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PERFORMANCE OF AGED DAM-RESERVOIR-FOUNDATION COUPLED SYSTEM WITH ABSORPTIVE RESERVOIR BOTTOM

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ABSTRACT

The paper deals with the responses of degraded dam-reservoir-foundation system with absorptive reservoir bottom. A two dimensional strong coupling methodology, in which dam and foundation are simulated by displacement and reservoir is by pressure is used to model the dam-reservoir-foundation system. The material properties of dam at different ages are predicted by an isotropic degradation index and a reflection coefficient is introduced to simulate the reservoir bottom absorption. From the present study, it is clear that the crest displacement increases with age of the dam. On the other hand, the principal stresses of dam and hydrodynamic pressure decrease with the age of dam. However, at later age of the dam, the effect of reservoir bottom absorption on the performance of dam-reservoir-foundation is quite less.

KEYWORDS: Dam-reservoir-foundation system, Long term ageing, Degradation index, Absorptive reservoir bottom, Hydrodynamic pressure

INTRODUCTION

Safety evolution of any structures is the prime importance to the designers. In design of a dam, the accurate prediction of behavior at any age during its design life is the challenging job. The responses of gravity dams become realistic if fluid-structure and soil-structure interactions are considered simultaneously. Zienkiewicz et al. (1983) evaluated responses of dam considering fluid-structure interaction. Similarly, concrete gravity dam was studied in frequency domain by Chandrashaker and Humar (1993). A hybrid finite and boundary element technique was used to analyze the dam-reservoir-foundation coupled system in time domain by Touheiand Ohmachi, 1993. Bayraktar et al., 2005; Bilici et al., 2009 and Wang, 2011found dynamic responses of dam-reservoir-foundation systems in frequency domain. Mirzabozorg et al. (2010) used spatially varying seismic excitation to obtain the responses of arch dams. In this study, the foundation is assumed to be massed and is modeled by infinite elements. Bayraktar et al. (2011) used 3D solid element to model the dam and foundation (SOLID 45) and eight-node 3D fluid element (FLUID 80) in ANSYS to model the dam, foundation and reservoir respectively. Similarly, Papazafeiropoulos et al. (2011),Banerjee et al. (2014) and Sarkar et al. (2007) investigated the performance of dam-reservoir-foundation system against different external excitations.

In most of the previous studies, the responses of dam-reservoir-foundation coupled system were investigated with constant elastic properties of concrete in the dam. However, due to heat transport, freeze-thaw action, dissolution processes such as calcium leaching and chemical expansive reactions such as alkali-silica reaction and due to continuous contact of dam with reservoir the strength of the concrete decreases with the age of the dam. Kuhl. D et al. (2004) proposed a coupled thermo-mechanical model to describe the interaction of calcium leaching and mechanical damage in cementitious materials. Authors used total porosity to simulate the degradation of the stiffness with the age of concrete. Gogoi and Maity (2007) and Burman et al. (2009) considered the gain of compressive strength of concrete as well as the deterioration of the concrete in their respective modeling study. Therefore, the behavior of dam-reservoir-foundation coupled system will be more exact if the long term degradation of concrete in dams is incorporated in the study.

The characteristic of reservoir bottom or accumulation of sediments at reservoir bottom may have some impact on the performance of dam-reservoir-foundation coupled system. In most of the cases the reservoir bottom is considered as rigid one. However, some researchers considered reservoir bottom absorption in their respective studies. Lotfi (1986) considered the sediment layer at reservoir bottom as viscoelastic and almost incompressible. Some researchers (Sharan, 1992; Chandrashaker and Humar, 1993; Tan and Chopra, 1995; Li et al., 1996; Hatami, 1997) used a damping boundary condition at the reservoir bottom to simulate the reservoir bottom absorption. Bougacha and Tassoulas (1991a, b) modeled the sediment as a two-phase medium i.e., fluid-filled and poroelastic solid. Dominguez et al. (1993) developed a boundary element approach for the dynamic analysis of continuous systems that consists of water, viscoelastic and fluid-filled poroelastic zones of arbitrary shapes. Based on the solution of the wave equation in a sediment layer of visco-elastic material with a constant thickness, Hatami (1997), and Chuhan et al. (2001) and Gupta and Pattanur (2007) proposed a wave reflection coefficient approach. A novel procedure for measuring the overall or average reflection coefficient at the reservoir boundaries was developed Ghanaat et al. (2000). Influence of sediment layers at the bottom of the reservoir on dynamic behavior of aged concrete dam-reservoir system was studied by Gogoi and Maity (2007).

It is apparent from the literatures referred above that the coupled effect of dam-reservoir-foundation system is important for accurate responses of concrete gravity dam at any age during its service life. Very few research works considering the three systems in a coupled way are observed using standard software such as ANSYS and ABAQUS. However, the effects of degradation of concrete in dam and reservoir bottom absorption cannot be incorporated in such software. In the present study, a computer code has been developed to obtain response of concrete gravity dam at different ages with absorptive reservoir bottom.

THEORETICAL FORMULATION

1. Finite Element Formulation of Dam and Soil Foundation

The equation of motion of a structure subjected to external forces can be written in standard finite element form as

$$M_d\ddot{u} + C_d\dot{u} + K_d u - M_d\ddot{u}_g = 0 \tag{1}$$

where, $M_d = \int\limits_{\Omega} N^T \rho N \, d\Omega$

and $K_d = \int\limits_{\Omega} B^T DB \, d\Omega$

Where, N and B are the shape function and strain displacement matrixes. ρ is the density. $\ddot{u}$, $\dot{u}$ and u are nodal accelerations, velocities and displacements, $\ddot{u}_g$ is the external acceleration. In present investigation dam and foundation have been discretised by two dimensional eight node rectangular elements and both the structures are assumed to be in a state of plane strain and Rayleigh damping is considered as structural damping.

2. Degradation Model for Concrete

The concept of degradation of concrete strength is based on the reduction of the net area capable of supporting stresses. The loss of rigidity of the material follows as a consequence of material degradation due to various environmental and loading conditions. In the present study, a parameter named as degradation index is introduced to consider the deterioration in elastic property of dam with age and this degradation index can be expressed as

$$d_{gj} = \frac{\Psi_j - \Psi_j^d}{\Psi_j} \tag{2}$$

Here, Ψ_j is tributary area of the surface in direction j; and Ψ_j^d is area affected by degradation. In a scale of 0 to 1, the orthotropic degradation index, $d_{gj} = 0$, indicates no degradation and $d_{gj} = 1$, indicates completely degraded material. The index $j = 1, 2$ corresponds with the Cartesian axes x and y in the two dimensional case. The effective constitutive relationship for plane strain analysis can be expressed as

$$\left[D_g\right]=\frac{E_d}{(1+\mu)(1-2\mu)}\begin{bmatrix}(1-\mu)\Lambda_1^2 & \mu\Lambda_1\Lambda_2 & 0\\ \mu\Lambda_1\Lambda_2 & (1-\mu)\Lambda_2^2 & 0\\ 0 & 0 & (1-2\mu)\Lambda_1^2\Lambda_2^2/\left(\Lambda_1^2+\Lambda_2^2\right)\end{bmatrix} \tag{3}$$

Where, $\Lambda_1 = (1-d_{g1})$ and $\Lambda_2 = (1-d_{g2})$. In the above equation, E_d is the elastic modulus of the material without degradation. If $d_{g1} = d_{g2} = d_g$, the isotropic degradation model is expressed as

$$\left[D_g\right]=\left(1-d_g\right)^2\left[D\right] \tag{4}$$

Where, $[D_g]$ and $[D]$ are the constitutive matrices of the degraded and un-degraded model respectively.

