w^2_{nx} = \frac{(a_x)^2}{r^2_x} \cdot \frac{Gg}{\rho} = \frac{(0.85)^2 \times 9.81 \times G.}{(0.564)^2 \times 1.8} = 12.7 \text{ G.}$$

$$w^2 n\phi = \frac{a^2\phi}{r^2\phi} \times \frac{G.g}{\rho} = \frac{(0.67)^2 \times G \times 9.81}{(0.6.4)^2 \times 1.8} = 6.0 \text{ G}$$

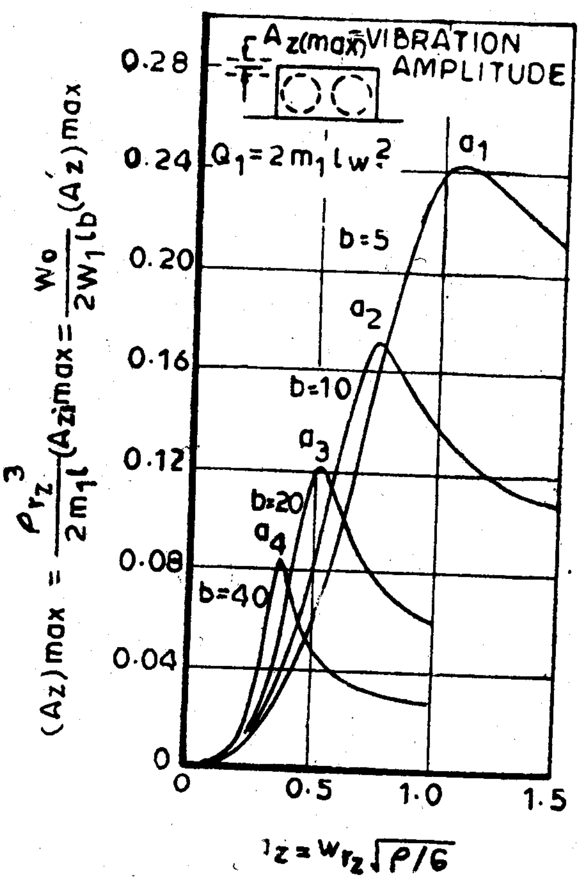

Fig. 2(a). For exciting force amplitude dependent upon the exciting frequency

2. Mass ratio

$$b_z = \frac{W}{r_o^3} = \frac{14.02}{1.91 \times (0.785)^3} = 15.12$$

3. Dimensionless frequency factor from Richarts, Fig. 2(a) For bz = 15.12

$$a_z = 0.59$$

4. Determination of G

$$a_z = 2\pi f_{nz} \sqrt{\frac{\rho}{Gg}} \; ; \quad G = \frac{4\pi^2 f^2_{nz} r_z^2}{a_z^2} \frac{\rho}{g}$$

f_{nz}, r_z, a_z being known, G can be computed. The values of G so obtained have been given in col. 4, Table IV below:

Table IV. Soil Constants for Silty Clay

Eccentricity factor $\varepsilon \times 10^{-3}$ cm	Observed natural frequency c.p.s.	C_u kg/cm³	G kg/cm²
1	2	3*	4
1.77	15.2	6.75 (2.98)	322.0
3.60	13.6	5.40 (2.38)	256.0
5.40	12.8	4.76 (2.10)	230.0
7.20	12.0	4 20 (1.86)	220.0

* Values in the brackets show values of Cu for standard 10 m² area.

Pauw's method

From the data reported by Kondner for the silty clay under consideration.
Compression modulus of the soil at the surface Eo = 740 kg/cm²
Compression modulus of the soil at depth of 8.85 m E = 1600 kg/cm²
Shear modulus of the soil at surface Go = 326 kg/cm²

Shear modulus of the soil at depth of 8.85 m G = 693 kg/cm²

Rate of increase of compression modulus with depth $\beta = \frac{1600-740}{8.85} = 0.9725$ kg/cm³

Rate of increase of shear modulus with depth $\beta' = \frac{693-326}{8.85} = 0.415$ kg.cm³

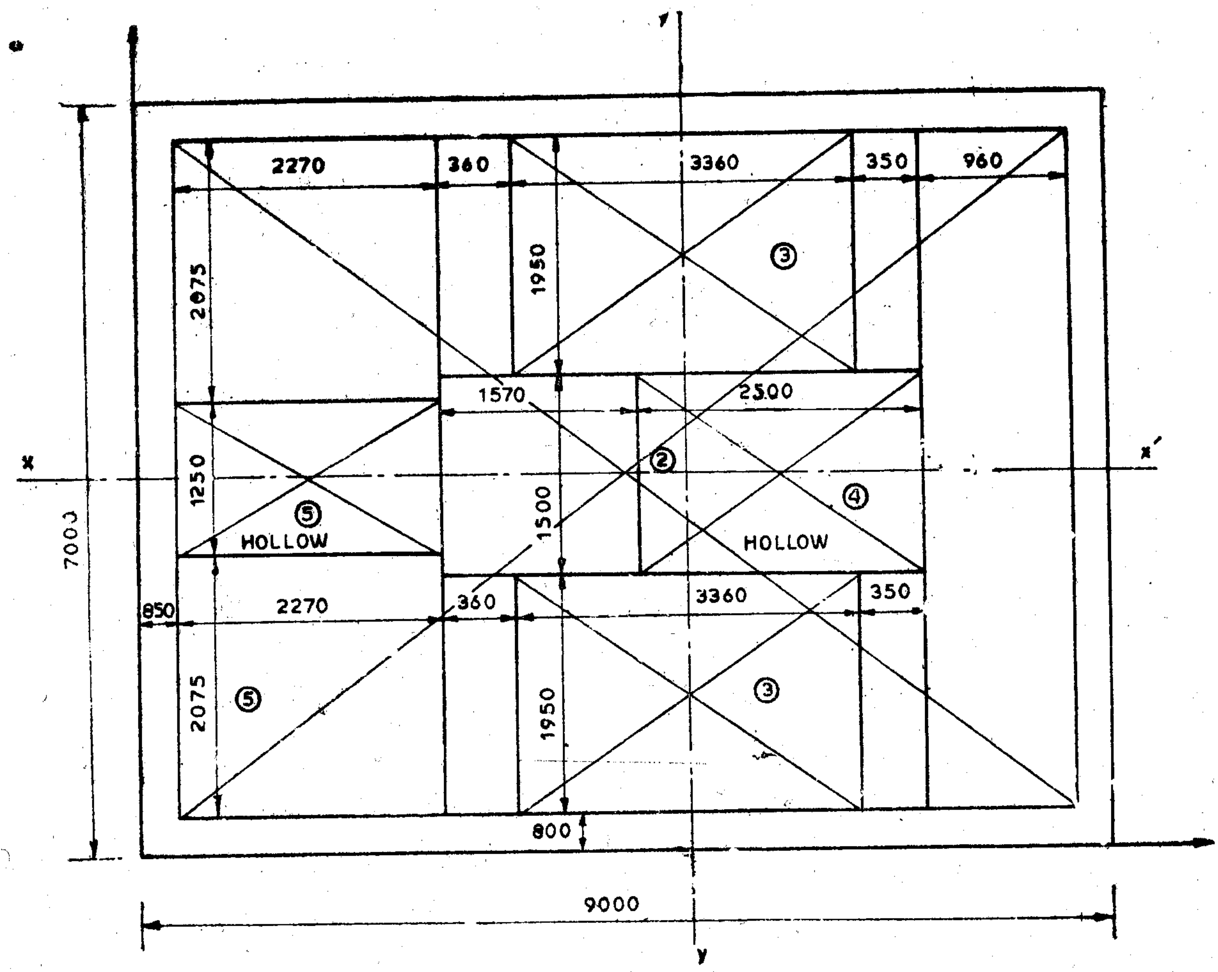

Fig. 3 (a) Plan

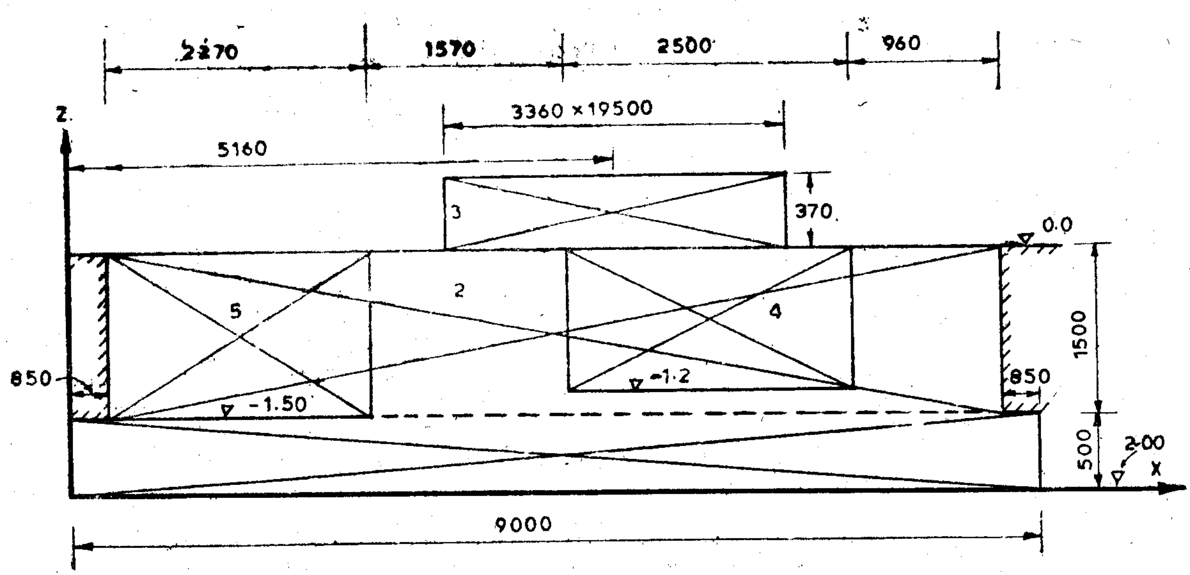

Fig. 3 b—Section along longitudinal axis section x x'

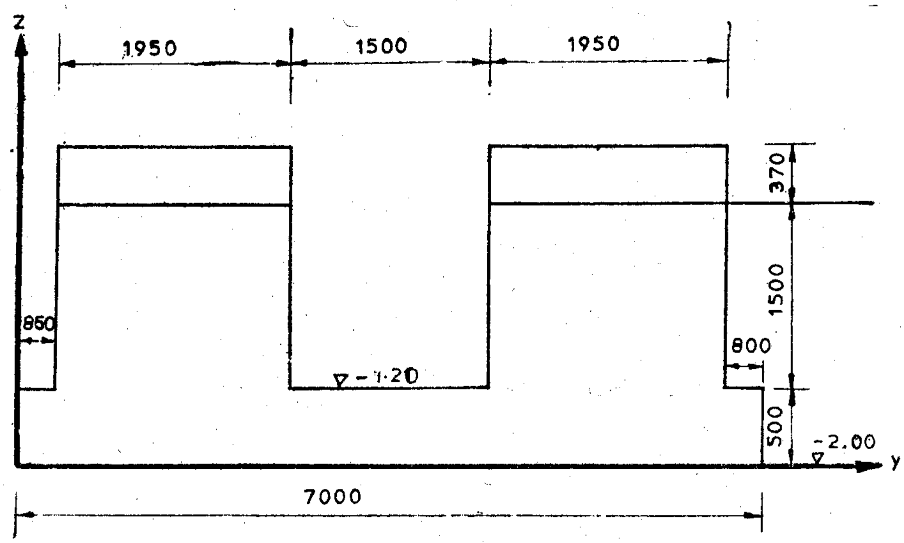

Fig 3 c—Section along the axis of main shaft section y y'

Design Examples

Design a foundation for a reciprocating horizontal compressor, the following data being given,

Operating speed of the engine	= 120 R.P.M.
Horizontal unbalance force PSinwt	= 2.5 Sin wt tonnes
Weight of compressor	=26.0 tonnes
Weight of motor	=14.8 tonnes
Density of soil	= 1.8 t/m³

The horizontal unbalance force acts at a height of 0.5 metre above the top surface of the foundation. The soil is sandy having bearing capacity of 1.5 kg/cm². Use soil constants obtained in Table III.

Barkan's Method

Cofficient of elastic uniform shear of soil C_τ = 0.85 kg/cm³. Use foundation of the type shown in Fig. 3.

