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70-TH ANNIVERSARY OF BIOT SPECTRUM

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ABSTRACT

The concept of the response spectrum method was formulated and proposed in 1932 for use in the analysis and design of earthquake-resistant structures. To commemorate the 70th anniversary of this event, this paper reviews Biot's seminal contributions, which created this method, and then briefly outlines the milestones in the general evolution of the response spectrum method. The techniques for computation of spectral amplitudes and the studies of spectral shapes are described for the period preceding the modern digital computer age. The role of response spectrum in influencing the development of design codes, and their transition from the use of static to dynamic methods of analysis are illustrated, using the examples from code development in California. Finally, the limitations of the linear response superposition method are considered, and future directions for the development of the earthquake-resistant design tools are suggested.

KEYWORDS: Response Spectrum, Linear Response, Spectral Shapes

INTRODUCTION

The year 2002 marks the 70th anniversary of the birth of the response spectrum method. It is also the 60th anniversary of Biot's fourth, and last paper on earthquake engineering, in which he presented the general principles of how to use the response spectrum method (RSM) in design. Finally, it is approximately the 30th anniversary of the general acceptance of the RSM, following the 1971 San Fernando, California earthquake. In commemoration of these anniversaries, this paper outlines and examines, the first ideas on how to formulate this method, its use during the past 30 years, its limitations, and the prospects for its future use.

The response of a structure to strong earthquake ground motion may be investigated by two different methods. One of these consists of constructing a model of a structure and calculating the exact dynamic response for an assumed motion of the foundation. This approach has been used frequently for the final design of important structures. The other approximate method is formulated in a manner that permits separation of the characteristics of particular structures from those of the earthquake, the latter being given by the "response spectra". This approach is used for the design of many earthquake-resistant structures, and it is often the main tool for preliminary design, before the final design is examined and tested by the first method. Because of the importance of this use of response spectra in design, and because spectra contain valuable information about the characteristics of recorded strong-earthquake ground motions, some basic facts about the use and evolution of the RSM will be outlined in the following sections.

Extension of the RSM to the non-linear response of structures has been studied extensively with varying degrees of success. In the following, we will cite only a few examples, leaving the complete review of such analyses for a future separate paper. Here we will focus on (1) the development of the RSM for "linear" response of structures, (2) its role in influencing current design methods and codes, and (3) its suitability for analysis of transient response to impulsive loading.

RESPONSE SPECTRUM

1. Historical notes

In the early 1930s, Professors Theodore von Kármán[1] and Maurice Biot[2] were very active in the theoretical dynamics aspects of what would later become known as the response spectrum method in earthquake engineering. These ideas were first outlined in 1932, and were fully developed by 1942. Today, 70 years later, this theory essentially remains intact, and still forms the core of the linear mechanics framework of earthquake engineering.

In his summary comments on the state of Earthquake Engineering, following the Seventh World Conference on Earthquake Engineering in Istanbul, Turkey, in 1980, Krishna (1981) stated: "Earthquake engineering as such could be considered to have been born with Biot's concept of a response of an idealized structure to ground motion". Here Krishna was referring to the second chapter of Biot's Ph.D. dissertation, defended at Caltech in 1932 and entitled "Vibration of Buildings during Earthquakes" (Biot, 1932). Biot's ideas, and further studies undertaken at the suggestion of Professor Theodore von Kármán, were refined further while Biot was a Research Fellow at Caltech in 1932 (Biot, 1933, 1934). In his 1933 paper, communicated on January 19, 1933, just 50 days before the first strong acceleration was recorded on March 10, 1933 in Long Beach, California, Biot stated: "The study of seismogram spectral distributions has not yet been made; it is, however, the author's opinion that this study would be of great importance for two reasons: (1) The peaks of spectral curves will reveal the presence of certain characteristic frequencies of the soil at a given location, (2) By applying the preceding theorem, the maximum effect of earthquakes on buildings will be easily evaluated…". Biot briefly returned to the subject of earthquake engineering, describing computation of response spectra by means of mechanical analyzer (Biot, 1941) and formulating the general theory and principles of response analysis and response spectrum superposition (Biot, 1942).

The response spectrum method remained in the academic sphere of research for about 40 years, only finally gaining wide engineering acceptance during the early 1970s. There were two main reasons for this. First, the computation of response to irregular ground motion led to "certain rather formidable difficulties" (Housner, 1947), and, second, there were only a few well-recorded accelerograms that could be used for response studies (Figure 1). All of this started to change in the mid-1960s, with appearance of digital computers and with commercial availability of strong-motion accelerographs (Trifunac and Todorovska, 2001). Before the digital computer age, the computation of response was extremely time consuming, and the results were so unreliable that many studies using response spectrum amplitudes from that period must be treated with caution (Trifunac et al., 2001a). By late 1960s and early 1970s, the digitization of accelerograms (Trifunac et al., 1973a) and the digital computation of ground motion and of the response spectra were developed completely and tested for accuracy (Trifunac and Lee, 1973, 1974). Then, in 1971, with the occurrence of the San Fernando, California, earthquake, the modern era of RSM was launched. This earthquake was recorded by 241 accelerographs, including more than 175 from the Los Angeles area, where a large number of instruments had been installed at various levels in high-rise buildings. By combining the data from the San Fernando earthquake with all previous strong motion records, it became possible to launch the comprehensive empirical scaling analyses of spectral amplitudes (Trifunac, 1976, 1978).

[1] Theodore von Kármán, born in Budapest, Hungary (1881-1963), engineer, applied scientist, teacher and visionary, was the first director of the Daniel Guggenheim Graduate School of Aeronautics at the California Institute of Technology, where he arrived in 1930 from Aachen, Germany. von Kármán had foresight, creativity, and a remarkable talent for getting people together across professional, national, and language barriers. He was one of the foremost leaders in the world of aviation and space technology (see, for example, von Kármán and Edson (1967)).

[2] Maurice A. Biot, born in Antwerp, Belgium (1905-1985), was an engineer, physicist, and applied mathematician. After graduating in electrical and mining engineering and philosophy, and receiving his D.Sc. degree (1931) from the University of Louvain (Belgium), he went to Caltech, where he received a Ph.D. in 1932, in Aeronautical Sciences. He was a student and then collaborator of Theodore von Kárnán with whom he wrote a classical textbook "Mathematical Methods in Engineering" (von Kármán and Biot, 1940). He taught briefly at Louvain, Harvard, Columbia, Caltech, and Brown Universities. As an independent scientific consultant, he worked for Shell Development, Cornell Aeronautic Laboratory, and Mobil Research. Biot published 179 articles, three books (*Mathematical Methods in Engineering*, with Theodore von Kárnán, McGraw Hill 1940; *Mechanics of Incremental Deformations,* Wiley, 1965; *Variational Principles in Heat Transfer*, Oxford, 1970), and he was holder of seven patents. Twenty-one papers written by Biot on the theory of porous media have been edited and reprinted by Tolsloy (1992). A man of great and unique talent, Biot worked without students and essentially alone.

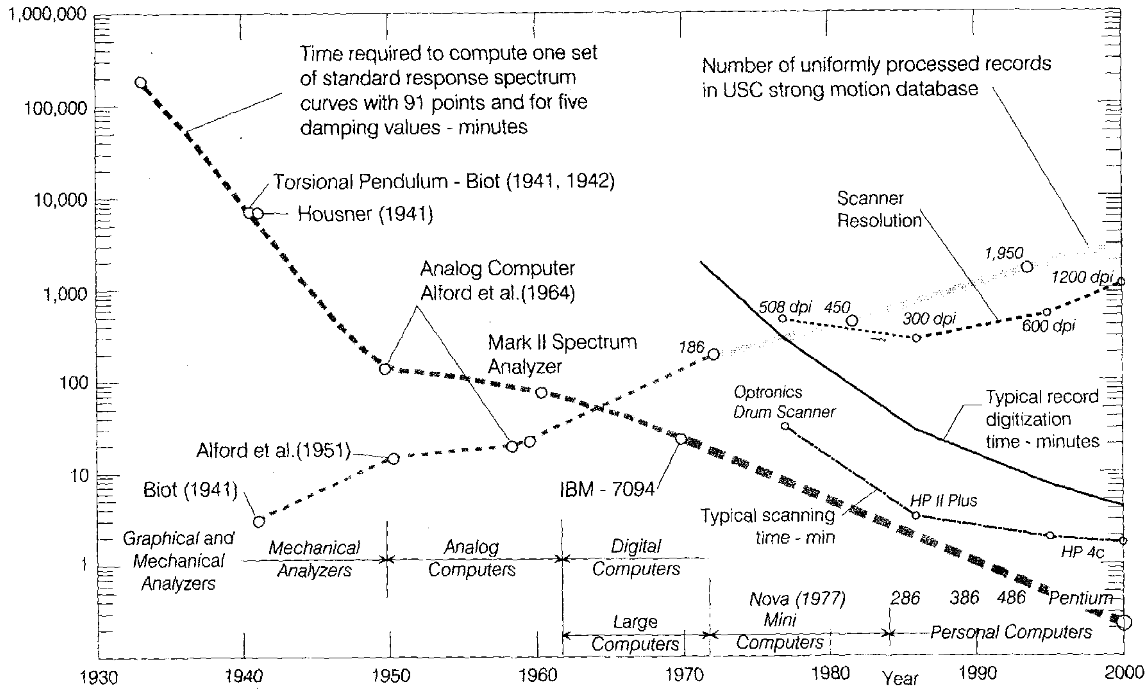

Fig. 1. Trends in the capabilities of accelerogram digitization and data processing, between 1970 and 2000: time required to compute one set of standard response spectrum curves (in minutes), and the cumulative number of accelerograms in strong-motion data bases (light dashed line for the period prior to 1970), and in the uniformly processed strong-motion data bases (wide gray line for the period after 1970)

2. Computation of Response Spectra

Computation of response spectra requires the solution of Duhamel's integral (Appendix A) and then selection of the maximum response. Prior to the age of digital computers, execution of these tasks was laborious and very time consuming. For example, before the 1940s, direct numerical integration (Martel and White, 1939) and semi-graphical procedures using Intergraph instruments (Hudson, 1956) had been used.