3. Evaluation of Degradation Index

In normal practice, it is assumed that all the cement particles in concrete get hydrated in 28 days and concrete gets its full compressive strength. But in reality, concrete gains some compressive strength with age beyond 28 days. On the other hand, the durability of concrete is considerably affected due to damage resulting from time variant external loading, moisture, heat transport, freez-thaw actions, chemically expansive reaction and chemical dissolution. Out of the wide range of environmental induced mechanisms, damage due to chemical and mechanical degradation is modeled here to get reasonable elasticity properties of concrete at its different ages.

4. Gain in Compressive Strength with Age

The gain of compressive strength of concrete can be predicted by curve fitting on 50 years of experiential data published by Washa et al. (1989). The authors proposed four different curves for different concretes. All the curves showed an increase in compressive strength roughly proportional to the logarithm of age during the first 10 years and very small variation thereafter. Gogoi and Maity (2007) carried out a least square curve fitting analysis on the set of compressive strength data published by Washa et al. (1989) and proposed following equation to predict the gain of compressive strength with passage of time in years.

$$f(t)=3.57\ln(t)+44.33 \tag{5}$$

Where $f(t)$ is the gain of compressive strength in SI unit, t is the age of concrete in years. The value of modulus of elasticity of concrete in SI unit is obtained by the expression proposed by Neville and Brooks (1987).

$$E_0=5000\sqrt{f(t)} \tag{6}$$

5. Degradation with Age

The compressive strength of concrete is expected to decrease with its age due to chemical and mechanical degradation and this degradation is measured in terms of degradation index. In present work, the degradation due to hygro-chemo-mechanical actions is implemented and the total porosity, ϕ which is the sum of the initial porosity ϕ_0, the porosity due to matrix dissolution ϕ_c and the apparent porosity ϕ_m is considered as the measurement of degradation index. Bangert et al. (2003) and Kuhl et al. (2004) have suggested the following relationship to relate these parameters.

$$\phi=\phi_0+\phi_c+\phi_m \tag{7}$$

The apparent mechanically induced porosity, φ_m considers the influence of mechanically induced micro-pores and micro-cracks on the macroscopic material properties of the porous material. It is obtained as

$$\phi_m=\left[1-\phi_0-\phi_c\right]d_g \tag{8}$$

Where, d_g is the scalar degradation parameter. The strain based exponential degradation function as proposed by Simo and Ju (1987) is given as

$$d_g = a_s - \frac{\kappa^0}{\kappa}\left[1 - \alpha_c + \alpha_c e^{\left(\beta_c\left[\kappa^0 - \kappa\right]\right)}\right] \tag{9}$$

Where, κ^0 and κ are values of strain that represents the initial threshold degradation and the internal variable defining the current damage threshold depending on the loading history. κ^0 is given by f_t/E_0, where f_t is the static tensile strength and E_0 is the elastic modulus of the un-degraded material before any mechanical loading is imposed. The value of d_g at any age can be determined from equation (9) that will vary with κ caused by the mechanical loading history. In the absence of degradation due to mechanical loading, $\kappa^0 = \kappa$, which makes $d_g = 0$ and hence ϕ_m=0. α_c and β_c are material parameters that can be obtained experimentally (Bangert et al., 2003). In equation (9), the value of a_s is considered to be 1.0 by Simo and Ju (1987) which is the maximum allowable degradation due to mechanically induced porosity.

6. Truncation Boundary for Soil Foundation

For finite element modeling of infinite soil foundation, one of the biggest chalange is to model the boundary domain accurately. The modeling of the truncation boundary should be such that it has capability to absorb the outgoing waves. The most commonly used absorbing boundary is presented in this section. This boundary condition is based on one dimensional wave propagation. However, in 2-D case, the absorbing boundary condition is formulated from eq. (10) and eq. (11) as follows.

$$\sigma(s) + \rho c_p \dot{u}(s) = 0 \tag{10}$$

$$\tau(s) + \rho c_s \dot{v}(s) = 0 \tag{11}$$

Where, σ and τ are normal and shear stresses at truncation boundary and u and v are normal and shear displacements. c_p and c_v represent the compression wave and shear wave velocity respectively and s denotes the coordinate of truncation boundary. In discretized form, the damper coefficient c_n and c_t in normal and tangential directions may be expressed as.

$$c_n = A_1 \rho c_p \tag{12}$$

$$c_t = A_2 \rho c_s \tag{13}$$

The dashpot coefficient c_s and c_p in two mutually orthogonal directions are given by the following expressions.

$$c_s = \sqrt{\frac{G}{\rho}} \tag{14}$$

$$c_p = \sqrt{\frac{E(1-\mu)}{(1+\mu)(1-2\mu)\rho}} \tag{15}$$

G is the shear modulus and is expressed as

$$G = \frac{E}{2(1+\mu)} \tag{16}$$

A_1 and A_2 are the applicable area and depend on the direction of wave propagation. Usually these two factors are considered to be equal to 1. However, White et al. (1977) proposed a ‘diffused’ version of expression of equation (12) and equation (13) and area A_1 and A_2 were evaluated by minimizing the ratio between the reflected energy and the incidental energy. For an isotropic medium this results

$$A_1 = \frac{8}{15\pi}(5 + 2s - 2s^2) \tag{17}$$

$$A_2 = \frac{8}{15\pi}(3 + 2s) \tag{18}$$

and s can be expressed as

$$s = \sqrt{\frac{(1-2\mu)}{2(1-\mu)}} \tag{19}$$

7. Formulation for Infinite Fluid Domain

Assuming fluid to be linearly compressible, inviscid and with small amplitude irrotational motion, the hydrodynamic pressure distribution due external excitation is given as

$$\nabla^2 p(x,y,t) = \frac{1}{c^2}\ddot{p}(x,y,t) \tag{20}$$

Where, c is the acoustic wave velocity in the water and ∇^2 is the Laplacian operator in two dimensions. The pressure distribution in the fluid domain is obtained by solving equation (20) with the following boundary conditions. A typical geometry of dam-reservoir-foundation system is shown in Figure 1.