1. Determination of combined C.G.

Density of concrete = 2.2 t/m³

Let x_0, y_0, z_0 denote co-ordinates of the centre of gravity of the whole system w.r.t. co-ordinate axes.

$$x_0 = \frac{111.76}{24.02} = 4.64 \; ; \quad y_0 = \frac{84.67}{24.02} = 3.53 \; ; \quad z_0 = \frac{28.34}{24.02} = 1.19$$

$$\% \text{ eccentricity in } x - \text{direction} = \frac{4.64-4.50}{9.0} \times 100 = 1.56\%$$

$$\% \text{ eccentricity in } y - \text{direction} = \frac{3.52-3.50}{7.0} \times 100 = 0.0281$$

Table V

Computation of Combined Center of Gravity

Element of the System	Dimensions of the Element			Mass t–m^{-1} sec^2 m_1	Coordinates of C–G of the element wrt x,y,x axes			Static moment of mass of element wrt x,y,z			Moment of Inertia of the element wrt axes passing through CG. t–m—sec^2	Distance between CG. of the element and comined CG. (m)		$m_1(x^2_{01}+z^2_{01})$
	a_{x1}	a_{y1}	a_{z1}		x_1	y_1	z_1	$m_1 x_1$	$m_1 y_1$	$m_1 z_1$	$\frac{m_1}{12}(a^2_{x1}+a^2_{z1})$	x_{01}	z_{01}	
Compressor	—	—	—	2.23	4.11	3.34	2.50	9.17	7.45	5.57	—	0.53	1.31	+4.36
Motor	—	—	—	1.47	5.94	3.68	2.50	8.71	5.40	3.68	—	1.30	1.01	+5.00
1	9.0	7.0	0.50	7.50	4.50	3.50	0.25	33.8	27.0	1.87	50.80	0.14	0.94	+6.75
2	7.30	5.40	1.50	13.30	4.50	3.50	1.25	59 80	46.5	16.60	61.50	0.14	0.06	+0 031
3(2nos)	3.36	1.95	0.370	1.50	5.16	3.50	2.185	7.75	5.25	3.28	1.43	0.52	0.995	+1.85
–4	2.50	1.50	1.20	—1.02	5.94	3.50	1.40	—6.05	—3.57	—1.46	—0.63	1.30	0.21	—1.73
–5	2.270	1.25	1.50	—0.96	1.475	3.50	1.25	—1.420	—3.36	—1.20	—0.59	3.175	0.06	—9.70
				24.02				111.76	84.67	28.34	112.51			+6.56

Static soil pressure $= \frac{W}{A} = \frac{24.02 \times 9.81}{9 \times 7} = 3.75 \text{ t/m}^2$

$= 0.375 \text{ kg/cm}^2 < 1.5 \text{ kg/cm}^2$

Operating frequency of the engine $w = 120$ R.P.M. $= 2$ c.p.s.

$= 12.6$ rad/sec.

$w^2 = 158 \text{ sec}^{-2}$

Height of the force axis above the combined C.G., $h = 2+0.5-1.19 = 1.31$ m

Exciting momet about C.G. of the combined system $M = 2.50 \times 1.31 = 3.28$ t-m

2. Moment of Inertia

(a) Base Area

$$I = \frac{7 \times 9^3}{12} = 425 \text{ m}^4$$

(b) $I_m = \frac{1}{12}(a_{x1}^2 + a_{z1}^2)\, m_1 + m_1 (x_{01}^2 + Z_{01}^2)$

$I_m = 112.51 + 6.56 = 119$ t-m-sec²

$I_{mo} = I_m + mL^2 = 119 + 24.02 \times 1.19^2 = 153$ t-m-sec² $(L = Z_o)$

$$\gamma = \frac{I_m}{I_{mo}} = \frac{119}{153} = 0.78$$

3. C_T for 63 m² area $= 0.85 \sqrt{\frac{10}{63}} = 0.34$ kg/cm³, $C\phi = 1.20$ kg/cm³

4. Natural frequency in sliding

$$w_{nx} = \sqrt{\frac{C_{TA}}{m}} = \sqrt{\frac{0.34 \times 10^3 \times 63}{24.02}} = 30 \text{ rad/sec., } f_{nx} = 4.8 \text{ c.p.s.}$$

5. Natural frequency in Rocking

$$w_{n\phi} = \sqrt{\frac{C\phi . I - WL}{I_{mo}}} = \sqrt{\frac{1.20 \times 10^3 \times 425 - (24.02 \times 9.81) \times 1.19}{153}} = 57.8 \text{ rad/sec.}$$

$$\therefore f_{n\phi} = 9.2 \text{ c.p.s.}$$

6. Natural frequency in combined mode

$$w^4_{nx\phi} - w^2_{nx\phi}\left(\frac{w^2_{n\phi} + w^2_{nx}}{\gamma}\right) + \frac{w^2_{nx} \,.\, w^2_{n\phi}}{\gamma} = 0$$

$$w^2_{nx\phi} - \frac{(30)^2 + (57.8)^2}{0.78} w^2_{nx\phi} + \frac{(30)^2 \times (57.8)^2}{0.78} = 0$$

$w^2_{nx\phi_{1,2}} = 2.71 \times 10^3 (1 \pm 0.69) \text{ sec}^{-2}$

$w_{nx\phi_1} = 29 \text{ sec}^{-1}$; $f_{nx\phi_1} = 4.6$ c.p.s.

$w_{nx\phi_2} = 67.8 \text{ sec}^{-1}$; $f_{nx\phi_2} = 10.8$ c.p.s.

7. Amplitudes

$\triangle(\omega^2) = m\, I_m (w_{nx\phi_1} - w^2)(w_{nx\phi_2} - w^2)$

$= 24.02 \times 119 \,((29)^2 - (12.6)^2)\,((67.8)^2 - (12.6)^2)$

$= 8.6 \times 10^9$

$$A_x = \frac{C_\phi . I - WL + C_T.AL^2 + I_m w^2}{\triangle(\omega^2)} \times P + \frac{C_T.AL.}{\triangle(\omega^2)} . M$$

$$= \frac{1.20 \times 425 - 236.19 + 0.34 \times 10^3 \times 63 \times (1.19)^2 + 119\ (12.6)^2}{8\ 6 \times 10^9} \times 2.5 +$$

$$\frac{0.34 \times 10^3 \times 63 \times 1.19}{8.6 \times 10^9} = 0.0143 + 0.0975 = 0.1118 \text{ mm.}$$

$$A\phi = \frac{C_T.AL}{\triangle\ (\omega^2)} . P + \frac{C_{TA} - mw^2}{\triangle\ (\omega^2)}\ M$$

$$= \frac{0.34 \times 10^3 \times 63 \times 1.19}{86 \times 10^9} \times 2.5 + \frac{0.34 \times 10^3 \times 63 - 24.02 \times (12.6)^2}{8.6 \times 10^9} \times 3.28$$

$$= 7.4 \times 10^{-6} + 6.2 \times 10^{-6} = 13.6 \times 10^{-6} \text{ radian}$$

$$Ax\phi = Ax + hA\phi = 0.1118 + 13.6 \times 10^{-6} \times 0.81$$

$$= 0.188 + 0.011 = 0.129 \text{ mm where } h = 2 - 1.19 = 0.81 \text{ m}$$

Richart's Method

$G = 1740\ t/m^2$

For the Foundation shown in Fig. 3.

1. Equivalent radii

$$r_x = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{63}{\pi}} = 4.48 \text{ m}$$

$$m\left(\frac{r^2\phi}{4} + \frac{h^2}{3}\right) = I_o = I_{mo} - \text{where } h = \text{height of block} = 2\text{m.}$$

$$24.02\left(\frac{r\phi^2}{4} + \frac{2^3}{3}\right) = 153\ \therefore\ \ r\phi = 4.9 \text{ m}$$

2. Mass ratio

$$bx = \frac{mg}{\rho\ (rx)^3} = \frac{24.02 \times 981}{1.8\ (4.48)^3} = 1.45$$

Inertia Ratio

$$b\phi = \frac{I_o \times q}{\rho(r\phi)^5} = \frac{153 \times 9.81}{1.8 \times (4.9)^5} = 0.3$$

3. Frequency Factors

From Richarts charts Fig. 2b and 2c.

$a_x = 1.4$; $a\phi = 1.4$

4. Natural frequency in sliding.

$$w^2_{nx} = \frac{a^2_x\ Gg}{\rho ro^2} = \frac{(1.4)^2 \times 1740 \times 9.81}{1.8 \times (4.48)^2} = 930 \text{ sec}^{-2}$$

$$w_{nx} = 30\ 5 \text{ sec}^{-1}\ ;\ f_{nx} = 4.8 \text{ c. p. s.}$$

5. Natural frequency in Rocking

$$w^2_{n\phi} = \frac{a^2_\phi \; Gg}{\gamma \; (r\phi)^2} = \frac{(1.4)^2 \times 1740 \times 981}{1.8 \times (4.9)^2} = 775 \text{ sec}^{-2}$$

$w_{n\phi} = 27.8 \text{ sec}^{-1}$; $f_{n\phi} = 4.35$ cp.s.

6. Natural Frequency in combined mode

$$w^4_{nx\phi} - \left[(w^2_{nx} + w^2_{n\phi}) + \frac{mz^2}{I_o} w^2_{nx} \right] w^2_{nx\phi} + w^2_{nx} \cdot w^2_{n\phi} = 0$$

$z = 1.19$

$$w^4_{nx\phi} - \left[(27.8)^2 + (30.5)^2 + \frac{24.02 \times (1.19)^2}{15^3} \times (30.5)^2 \right] w^2_{nx\phi} + (27.8)^2 \times (30.5)^2 = 0$$

$w^2_{nx\phi 1} = 725 \text{ sec}^{-2}$; $w_{nx\phi 1} = 26.8 \text{ sec}^{-1}$; $f_{nx\phi 1} = 4.2$ c.p,s.

$w^2_{nx\phi 2} = 1425 \text{ sec}^{-2}$: $w_{nx\phi 2} = 37.5 \text{ sec}^{-1}$; $f_{nx\phi 2} = 6$ c.p.s.

7. Amplitudes

Amplitude factors

$As = 0.27$; $Ar = 1.3$

$$A_x = \frac{P}{Gr_x} \times As = \frac{2.5}{(1740 \times (4.9)^3} \times 0.27 = 0.08 \text{ mm}$$

$$A\phi = \frac{M \times Ar}{G \times r\phi^3} = \frac{3.28 \times 13}{1740 \times (4.9)^3} = 2.1 \times 10^{-5} \text{ radian}$$

$$A_x\phi = A_x + hA\phi = 0.08 + (2.1 \times 10^{-5}) \times 0.8 \times 10^{-3}$$
$$= 0.08 + 0.0168 = 0.0068 \text{ mm}$$

Pauw's Method

From Table III, using smallest values

$\beta = 3.87$ kg/cm³

$\beta' = 1.42$ kg/cm³

Assume $\alpha = 1$

For the foundation shown in Fig. 2,

$a = 9$ m ; $b = 7$ m

$$q = \frac{24.02 \times 9.81}{63} = 3.75 \text{ t/m}^2; \; h = \frac{q}{\rho} = \frac{3.75}{1.8} = 2.08 \text{ m}$$

$$S = \frac{\alpha h}{b} = \frac{1 \times 2.08}{7} = 0.297; \; r = \frac{a}{b} = \frac{9}{7} = 1.29$$

1. Spring constants

From Pauw's charts ; for above values of S and r.

$$\frac{\gamma_x^{xy}}{r} = 0\,95 \; ; \; \gamma_x^{xy} = 1.29 \times 0.95 = 1.23$$

$$k_x^{xy} = \beta' b^2 \gamma_x^{xy} = 1.42 \times (10)^3 \times (7)^2 \times 1.23 = 85100 \text{ t/m}$$

$$\frac{\gamma_{xy}^{xy}}{r} = 0.18 \;;\; k_{xz}^{xy} = \beta b^4 \gamma_{xz}^{xy} = 3.87 \times 10^3 \times (7)^4 \times 1.29 \times 0.18 \text{ t/m}$$
$$= 2160000 \text{ t/m}$$

2. Apparent Soil mass

$$\frac{Cm}{r} = 0.62 \;;\; Cm = 0.62 \times 1.29 = 0.795$$

$$ms = \frac{\rho\, b^3\, Cm}{g a} = \frac{1.8 \times 7^3}{9.81 \times 1} \times 0.795 = 63.0 \text{ t-m}^{-1}\text{-sec}^2$$

3. Apparent mass moment of Inertia of soil I_s

$$\frac{C_i}{r^3} = 0.21 \;;\; C_i = (1.29)^3 \times 0.21 = 0.451$$

$$I_s = \frac{C_i\, \rho\, b^5}{12\, g a} = \frac{0.451 \times (7)^5 \times 1.8}{19 \times 9.81 \times 1} = 116 \text{ t-m-sec}^2$$

Height of C.g. of the soil mass and foundation and machine above the base

$$Z = \frac{24.02 \times 1.19}{63.0 + 24.02} = 0.328 \text{ m}$$

$$I_{ms} = I_m + m\,(Z_{01} - Z)^2 + I_s + m_s\,(Z)^2$$
$$= 119 + 24.02\,(1.19 - 0.328)^2 + 116 + 63\,(0.228)^2 = 256 \text{ t-m-sec}^2$$

4. Natural Frequency in Sliding

$$w_{nx} = \sqrt{\frac{kx}{m + m_s}} = \sqrt{\frac{85100}{24.02 + 63.0}} = 31.3 \text{ sec}^{-1}$$

$$f_{nx} = 5 \text{ c.p.s.}$$

5. Natural Frequency in Rocking.

$$w_{n\phi} = \sqrt{\frac{k_{xz}}{I_{ms}}} = \sqrt{\frac{2160000}{256}} = 92 \text{ rad/sec.}$$

$$f_{n\phi} = 14.5 \text{ c.p.s.}$$

6. Natural Frequency in Combined Mode

$$w^2_{nx\phi_{1,2}} = \frac{1}{2}\left[\left(\frac{kx}{m} + \frac{Z^2\, k_x + k_{zx}}{I_{ms}}\right) \pm \sqrt{\left(\frac{kx}{m} + \frac{z^2\, kx + kzx}{I_{ms}}\right)^2 - \frac{4kx\, kxz}{m\, I_{ms}}}\right]$$

$$w^2_{nx\phi_{1,1}} = \frac{1}{2}\left[\frac{85100}{87.02} + \frac{(0.328)^2 \times 85100 + 2160000}{256}\right.$$
$$\left. \pm \sqrt{\left(\frac{85100}{87.02} + \frac{(0.328)^2 \times 85100 + 2160000}{256}\right)^2 - \frac{4 \times 85100 \times 2160000}{86.02 \times 256}}\right]$$
$$= 4715\,(1 \pm 0.90)$$

$w_{nx\phi_1} = 21.7\ sec^{-1}$; $f_{\eta x\phi_1} = 3.5$ c.p.s.