The first use of a mechanical analyzer for finding oscillator response to an earthquake motion was by Frank Neumann (Neumann, 1936, 1937) of the U.S. Coast and Geodetic Survey in 1936. In this work, the earthquake displacement curve, obtained by double integration of an accelerogram, was used to govern the motion of a torsional pendulum (see discussion by M.P. White (White, 1942)). Response spectra were evaluated mechanically at Stanford University, as follows: "The accelerogration record was integrated twice to give ground displacements. A cam cut in the pattern of these displacements actuated a shaking table upon which a simple oscillator was placed. Because the displacement of such an oscillator is equal to $x - \omega_n \int_0^{t_1} \ddot{z} \sin \omega_n (t_1 - t) dt$ (see Appendix A), the maximum recorded displacements multiplied by ω_n give the required value of V" (pseudo spectral velocity) (Housner, 1941a; Hoff, 1942).

White and Byrne (1939) suggested a method by which an accelerogram might be used directly to actuate a mechanical analyzer. This principle is the same as the one later used by Biot (1941, 1942) and Housner (1941a, 1941b).

The first practical method for computation of spectral amplitudes was based on the torsional pendulum analog (Savage, 1939; Biot, 1941). In this method, an oscillator is represented by an eccentric mass supported by stretched wire, one end of which is forced to twist through angles proportional to the acceleration amplitude, versus time (Biot, 1941; Alford et al., 1964). The most time-consuming difficulty associated with the use of a torsional pendulum was the inconvenience of changing the natural period of torsional response. Gross changes in period were made by using torsional wire of different diameters. Fine changes were made by selecting the separation distances of the masses on the inertia bar. Damping was also difficult to control. At first, it was thought to be zero, but later it was discovered to be in the range of a few percent of critical. The damping in the torsional pendulum came from the internal friction of the torsional spring and from air damping of inertia bar (Alford et al., 1964). With Biot's torsional pendulum at Columbia University, it took about 8 hours to construct one spectrum curve consisting of about 30 points (Biot, 1942). At Caltech, it took about 15 minutes to construct one spectrum point (Alford et al., 1964). Prorating these durations to computation of spectra at 91 period points for 5 damping values results in a period of about 7,000 minutes (167 hours; see Figure 1). At the Earthquake Research Institute of Tokyo University, a moving coil galvanometer element was used as the mechanical torsional system (Takahasi, 1953). It had a torsional element with fixed frequency, and the period changes were effected by changing the speed of the film drive mechanism in the ground motion generator. By energy input into the torsional system, through a feedback loop, effective zero damping of the system was made possible.

The idea of using analog computers for computation of response spectra can be traced back to 1934: "The direct computation of … spectra might be tedious, but automatic electrical methods can be easily imagined, such as a photographic record passing in front of a photo-electric cell acting upon a tuned circuit" (Biot, 1934). As will be seen from the following, this idea was finally implemented during the 1950s (Alford et al., 1964; Caughey et al., 1960).

In the late 1940s, an analog computer technique was proposed for solving the response of a single-degree-of-freedom system to arbitrary excitation $F(t) = -M\ddot{z}$ (Griner et al., 1945; McCann, 1946), as shown below:

$$M\ddot{x} + C\dot{x} + Kx = -M\ddot{z} \tag{1}$$

via its electrical analog

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = E \tag{2}$$

where M, C, and K are mass, damping coefficient, and stiffness, respectively, x is relative displacement of M, and z is absolute ground displacement. In Equation (2), L is inductance, R is resistance, and C is capacitance. E is applied variable voltage, and q is electrical charge (Figure 2 and Table 1). The voltage input was introduced through a photoelectric cell, which scanned a rotating film disc (Figure 3).

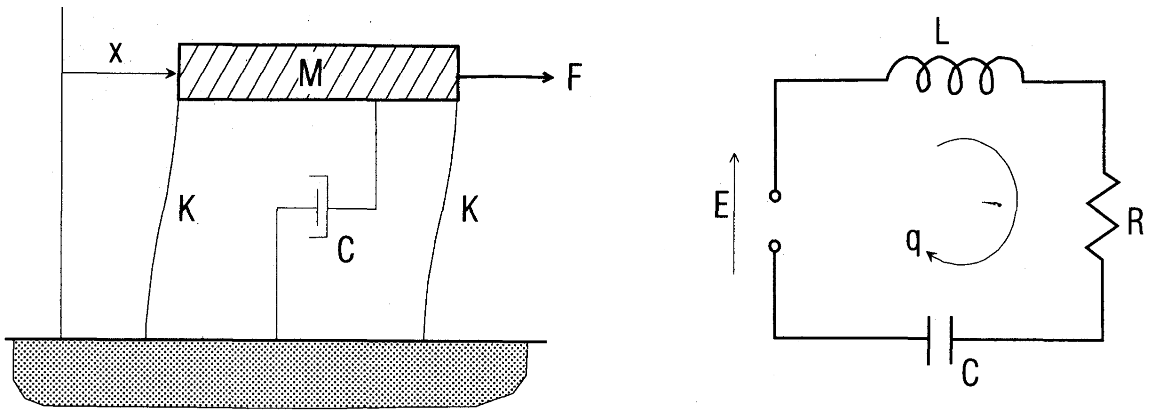

Fig. 2 Basic electro-mechanical analog (redrawn from Alford et al. (1964))

The significance of the analog computer was that it enabled, for the first time, systematic calculation of response spectra with assigned damping values. It was about 30 times faster than the torsional pendulum analog (Figure 1). Crede et al. (1954) showed how a commercial electronic differential analyzer could be used for the determination of response spectra. A special-purpose spectrum analyzer using electronic operation techniques was described by Morrow and Riesen (1956). A small special-purpose analog computer system, Mark II, designed for computation of response spectra was developed in 1954 and tested through the mid-1950s (Caughey et al., 1960). Using this electric analog, response spectra were calculated for a series of strong motion earthquakes in the western United States (Hudson, 1956).

Table 1 (from Alford et al. (1964))

[Mechanical-Electrical Relations for Analog]

Mechanical System		**Electrical Analog**	
M =	mass of system	L =	inductance = $\frac{a}{N^2} m$
K =	spring constant	C =	capacitance = $\frac{1}{ak}$
C =	damping constant	R =	resistance = $\frac{ac}{N}$
τ =	period of vibration	τ^1 =	simulated period = $\frac{\tau}{N}$
F =	exciting force	E =	applied voltage
x =	displacement	q =	electrical charge
v =	velocity	i =	current

$$x = a\frac{F}{E}q$$

N = time-scale change factor
a = impedance change factor

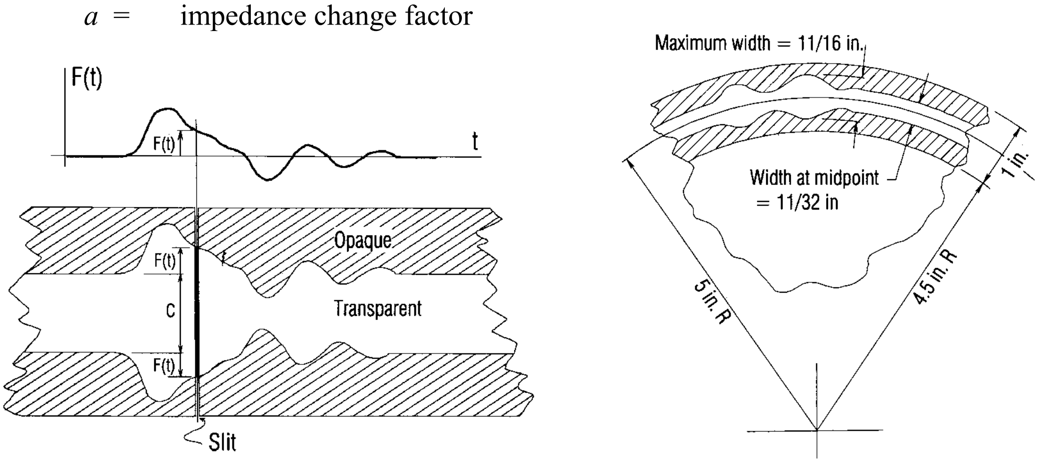

Fig. 3 Left: Transformation of input function to variable-width film trace, Right: Dimensions of the standard film disk record for the function generator (redrawn from Caughey et al. (1960))

In the early 1960s, the methods for computation of response spectra started to change, following the general availability of digital computers. Digitized accelerograms could be used in Duhamel integral and integration being performed numerically. Assuming that acceleration data can be approximated by piece-wise straight line segments between equally spaced points in time, the Duhamel integral can be integrated exactly over each time interval, thus reducing numerical integration to a sequential application of 2 × 2 matrices and two 2-component vectors. This required eight multiplications and six additions for each time step, or 14 N operations for an accelerogram defined by N points (Nigam and Jennings, 1968; see Appendix B).

Through the 1980s, proposals were made to speed up these calculations by using digital filter simulations of response (Lee, 1984; Beck and Park, 1984; Beck and Dowling, 1988). Lee (1990) showed that by using the digital impulse and step invariant simulations of a continuous system, calculations of response displacement and velocity can be reduced to two multiplications and three additions per time step and to two multiplications and 4 additions per time step if displacement, velocity, and acceleration responses are computed simultaneously.

The speed of digital computers has increased remarkably since 1970 (Figure 1), eliminating the need to increase the speed of response calculation. Therefore, in USC's Strong Motion Data Processing Laboratory, we have chosen to continue with the use of the exact response calculations, based on equally spaced points in time and linear interpolation of acceleration between the digitized points (Trifunac and Lee, 1973, 1979; Lee and Trifunac, 1990).