i) At surface I

Considering the effect of surface wave of the fluid, the boundary condition of the free surface is taken as

$$\frac{1}{g}\ddot{p} + \frac{\partial p}{\partial y} = 0 \tag{21}$$

ii) At surface II

At dam-reservoir interface, the pressure should satisfy

$$\frac{\partial p}{\partial n}(0,y,t) = \rho_f a e^{i\omega t} \tag{22}$$

Where $ae^{i\omega t}$ is the horizontal component of the ground acceleration in which, ω is the circular frequency of vibration and $i = \sqrt{-1}$, n is the outwardly directed normal to the element surface along the interface. ρ_f is the mass density of the fluid.

iii) At surface III

In the present study, the reservoir bottom absorption is modeled according to the technique proposed by Hall and Chopra (1982). Neglecting the vertical acceleration at reservoir, the pressure should satisfy the following condition.

$$\frac{\partial p}{\partial n}(x,0,t) = -q\dot{p}(x,0,t) \tag{23}$$

Assuming a time harmonic behavior of pressure $p(x,0,t) = p_0(x,0,t)\,e^{i\omega t}$, the equation (23) may be expressed as:

$$\frac{\partial p}{\partial n}(x,0,t) = i\omega q p(x,0,t) \tag{24}$$

Where, n is the outwardly directed normal to the element surface and q is a coefficient expressed as:

$$q = \frac{1}{C}\left(\frac{1-\alpha}{1+\alpha}\right) \tag{25}$$

α is the frequency independent reflection coefficient.

iv) At surface IV

In case of finite fluid domain, this surface is considered to be rigid and thus the boundary condition in this case will become as follows:

$$\frac{\partial p}{\partial n}(L,y,t) = 0.0 \tag{26}$$

Where, L is the distance between structural surface and *surface IV*. In case of infinite fluid domain, the domain needs to be truncated at a suitable distance for the finite element analysis. The truncation boundary as proposed by Gogoi and Maity (2006) has been implemented for the finite element analysis of infinite reservoir.

8. Finite Element Formulation for Fluid domain

By using Galerkin approach and assuming pressure to be the nodal unknown variable, the discretised form of equation (20) may be written as

$$\int_{\Omega} N_{rj}\left[\nabla^2 \sum N_{ri} p_i - \frac{1}{c^2}\sum N_{ri}\ddot{p}_i\right] d\Omega = 0 \tag{27}$$

Where, N_{rj} is the interpolation function for the reservoir and Ω is the region under consideration. Using Green's theorem equation (27) may be transformed to

$$-\int_{\Omega}\left[\frac{\partial N_{rj}}{\partial x}\sum\frac{\partial N_{ri}}{\partial x}p_i + \frac{\partial N_{rj}}{\partial y}\sum\frac{\partial N_{ri}}{\partial y}p_i\right]d\Omega - \frac{1}{c^2}\int_{\Omega} N_{rj}\sum N_{ri}\, d\Omega\ \ddot{p}_i + \int_{\Gamma} N_{rj}\sum\frac{\partial N_{rj}}{\partial n}d\Gamma\ \ p_i = 0 \tag{28}$$

in which i varies from 1 to total number of nodes and Γ represents the boundaries of the fluid domain. The last term of the above equation may be written as

$$\{F\} = \int_{\Gamma} N_{rj}\frac{\partial p}{\partial n}d\Gamma \tag{29}$$

The whole system of equation (28) may be written in a matrix form as

$$\left[\bar{E}\right]\left\{\ddot{P}\right\} + \left[\bar{G}\right]\left\{P\right\} = \left\{F\ \ \right\} \tag{30}$$

Where,

$$\left[\bar{E}\right] = \frac{1}{C^2}\sum\int_{\Omega}\left[N_r\right]^T\left[N_r\right]d\Omega \tag{31}$$

$$\left[\bar{G}\right] = \sum\int_{\Omega}\left[\frac{\partial}{\partial x}\left[N_r\right]^T\frac{\partial}{\partial x}\left[N_r\right] + \frac{\partial}{\partial y}\left[N_r\right]^T\frac{\partial}{\partial y}\left[N_r\right]\right]d\Omega \tag{32}$$

$$\left[F\right] = \sum\int_{\Gamma}\left[N_r\right]^T\frac{\partial p}{\partial n}d\Gamma = \left\{F_f\right\} + \left\{F_{fs}\right\} + \left\{F_{fb}\right\} + \left\{F_t\right\} \tag{33}$$

Here the subscript f, f_s, f_b, and t stand for the free surface, fluid-structure interface, fluid-bed interface and truncation surface respectively. If the boundary conditions mentioned above are implemented, the equation (30) may be modified as

$$\left[E\right]\left\{\ddot{P}\right\} + \left[A\right]\left\{\dot{P}\right\} + \left[G\right]\left\{P\right\} = \left\{F_r\right\} \tag{34}$$

Where,

$$\left[E\right] = \left[\bar{E}\right] + \frac{1}{g}\left[R_f\right] \tag{35}$$

$$\left\{F_r\right\} = -\rho\left[R_{fs}\right]\left\{a\right\} \tag{36}$$

$$\left[A\right] = \frac{1}{C}\left[R_t\right] \tag{37}$$

$$\left[G\right] = \left[\bar{G}\right] + \zeta\left[R_t\right] - i\omega q\left[R_{fb}\right] \tag{38}$$

In which,

$$\left[R_f\right] = \sum \int_{\Gamma_f} \left[N_r\right]^T \left[N_r\right] d\Gamma \tag{39}$$

$$\left[R_{fs}\right] = \sum \int_{\Gamma_{fs}} \left[N_r\right]^T \left[T\right]\left[N_d\right] d\Gamma \tag{40}$$

$$\left[R_{fb}\right] = \sum \int_{\Gamma_{fb}} \left[N_r\right]^T \left[N_r\right] d\Gamma \tag{41}$$

$$\left[R_t\right] = \sum \int_{\Gamma_t} \left[N_r\right]^T \left[N_r\right] d\Gamma \tag{42}$$

Where,

$[T]$ is the transformation matrix at fluid structure interface and N_d is the shape function and ς is a coefficient as expressed by Gogoi and Maity (2006).