$w_{nx\phi_2} = 94.8\ sec^{-1}$; $f_{nx\phi_2} = 15.1$ c.p.s.

7. Amplitudes

Assume damping factor = 0.20

$$Ax = \frac{P}{kx\sqrt{\left(1-\left(\frac{w}{w_{nx}}\right)^2\right)^2 + \left(2\xi\frac{w}{w_{nx}}\right)^2}}$$

$$= \frac{2.5}{85100\sqrt{(1-(0.4)^2)^2+(2\times 0.2\times 0.4)^2}} = 0.03 \text{ mm}$$

$$A\phi = \frac{Q}{kxz\sqrt{\left(1-\left(\frac{w}{w_{n\phi}}\right)^2\right)^2 + \left(2\xi\frac{w}{w_{n\phi}}\right)^2}}$$

$$= \frac{3.28}{2160000\sqrt{(1-0.138^2)^2+(2\times 0\ 2\times 0.138)^2}} = 1.77\times 10^{-6} \text{ radian}$$

$$Ax\phi = Ax + h\,A\,\phi$$

$$= 0.03 + (0.81\times 1.77\times 10^{-6})\times 10^3$$

$$= 0.03 + 0.014 = 0.0443 \text{ mm}$$

Design a foundation for a horizontal reciprocating engine with following data.

Speed of the engine	= 200 R.P.M.
Total weight of engine	= 7000 kg
Weight of reciprocating parts	= 45 kg
Weight of eccentrically rotating parts	= 30 kg
Length of connecting rod	= 100 cms
Crank radius	= 30 cm
Height of horizontal unbalance force above the top surface of foundation	= 30 cm
Soil-Uniform Silty clay	
Allowable bearing capacity	= 1.0 kg/cm²

Use soil constants determined in Table IV

Solution

$$P = \frac{W_p}{g} rw^2 \left(\cos wt \frac{r}{l} \cos 2\,wt\right) + W_c\, rw^2 \cos wt$$

$$(P)_{max} = \frac{W_p}{g} rw^2 \left(1+\frac{r}{l}\right) + \frac{W_c}{g} rw^2,\ w = \frac{200}{60}\times 2\pi = 21 \text{ rad/sec}$$

$$= \frac{0.045}{9.81}\times 1 \times (21)^2\,(1+0.3) + \frac{0.03}{9.81}\times 0.3\,(21)^2 = 1.2 \text{ t.}$$

Barkan's Method

Adopt a block foundation shown in Fig 4.

Density of concrete $= 2.2\ t/m^2$

Wt. of foundation block $= (4\times4\times0.5 + 3\times3\times1)\times2.2$
$= 37.4$ t.

$$\text{Soil pressure} = q = \frac{44.4}{4\times4} = 2.78\ t/m^2$$
$$= 0.278\ kg/cm^2 < 1.0\ kg/cm^2$$

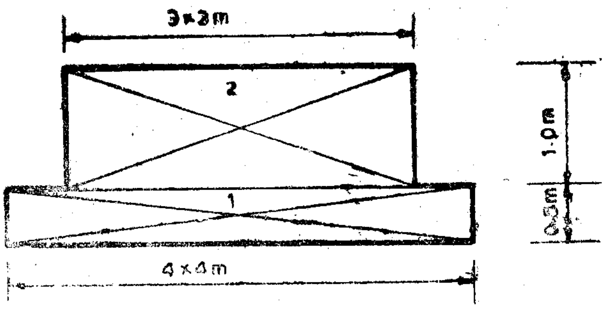

Fig. 4. Cross-section of the foundation block

1. Hight of C–G. above base of foundation

$$Z = \frac{3\ 57}{4.345}\ 0.825\ m$$

Exciting Moment $= M = 1.2\times0.975 = 1.17$ t–m

2. Moment of Inertia

$$I = \frac{4\times4^3}{12} = 21.3\ m^4$$

$$I_m = 3.95 + 1.68 = 5.63\ \text{t–m–sec}^2$$

$$I_{mo} = I_m + m\,L^2 = 5.63 + \frac{4.44}{9.81}\,(0.825)^2$$
$$= 8.58\ \text{t–m–sec}^2$$

$$\gamma = \frac{I_m}{I_{mo}} = \frac{5.63}{8.58} = 0.65$$

3. Soil Constants

Cu for 10 m^2 area (Table IV) $= 1.68\ kg/cm^3$

Cu for 16 m^2 area $= 1.86\sqrt{\frac{10}{16}} = 1.47\ kg/cm^3 = 1.47\ t/m^3$

$C_T = 0.735 \times 10^3\ t/m^3$

$C\phi = 2.5 \times 10^3\ t/m^3$

Table V Computation of Centre of Gravity

Element	Dimension of element			Mass t–m^{-1} –sec^2	Cordinates of C.G. of the element w.r.t. x,y,z axes (m)			Static moment w.r.t. z–axis t–m	Mass moment of inertia of element about an axis through its C.G.	Distance between C.G. of the element and combined C.G.		
A	a_{x1}	a_{y1}	a_{z1}	m_1	x_1	y_1	z_1	$m_1 z_1$	$\frac{m_1}{12}(a_{x1}^2+a_{z1}^2)$	x_{01}	z_{01}	$m_1(x_{01}^2+z_{01}^2)$
Enine	—	—	—	0.715	2	2	1.80	1.28	—	0	0.975	0.680
1	4.0	4.0	0.5	1.79	2	2	0.25	0.45	2.42	0	0.725	0.945
2	3.0	3.0	1.0	1.84	2	2	1.0	1.84	1.53	0	0.175	0.056
				4.345				3.57	3.95			1.681

4. Natural frequency in Sliding

$$w_{nx} = \sqrt{\frac{C_T A}{m}} = \sqrt{\frac{0.735 \times 10^3 \times 16}{4.35}} = 54.8 \text{ sec}^{-1}$$

$$f_{nx} = 8.5 \text{ c.p.s. } ; \quad w^2_{nx} = 3.0 \times 10^3 \text{ sec}^{-2}$$

5. Natural frequency in Rocking

$$w_{n\phi} = \sqrt{\frac{C\phi . I - WL}{I_{mo}}} = \sqrt{\frac{2.5 \times 10^3 \times 21.3 - 44.4 \times 0.825}{8.58}} = 78 \text{ sec}^{-1}$$

$$f_{n\phi} = 12.4 \text{ c/s } ; \quad w^2_{n\phi} = 6.1 \times 10^3 \text{ sec}^{-2}$$

6. Natural frequency in combined mode

$$w^4_{nx\phi} - \frac{w^2_{n\phi} + w^2_{nx}}{\gamma} w^2_{nx\phi} + \frac{w^2_{nx} + w^2_{n\phi}}{\gamma} = 0$$

$$w^4_{nx\phi} - \frac{6.1 \times 10^3 + 3.0 \times 10^3}{0.65} w^2_{nx\phi} + \frac{6.1 \times 3.0 \times 10^6}{0.65} = 0$$

$$w^2_{nx\phi_1} = 2.415 \times 10^3 \text{ sec}^{-2} ; \; w_{nx\phi_1} = 49.1 \text{ sec}^{-1} ; \; f_{nx\phi_1} = 7.8 \text{ c.p.s.}$$

$$w^2_{nx\phi_2} = 11.58 \times 10^3 \text{ sec}^{-2} ; \; w_{nx\phi_2} = 108 \text{ sec}^{-1} ; \; f_{nx\phi_2} = 17.2 \text{ c.p.s.}$$

7. Amplitudes

$$\Delta(\omega^2) = m \, I_m \, (w^2_{nx\phi_1} - w^2)(w^2_{nx\phi_2} - w^2)$$

$$= 4.35 \times 5.63 \, (2.415 \times 10^3 - 441)(11.58 \times 10^3 - 441) = 5.4 \times 10^8$$

$$A_x = \frac{C\phi . I - WL + C_T.AL^2 + I_m w^2}{\Delta(\omega^2)} \times P + \frac{C_T.AL}{\Delta(\omega^2)} . M$$

$$= \frac{2.5 \times 10^3 \times 21.3 - 44.4 \times 0.825 + 0.77 \times 10^3 \times 16 \times 0.8 - 5.63(21)^2}{5.4 \times 10^8} \times 1.2 +$$

$$\frac{0.77 \times 10^3 \times 16 \times 0.825}{5.4 \times 10^8} \times 1.17 = 0.152 \text{ mm.}$$

$$A\phi = \frac{C_T.AL}{\Delta(w^2)} .. P + \frac{C_T AL - mw^2}{\Delta(w^2)} M$$

$$= \frac{0.77 \times 10^3 \times 16 \times 0.825}{5.4 \times 10^8} \times 1.2 + \frac{0.77 \times 10^3 \times 16 - 4.35 \times 441}{5.4 \times 10^2} \times 1.17$$

$$= 4.5 \times 10^{-5} \text{ radian}$$

$$Ax\phi = A_x + h\, A\phi = 0.152 + 0.675 \times 4.15 \times 10^{-5} \times 10^3$$

$$= 0.152 + 0.0302 = 0.182 \text{ mm}$$

Richart's Method

Try Foundation shown in Fig. 4.

Adopt value of $G = 2200 \text{ t/m}^2$ (From Table IV)

1. Equivalent radii

$$rx = \sqrt{\frac{16}{\pi}} = 2.26 \text{ m}$$

$$m\left(\frac{r^2\phi}{4} + \frac{h^2}{3}\right) = I_{mo}$$

$$4.35\left(\frac{r\phi^2}{4} + \frac{1.5^2}{3}\right) = 8.58 \; ; \quad r\phi = 2.20 \text{ m}$$

2. Mass ratio

$$bx = \frac{mg}{\rho \, rx^3} = \frac{4.35 \times 9.81}{1.91 \times (2.26)^3} = 2$$

3. Inertia ratio

$$b\phi = \frac{I_{mo} \times g}{\rho (r\phi)^5} = \frac{8.58 \times 9.81}{1.91 \times (2.2)^5} = 1.0$$

4. Frequency Factors

From Richarts charts (Fig. 2)

$a_x = 1.7$; $a\phi = 1.38$

5. Natural frequency in sliding.

$$w_{nx} = \frac{a_x}{r_x}\sqrt{\frac{G.g}{\rho}} = \frac{1.7}{2.26}\sqrt{\frac{9.81 \times 2200}{1.91}} \; 80 \text{ sec}^{-1}$$

$f_{nx} = 12.7$ c.p.s.

6. Natural frequency in rocking

$$w_{n\phi} = \frac{a\phi}{r\phi}\sqrt{\frac{g\,G}{\rho}} = \frac{1.38}{2.2}\sqrt{\frac{9.81 \times 2200}{1.91}} = 67 \text{ sec}^{-1}$$

$f_{n\phi} = 10.7$ c.p.s.

7. Natural frequency in combined mode

$$w^4_{nx\phi} - \left[(w^2_{nx} + w^2_{n\phi}) + \frac{m}{I_{mo}} Z^2 \, w^2_{nx}\right] w^2_{nx\phi} + w^2_{nx} \, . \, w^2_{n\phi} = 0$$

$$w^4_{nx\phi} - \left[(84)^2 + (67)^2 + \frac{4.35}{8.58} (0.825)^2 \times (84)^2\right] w^2_{nx\phi} + (84)^2 \times (67)^2 = 0$$

$w^2_{nx\phi_1} = 2800 \text{ sec}^{-2}$: $w_{nx\phi_1} = 53 \text{ sec}^{-1}$; $f_{nx\phi_1} = 8.45$ c.p.s.

$w^2_{nx\phi_2} = 11200 \text{ sec}^{-2}$: $w_{nx\phi_2} = 106 \text{ sec}^{-1}$; $f_{nx\phi_2} = 16.9$ c.p.s.