In summary, before introduction of the torsional pendulum analog, computation of response spectra was so long and difficult that spectra of only several recorded accelerograms, for "zero" damping, could be considered (Housner, 1941a, 1941b). Between 1940 and 1950, the torsional pendulum method (Biot, 1941) "was a big advance, because it was about 30 times quicker than doing it graphically" (EERI, 1997). The introduction of the analog computer method in the early 1950s reduced the time of computing spectral amplitudes by about 60 times, but the conversion of recorded accelerograms into film disk records (Alford et al., 1964; Caughey et al., 1960), the selection and calibration of required electrical constants, and the reading of maximum responses from an oscilloscope complicated and delayed the process. With the introduction of digital computers in the 1960s, it became necessary to convert analog film records of acceleration into digital points, and until late 1970s, this slowed down the process considerably. Since the early 1990s, digitization of analog records has become fast, efficient, and accurate. At present, it is the organization of the whole process and the archiving and distribution of data that limit the speed, with which response spectra can be obtained in their final form.

Figures 4, 5, and 6 illustrate the accuracy of different old methods of computing response spectra. Figure 4 compares relative velocity spectra computed by (1) Biot, using torsional pendulum at Columbia University (Housner, 1941a); (2) Housner, using torsional pendulum at Caltech (Housner, 1941a); (3) Alford et al. (1964), using an analog computer; and (4) modern digital computing (Trifunac and Lee, 1973b, from digitized accelerogram of Helena, Montana, earthquake of 1935; Trifunac et al., 1973b). It can be seen that all spectra follow the same broad trends (large amplitudes near 0.2 s to 0.4 s and near 1 s, and small amplitudes near 0.5 to 0.6 s), but the local peaks fluctuate in a random manner. For most periods, zero damped spectral amplitudes, computed by modern digital methods, are smaller than all three "old" spectra. This may be related to the selection of scaling constants used in electrical analog computing and in the two analyses based on the torsional pendulum. Two spectral curves, for $\zeta = 0.1$ and 0.2, computed by Alford et al. (1964) have amplitudes and trends similar to those of digitally computed spectra, but their details do not agree.

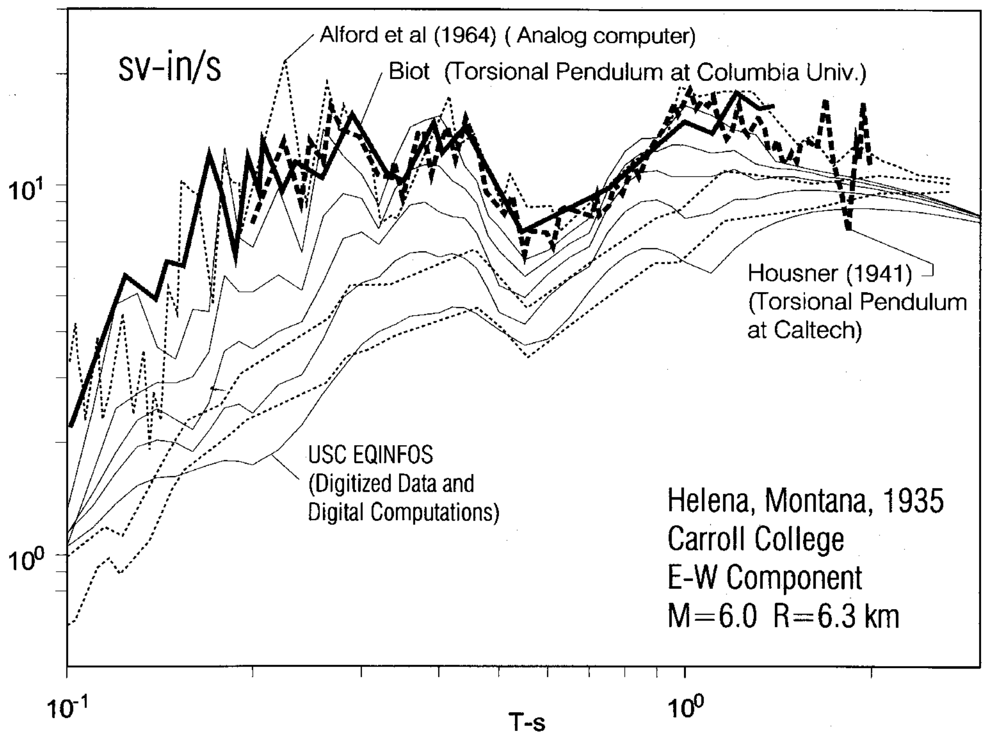

Fig. 4 Comparison of relative velocity response spectrum amplitudes for E-W component of strong motion recorded at Carroll College, during Helena, Montana earthquake of 1935 (spectra computed by torsional penduli of Biot and Housner (damping not specified) are compared with spectra computed by analog computer (damping values 0, 0.10, and 0.20), and digital computer (for five damping values 0, 0.02, 0.05, 0.10, and 0.20; Lee and Trifunac, 1987))

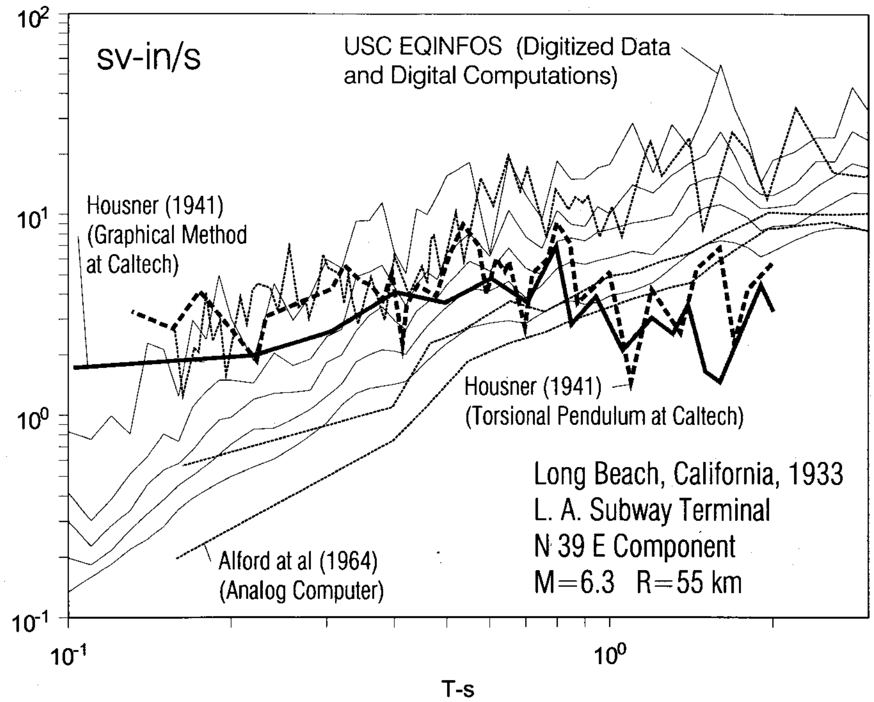

Fig. 5 Comparison of relative velocity response spectrum amplitudes for N39E component of strong motion recorded at Los Angeles Subway Terminal, during Long Beach, California earthquake of 1933 (spectra computed by graphical method (zero damping), torsional pendulum (damping not specified), analog computer (damping values 0, 0.10, and 0.20), and digital computer (for five damping values 0, 0.02, 0.05, 0.10, and 0.20; Lee and Trifunac, 1987))

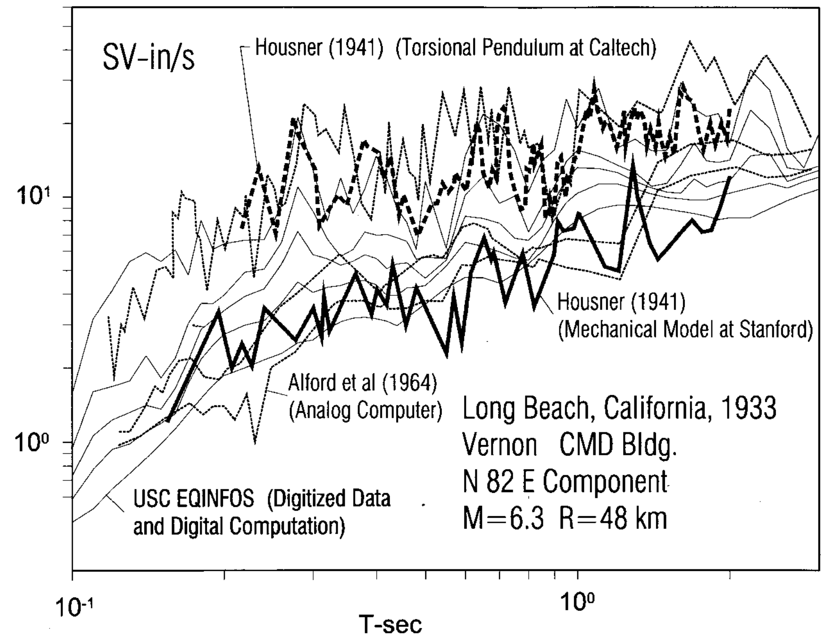

Fig. 6 Comparison of relative-velocity response spectrum amplitudes for N82E component of strong motion recorded at Vernon, CMD Building, during Long Beach, California earthquake of 1933 (spectra computed by mechanical model at Stanford (damping not specified), torsional pendulum at Caltech (damping not specified), analog computer (three damping values 0, 0.10, and 0.20), and digital computer (for five damping values 0, 0.02, 0.05, 0.10, and 0.20; Lee and Trifunac, 1987)).

Figures 5 and 6 reveal far worse agreement. In both figures, the spectra computed by Alford et al. (1964) follow the trend of modern results, but detailed comparison shows serious discrepancies at various periods. For the Vernon record (Trifunac et al., 1973b), spectra computed by torsional pendulum at Caltech (Housner, 1941a) have a correct overall trend, but local peaks do not correlate with modern calculations. The spectra computed by the mechanical method at Stanford have the wrong amplitudes and are depleted of high-frequency amplitudes. The spectra computed by the graphical method and by torsional pendulum at Caltech (Housner, 1941a) for the Los Angeles subway terminal record (Trifunac et al., 1975), shown in Figure 5, have erroneous amplitudes and trends. This difference is so large that the explanation probably involves an erroneous selection of scaling constants or possibly the use of an incomplete record. These major discrepancies, together with other similar discrepancies in spectral amplitudes reported by Trifunac et al. (2001a), show that the accuracy and reliability of the methods used to compute response spectra prior to the introduction of digital computer methods was so bad that all pre-1960s spectra must be considered with great caution. These examples also show that the estimates of the discrepancies in "old" computations of spectral amplitudes of "25 percent or more", with accuracy "to within 10 percent" using an analog computer, as claimed by Alford et al. (1964), are not realistic. The speed, efficiency, and accuracy of computing response spectrum amplitude by analog computer were apparently offset by noise introduced into the process while (1) making the film disc records (Caughey et al., 1960) and (2) reading the peak response from a cathode ray tube display. It is unfortunate that there are no published reports on analyses and properties of these errors.