9. Formulation of Dam-Reservoir-Foundation Coupled System

In this section equilibrium equations dam, reservoir and soil foundation are coupled in such a way they act as a single system and the coupling procedure is as follows

The discrete dam equation with damping may be written as:

$$M_d\ddot{u}_d + C_d\dot{u}_d + K_d u_d - Qp - M_d\ddot{u}_g = 0 \tag{43}$$

The coupling term $[Q]$ in equation (43) arises due to the acceleration and pressure at the dam-reservoir interface and can be expressed as:

$$\int_{\Gamma_{dr}} N_{dr}^T npd\Gamma = \left(\int_{\Gamma_{dr}} N_{dr}^T nN_{dr} d\Gamma \right) p = Qp \tag{44}$$

Where, n is the direction vector of the normal to the dam-reservoir interface and, N_{dr} is the shape function of bar element at the dam-reservoir interface. Similarly, discretized fluid equation may be written as:

$$E\ddot{p} + A\dot{p} + Gp + Q^T\ddot{u}_d - F_r = 0 \tag{45}$$

Now, the system of equation (43) and (45) are coupled in a second-order ordinary differential equations, which defines the equation for coupled dam-reservoir system completely. The equation (43) and (45) may be written as a set:

$$\begin{bmatrix} E & Q^T \\ 0 & M_d \end{bmatrix} \begin{Bmatrix} \ddot{p} \\ \ddot{u}_d \end{Bmatrix} + \begin{bmatrix} A & 0 \\ 0 & C_d \end{bmatrix} \begin{Bmatrix} \dot{p} \\ \dot{u}_d \end{Bmatrix} + \begin{bmatrix} G & 0 \\ -Q & K_d \end{bmatrix} \begin{Bmatrix} p \\ u_d \end{Bmatrix} = \begin{Bmatrix} F_r \\ M_d\ddot{u}_g \end{Bmatrix} \tag{46}$$

The responses of dam considering the effect of reservoir adjacent to it may be obtained from equation (46) and this equation is the equilibrium equation for dam-reservoir coupled system. However, the equation (46) does not include the elasticity of elastic foundation. The dynamic equilibrium equations for coupled dam-reservoir-foundation may be developed in the following way.

$$\begin{bmatrix} E & Q_d^T & Q_c^T & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc} & M_{cf} \\ 0 & 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{P} \\ \ddot{U}_d \\ \ddot{U}_c \\ \ddot{U}_f \end{Bmatrix} + \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & C_{dd} & C_{dc} & 0 \\ 0 & C_{cd} & C_{cc} & C_{cf} \\ 0 & 0 & C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{P} \\ \dot{U}_d \\ \dot{U}_c \\ \dot{U}_f \end{Bmatrix} + \begin{bmatrix} G & 0 & 0 & 0 \\ -\begin{bmatrix} Q_d \\ Q_c \end{bmatrix} & K_{dd} & K_{dc} & 0 \\ 0 & K_{cd} & K_{cc} & K_{cf} \\ 0 & 0 & K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} P \\ U_d \\ U_c \\ U_f \end{Bmatrix} = -\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc} & M_{cf} \\ 0 & 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} F_r \\ \ddot{U}_d^g \\ \ddot{U}_c^g \\ \ddot{U}_f^g \end{Bmatrix} \tag{47}$$

Here "added motion" approach is used and the subscripts ''d, f and c'' stand for the nodes within the dam, nodes within the foundation and nodes at the junction of dam and foundation respectively and $\ddot{U}_g$ is the ground acceleration vector. The mass, damping and stiffness at the common junction are the sum of the contributions from the dam (d) and foundation (f), and are expressed in equation (47) and the coupling matrix is rearranged as $Q = \left[[Q_d] \quad [Q_c] \right]^T$. Where, $[Q_d]$ associates with the nodes of dam other than common nodes and $[Q_c]$ is associated with the common nodes at the junction of dam and foundation as shown in Figure 1. The absolute responses of dam-reservoir-foundation are considered to be the sum of two parts, viz. free field response and added part of the responses and expressed in equation (50)

$$M_{cc} = M_c^d + M_c^f \quad C_{cc} = C_c^d + C_c^f \quad and \quad K_{cc} = K_c^d + K_c^f \tag{48}$$

$$\begin{Bmatrix} \ddot{P} \\ \ddot{U}_d \\ \ddot{U}_c \\ \ddot{U}_f \end{Bmatrix} = \begin{Bmatrix} \ddot{P}^{ff} \\ \ddot{u}_d^{ff} \\ \ddot{u}_c^{ff} \\ \ddot{u}_f^{ff} \end{Bmatrix} + \begin{Bmatrix} \ddot{P}^a \\ \ddot{u}_d^a \\ \ddot{u}_c^a \\ \ddot{u}_f^a \end{Bmatrix} \quad \begin{Bmatrix} \dot{P} \\ \dot{U}_d \\ \dot{U}_c \\ \dot{U}_f \end{Bmatrix} = \begin{Bmatrix} \dot{P}^{ff} \\ \dot{u}_d^{ff} \\ \dot{u}_c^{ff} \\ \dot{u}_f^{ff} \end{Bmatrix} + \begin{Bmatrix} \dot{P}^a \\ \dot{u}_d^a \\ \dot{u}_c^a \\ \dot{u}_f^a \end{Bmatrix} \; and \; \begin{Bmatrix} P \\ U_d \\ U_c \\ U_f \end{Bmatrix} = \begin{Bmatrix} P^{ff} \\ u_d^{ff} \\ u_c^{ff} \\ u_f^{ff} \end{Bmatrix} + \begin{Bmatrix} P^a \\ u_d^a \\ u_c^a \\ u_f^a \end{Bmatrix} \tag{49}$$

Where, superscripts "f and a" define free field and added part response respectively. Now, putting the equation (47) in equation (49), the following equation is obtained.

$$\begin{bmatrix} E & Q_d^T & Q_c^T & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc} & M_{cf} \\ 0 & 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{P}^a \\ \ddot{u}_d^a \\ \ddot{u}_c^a \\ \ddot{u}_f^a \end{Bmatrix} + \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & C_{dd} & C_{dc} & 0 \\ 0 & C_{cd} & C_{cc} & C_{cf} \\ 0 & 0 & C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{P}^a \\ \dot{u}_d^a \\ \dot{u}_c^a \\ \dot{u}_f^a \end{Bmatrix} + \begin{bmatrix} G & 0 & 0 & 0 \\ -\begin{bmatrix} Q_d \\ Q_c \end{bmatrix} & K_{dd} & K_{dc} & 0 \\ 0 & K_{cd} & K_{cc} & K_{cf} \\ 0 & 0 & K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} P^a \\ u_d^a \\ u_c^a \\ u_f^a \end{Bmatrix} = R + F \tag{50}$$

where,

$$R=-\begin{bmatrix} E & Q_d^T & Q_c^T & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc} & M_{cf} \\ 0 & 0 & M_{fc} & M_{ff} \end{bmatrix}\begin{Bmatrix} \ddot{P}^{ff} \\ \ddot{u}_d^{ff} \\ \ddot{u}_c^{ff} \\ \ddot{u}_f^{ff} \end{Bmatrix}-\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & C_{dd} & C_{dc} & 0 \\ 0 & C_{cd} & C_{cc} & C_{cf} \\ 0 & 0 & C_{fc} & C_{ff} \end{bmatrix}\begin{Bmatrix} \dot{P}^{ff} \\ \dot{u}_d^{ff} \\ \dot{u}_c^{ff} \\ \dot{u}_f^{ff} \end{Bmatrix}- \begin{bmatrix} G & 0 & 0 & 0 \\ -\begin{bmatrix} Q_d \\ Q_c \end{bmatrix} & K_{dd} & K_{dc} & 0 \\ 0 & K_{cd} & K_{cc} & K_{cf} \\ 0 & 0 & K_{fc} & K_{ff} \end{bmatrix}\begin{Bmatrix} P^{ff} \\ u_d^{ff} \\ u_c^{ff} \\ u_f^{ff} \end{Bmatrix} \tag{51}$$

and

$$F=-\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc} & M_{cf} \\ 0 & 0 & M_{fc} & M_{ff} \end{bmatrix}\begin{Bmatrix} F_r \\ \ddot{U}_d^g \\ \ddot{U}_c^g \\ \ddot{U}_f^g \end{Bmatrix} \tag{52}$$