8. Amplitude factors

$A_s = 0.5$; $A_r = 1.2$

Amplitudes

$$A_x = \frac{P \times A_s}{G\, r_x^3} = \frac{1.2 \times 0.5}{2200 \times (2.26)^3} = 0.12 \times 10^{-3}\ \text{m} = 0.12\ \text{mm}$$

$$A\phi = \frac{A_r \times M}{G \times (r\phi^3)} = \frac{1.2 \times 1.17}{2200 \times (2.2)^3} = 0.06 \times 10^{-3}\ \text{radian}$$

$$A_x\phi = A_x + h\, A\phi = 0.12 + 0.06 \times 10^{-3} \times 0.675 \times 10^3 = 0.1605\ \text{mm}$$

Pauw's Method

Use the same foundation as above,

The values of β and β′ to be used are as given in (Table IV)

$$\beta = 0.97\ \text{kg/cm}^3 = 970\ \text{t/m}^3$$
$$\beta' = 0.42\ \text{kg/cm}^3 = 420\ \text{t/m}^3$$
$$E_o = 740\ \text{kg cm}^2$$

Assume $\alpha = 1$

$$q = \frac{44.4}{4 \times 4} = 2.78\ \text{t/m}^2;\ h = \frac{E_o}{\beta} + \frac{q}{\rho} = \frac{740}{97.0} + \frac{2.78}{1.91} = 0.9\ \text{m}$$

$$r = \frac{a}{b} = 1;\ S = \frac{ah}{b} = \frac{1 \times 9}{4} = 2.25$$

1. Spring constants

$$\frac{\gamma_x^{xy}}{r} = 1.8;\ \gamma_x^{xy} = 1.8,\ k_x^{xy} = \beta^1 b^2\, \gamma_x^{xy} = 420 \times 16 \times 1.8 = 12100\ \text{t/m}$$

$$\gamma_{xz}^{xy} = 0.48;\quad k_{xz}^{xy} = \beta b^4\, \gamma_{xz}^{xy} = 970 \times 256 \times 0.48 = 120000\ \text{t/m}$$

2. Apparent Soil mass

$$\frac{Cm}{r} = 1.8;\quad Cm = 1.8;$$

$$ms = \frac{\rho\, b^3}{g a} Cm = \frac{1.91 \times 4^3}{9.81 \times 1} \times 1.8 = 22.4\ \text{t-m}^{-1}\text{-sec}^2$$

3. Apparent mass moment of Inertia of soil

$$\frac{C_1}{r} = 0.45;\ C_1 = 0.45$$

$$I_s = \frac{\rho\, b^5}{12\, g a} C_1 = \frac{1.91 \times (4)^5}{12 \times 9.81 \times 1} \times 0.45 = 7.2\ \text{t-m-sec}^2$$

4. Height of Combined C.G. of the soil and foundation etc.

$$\bar{Z} = \frac{4.35 \times 0.825}{4.35 + 22.4} = 0.134\ \text{m}$$

$$I_{ms} = \frac{1.79}{12}(4^2+0.5^2) + 1.79\,(0.116)^2 + 1.84\,(3^2+1^2) + 1.84\,(0.866)^2 + 0.715\,(1.66)^2 + 7.2 + 22.4\,(0.134)^2 = 14.95 \text{ t-m-sec}^2$$

5. Natural Frequency in Sliding

$$w_{nx} = \sqrt{\frac{k_x}{m+ms}} = \sqrt{\frac{12100}{4.35+22.4}} = 67.2 \text{ sec}^{-1}$$

$$f_{nx} = 10.7 \text{ c p.s.}$$

6. Natural Frequency in Rocking.

$$w_{n\phi} = \sqrt{\frac{k_{xz}}{I_{ms}}} = \sqrt{\frac{120000}{14.95}} = 90 \text{ sec}^{-1}$$

$$f_{n\phi} = 14.3 \text{ c.p.s.}$$

7. Natural frequency in combined mode

$$w^2_{nx\phi1,2} = \frac{1}{2}\left[\left(\frac{kx}{m} + \bar{Z}^2\,\frac{k_x+k_{zx}}{I_{ms}}\right) \pm \sqrt{\left(\frac{k_x}{m} + \bar{z}^2\,\frac{k_x+k_{zx}}{I_{ms}}\right)^2 - \frac{4kx\,kxz}{m\,I_{ms}}}\right]$$

$$w^2_{nx\phi1,2} = \frac{1}{2}\left[\left(\frac{12100}{26.7} + \frac{(0.134)^2 \times 12100 + 120000}{14.95}\right) \pm \sqrt{\left(\frac{12100}{26.7} + \frac{(0.134)^2 \times 12100 + 120000}{14.95}\right)^2 - \frac{4 \times 120000 \times 12100}{14.95 \times 26.7}}\right]$$

$w_{nx\phi1} = 3700 \text{ sec}^{-2}$; $w_{nx\phi_1} = 60.8 \text{ sec}^{-1}$; $f_{nx\phi1} = 9.65$ c.p.s.

$w_{nx\phi_2} = 9553 \text{ sec}^{-2}$; $w_{nx\phi2} = 98 \text{ sec}^{-1}$; $f_{nx\phi2} = 15.6$ c.p.s.

8. Amplitudes

$$Ax = \frac{P}{k_x\sqrt{\left(1-\left(\frac{w}{w_{nx}}\right)^2\right)^2 + \left(2\xi\,\frac{w}{w_{nx}}\right)^2}}$$

Assume $\xi = 0.2$; $\frac{w}{w_{nx}} = \frac{21}{67.8} = 0.312$

$$= \frac{1.2}{12100\sqrt{(1-(0.312)^2)^2+(2\times0.2\times0.312)^2}} = 0.1\times10^{-5} \text{ m} = 0.10 \text{ mm}$$

$$A\phi = \frac{M}{k_{xz}\sqrt{\left(1-\left(\frac{w}{w_{n\phi}}\right)^2\right)^2 + \left(2\zeta\,\frac{w}{w_{n\phi}}\right)^2}}$$

$$\frac{w}{w_{n\phi}} = \frac{21}{90}$$

$$= \frac{1.17}{120000\sqrt{\left(1-\left(\frac{21}{90}\right)^2\right)^2+\left(2\times0.2\times\frac{21}{90}\right)^2}} = 1.02\times10^{-5} \text{ radian}$$

$$Ax\phi = Ax + h\,A\phi = 0.1\times10^{-3} + 1.02\times10^{-5}\times0.675$$
$$= 0.107 \times 10^{-3}\text{ m} \quad = 0.107 \text{ mm}$$

Comparison of Results

Example A

Table VI

Method	Barkan	Pauw	Richart
Natural frequency in sliding $f_{n\phi}$ c p.s.	4.8	5.0	4.8
Natural frequency in rocking $f_{n\phi}$ c.p s.	9.2	14.5	4.35
Natural frequency in combined mode $f_{nx\phi_1}$, $f_{nx\phi_2}$ c.p.s.	4.6, 10.8	3.5, 15.1	4.2, 6.0
Amplitude in sliding A_x mm	0.1118	0.03	0.08
Amplitude in rocking $A\phi$ radian	13.6×10^{-6}	1.77×10^{-6}	2.1×10^{-5}
Amplitude in combined mode $A_{x\phi}$	0.129	0.0443	0.0968

Example B

Table VII

Method	Barkan	Pauw	Richart
Natural frequency in sliding f_{nx} c.p s.	8.5	10.7	13.4
Natural frequency in rockin $f_{n\phi}$ c p.s.	12.4	14.3	10.7
Natural frequency in combined mode, $f_{nx\phi_1}$, $f_{nx\phi_2}$ c.p.s.	7.8, 17.2	9.65, 15.6	8.45, 16.9
Amplitude in sliding A_x mm	0.152	0.100	0.120
Amplitude in rocking A_ϕ radian	4.5×10^{-5}	1.02×10^{-5}	6.0×10^{-5}
Amplitude in combined mode $Ax\phi$ mm	0.182	0.107	0.1605

Conclusions

A comparison of natural frequencies and amplitudes obtained in the two design problems for typical foundation shows that these three methods give results which are sufficiently in agreement. The slight difference is due to assumptions made in particular type of analysis.

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Paper No. 77

Bulletin of the Indian Society of Earthquake Technology, Vol. 6, No. 3, pp. 137-146, June, 1969

BEHAVIOUR OF CELLULAR BRIDGE PIERS UNDER DYNAMIC LOADS

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Synopsis

In this paper an experimental and theoretical study is made to find the effect of different filling materials in cellular bridge piers on their behaviour under free and forced vibrations. Four different fills, which can be commonly made use of, are tried and their effect on damping and natural frequency of the pier is observed under low water level and high flood level conditions. Some conclusions are drawn as a result of this study which will be useful in practice and for planning further investigation.

1. Introduction

In an earlier paper by the authors(1), the dynamic characteristics of masonry piers and their behaviour under free and forced vibration conditions were discussed. In order to rationalize the aseismic design of the bridge sub-structure, an attempt is made in this paper to study the behaviour of the cellular piers with different fills when subjected to dynamic loads For this purpose an experimental steel model of a reinforced concrete balanced cantilever bridge supported on cellular piers founded on rock was subjected to free and forced vibrations.

2. Details of the Model

Details of the various elements of the model bridge are given below :

2.1. *Super-structure* : Super-structure for the model consisted of one main span of 270 cm length with overhangs of 45 cm length on either side of a box type balanced cantilever bridge. It was made of steel plates welded together as shown in Fig. 1. At the two ends of the main span bearing plates were welded underneath the soffit slab so as to fix the roller and rocker bearings as shown in Fig. 2(a) and 2(b).

2.2. *Piers* : Model piers were chosen of circular cellular cross section made of steel. The main dimensions of the piers were : internal diameter 15 6 cm, height 75 cm, thickness of the steining 0.35 cm. The piers were fixed to a 45 cm × 45 cm × 45 cm size 1:2:4 cement concrete foundation through 4 nos. 16 mm dia anchor bolts and a steel plate as shown in Fig. 1.

2.3. *Bearings* : For supporting the superstructure on top of piers, a rocker steel bearing was provided on top of one pier and a roller steel bearing on top of the other pier. Bearings were made of a shaft with two wheels on either end of it. Ball bearing arrangement was made between the shaft and the wheels so as to allow free rotation of the two bearings.

2.4. *Filling Materials* : The following most commonly used filling materials were used to fill the cellular space in the piers.

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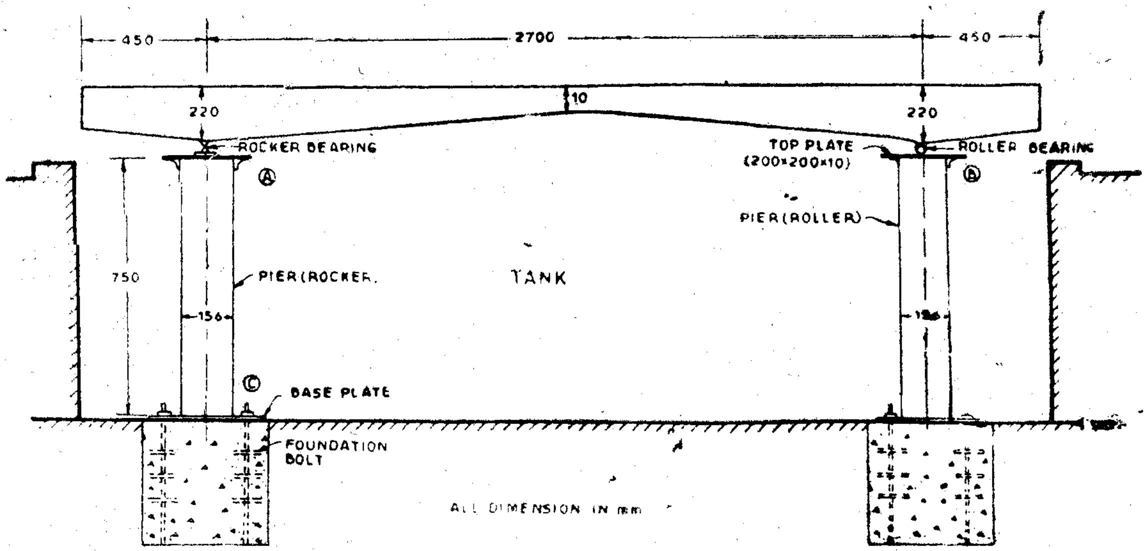

Fig. 1. Model Bridge

(1) Dry sand of density 1.48 gm/cc
(2) Stone ballast of density 1.60 gm/cc
(3) Saturated sand having density of 1.9 gm/cc
(4) Water with density of 1.00 gm/cc

Besides, the piers were also tested with no filling materials inside.

To simulate the high flood conditions, water was filled all round upto a height of 65 cm and to represent the low water conditions no water was filled outside.