Biot (1933) had unmistakable vision when he stated that "the peaks of spectral curves will reveal the presence of certain characteristic frequencies of the soil at given locations". With digitized data and modern digital processing, today, response spectra can be used for such studies (Trifunac et al., 1999a). Unfortunately, as illustrated by Figures 4, 5, and 6, the accuracy of the "old" methods for response calculation was not adequate for such analyses.

3. Preparation of Strong-Motion Accelerograms for Processing

In the old graphic method, the record was first drawn on a large scale. This was followed by multiplications and integration, or the use of an Intergraph instrument.

The introduction of the torsion pendulum (Biot, 1941, 1942) presented an advantage in that it was not necessary to convert the recorded accelerogram to a different analog record. A point of suspension of a torsional pendulum was given angular displacements by rotating its arm through an angle proportional to recorded acceleration. The accelerogram was placed on a table traveling with constant velocity, while the arm, connected to the torsional wire at one end and a tracer on the other, was forced to follow the acceleration trace.

The introduction of analog computers required conversion from inertial force $-M\ddot{z}$ in the mechanical system into applied voltage $E(t)$ in the electrical analog. This was accomplished by designing a "plotting table," to prepare film disc records, which were then used in a forcing function generator to produce $E(t)$ (Figure 3). According to Caughey et al. (1960): "The earthquake ground acceleration record drawn to a suitable scale is wrapped around a drum, which is slowly rotated by an electric motor around a vertical axis. The curve is manually traced by a follower mechanism, which is converted through a selsyn system to a shutter, thus exposing a photographic negative, which rotates along with the drum. In this way, a variable-width film trace is produced…where the overall slit width is seen to be equal to a constant plus twice the acceleration function. A similar slit system, along with a light source and a photocell, is then used in the function generator to reproduce the original ground acceleration curve" (see Figure 3).

With the appearance of digital computers, it became necessary to convert analog acceleration traces into a sequence of digital points representing acceleration versus time. The first digitization system in California capable of digitizing large number of accelerograms and converting the digitized data into computer punch cards was described by Hudson (1979). Similar hand-operated digitization tables were in operation at that time in Japan, the Soviet Union, New Zealand and Yugoslavia. This digitization method was accurate but time consuming. Digitization, plotting to verify the accuracy of digitization, and the conversion of data from computer punch cards to files on magnetic tapes took, on average, four days per record. However, subsequent data processing was relatively fast and efficient. It consisted of: (1) preparation of scaled digitized accelerograms (Volume I; Hudson et al., 1969); (2) instrument and baseline correction, followed by computation of velocity and displacement curves (Volume II; Trifunac et al., 1971); (3) computation of response spectra (Volume III; Trifunac et al., 1972a); and (4) computation of Fourier amplitude spectra using a fast Fourier algorithm (Volume IV; Trifunac et al., 1972b).

The next major step forward in digitization of strong-motion accelerograms occurred in 1978, when the first automatic system for digitization (based on a rotating drum scanner by Optronics, controlled by a Nova mini computer) was developed (Trifunac and Lee, 1979) at the University of Southern California. With this system, digitization of a typical accelerogram was reduced to one or two hours. Because checking for the accuracy of digitization now became an integral part of running the digitization software, and because the digitized and corrected data resided on the same computer disk, this system was about 50 times faster than the hand-operated digitizers. During late 1980's, with the development of high-resolution flat bed scanners (HP II with 300 dpi resolution, and HP 4C with 600 dpi resolution) and the commercial availability of fast personal computers, automatic digitization was converted to operate on this new hardware (Lee and Trifunac, 1990; Trifunac et al., 1999a). At present, it takes less than 15 minutes to digitize and prepare the Volume I data of good quality accelerograms that are less than 28 s long. This is about 380 times faster than with the hand-operated digitizers of the late 1960s and early 1970s (Hudson, 1979).

4. Data Distribution

Before the 1970s, the number of digitized and processed accelerograms and their response spectra was small (less than about 100), and the data distribution could easily be organized through personal contacts and mail. During the 1970s and through early 1980s, magnetic tapes with data could be ordered from major centers performing data processing (United States Geological Survey, or USGS; the California Division of Mines and Geology, or CDMG; and the University of Southern California's Strong Motion Group). These groups also contributed their data for archiving and distribution by the National Geophysical Data Center in Boulder, Colorado.

With the development of the Internet and the creation of specialized web-sites dealing with strong-motion data, it is possible today to download large volumes of strong-motion data at no cost. Useful links to such web-sites that offer such data can be found at http:/www.usc.edu/dept/civil_eng/Earthquake_eng/.

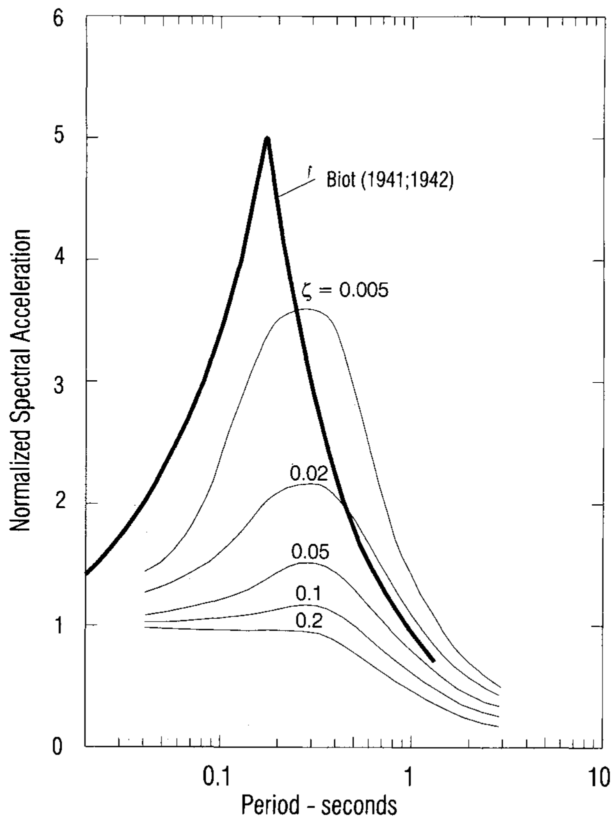

Fig. 7 Comparison of Biot (1941, 1942) "standard spectrum" (heavy line) with average spectrum of Housner (1959, 1970)

SHAPE OF "STANDARD" ELASTIC RESPONSE SPECTRUM

1. Fixed-Shape Response Spectra

In his 1934 paper, Biot stated: "If we possessed a great number of seismogram spectra we could use their envelope as a standard spectral curve for the evaluations of the probable maximum effect on buildings." In Biot (1941), he continued: "These standard curves … could be made to depend on the nature and magnitude of the damping and on the location. Although the previously analyzed data do not lead to final results, we … conclude that the spectrum will generally be a function decreasing with the period for values of the latter greater than about 0.2 s. A standard curve for earthquakes of the Helena and Ferndale … for values $T > 0.2$ s, could very well be the simple hyperbola $A = \frac{0.2g}{T}$ and for $T < 0.2$ s, $A = g(4T + 0.2)$, where T is the period in seconds and g the acceleration of gravity. This standard spectrum is plotted in Figures 7 through 10. Whether this function would fit other earthquakes can only be decided by further investigations".

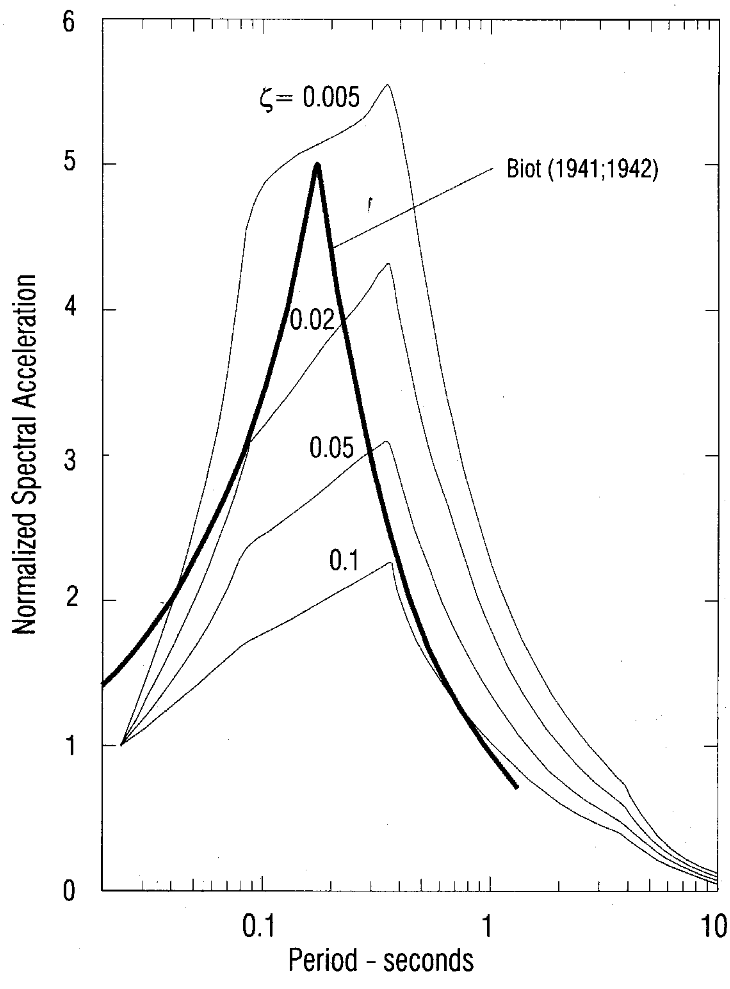

Fig. 8 Comparison of Biot (1941, 1942) "standard spectrum" (heavy line) with regulatory guide 1.60 spectrum (USAEC, 1973)

Fifteen years later, Housner averaged and smoothed the response spectra of three strong-motion records from California (El Centro, 1934, $M = 6.5$; El Centro, 1940, $M = 6.7$; and Tehachapi, 1952, $M = 7.7$), and one from Washington (Olympia, 1949, $M = 7.1$). Housner then proposed the use of his average spectrum in design projects (Figure 7; Housner, 1959, 1970).