Free field responses are obtained by analyzing the foundation. Since the analysis is now for the foundation part only, the corresponding values of the displacement, velocity and acceleration for the dam and reservoir domain are taken as zero. This involves the introduction of the following change of variables:

$$\begin{Bmatrix} \ddot{P} \\ \ddot{U}_d \\ \ddot{U}_c \\ \ddot{U}_f \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ \ddot{u}_c^{ff} \\ \ddot{u}_f^{ff} \end{Bmatrix}+\begin{Bmatrix} \ddot{P}^a \\ \ddot{u}_d^a \\ \ddot{u}_c^a \\ \ddot{u}_f^a \end{Bmatrix},\quad \begin{Bmatrix} \dot{P} \\ \dot{U}_d \\ \dot{U}_c \\ \dot{U}_f \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ \dot{u}_c^{ff} \\ \dot{u}_f^{ff} \end{Bmatrix}+\begin{Bmatrix} \dot{P}^a \\ \dot{u}_d^a \\ \dot{u}_c^a \\ \dot{u}_f^a \end{Bmatrix} \text{ and } \begin{Bmatrix} P \\ U_d \\ U_c \\ U_f \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ u_c^{ff} \\ u_f^{ff} \end{Bmatrix}+\begin{Bmatrix} P^a \\ u_d^a \\ u_c^a \\ u_f^a \end{Bmatrix} \tag{53}$$

Therefore, if the foundation domain is subjected to earthquake motion, the free-field responses are obtained by solving the following expression.

$$\begin{bmatrix} M_{cc} & M_{cf} \\ M_{fc} & M_{ff} \end{bmatrix}\begin{Bmatrix} \ddot{u}_c^f \\ \ddot{u}_f^f \end{Bmatrix}+\begin{bmatrix} C_{cc} & C_{cf} \\ C_{fc} & C_{ff} \end{bmatrix}\begin{Bmatrix} \dot{u}_c^f \\ \dot{u}_f^f \end{Bmatrix}+\begin{bmatrix} K_{cc} & K_{cf} \\ K_{fc} & K_{ff} \end{bmatrix}\begin{Bmatrix} u_c^f \\ u_f^f \end{Bmatrix}=-\begin{bmatrix} M_{cc} & M_{cf} \\ M_{fc} & M_{ff} \end{bmatrix}\begin{Bmatrix} U_c^g \\ U_f^g \end{Bmatrix} \tag{54}$$

After obtaining the free field response the interaction force R is calculated using equation (55) in the following simplified manner:

$$R=-\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc}^d & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ \ddot{u}_c^f \\ 0 \end{Bmatrix}-\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{dd} & C_{dc} & 0 \\ 0 & C_{cd} & C_{cc}^d & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ \dot{u}_c^f \\ 0 \end{Bmatrix}-\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & K_{dd} & K_{dc} & 0 \\ 0 & K_{cd} & K_{cc}^d & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ u_c^f \\ 0 \end{Bmatrix} \tag{55}$$

After obtaining the interaction forces R, the added responses of the dam, foundation and reservoir domain are calculated using equation (50). These calculated responses are added to the free field responses to obtain the absolute responses of the coupled dam- reservoir-foundation system.

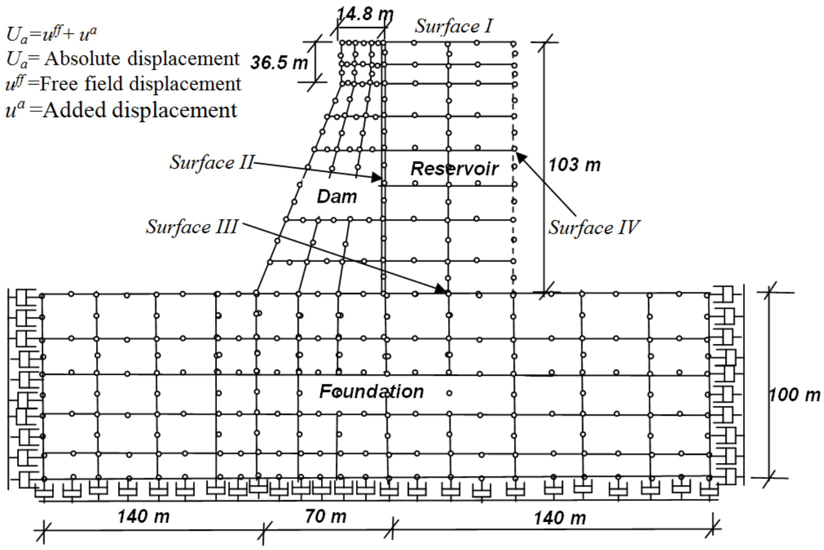

Fig. 1 Dam-Reservoir-Foundation system

RESULTS AND DISCUSSIONS

In the present study, dam-reservoir-foundation coupled system is assumed to be in a state of plane strain. A rigorous study is carried out to study the effect of hygro-chemo-mechanical degradation of concrete in dam on the dynamic responses of the dam-reservoir-foundation system with absorptive reservoir bottom.

1. Validation of the Present Algorithm

The developed Eulerian-Lagrangian coupling methodology for dam-reservoir-foundation system is validated with a bench mark problem carried out by Papazafeiropoulos et al. (2011). The geometry and material properties of this system are as considered by Papazafeiropoulos et al. (2011). This validation study is carried out for two different modulus of elasticity of foundation. At the truncation surface of infinite reservoir and soil foundation, suitable non reflecting boundary conditions as proposed by Papazafeiropoulos et al. (2011) are considered. The hydrodynamic pressure along the face of the dam due to harmonic excitation at resonance is presented in Figure 2. The results obtained from the present method are almost similar to the results obtained by Papazafeiropoulos et al. (2011) for $E_d/E_s = 1$. However, a little variation of the results is observed for $E_d/E_s = 500$ and this variation are due to the different mesh size and element used in discretization of dam-reservoir-foundation system.