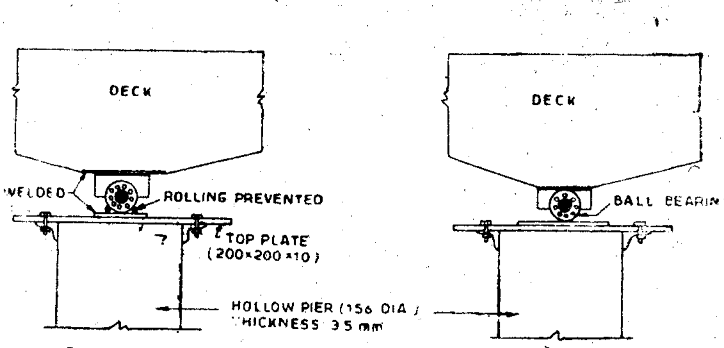

Fig. 2. Details at A,B (*vide* Fig. 1)

3. Tests

The following tests were carried out on the model :—

3.1 *Pull test* : In order to determine the degree of fixity at the base and the effect of the various filling materials on the stiffness of a pier, static horizontal load was applied at the top of the pier. The deflections at top were measured for different horizontal pulls under different conditions of fill material. The load deflection curves are plotted in Fig. 3. It is observed that there is only a slight effect of the various fills on the stiffness of the piers and that the deflections in each case are in excess than those obtained theoretically by considering the pier to be rigidly fixed at the base. It is found from the curve of empty tank that the base is having a stiffness against rotation equal to 21.5×10^6 kg–cm/radian in place of being completely rigid. Subtracting the effect of the base rotation from the deflection the stiffness of the pier under various filling conditions is obtained as follows :

(a) Empty or with water	2550 kg/cm
(b) Dry sand or ballast	2670 kg/cm (about 5% increase)
(c) Saturated sand	2840 kg/cm (about 11% increase)

1	2	3	4	5
17	XX XX	23.6	12.872	45.50
18	——— ———	2.71	2.469	8.90
19	–X— ———	3.18	2.936	7.68
20	—XX ———	3.36	2.978	11.39
21	XXX ———	3.47	2.995	13.70
22	——— —X—	4.81	3.957	17.72
23	—X— –X—	9.61	9.476	1.39
24	——X —X—	9.25	9.069	1.96
25	–XX –X–	13.10	9.277	29.20
26	–X— X–X	15.80	11.038	30.20
27	—X– —XX	15.38	10.479	32.00
28	—XX –XX	17.05	11.765	31.00
29	—XX XX–	17.60	13.846	21.35
30	XXX —XX	18.10	13.093	27.60
31	—X— XXX	15.80	11.926	24.50
32	—XX XXX	18.65	15.669	16.00
33	XXX XXX	20.05	17.037	15.00

Note : (minus sign indicates that the measured frequency is lower than the corresponding computed frequency).

Acknowledgements

The authors acknowledge with gratitude the help rendered by the Earthquake Research School Workshop and Laboratories in conducting the experiments.

The authors are also very thankful to Mr. N.V. Raman, S.E.R C. Roorkee, for the use of his program. Help rendered by the staff of the S.E.R.C is also gratefully acknowledged.

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3.2 *Free Vibration tests* : For carrying out free vibration test a known horizontal pull was applied along the axis of the bridge at the top of the pier and released suddenly by clutch-plate arrangement as shown in Fig. 4(a) and (b). The vibrations were recorded by making use of an acceleration pick-up fixed at 16 cm height from the base along the axis of the pier. The records were obtained on Brush ink-writing oscillograph by connecting the pick-up with D. C. amplifier and recorder as shown in Fig. 4. The following observations were recorded :—

(1) Vibration of pier along axis of the bridge.

(2) Horizontal vibration of the deck.

(3) Vertical vibration at the centre of the deck.

The values of natural fundamental frequencies and average damping over 5 cycles for the same maximum first amplitude of 10 mm are given in Table 1 for various filling materials and water conditions.

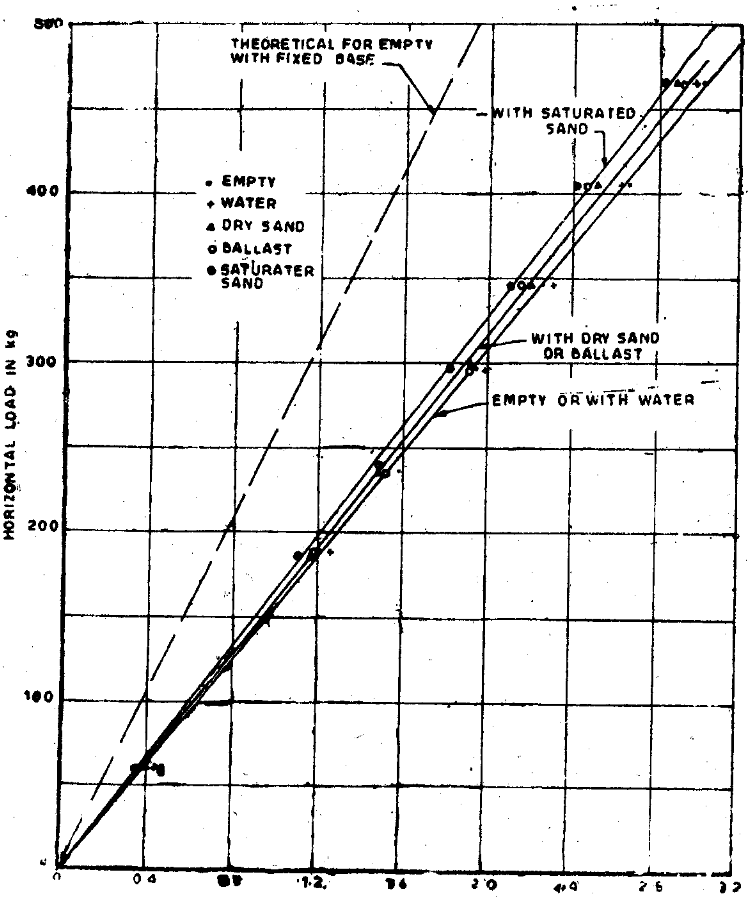


Fig. 3. Static load deflection curves

The values of maximum horizontal acceleration at the end of the super-structure and maximum vertical acceleration at the centre of the deck are given in Fig. 5 for various cases.

3 3 *Forced Vibration tests* : The forced vibration tests have been conducted by imparting horizontal accelerations to the deck by means of a mechanical oscillator mounted over the deck as shown in Fig. 4 (c) The speed of the motor was controlled at the desired rate with the help of a speed control unit so that a sinusoidal periodic force of various desired frequencies and amplitudes could be imparted at the top of the bridge model. The horizontal motion was applied along the longitudinal axis of the bridge and records were obtained for the vibration of pier with the help of a strain gauge fixed at 15 cm height from the base and horizontal acceleration of the deck with the help of an acceleration pick-up fixed on top.

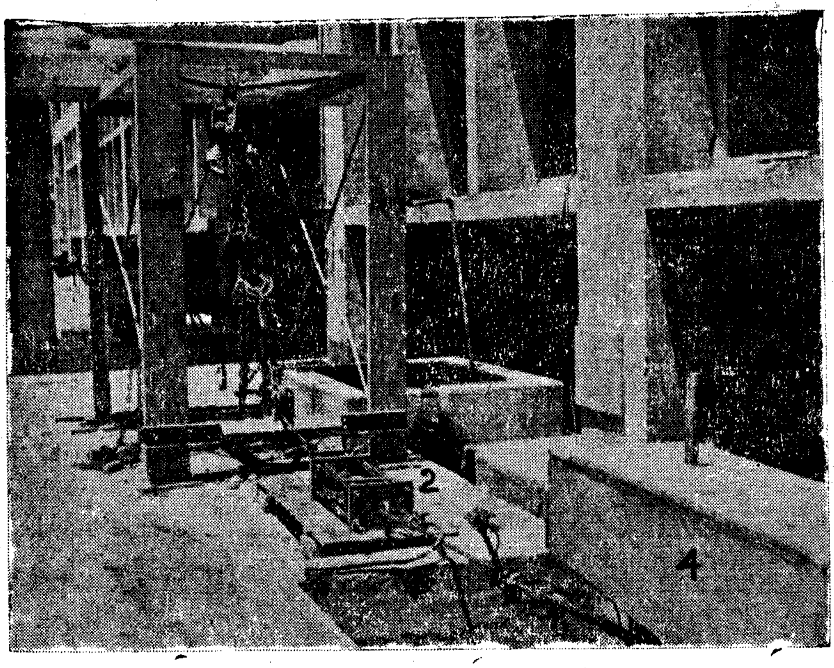
Resonance curves are drawn taking A/ω^2 as ordinate and ω/p as abscissa for empty and ballast filled conditions where ω is the circular frequency of the oscillator, p is the natural frequency and A is the strain in micro cm/per cm. These are shown in Fig. 6 (a) and (b).

Table 1

Natural Frequencies and Damping from Free Vibration Test (Rocker Pier with Superstructure)

Horizontal pull = 238 kg.

Type of filling	Frequencies of pier in cps		Damping % of critical		Horizontal frequencies of deck in cps.		Vertical frequencies at centre of the deck in cps.	
	No water outside	Water outside	No water outside	Water outside	No water outside	Water outside	No water outside	Water outside
Empty	11.25	11.00	2.60	2.75	11.25	11.00	32.0	30
Dry sand	11.25	11.00	3.00	2.98	11.25	11.00	31.5	30
Ballast	11.25	10.50	2.96	3.10	11.25	10.50	30.0	31
Saturated sand	11.00	10.25	2.70	2.90	11.00	10.25	31.0	32
Water	11.25	10.75	2.65	2.83	11.25	10.75	28.5	29