In engineering design work, the fixed shapes of Housner and Newmark spectra, normalized to unit peak acceleration, are scaled by selecting the "design" peak acceleration. This procedure, which was first systematically used in the design of nuclear power plants, emerged as the "standard" scaling procedure in the late 1960s and early 1970s, and it is still used today.

2. Site-Dependent Spectral Shapes

In one of the first studies to consider the site-dependent shape of spectra, Hayashi et al. (1971) averaged spectra from 61 accelerograms in three groups (A – very dense sands and gravels; B – soils with intermediate characteristic, and C – very loose soils), and showed that the soil site condition has an effect on the shape of average response spectra. This was later confirmed by Seed et al. (1976), who considered 104 records and four site conditions (rock, stiff soil, deep cohesionless soil, and soft to medium clay and sand; Figure 9).

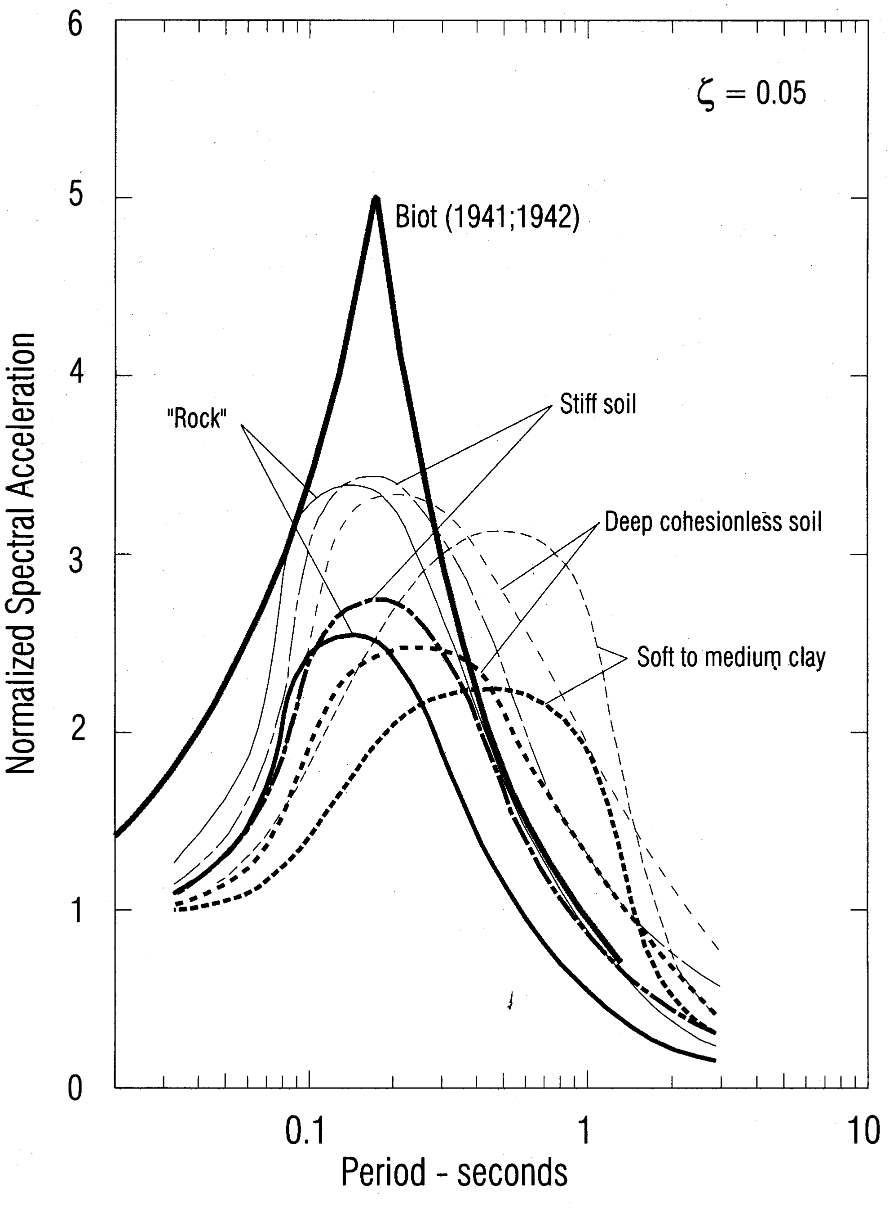

Fig. 9 Comparison of Biot (1941, 1942) "standard spectrum" (heavy line) with average (heavy lines) and average plus standard deviation spectra (light lines) of Seed et al. (1976) for four soil site conditions

Mohraz et al. (1972) suggested that the peak ground displacement, d, and peak ground velocity, v, were: $d = 36$ in and $v = 48$ in/s for "alluvium" sites, and $d = 12$ in and $v = 28$ in/s for "rock" sites, both corresponding to a 1g peak ground acceleration. However, because of the small number of recorded accelerograms on rock in 1972, conclusive recommendations on how to describe the dependence of spectra on site conditions were not possible at that time.

A major and persistent problem in the evaluation of site-dependent spectra of strong earthquake motion is the lack of generally accepted procedures on how to characterize a site. Gutenberg (1957) studied the amplification of weak earthquake motions in the Los Angeles area, and published the results on average trends and amplifications of peak wave motions in sedimentary basins for periods of motion longer than about 0.5 s. His site characterization could be termed "geological," because he considered the "site" on the scale of kilometers and used the term "rock" to represent geological basement rock. Twenty years later, Gutenberg's results were shown to be in excellent agreement with the empirical scaling of Fourier amplitude spectra of strong-motion accelerograms of 186 records (Trifunac, 1976). While it is clear today that both geotechnical and geological site characterizations must be considered simultaneously (Trifunac, 1990; Lee and Trifunac, 1995), there is so far no general consensus on how to do this.

3. Site-, Magnitude-, and Distance-Dependent Spectra

The occurrence of the San Fernando, California earthquake of 1971 and the large number of new recordings, it contributed to the strong-motion database (Hudson, 1976), opened a new chapter in the empirical studies of response spectra. For the first time, it became possible to consider multi-parametric regressions and to search for the trends in recorded strong-motion data. It became possible to show how spectral amplitudes and spectrum shape change, not only with local soil and geologic site conditions, but also with earthquake magnitude and source-to-station distance (Figure 10; Trifunac, 1978). During the following 20 years, the subsequent regression studies evolved into advanced empirical scaling equations, contributing numerous detailed improvements and producing a family of advanced, direct scaling equations for spectral amplitudes in terms of almost every practical combination of scaling parameters. The literature on this subject is voluminous, and its review is beyond the scope of this paper, but readers can find many examples and a review of this subject in Lee (2002).

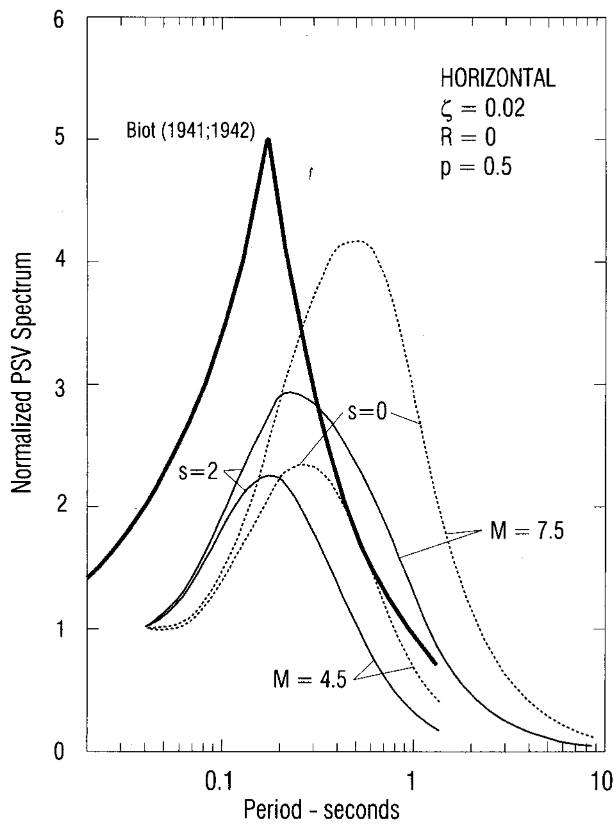

Fig. 10 Comparison of Biot (1941, 1942) "standard spectrum" (heavy line) with spectral shapes, which depend upon magnitude (M = 4.5 and 7.5) and geological site conditions (s = 2 for basement rock and s = 0 for sediments), for average spectral amplitudes (p = 0.5), at zero epicentral distance (R = 0) and for 2 percent of critical damping (ζ = 0.02; Trifunac, 1978)

Figures 7 through 10 compare Biot's "standard" spectrum shape with other examples of fixed (Figures 7, 8, and 9) and variable (Figure 10) spectral shapes. These comparisons are only qualitative, because the methods used in their development and the intended use of the spectral shapes differ. Biot's spectrum was originally thought to correspond to zero damping, but it was later discovered that it has small variable damping, probably less than 3 percent of critical. It was based on the spectra of two earthquakes only (Helena, Montana, 1935, $M = 6.0$, and Ferndale, California, 1934, $M = 6.4$). Housner (Figure 7), NRC (Figure 8), and Seed et al. (Figure 9) spectra were based on progressively larger numbers of recorded accelerograms (4, 33, and 104, respectively) and on recordings during large earthquakes. Therefore, they have broader spectral shapes. The variable shape spectrum, shown in Figure 10, shows only the dependence of spectral shape (normalized to 1g acceleration) on magnitude and geological site conditions, but it shows clearly how the spectra broaden with increasing magnitude and how larger magnitudes contribute larger long-period spectral amplitudes.