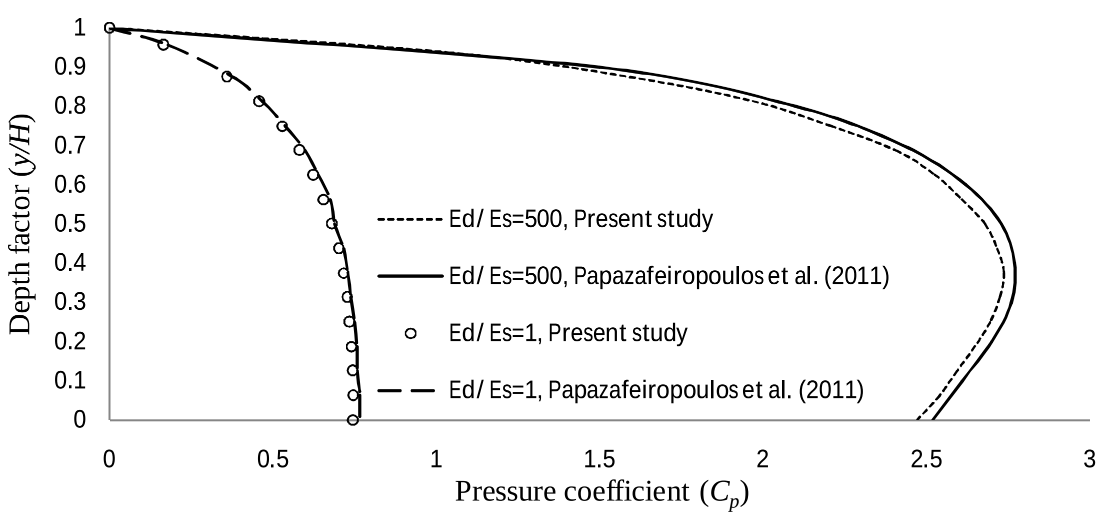

Fig. 2 Hydrodynamic pressure along the upstream face of the dam

2. Effect of Sediment on Reflection Coefficient

The reflecting characteristic of reservoir bottom may alter the hydrodynamic pressure on adjacent dam. Highly reflective reservoir bottom $(\alpha = 1.0)$ yields exaggerated hydrodynamic pressure. In practical situation, a fully absorptive reservoir bottom, i.e., $(\alpha = 0.0)$ may not exists. The value of 'α' depends on the density of the sediment accumulated at the reservoir bottom. In this section, the effect of sediment density on the reflection coefficient is studied. Here, a sediment layer having an elastic modulus of 35000 MPa (Leurer 1997) is considered. The variation of reflection coefficient with density is plotted in Figure 3. The results show a steep increase in reflection coefficient for sediment densities lower than 1000 kg/m^3. Therefore, for all the practical problems, with sediment density more than 1000 kg/m^3, the value of α may be considered in the range of 0.5 to 1.

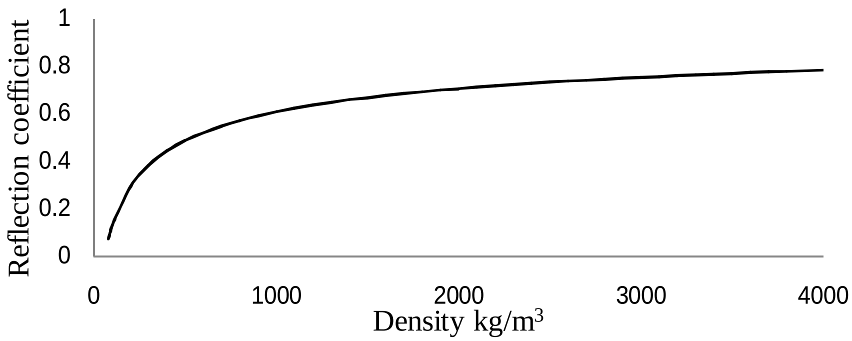

Fig. 3 Effect of sediment density on reflection coefficient

The effect of depth of sediment on the refection coefficient is also studied. In this study, it is assumed that the sediment is not flushed out at regular intervals and the depth of sediment layer will increase with the age of the reservoir. The reflection coefficient considering increasing depth of sediment layer with age is calculated. This study is carried out for different properties of the underlying strata for a period of 100 years. The sediment layer is considered to lie (i) over a rigid stratum (RS) (ii) over a flexible stratum (FS). The reflection coefficients obtained in the present analysis are compared with the frequency dependent reflection coefficients obtained by Hatami (1997) at an attenuation constant, $b = 0.25$. The material parameters considered in the present analysis is same as in the reference Hatami (1997). Elastic modulus of sediment layer=4500 MPa and the wave propagation velocity in sediment is considered to be 1500 m/sec. The reflection coefficients are evaluated considering a rate of sedimentation as 0.15 m/year and 0.40 m/year. It is observed from Figure 4 that for sediment layer lying over a rigid stratum (RS), the reflection coefficient varies from 1.0 to 0.85 and 1.0 to 0.71 at sedimentation rates of 0.15m/year and 0.40 m/year respectively within a period of 100 years.

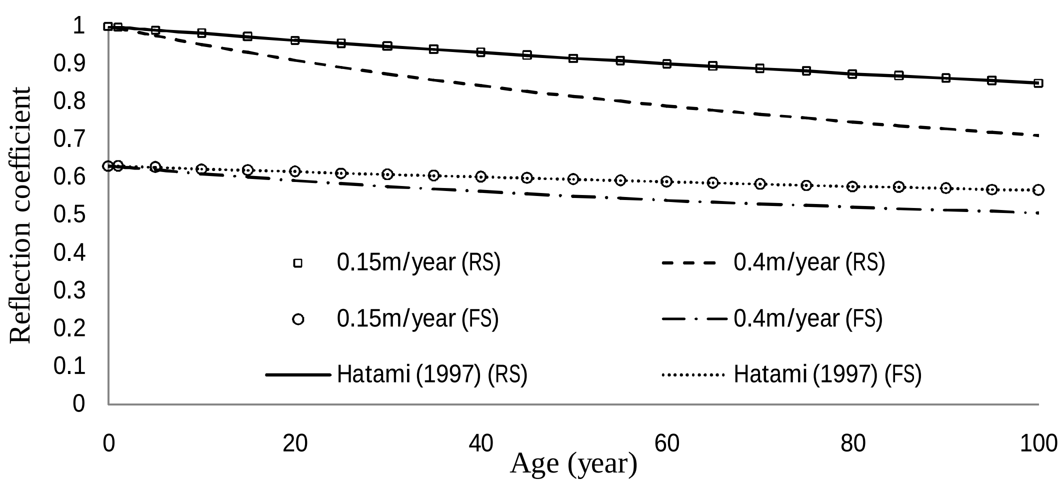

Fig. 4 Variation of reflection coefficient with age

In case of flexible underlying stratum, the elastic modulus of the underlying layer is considered to be 5.0 E_{s1}, where E_{s1} is the elastic modulus of overlying sediment layer and the exciting frequency is considered to the fundamental frequency of dam-reservoir-foundation coupled system. It is observed from Figure 4 that for sediment layer overlying a flexible stratum (FS), the reflection coefficient ranges from 0.63 to 0.57 for a sedimentation rate of 0.15 m/year. At a sedimentation rate of 0.40 m/year, the reflection coefficient ranges from 0.63 to 0.51. The decrease in the reflection coefficient with age is significant when the sediment layer is considered to be deposited over a rigid stratum than when the sediment layer is considered to be deposited over a flexible stratum. These results are in agreement with those published by Hatami 1997.