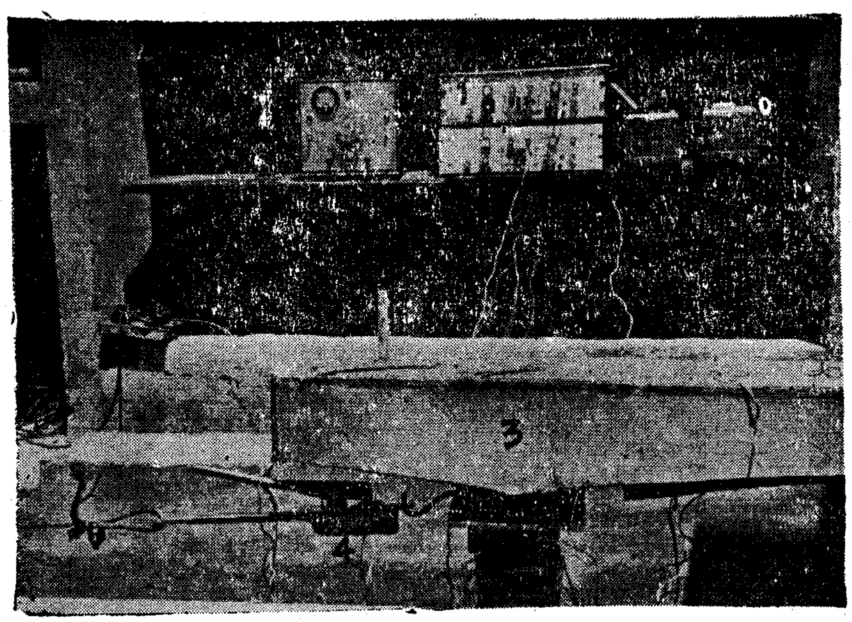


(a) Load Application

1. Chain pulley, 2. Tension Proving Ring
3. Release Cluth, 4. Bridge Deck

(c) Forced Vibration Test

1. Speed Control Unit
2. Universal Amplifiers
3 Motor
4 Lazan Oscillator
5 Bridge Deck



(b) Measuring Equipment

1. Universal Amplifiers, 2. Recording Oscillograph
3. Bridge Deck, 4. Release Clutch

Fig. 4. Details of Testing Arrangement

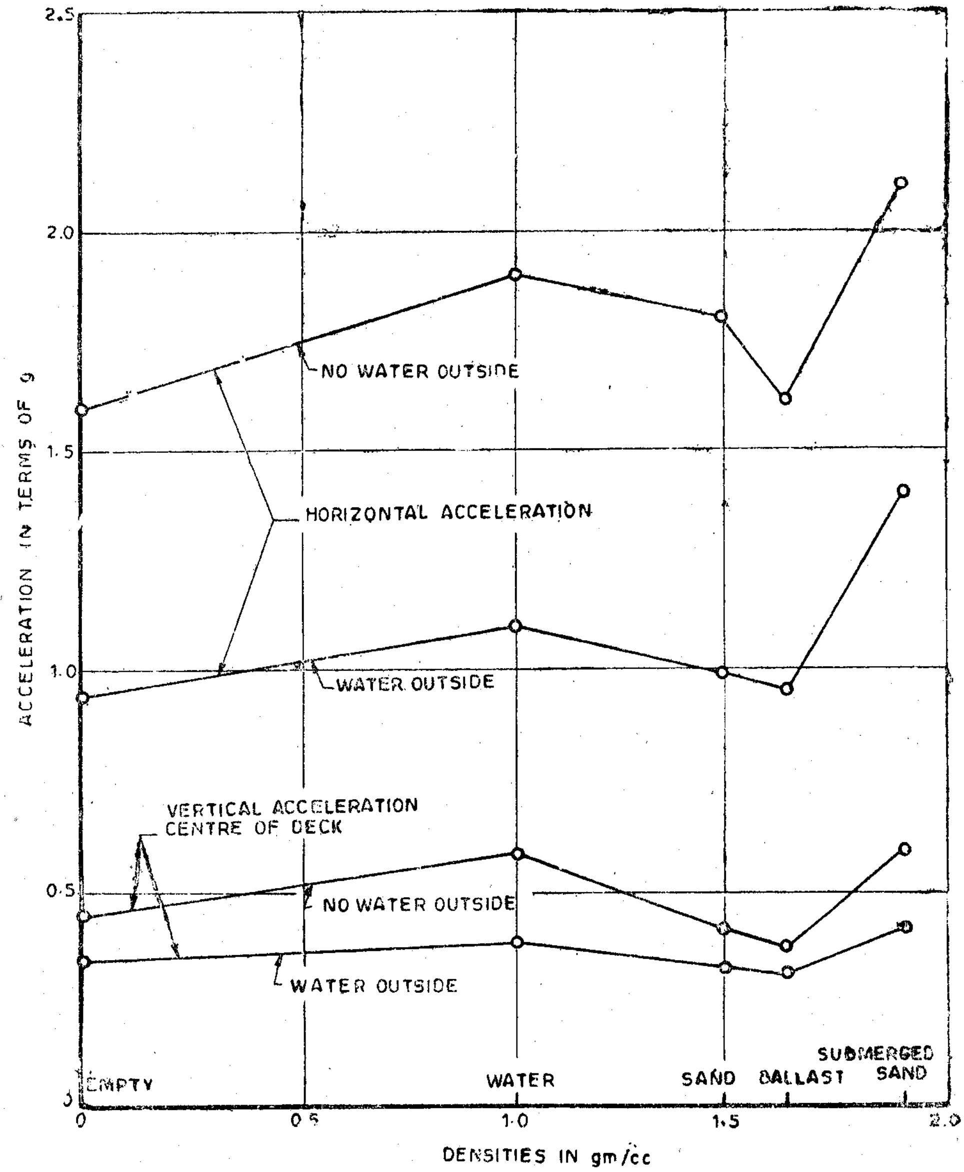


Fig. 5. Accelerations of Deck VS Density of Filling Material

4. Discussion of the Test Results

From the free vibration test results given in Table 1, it is observed that the fundamental frequency of the rocker pier is very little affected by the presence of the various filling materials. It is because of no appreciable change taking place either in the effective mass or stiffness of the pier due to the presence of various fills. In the case of saturated sand fill, the reduction in frequency is rather contrary to the increase in stiffness observed earlier. It appears that the dynamic pressures in the pore-water may have decreased the stiffness observed in static tests. The damping values are found to be maximum in case of dry sand and ballast. The natural frequency is decreased and damping is increased when water is filled outside as compared with the case when no such water was present. The decrease in frequency in water may be attributed to the virtual mass of water vibrating with the pier since there in no change in stiffness. Natural frequency for the vertical acceleration of the deck is almost the same in all conditions and equal to the fundamental frequency of the deck.

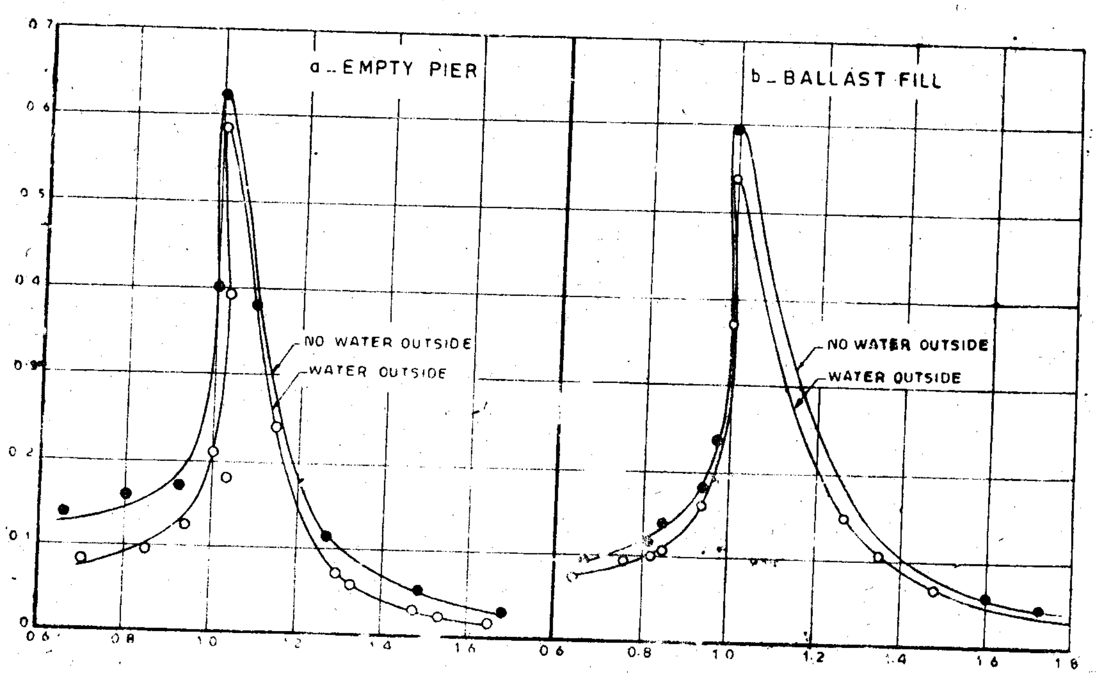


Fig. 6. Resonance test results for empty and ballast filled pier

From Fig. 5, it is obvious that the horizontal and vertical accelerations are minimum when ballast is used as a filling material. The accelerations are considerably reduced in each case when water is present outside. The average reduction in the horizontal accelerations is 37% and in the vertical accelerations of the deck 24%.

From forced vibration test results presented in Fig. 6 (a and b) it is obvious that resonant amplitude is less in case of water outside condition than that obtained in no water outside condition.

During forced vibration tests, it was observed that the superstructure got considerably accelerated in the vertical direction as well although the forced vibrations were imparted along the longitudinal axis of the bridge. At resonance the whole deck along with the oscillator tended to jump on the piers.

5. Theoretical Analysis

For the purpose of theoretical analysis, the pier is treated as a vertical cantilever either fixed or having some flexibility at the base. Depending upon the ratio of height to diameter of the pier both bending and shearing deformations may be significant. Therefore, natural frequencies are calculated considering both types of deformations. But the rotatory inertia is neglected since it will be small in such stiff structures The frequencies for the rocker pier have been computed numerically by Myklested—Prohl method (2,3,4) and are given in Table 2 for various values of rotational spring stiffness at the base. It is observed that the experimental and theoretical values of the fundamental frequencies are in agreement with each other for a value of rotational spring constant at base equal to 13.0×10^6 kg cm/radian. It will be recalled that the rotational spring constant was earlier found as 21.5×10^6 kg cm/radian by the static pull test. The dynamic value is perhaps reduced due to the mass of the foundation that may also be vibrating with the pier causing a reduction of the observed frequency so that it became equal to the calculated value with a smaller base spring stiffness.

Table 2

Natural Frequencies for Rocker Pier

Rotational Spring Stiffness $\times 10^6$ kg. cm	Frequency in		
	First Mode c/s	Second Mode c/s	Third Mode c/s
5.0	7.551	726.1	2206
10.0	10.117	742.1	2214
13.0	11.197	750.4	2218
16.0	12.078	758.0	2222
20.0	13.037	767.3	2227
25.0	13.993	777.65	2233
35.0	15.993	794.9	2243
50.0	16.776	814.9	2255

Natural frequencies for the rocker pier have also been computed when the cellular space is filled with various filling materials and for both 'water outside' and 'no water outside' conditions. The rotational stiffness of the base is considered equal to 13.0×10^6kg-cm/rad. as obtained above. While calculating the natural frequencies of vibration with water condition outside it is assumed that an equivalent mass of water of the enveloping cylinder is added to the mass of the pier upto the height of water outside[(5)]. In the circular section the enveloping cylinder has its diameter equal to the external diameter of the vibrating system. The values of the frequencies so calculated are given in Table 3. It is observed that there is no appreciable change in the fundamental frequency of the rocker pier with different filling materials. The fundamental frequency is slightly less when water outside condition exists. This is also confirmed by experimental test results. However, the higher mode frequencies are very much reduced by the presence of the various filling materials. Furthermore, at high flood level condition, the frequencies are considerably reduced.

Table 3

Theoretical Natural Frequencies

Type of filling	First mode		Second mode		Third mode	
	No water outside c/s	Water outside c/s	No water outside c/s	Water outside c/s	No water outside c/s	Water outside c/s
Empty	11.197	11.134	750	482	2218	1465
Dry sand	11.109	11.064	426	354	1257	1058
Ballast	11.084	11.059	408	343	1201	1024
Submerged sand	11.063	11.029	383	328	1128	978
Water	11.126	11.092	475	380	1402	1140

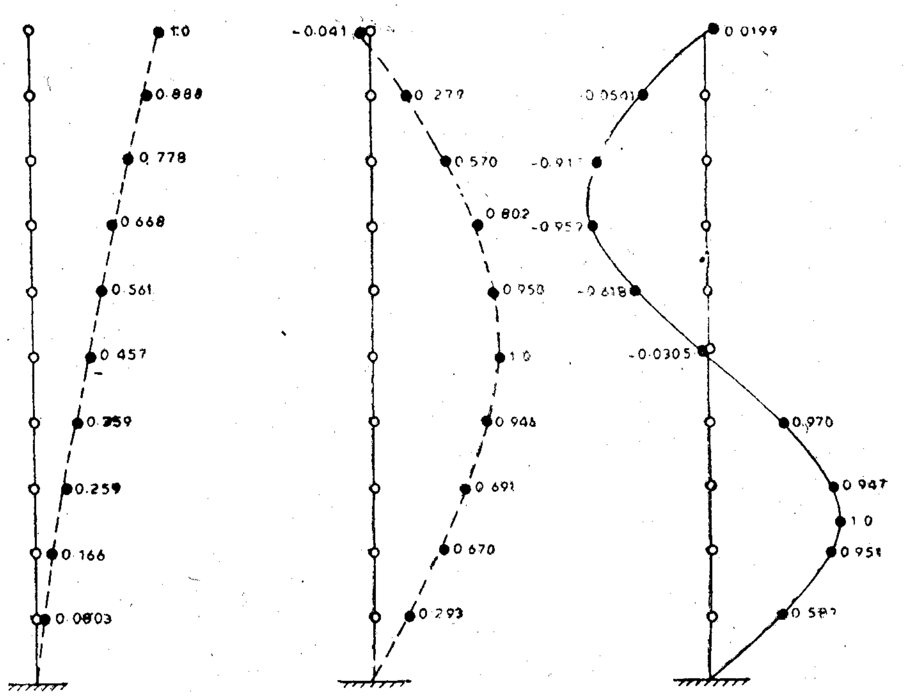


Fig. 7. Mode shapes for rocker pier in empty condition

It may also be noticed that higher mode frequencies are far removed from the fundamental. In fact because of heavy mass at the top, the pier tends to develop a node at its top as observed from the mode shapes shown in Fig. 7. Therefore, in dynamic response of the rocker pier, the fundamental mode only will be of significance.

6. Conclusions

The tests are not exhaustive enough to permit general conclusions. The pier model tested was rather stiff having fundamental frequency of about 11 cps. The conclusions as presented hereunder are therefore of qualitative nature and must be verified for flexible piers :

(1) Presence of the filling material in the cellular space of the pier has only a small effect on the stiffness of the pier and no noticeable influence on the fundamental frequency of the rocker pier. The damping is however slightly increased.

(2) For reducing the dynamic response of the pier and superstructure, ballast turns out as the best material for filling when choice is from amongst dry sand, ballast, saturated sand and water.

(3) The pier has exhibited better response to dynamic loading when water is present outside representing highest flood level condition. There is slight reduction in frequency indicating addition of virtual mass and increase in the damping value over those obtained under 'no water condition'. The actual amount of virtual mass remains to be determined.

(4) The base of the pier is found apparently more flexible in case of dynamic test than the static pull test perhaps due to the vibration of mass of foundation. This effect will be significant in design and must be investigated further.

(5) For the rocker pier, the higher mode frequencies are rather far removed from the fundamental. Therefore, for dynamic calculations, the fundamental mode only is of significance.

Acknowledgement

The assistance rendered by K.L. Mokha, Gradutate Student, in carrying out these studies is gratefully acknowledged. More camplete report of this work is presented in his Master of Engineering thesis[6] written under the guidance of the authors.

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Paper No. 78

Bulletin of the Indian Society of Earthquake Technology, Vol. 6, No. 3, pp. 147-155, Sept. 1969

A METHOD FOR ESTIMATING RESPONSE OF STRUCTURES DURING EARTHQUAKES

Brijesh Chandra*

Synopsis

A method has been developed to estimate seismic forces on structures at a site during future earthquakes. A set of new relationships has been obtained which relate the exciting potential of shock with its distance from the epicentre. The method takes into account the properties of ground motion and the dynamic behaviour of structures.

Introduction

Aseismic design of structures is a complex problem. For average structures the analysis and design is based on arbitrarily chosen static coefficients. However, for all important structures, it is essential that a dynamic analysis using the response spectrum technique[1] is carried out. This requires an accelerogram of expected earthquake shock at the site. If an accelerogram of past earthquake at the site is available it is the most reliable base to derive the response of structures precisely. If not, the data obtained elsewhere in similar conditions must be usefully utilised and taken for guidance in such computations. Realizing this, a detailed study has been made of the accelerograms that have been obtained so far during strong motion shocks at various places. Based on this study, a method has been developed for computing structural response at a site during future shocks. The same is described in following paragraphs.