BIOT SPECTRUM AND EVOLUTION OF CODES

Work on developing building codes began in Italy in 1908, following the Massina disaster in which more than 100,000 persons were killed; in Japan following the 1923 Tokyo disaster, in which more than 150,000 perished; and in California after the Santa Barbara earthquake of 1925 (Freeman, 1932; Suyehiro, 1932). In 1927, the "Palo Alto Code", developed with the advice of Professors Willis and Marx of Stanford University, was adopted in Palo Alto, San Bernardino, Sacramento, Santa Barbara, Klamath, and Alhambra, all in California. It specified the use of a horizontal force equivalent to 0.1 g, 0.15 g and 0.2 g acceleration on hard, intermediate, and soft ground, respectively.

"Provisions against Earthquake Stresses", contained in the Proposed U.S. Pacific Coast Uniform Building Code, was prepared by the Pacific Coast Building Officials Conference and adopted at its 6th Annual Meeting in October, 1927, but these provisions were not generally incorporated into municipal building laws (Freeman, 1932). The code recommended the use of horizontal force equivalent to 0.075, 0.075, and 0.10 g acceleration on hard, intermediate, and soft ground, respectively. Following the 1933 Long Beach earthquake, the Field Act was implemented. Los Angeles and many other cities adopted an 8 percent g base shear coefficient for buildings and a 10 percent g for school buildings. In 1943, the Los Angeles Code was changed to indirectly take into account the natural period of vibration.

San Francisco's first seismic code ("Henry Vensano" code) was adopted in 1948, with lateral force values in the range from 3.7 to 8.0 percent of g, depending upon the building height (EERI Oral History Series (EERI, 1994a, 1994b)). Vensano code called for higher earthquake coefficients than were then common in Northern California, and higher than those prescribed by the Los Angeles 1943 code. Continued opposition by San Francisco area engineers led to a general consensus-building effort, which resulted in the "Separate 66" report in 1951. "Separate 66" was based on Maurice Biot's response spectrum calculated for the 1935 Helena, Montana earthquake (EERI Oral History Series (EERI, 1997); ASCE (1951)).

In Los Angeles, until 1957 (for reasons associated with urban planning, rather than earthquake safety, and to prevent development of downtown "canyons"), no buildings higher than 150 feet (13-story height limit) could be built. In 1957, the fixed height limit was replaced by the limit on the amount of floor area that could be built on a lot. After the San Fernando, California earthquake of 1971, Los Angeles modified the city code in 1973 by requiring dynamic analysis for buildings over 16 stories high (160 feet).

In 1978, the Applied Technology Council (ATC) issued its ATC-3 report on the model seismic code for use in all parts of the United States. This report, written by 110 volunteers working in 22 committees, incorporated many new concepts, including "more realistic ground motion" intensities. Much of the current Uniform Building Code was derived from ATC-3 report.

LIMITATIONS OF THE RESPONSE SPECTRUM METHOD

Biot's mathematical formulation of the response of structures uses the vibrational approach, in which the solution is represented by superposition of characteristic functions (mode shapes) of the problem. Physically, characteristic functions (mode shapes) represent standing waves that have been created by constructive interference of the waves incident to and reflecting from the boundaries of the model. All other wave energy does enter the structure, but after some time, it dies out due to destructive interference, scattering transmission and refraction, and propagation out of the structure.

1. Low–Pass Filtering Effects

In practical applications, and for most structures, the mode participation factors (Appendix A) for the lowest frequencies are usually the largest. In applications using detailed models (lumped mass, finite elements, finite differences, etc.), the contributions of high-frequency modes are routinely neglected because these contributions to the response can be shown to be very small. This is equivalent to low-pass filtering of the computed motions, and it results in reduction of the transient peak response amplitudes. In applications that consider only the fundamental mode of vibration, this low-pass filtering effect is the largest.

2. Short, Impulsive Excitation

It can be shown that the modal approach is not appropriate to represent "early" transient response, particularly for excitation consisting of high-frequency pulses with durations shorter than the travel time, t, of an incident wave to reach the top of the building ($t < H/\beta$; H and β are the building height and the velocity of shear waves in the building). As modes of vibration result from the constructive interference of the incoming wave and the wave reflected from the top of the building, the building “starts vibrating” in the first mode, only after time $t = 2H/\beta$ has elapsed from the time the shaking starts. Although, in principle, the representation of the response as a linear combination of the modal responses is complete and therefore can be used to represent any response, short, impulsive excitation would require the considerations of many modes (infinitely many for a continuous model), which is impractical. Thus, the wave propagation methods are more natural for representation of the "early" transient response, and should be explored further and used to solve problems in which the use of the modal approach is limited.

Wave propagation models of buildings have been used to study the physics of the earthquake response problem, but they are only beginning to be verified against actual observations. Continuous, 2-D wave propagation models (homogeneous, horizontally layered, and vertically layered shear plates) have been used to study the effects of travelling waves in the response of long buildings (Todorovska et al., 1988, 2001a, 2001b; Todorovska and Trifunac, 1989, 1990a, 1990b; Todorovska and Lee, l989; Trifunac et al., 2003). Discrete-time, 1-D wave propagation models have been used to study the seismic response of tall buildings (Safak, 1999).

3. Soil-Structure Interaction

In general, the response spectrum method cannot be used for evaluation of the relative response of structures supported by flexible or multiple foundations and in the presence of non-linear deformations in the soil. The complex role that flexible soil plays in the response of structures to incident wave excitation, has been recognized and studied since 1930s (Suyehiro, 1932; Sezawa and Kanai, 1935, 1936). In an unpublished note, Biot (l941b) states: “the problem is extremely complex because it involves a complete knowledge of the propagation and properties of the seismic waves in the strongly heterogeneous surface layers of the earth, as well as their diffraction and reflection by objects built on the surface…. In the present investigation, we have attempted to answer the following question: What is the influence of the elasticity of the ground on the rocking motion of a building? How resistant is the surrounding soil to the rocking displacement of a foundation; what are the factors influencing this rigidity, and can we expect this effect to have a practical influence in the action of earthquakes on buildings? The problem is simplified by neglecting the radiation of elastic wave due to the rocking.” The ideas and equations from this unpublished note appear in abridged form in Section V, entitled “Influence of Foundation on Motion of Blocks” in Biot (1942) paper.

Between 1970 and 1980, the research on soil-structure interaction grew steadily. Important theoretical problems were solved, and key full-scale experiments were conducted (Todorovska, 2002). However, soil structure interaction is rarely considered in the routine design of engineered structures, and when it is considered, it is based on the most elementary models.

A common assumption in many models that consider the soil-structure interaction effects is that the foundation is rigid. This reduces the number of degrees of freedom of the model and gives good approximations of response for ground motions composed of long wavelengths relative to the foundation dimensions (Lee, 1979). For short wavelengths, this assumption can result in nonconservative estimates of the relative deformations in the structure (Trifunac, 1997; Trifunac and Todorovska, 1997), and, in general, such an assumption can be expected to result in excessive estimates of scattering of the incident wave energy and in excessive radiation damping (Todorovska and Trifunac, 1990b, 1991, 1993). The extent to which this simplifying assumption is valid, depends upon the stiffness of the foundation system relative to that of the soil and on the overall rigidity of the structure (Hayir et al., 2001; Todorovska et al., 2001b; Trifunac et al., 1999c).

Rigid foundation models are usually combined with lumped-mass, discrete representations of the structure. The entire system is then described by a system of differential equations, and the solution is given in terms of the motion of different building floors. A soil-rigid foundation-lumped-mass structural model is usually limited to representation of one-dimensional (1-D) models, and offers useful approximation for the lower-frequency modes of relative response. The response spectrum superposition method can be used in deterministic or in probabilistic form (Gupta and Trifunac, 1987a, 1989, 1990) with such models.

The other extreme is to neglect the stiffness of the foundation system, ignore the soil-structure interaction, and assume that the wave energy in the soil drives the building according to the principles of wave propagation. This approximate approach underestimates the scattering of the incident wave energy by the foundation and overestimates the energy entering the structure (Trifunac et al., 2001b).

As the soft soil surrounding the foundation begins to experience non-linear deformations for much smaller levels of shaking than the structure, the soil-structure system experiences the significant shifting of system frequencies, typical of non-linear softening spring behaviour (Trifunac et al., 2001c, 2001d). Because this occurs most of the time, ignoring soil-structure interaction and interpreting response solely through the response spectrum method can result in gross misrepresentation of the response within the structure.

4. Non-linear Systems

By definition, response spectrum amplitude corresponds to the peak response of the single-degree-of-freedom (SDOF) system, irrespective of the length of the excitation and the number and sign of the other peaks of response. This limitation is particularly important when linear response spectra are modified to describe the response of non-linear hysteretic systems. For linear systems, statistics of ordered peaks can be employed to describe the expected amplitudes of many peaks (Gupta and Trifunac, 1987b, 1987c, 1988, 1992; Udwadia and Trifunac, 1974), but the analogous representation for non-linear systems has not been developed thus far. Formulation of new design criteria based on the power of incident wave energy (demand) and the ability of structures to absorb that power (capacity) offers a rational way to consider amplitudes and durations of the pulses of incident motion, but this approach abandons the response spectrum method (Trifunac et al., 2001b).

GENERALIZATION OF RESPONSE SPECTRUM METHOD TO DIFFERENTIAL MOTIONS

Common use of the response spectrum method implicitly assumes that all points of building foundations move synchronously and with the same amplitudes. This implies that the wave propagation in the soil can be neglected. Unless the structure is long (e.g., a bridge with long spans, a dam, a tunnel) or "stiff" relative to the underlying soil, these simplifications are justified and can lead to selection of approximate design forces. Simple analyses of two-dimensional models of long buildings suggest that when $a/\lambda < 10^{-4}$, where a is the wave amplitude and λ is the corresponding wavelength, the wave propagation effects on the response of simple structures can be neglected (Todorovska and Trifunac, 1990a, 1990b).