3. Seismic Response of Aged Dam-Reservoir-Foundation system with Absorptive Reservoir Bottom

The presence of sediments at the bottom of the reservoir will make the reservoir bed absorptive. In this study, it is assumed that the reflection coefficient will remain constant at all ages, as the reservoir sediment is flushed out at regular intervals to maintain a constant depth of the sediment layer at reservoir bottom. The reflection coefficient is considered to be 0.5. The geometry properties for dam, reservoir and foundation are shown in Figure 1. The analysis is carried out to obtain the seismic response of the degraded Koyna dam (Figure 1) at the age of 25 years and 50 years with absorptive reservoir bottom and the results are plotted in graphical forms (Figures 5-10). The degradation indices considering a design life of 100 years at the 25th and 50th year are 0.191 and 0.347 respectively. It was observed from Figures 5-6 that, the crest displacement of the dam at both the ages of dam due to Koyna earthquake is reduced significantly when the absorptive of reservoir bottom is considered and this reduction is almost 24.76% and 11.86% for the age of 25 year and 50 year respectively. But Figures 5 and 6 show that though the crest displacement of the dam increases with an increase in age, the significance of absorption at the reservoir bed reduces comparatively.

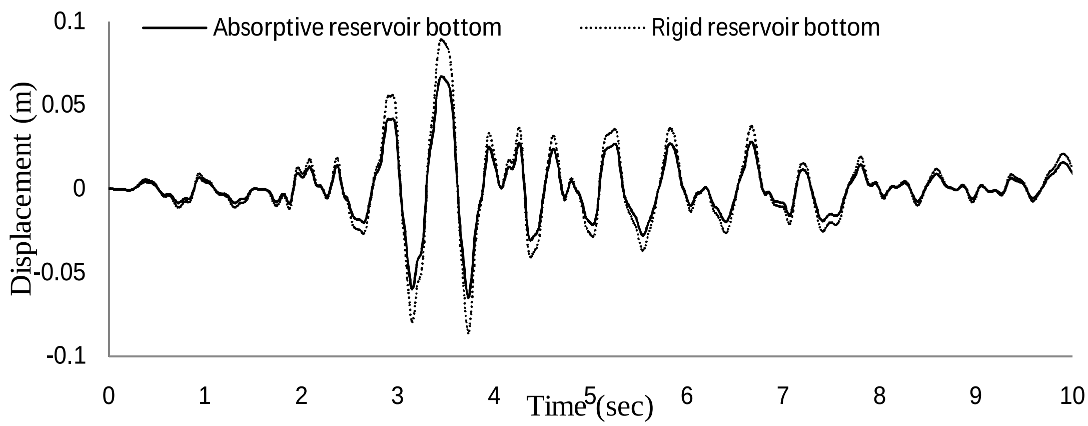

Fig. 5 Crest displacement of the dam for different reservoir beds at the age of 25 years

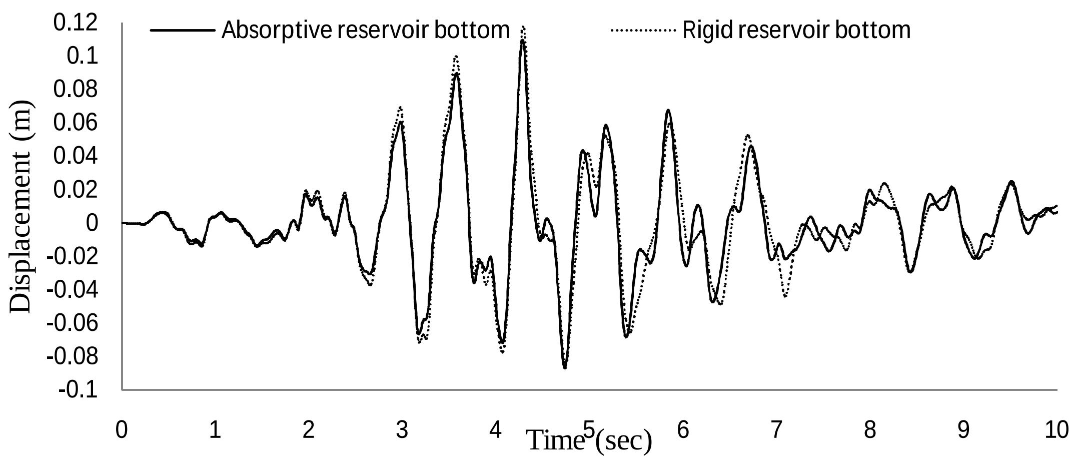

Fig. 6 Crest displacement of the dam for different reservoir beds at the age of 50 years

The hydrodynamic pressures coefficient at the heel of the dam due to Koyna earthquake in the 25th and 50th year are plotted in Figures 7 and 8 respectively. From these two graphical results it is observed that the magnitude of hydrodynamic pressure coefficient decreases with the increase of degradation of the dam. The hydrodynamic pressure coefficient obtained at different ages show that the significance of the absorptive reservoir bottom is more pronounced at an early age.