Response of Structures During Earthquake

Considering an idealized structure (Fig. 1), with stiffness coefficient 'K', mass 'M' and damping 'C', subjected to ground motion represented by y, the equation of motion of the system could be written as follows :

$$M\ddot{x} + C(\dot{x} - \dot{y}) + K(x-y) = 0 \qquad (1)$$

where x is the absolute displacement of mass M as shown in Fig. 1, and dots represent differentiation with respect to time.

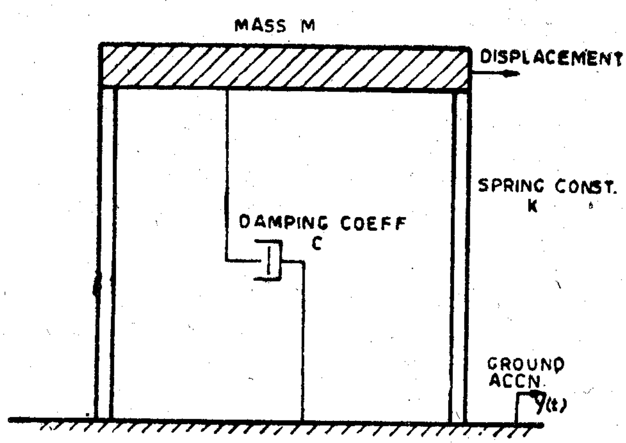

Fig. 1. The Idealised Structure

Subtracting $M\ddot{y}$ from both sides of eqn. (1) and calling $z = (x-y)$, we have,

$$M\ddot{z} + C\dot{z} + Kz = -M\ddot{y} \qquad (2)$$

Solution of eqn. (2) is well known[1] and is given by following :

$$z = \frac{1}{p\sqrt{1-\zeta^2}} \int_0^t \ddot{y}(\tau)\, e^{-p\zeta(t-\tau)} \sin p\sqrt{1-\zeta^2}\,(t-\tau)\, dt \qquad (3)$$

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where,

$p = \sqrt{K/M}$ = undamped circular frequency of system,

$\zeta = C/2Mp$ = critical damping ratio, and t, τ are time parameters.

For small values of ζ, as is present in most structures, the maximum value of z is obtained as

$$z_{max} = \frac{S_v}{p}$$

where,

$$S_v = \left[\int_0^t y(\tau)\, e^{-p\zeta(t-\tau)} \sin p\sqrt{1-\zeta^2}\,(t-\tau)\, d\tau \right]_{max} \qquad (5)$$

This quantity S_v, popularly known as spectrum velocity, is the most important parameter is defining response of structures. With the value of z_{max} known, the maximum force caused on the structure can be worked out as

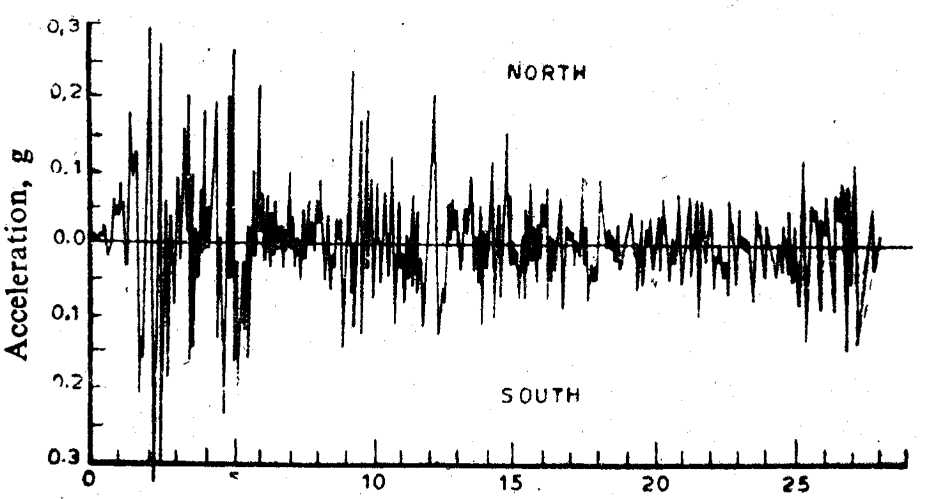
$$F_{max} = K \, . \, z_{max},$$

$$\text{or} \quad F_{max} = \frac{K}{p} S_v \qquad (6)$$

An examination of eqn (5) would show that S_v depends on the ground accelerations as function of time, and the properties of the structure viz. p and ζ. For a particular earthquake accelerogram therefore, a set of values of S_v will be obtained for various p and ζ values. It is customary to plot S_v against the undamped natural period, T, of the system ($T=2\pi/p$). This plot is termed as "velocity spectra". Fig 2 shows the accelerogram of one of the components of El Centro shock of May 18, 1940. The velocity spectra of this accelerogram is also shown in the same figure.

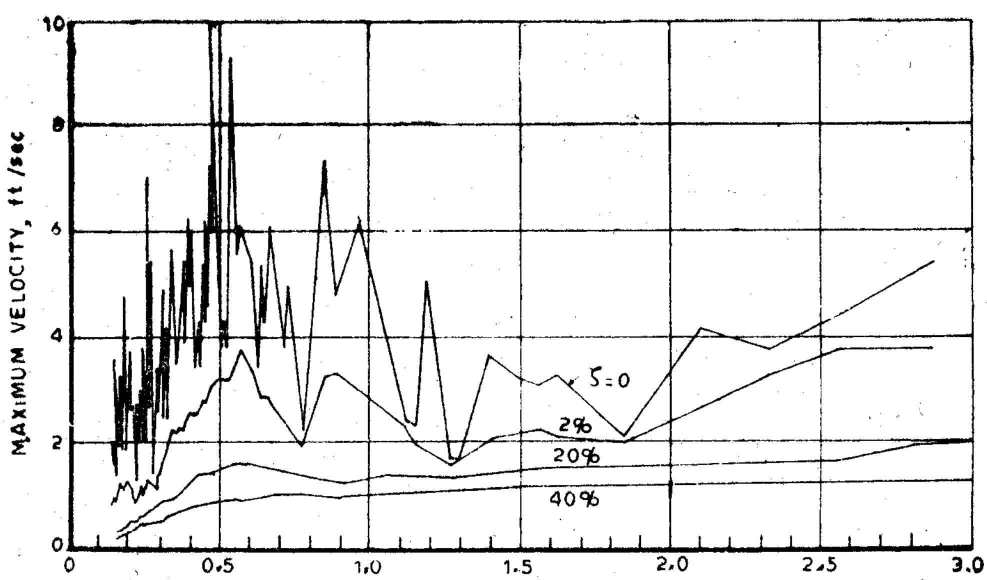
It is now recognized[2,3] that the general shape of spectra is same for all earthquakes although the absolute values of S_v are different in all cases. Based on this, Housner[3] has proposed what are called ' average spectrum curves". Multiplying factors are given for some important Californian earthquakes for which response could be obtained directly. This was a very important step towards simplifying response computations. However, the determination of such a multiplying factor remained to be decided for various sites. A method to work out this factor was developed by Jai Krishna[4] using the magnitude-distance-acceleration relationships given by Gutenderg and Richter[5]. Multiplying factor could then be calculated in proportion to the peak acceleration expected at the site. It is now realized that the peak acceleration alone does not represent the exciting potential of an earthquake shock. The frequencies associated with the acceleration pulses are as important as the peak accelerations. This fact carries special significance because accelerograms recorded at different distances from epicentre of a shock will contain different frequencies. The higher frequency components die out within a small distance from the epicentre and an accelogram at a greater distance will have the low frequency components—of course, the amplitudes of acceleration pulses will be much smaller in this case. Therefore, the question of a multiplying factor should be viewed with respect to the response spectrum rather than the peak accelerations alone. In fact, the excitation potential of an earthquake shock at a site would be better represented by the quantity spectral intensity (SI) which gives a quantitative idea regarding the spectral response of structures having periods varying from 0.1 sec to 2.5 sec. Mathematically,

$$SI = \int_{0.1}^{2.5} S_v(T, \zeta) . \, dT \qquad (7)$$



Duration sec

(a) Accelerogram



Undamped Natural Period, sec

(b) Velocity Spectra

Fig. 2. Acceleregram and Velocity Spectra of El Centro, May 18, 1940 Shock (Component N-S)

For the purpose of comparing the potential of various shocks, it is desirable that only undamped spectral intensities should be worked out[3,6]. This has been used in developing the new method which is described below.

Outline of the Proposed Method

The method presented here is an attempt to provide answer to very serious problem of estimating structural response at a site during expected earthquakes in future. The method is developed after a study of response spectra of a number of strong motion shock that have occured in the past. The following are the main assumptions in the method :

1. That acceleration—time record of earthquakes at any distance, D, from epicentre, is a function of the magnitude, M, and depth of focus, h, of the earthquake beside the quantity D itself.
2. That the frequencies present in a particular accelerogram at a site are functions of the distance, D, of the site from epicentre. Low frequencies are present if D is large.
3. That undamped spectral intensities of an accelerogram (SI_0) represent the exciting potential of an earthquake and is to be used to compare the various shocks.
4. That the general ground conditions are similar everywhere, and firm foundations would be available. For unusally loose foundation conditions, special study is called for, and is not included here.

Solution of the problem

The problem is divided into following parts :

1. Determination of undamped spectral intensity of an earthquake with the peak ground acceleration as unity. This will be referred to as normalized spectral intensity hereinafter and will be denoted by $(SI_0)_n$. A relationship between $(SI_0)_n$ and distance D from epicentre is sought.
2. Determination of peak ground acceleration 'a' expected at a site.
3. Exciting potential of earthquake, Q, could then be worked out at any place from the following equation,

$$Q = a. \ (SI_0)_n \tag{8}$$

4. Q, thus calculated, may be interpreted as a multiplying factor for the standard spectra.

Regarding item (2) above, detailed work has been done earlier and the author was associated in development of relationship expressing maximum ground acceleration a' as a function of magnitude 'M' and depth of focus 'h' and distance D, from epicentre. The relationship is discussed in great detail in another publication[7]. In this 'a' as fraction of acceleration due to gravity is expressed as

$$\frac{a}{g} = \frac{2.925 \dfrac{10^{(M-5)}}{h}}{1+4.5 \dfrac{10^{(M-5)}}{h}} \, e^{-0.26 \, (D/h)^{3/2}} \tag{9}$$

This is shown graphically in Fig. 3 for a focal depth h = 15 miles. Eqn. (9) has been found to give good correlation with the actually recorded values of 'a' in strong motion earthquakes. It is therefore proposed to use this relationship in the present work.

Relationship between $(SI_0)_n$ and distance 'D'

In order to develop such a relationship, a study has been made of the available data from the sixteen well recorded strong motion shocks. These shocks were recorded on firm

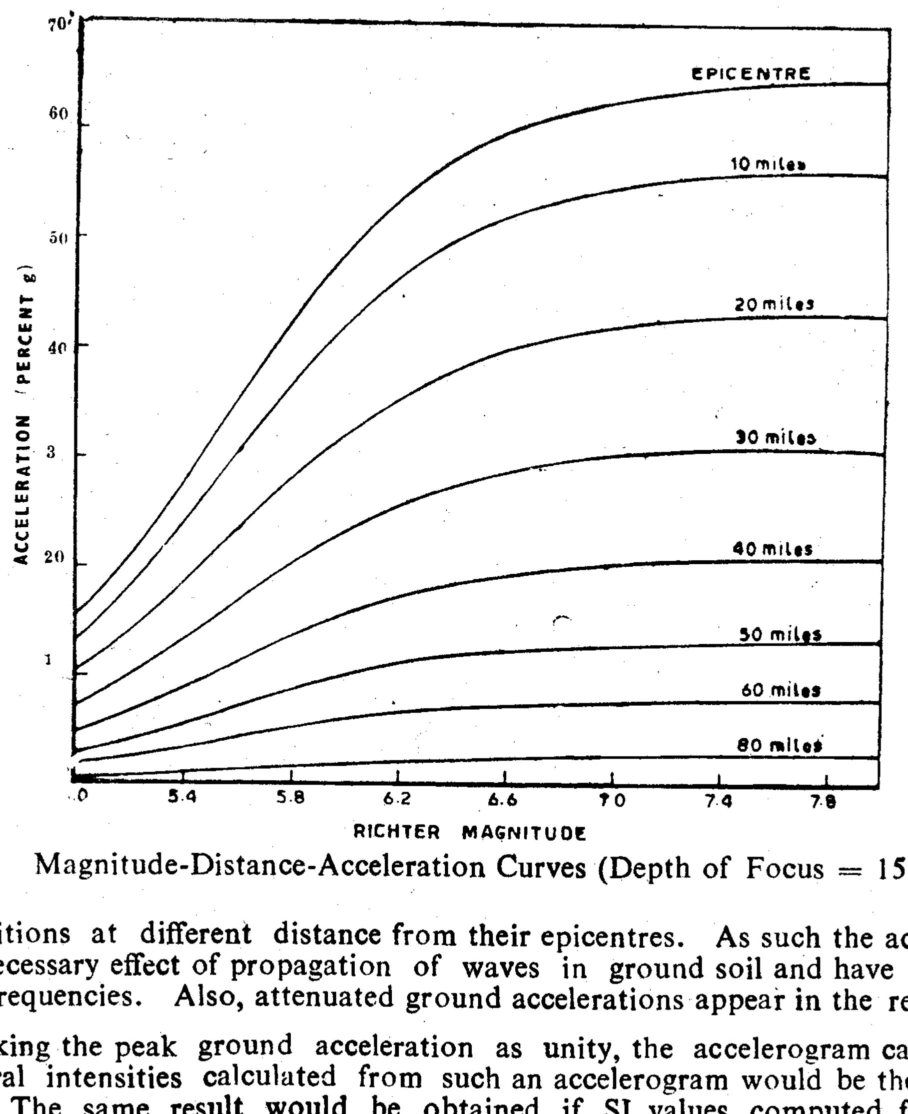


Fig. 3. Magnitude-Distance-Acceleration Curves (Depth of Focus = 15 miles)

ground conditions at different distance from their epicentres. As such the accelerograms contain the necessary effect of propagation of waves in ground soil and have components of different frequencies. Also, attenuated ground accelerations appear in the records.