Figures 11(a) and 11(b) illustrate the "short" waves propagating along the longitudinal axis of a "long" building or a multiple-span bridge. For simplicity, the incident wave motion has been separated into out-of-plane motion (Figure 11(a)), consisting of SH and Love waves, and in-plane motion (Figure 11(b)) consisting of P, SV, and Rayleigh waves. The in-plane motion can further be separated into horizontal (longitudinal), vertical, and rocking components, while out-of-plane motion consists of horizontal motion in a transverse direction and torsion along the vertical axis. Trifunac and Todorovska (1997) analyzed the effects of the horizontal in-plane component of differential motions, and showed how the response spectrum method can be modified to include the first-order effects of differential motion on individual columns.

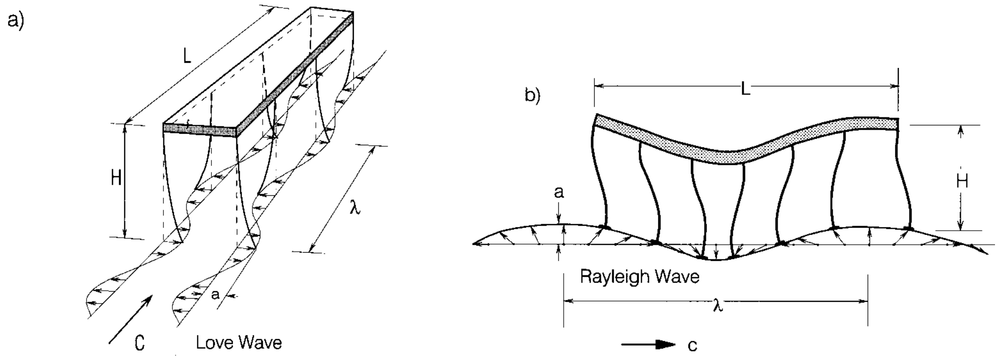

Fig. 11 (a) Deformation of columns in a two-degree-of-freedom system, during out-of-plane response, excited by Love waves, (b) Deformation of columns in a long structure during in-plane response and excited by Rayleigh waves

Designating by $SDC(T, \delta, \zeta, \tau)$, the relative displacement spectrum for column deformations, where T is the period of the equivalent single-degree-of-freedom system, ζ is its fraction of critical damping, δ is the ratio of the peak relative response of the first floor to $SD(T,\zeta)$, and $\tau = Ax/\beta_{av}$ is the travel time between "central" point R of all columns and of a given column at distance x (A is scaling parameter ~ 1, and β_{av} is the average shear wave velocity in the top 30 m of soil) for seismic waves propagating along the surface, it can be shown that for in-plane motions (Figure 11(b))

$$SDC(T,\delta,\varsigma,\tau) \sim \left\{ [\delta\, SD(T,\varsigma)]^2 + (v_{\max}\tau)^2 \right\}^{1/2} \tag{3}$$

where $SD(T,\zeta)$ is the relative displacement spectrum (e.g., see Figure 12), and $v_{\max}$ is the peak ground velocity associated with the corresponding excitation.

An example of $SDC(T, \delta, \zeta, \tau)$ for strong motion recorded at USC Station #53 (S16W component) during the Northridge earthquake is shown in Figure 12 for τ = 0.001 through 0.1 s, δ = 1, and ζ = 0.05.

In Equation (3), $SD(T,\zeta)$ is representative of relative column displacement caused by inertial forces, while $v_{\max}\tau$ approximates the maximum relative column displacement arising from pseudo-static deformations in the soil associated with wave passage. It can be seen that for long structures (large τ), pseudo-static deformation of columns can be large and can dominate in contribution to $SDC(T, \delta, \zeta, \tau)$ for intermediate and short periods of oscillators ("stiff" structures).

For out-of-plane motion (Figure 11(a)), and ground motion consisting of "long" waves, *SDC* must be calculated for a two-degree-of-freedom system, with translational period T, torsional period T_T, and their respective fractions of critical damping ζ and ζ_T. For $T \sim T_T$ and $\zeta \sim \zeta_T$, it can be shown that

$$SDC(T, T_T, \varsigma, \varsigma_T, \tau, \delta) \approx \left\{ [\delta\, SD(T,\varsigma)]^2 + 2(v_{\max}\tau)^2 \right\}^{1/2} \tag{4}$$

The SDC spectrum for in-plane motion is illustrated in Figure 12 for horizontal component S16W of a recording in the near field of the Northridge California earthquake of January 17, 1994. The results indicated that during this earthquake, the increase in the shear forces for peripheral columns (on individual foundations), caused by differential ground motion, was significant, so that one must consider this effect in the design of new structures and in retrofitting of existing structures. This shows that for high-frequency (stiff) structures, with moderate to large horizontal dimensions, the shear forces and the associated bending moments in the peripheral columns will exceed the estimates based on the relative displacement spectra $SD(T,\zeta)$ by factors that can be large.

In Figure 12, we also compare the computed $SD(T,\zeta)$ with "standard" spectral shapes of Biot, Housner, and Seed. While all of these shapes agree favourably with $SD(T,\zeta)$, for this particular recording, Biot's spectrum overestimates the classical $SD(T,\zeta)$ spectrum and is more conservative than the other two, if we consider the SDC spectra.

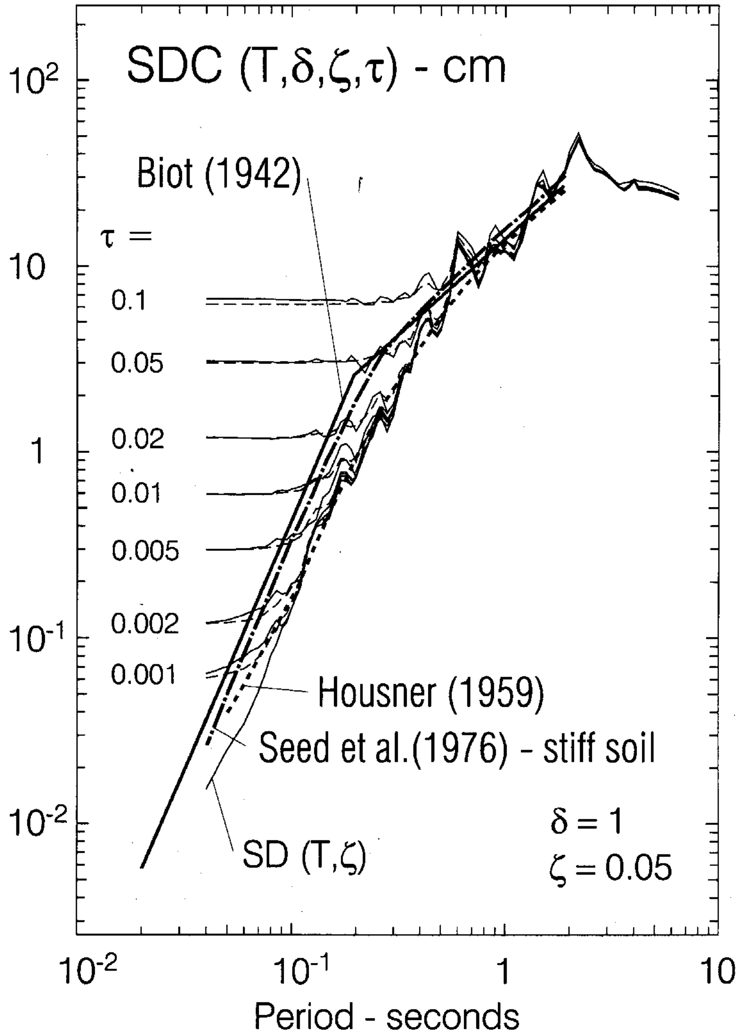

Fig. 12 Relative displacement spectrum for columns, $SDC(T, \delta, \zeta, \tau)$, for S16W component of acceleration recorded at USC station #53 of the Los Angeles Strong Motion Network (Trifunac and Todorovska, 2001a), during Northridge, California earthquake of January 17, 1994 (M = 6.7), at epicentral distance of 6 km, for ζ = 0.05 and δ = 1 (one story building) (the solid lines correspond to SDC spectra computed exactly, and the dashed lines to the approximation, given by Equation (3); "standard" spectrum shapes of Biot (1942), Housner (1959), and Seed et al. (1976), normalized to agree with recorded motions at long periods, are shown for comparison; peak amplitudes of strong motion at this site were 12.4 cm, 59.8 cm/s, and 381 cm/s^2)

DISCUSSION AND CONCLUSIONS

Before the digital computer age, computation of response spectra of strong-motion accelerograms was difficult and time-consuming, and the results had very uncertain accuracy. This, in combination with a very small number of available recorded accelerograms, made it impossible to carry out empirical studies on the scaling of spectral amplitudes. Also, it was difficult to explore the governing laws, and to link the physical nature of the earthquake source mechanism with the amplitudes and shape of the response spectrum. It was primarily for these reasons that the response spectrum method was confined largely to the realm of academic research for almost 40 years (1932 to ~1972).

As mentioned earlier, all of this changed during the early 1970s. Not only did digital computers become widely available, the number of recorded strong-motion accelerograms also grew rapidly. Since the early 1980s, it became possible to carry out sophisticated and complex regression analyses of the recorded data, to search for intricate and detailed properties of the physical nature of strong ground motion, and to discover how this nature affects the response spectrum amplitudes. Today, we understand all of the principal factors which determine the overall amplitudes and shapes of the response spectra in Southern California (Trifunac, 1993, 1995a, 1995b). In future, when sufficient strong-motion data has been recorded in other seismically active areas of the world, it will be possible to develop such area-specific empirical scaling equations in these locations as well, with comparable details and sophistication.

In spite of the voluminous published work on generalizations of the response spectrum method to the response of structures experiencing non-linear deformations, no general method has been developed thus far. To guide the development of future methods for the design of earthquake-resistant structures undergoing large non-linear response, it will be necessary to record the responses of many structures experiencing non-linear deteriorating response. This will require a far more comprehensive instrumentation of structures than is available today. It will also require development and deployment of new instrumentation systems capable of recording permanent displacements and permanent rotations at many locations in the soil-structure systems (Trifunac and Todorovska, 2001a, 2001b).