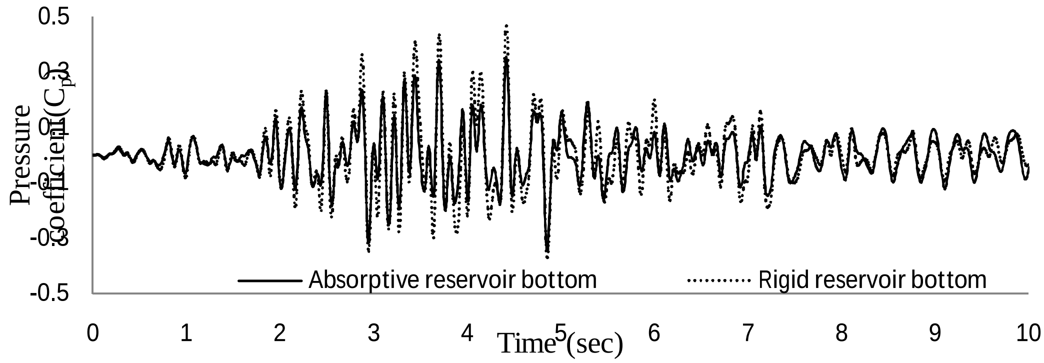

Fig. 7 Hydrodynamic pressure for different reservoir beds at the age of 25 years

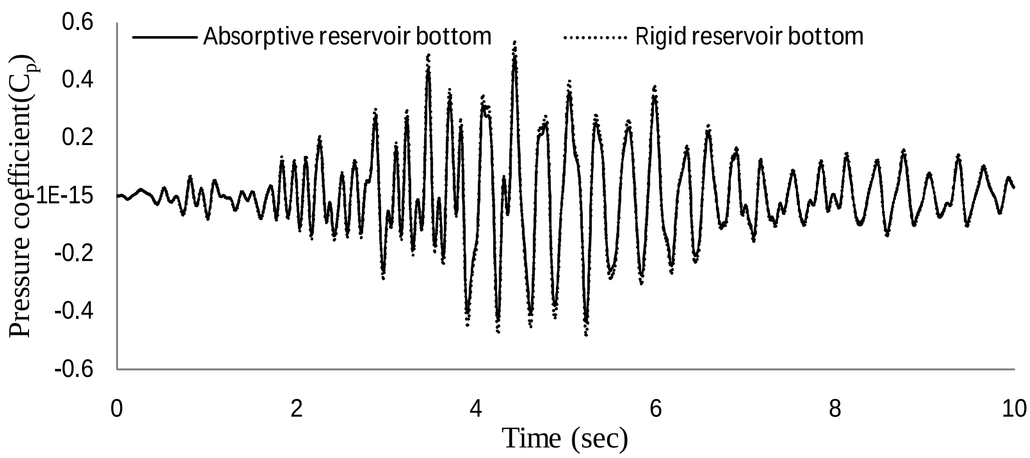

Fig. 8 Hydrodynamic pressure for different reservoir beds at the age of 50 years

The major principal stresses at notch of the degraded dam at 25th and 50th year are plotted in Figure 9 and 10 respectively. The maximum principal stresses at the notch are observed as 7.42×10^6 MPa and 5.78×10^6 MPa for rigid and absorptive reservoir bottom respectively at the age of 25 years. Similarly, these stresses values are 5.80×10^6 MPa and 5.41×10^6 MPa respectively at the age of 50 years. On comparing with the stresses obtained at notch of the dam, it is observed that the stresses changes significantly due to the effect of degradation. However, at later age of the dam, the difference in stresses for different conditions at reservoir bottom is not much.

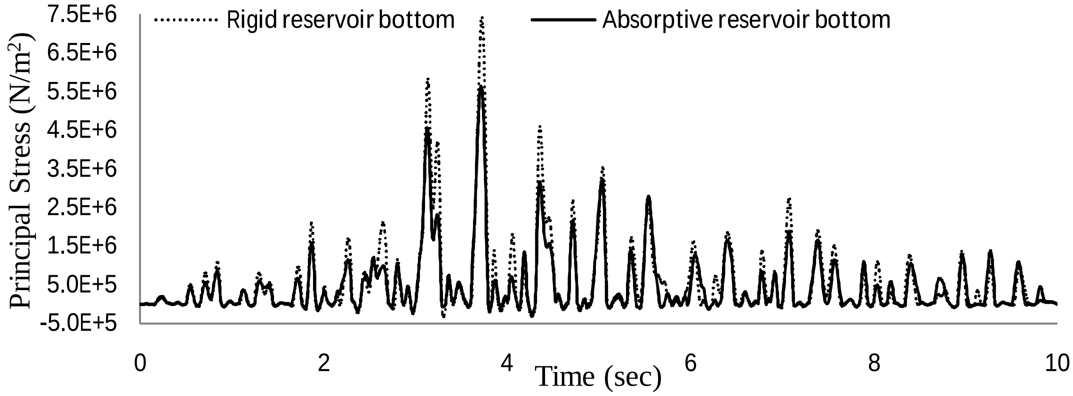

Fig. 9 Major principal stress at notch of the dam for different reservoir bed at the age of 25 years

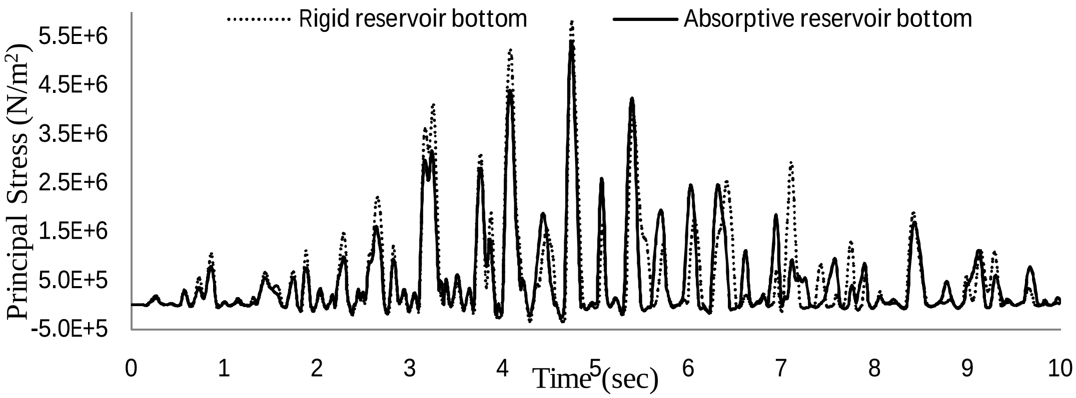

Fig. 10 Major principal stress at notch of the dam for different reservoir bed at the age of 50 years

CONCLUSIONS

A dynamic analysis of concrete gravity dam is presented herein to predict the behavior of concrete gravity dam at different ages of dam with absorptive reservoir bottom. In the present study, displacement and pressure based finite element are used to model dam, foundation and reservoir respectively. Degradation due to mechanical loading and chemical reaction within the concrete in dam is considered. Again, a reflection coefficient is introduced to simulate the absorptive condition of reservoir bottom and these effects on the performances of dam-reservoir-foundation coupled system are studied. From the present study, it is noted that the reflection coefficient due to accumulation of sediment at reservoir bottom decreases with the age of the dam and this decrease in the reflection coefficient is significant when the sediment layer is considered to be deposited over a rigid stratum than the sediment layer over a flexible stratum. The responses of the degraded dam-reservoir-foundation coupled system under seismic excitation are determined for different absorbing conditions of reservoir bottom. The crest displacement of dam at different ages of dam due to earthquake excitation is reduced significantly when the absorptive of reservoir bottom is considered. The results further show the increase of crest displacement with the increase of age of dam. However, the changes in the crest displacement due to absorptive reservoir bottom are decreased with the age of dams. Similar, trends are observed in case of hydrodynamic pressure within the reservoir and principal stress in dam and foundation. Therefore, it is concluded that the significance of reservoir bottom absorption due to accumulation of sediment decreases with the increase of age of dams.

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