By making the peak ground acceleration as unity, the accelerogram can be normalized. Spectral intensities calculated from such an accelerogram would be the normalized values $(SI)_n$. The same result would be obtained if SI values computed from original accelerogram, are divided by the peak recorded acceleration.

Table 1 lists the sixteen shocks alongwith the pertinent data regarding the shock viz. magnitude, depth of focus, distance from epicentre and the maximum recorded acceleration. Average SI_0 values for these shocks are available[3,8] and are tabulated in the same table. From this, $(SI_0)_n$ values are computed. A plot of $(SI_0)_n$ values against the distance from epicentre, shown in Fig. 4, reveals

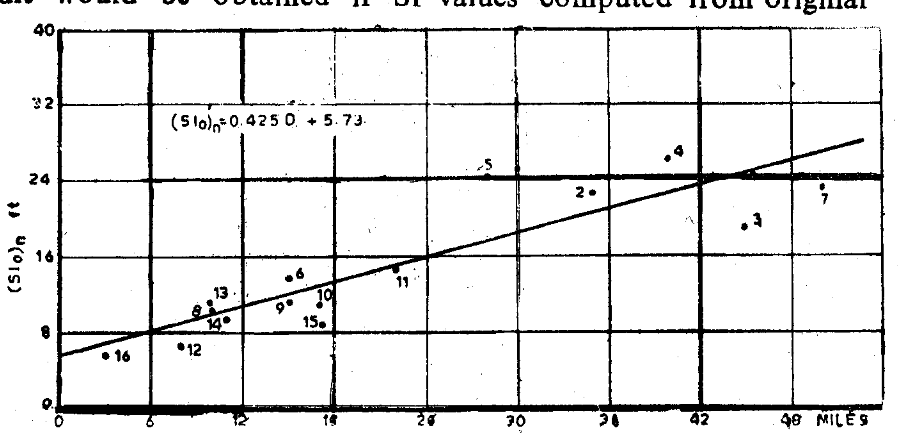


Fig. 4. Variation of normalized undamped intensity with distance from epicenter. Numbers on the dots refer to the serial no. of the shock. (Table 1)

Table 1. Comparative Study of Well Recorded Shocks

Sl. No.	Name of the Station and data of shock	M	D	h	a Max. accn. recorded x g	Average SI_o	SI_o/a $=(SI_o)_n$
1	El Centro May 18, 1940	7.1	30	15	0.33	8.35	25.3
2	El Centro Dec. 30, 1934	6.5	35	15	.26	5.88	22.6
3	Olympia April 13, 1949	7.1	45	45	.31	5.82	18.75
4	Taft July 21, 1952	7.7	40	15	.18	4.69	26.1
5	Vernon May 10, 1933	6.3	28	15	.19	4.62	24.3
6	Santa Barbara June 30, 1941	5.9	15	19	.24	3.29	13.7
7	Ferendale Oct. 3, 1941	6.4	50	15	.13	2.99	23.0
8	Hollister March 9, 1949	5.3	10	15	.23	2.36	10.5
9	Helena Oct. 31, 1935	6.0	15	25	.16	1.82	11.38
10	Vernon Oct. 2, 1933	5.3	17	15	.12	1.32	11.0
11	L.A. Subway Term. Oct. 2, 1933	5.3	22	15	.065	0.96	14.75
12	S.F. Golden Gate March 22, 1957	5.3	7.8	7	.13	0.84	6.46
13	S.F. State Bldg. March 22, 1957	5.3	9.8	7	.10	1.12	11.2
14	S.F. Alexander Bldg. March 22, 1957	5.3	10.8	7	.05	0.48	9.6
15	S.F. Oakland March 22, 1957	5.3	17.2	7	.05	0.38	7.6
16	Koyna Dec. 11, 1967	6.5	3	5	.63	3.72	5.9

very significant trends. A straight line fitted by the method of least squares yields the following relationship

$$(SI_o)_n = 0.425 \ D + 5.73 \tag{10}$$

Eqn. (10) brings out that structural response would be higher at larger distances provided that the peak accelerations are equal. In other words, accelerograms with lower frequency components would yield a higher structural response compared to its counterpart with higher frequencies. This could be explained with reference to the resonance curves (Fig. 5). Ruling out possibility of resonance, it may be seen that for a particular natural frequency p, the response of system is less at higher values of forcing frequency (w) than that at a lower frequency. It may be mentioned that we are referring to the w/p values greater than one since structural periods of interest range from 0.1 sec. to about 2.5 secs., and frequencies associated with earthquakes are generally higher than these. The trends indicated by eqn. (10) are therefore justified.

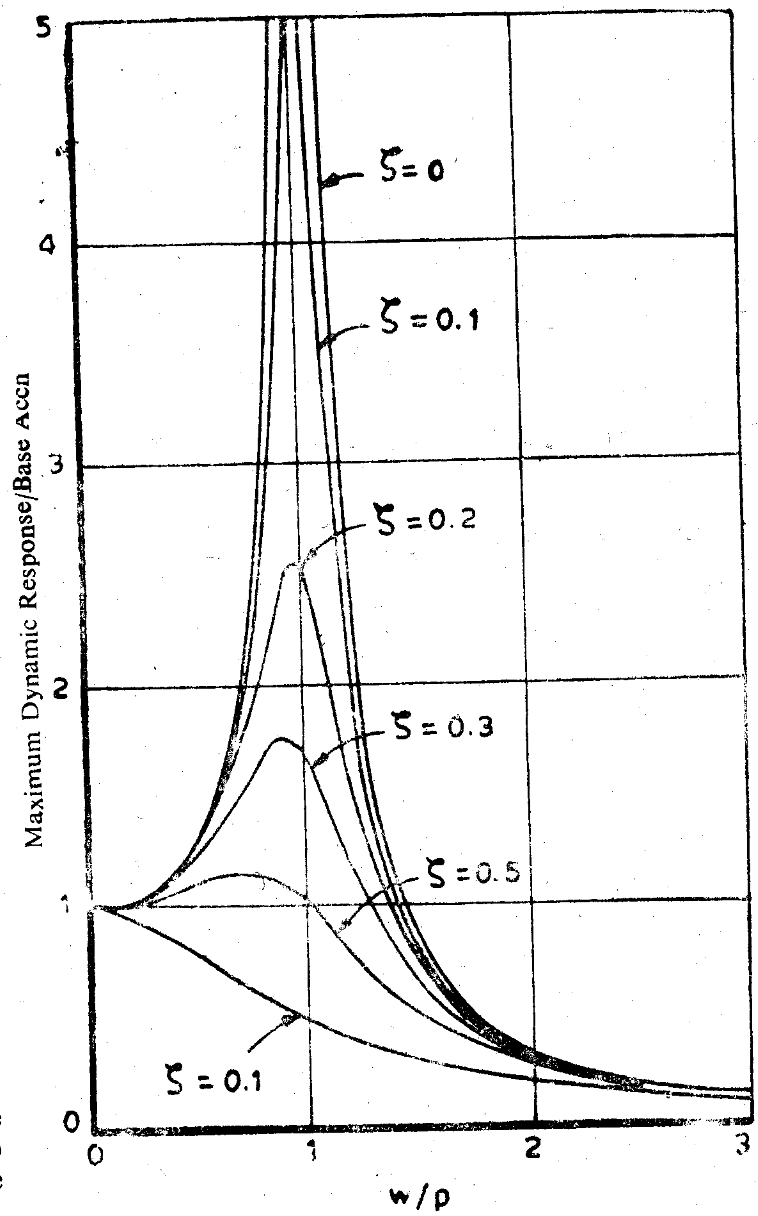


Fig. 5. Maximum dynamic response factor for damped Systems for sinusoidal loading

With the values of $(SI_o)_n$ as obtained from eqn. (10) and the value of 'a' obtained from eqn. (9), exciting potential of earthquake, Q could be worked out using eqn. (8).

The Standard Spectra

For the purpose of obtaining response parameter S_v, a standard spectra must be defined in such a way that the exciting potential 'Q' of an earthquake may be used to obtain the multiplying factor. Also, the standard spectra must take care of the fact that in different shocks the peaks will have random distribution with respect to 'period' parameter. Housner's average spectra satisfies these requirements. These are obtained by averaging the spectrum values of the eight components of the four strongest ground motions recorded (El Centro 1934, El Centro 1940, Olympia 1949 and Taft 1952) and turn out in a neat smooth shape as shown in Fig. 6. Multiplying factors for these shocks have been assigned as 1.9, 2.7, 1.9 and 1.6 respectively.

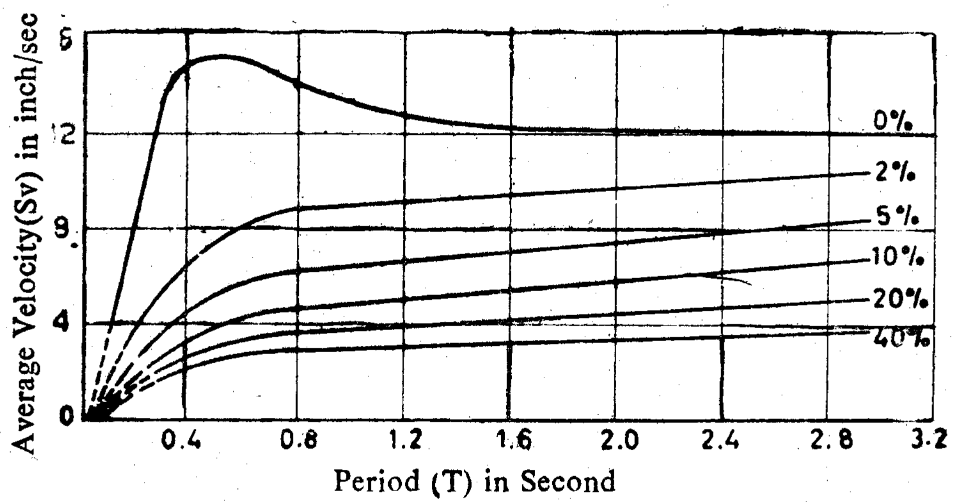


Fig. 6. The Standard Spectra

Observing that the average SI_0 values of these four shocks are 5.88, 8.35, 5.82 and 4.69 respectively, which are in about the same ratio as the multiplying factors proposed by Housner, it is conclusively established that the SI_0 values or Q values are directly proportional to the multiplying factor. Therefore, the average spectra (Fig. 6) can be very usefully and conveniently adopted as the standard spectra.

For determining the multiplying factor N for any earthquake, the following relationship may be used :

$$N = \frac{Q}{8.35} \times 2.7 \tag{11}$$

Illustrative Example

It is desired to determine N for a site which is situated 50 miles from the epicentre of a possible earthquake with magnitude 6.2 and depth of focus 15 miles.

Step 1. Using eq. (9) for M 6.2, D = 50, h = 15, a/g works out as 0.10.

Step 2. Using eq. (10) for D = 50, $(SI_0)_n$ work out as 26.98.

Step 3. Using eq. (8), Q works out to be 2.698

Step 4. Using eq. (11) N work out as 0.871.

This factor N should be used as the multiplying factor for the spectral values obtained from Fig. 6, for the appropriate period and damping.

Conclusion

Eqns. 8–11 presented in this paper should be used to determine the multiplying factor for the standard spectra shown in Fig. 6. Response of any structure could then be found easily from this, by picking up the ordinate for the appropriate period and damping.

Acknowledgement

Author is very thankful to Dr. Jai Krishna, Professor and Director, S.R.T.E.E., Roorkee, for the valuable suggestions made by him in this study, and for giving permission to publish this paper.

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1. Dr. Shamsher Prakash
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   Engineering, Building Research Institute
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Dear Member,

A notice for amending the constitution of Indian Society of Earthquake Technology was issued vide this office letter No. ISET/EC/ dated 20th July, 1969. The last date for return of the Ballot paper in this office was 15th August, 1969.

In all 55 ballot papers were returned and all have supported the amendment proposed in the letter.

It is therefore, my pleasant duty to inform all the members of the Society that the amendment of the constitution has been carried over and new copies of the constitution are available in this office which can be requested.

Yours sincerely,

Shamsher Prakash
Secretary

NOTICE

Sri Rama Rao Stanam had become a member of the Indian Society of Earthquake Technology last year, but unfortunately his address has been misplaced and is not available in our file. If any member is aware of his address, he may kindly convey the same to the Secretary, I.S.E.T.