In attempts to overcome the limitations of the response spectrum method, it is to be expected that in future its use will be reduced eventually to the design of the class of structures that are anticipated to experience only linear response amplitudes. For performance-based design of structures, which are expected to experience "controlled" non-linear responses, it will be necessary to develop new design principles. These new methods will most likely be based on equating the maximum power demand with the design capacity of the structure to absorb given energy per unit of time. In the meantime, Biot's response spectrum method, which is so deeply and so ubiquitously interwoven into all aspects of earthquake engineering, will continue to be the central guiding concept in earthquake-resistant design.

APPENDIX A: AN OUTLINE OF THE RESPONSE SPECTRUM METHOD

1. Single-Degree-of-Freedom System

A single-degree-of-freedom system corresponding to a one-story structure is schematically shown in Figure A.1. For vibration $z(t)$ of its base, the differential equation of linear motion is

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{z} \tag{A.1}$$

where x is the relative displacement between the structure and its base, and $-m\ddot{z}$ represents the inertial force applied to the mass *m*, supported on two columns, which has the equivalent spring constant *k*. Energy dissipation is assumed to be of viscous nature, with damping force proportional to the relative velocity between the mass and its foundation, and c being the damping constant.

The natural frequency ω and the fraction of critical damping ζ are defined as

$$\omega_n^2 = \frac{k}{m} \tag{A.2}$$

and

$$\zeta = \frac{c}{2\sqrt{km}} \tag{A.3}$$

Equation (A.1) then becomes

$$\ddot{x} + 2\omega_n\zeta\,\dot{x} + \omega_n^2 x = -\ddot{z} \tag{A.4}$$

For any general motion of the support, $z(t)$, the relative displacement $x(t)$ can be computed from the Duhamel integral (Biot, 1932, 1933, 1934; von Karman and Biot, 1940). For zero initial conditions, the expression for $x(t)$ takes the form

$$x(t) = \frac{-1}{\omega_n \sqrt{1-\zeta^2}} \int_0^t \ddot{z}(\tau) e^{-\zeta\omega_n (t-\tau)} \sin \omega_n \sqrt{1-\zeta^2}(t-\tau) d\tau \tag{A.5}$$

Thus, the linear relative response of the structure is characterized by its natural period $T_n = \frac{2\pi}{\omega_n}$, the fraction of critical damping ζ, and the nature of the base acceleration, $\ddot{z}(\tau)$.

The relative displacement $x(t)$ is important for earthquake-resistant design because the strains in the structure are directly proportional to the relative displacements. The total shear force V_B (Figure A.1), for example, exerted by the columns on the ground is

$$V_B(t) = kx(t) \tag{A.6}$$

The exact relative velocity $\dot{x}(t)$ follows directly from (A.5):

$$\dot{x}(t) = -\int_0^t \ddot{z}(\tau) e^{-\zeta\omega_n (t-\tau)} \cos \omega_n \sqrt{1-\zeta^2}(t-\tau) d\tau + \frac{\zeta}{\sqrt{1-\zeta^2}} \int_0^t \ddot{z}(\tau) e^{-\zeta\omega_n (t-\tau)} \sin \omega_n \sqrt{1-\zeta^2}(t-\tau) d\tau \tag{A.7}$$

The absolute acceleration $\ddot{y}(t)$ of the mass *m* is obtained by further differentiation of $\dot{x}(t)$, noting that $\ddot{y}(t) = \ddot{x}(t) + \ddot{z}(t)$. It is

$$\ddot{y}(t) = \omega_n \frac{\left(1-2\zeta^2\right)}{\sqrt{1-\zeta^2}} \int_0^t \ddot{z}(\tau) e^{-\zeta\omega_n (t-\tau)} \sin \omega_n \sqrt{1-\zeta^2}(t-\tau) d\tau + \frac{\zeta}{\sqrt{1-\zeta^2}} \int_0^t \ddot{z}(\tau) e^{-\zeta\omega_n (t-\tau)} \cos \omega_n \sqrt{1-\zeta^2}(t-\tau) d\tau \tag{A.8}$$

The absolute acceleration $\ddot{y}$ is important for experimental measurements, because it is the quantity, which is most simple to measure during strong, earthquake-induced vibrations. That is, an accelerograph located at that point records a close approximation to $\ddot{y}(t)$. The absolute acceleration also defines the seismic force on the mass *m* (Figure A.1).

It may be concluded that of primary interest, for engineering applications, are the maximum absolute values of $x(t)$, $\dot{x}(t)$, and $\ddot{y}(t)$ experienced during the earthquake response. These quantities are commonly defined as

$$SD \equiv \left|x(t)\right|_{\max} \tag{A.9}$$

$$SV \equiv \left|\dot{x}(t)\right|_{\max} \tag{A.10}$$

$$SA \equiv \left|\ddot{y}(t)\right|_{\max} \tag{A.11}$$

Plots of SD, SV and SA versus the undamped natural period of vibration $T_n = \frac{2\pi}{\omega_n}$, for various fractions of critical damping ζ, are called earthquake response spectra.

In typical engineering structures, the fraction of critical damping ζ is small. It is approximately 2% - 8% for buildings and 5% – 10% for soil structures. Therefore, $\sqrt{1-\zeta^2} \approx 1$, and the terms of order ζ, and ζ^2 in Equations (A.7) and (A.8) may be neglected. Furthermore, if the cosine term is replaced by a sine term in Equation (A.7), the following approximate relationships exist between the spectral quantities defined in Equations (A.9), (A.10), and (A.11):

$$SD \approx \frac{T}{2\pi} SV \tag{A.12}$$

and

$$SA \approx \frac{2\pi}{T} SV \tag{A.13}$$

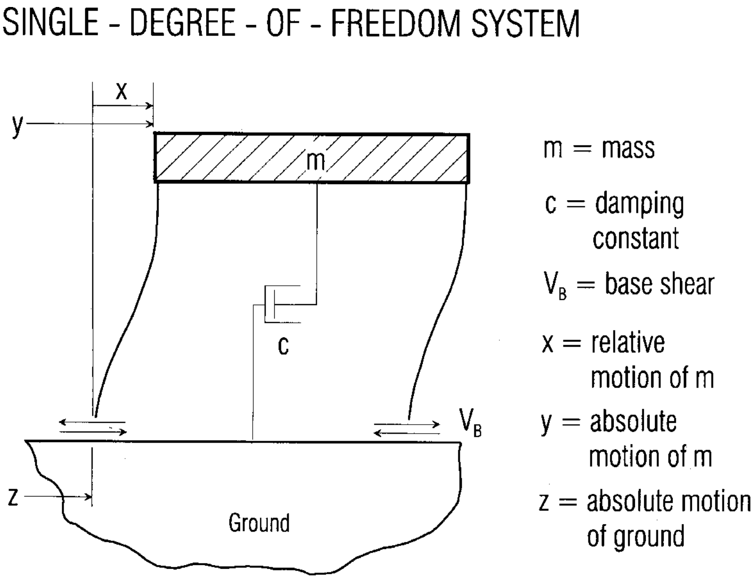

Fig. A.1 Single-degree-of-freedom system

For earthquake-like excitations, these approximations can be made plausible (Hudson, 1962). For engineering applications, it is convenient to use the following approximations

$$PSV = \frac{2\pi}{T} SD \tag{A.14}$$

and

$$PSA = \left(\frac{2\pi}{T}\right)^2 SD \tag{A.15}$$

because SD, PSV and PSA can be conveniently plotted on the common tripartite logarithmic plot versus period. In the engineering literature, PSV and PSA are frequently referred to as "pseudo velocity" and "pseudo absolute acceleration".

2. Fourier Spectra and Response Spectra

The Fourier spectrum of an input acceleration shows the significant frequency characteristics of recorded motion. For an accelerogram differing from zero in the time interval $0 < \tau < T$, the Fourier spectrum is defined as

$$F(\omega) = \int_0^T \ddot{z}(\tau) e^{-i\omega\tau} d\tau \tag{A.16}$$

The Fourier amplitude spectrum is then given by the square root of the sum of the squares of the real and imaginary parts of $F(\omega)$:

$$FS \equiv |F(\omega)| = \left\{ \left[\int_0^T \ddot{z}(\tau) \cos\omega\tau \, d\tau \right]^2 + \left[\int_0^T \ddot{z}(\tau) \sin\omega\tau \, d\tau \right]^2 \right\} \quad \text{(A.17)}$$

For an undamped oscillator, there is a close relationship between the Fourier amplitude spectrum and the exact relative velocity response spectrum (Kawasumi, 1956; Rubin, 1961; Hudson, 1962). For $\zeta = 0$, Equation (A.7) (dropping the subscript n on ω_n) reduces to

$$\dot{x}(t) = -\int_0^t \ddot{z}(\tau) \cos\omega(t-\tau) d\tau \quad \text{(A.18a)}$$

Expanding,

$$\dot{x}(t) = -\cos\omega\tau \int_0^t \ddot{z}(\tau) \cos\omega\tau \, d\tau + \int_0^t \ddot{z}(\tau) \sin\omega\tau \, d\tau \quad \text{(A.18b)}$$

From definition, as in Equation (A.10), and from Equation (A.18b), it follows that

$$SV = \left\{ \left[\int_0^{t_{\max}} \ddot{z}(\tau) \cos\omega\tau \, d\tau \right]^2 + \left[\int_0^{t_{\max}} \ddot{z}(\tau) \sin\omega\tau \, d\tau \right]^2 \right\}^{1/2} \quad \text{(A.19)}$$

where $0 \le t_{\max} \le T$, and $t_{\max}$ is the time at which the maximum response occurs.

Equations (A.17) and (A.19) show the similarity between the Fourier amplitude spectrum and the exact relative velocity spectrum. In the special case in which the time $t_{\max}$ coincides with the total duration of the earthquake, T, the two spectra SV and FS become identical. In general, for $0 \le t_{\max} \le T$ and $\zeta = 0$, SV is always greater than FS. In physical terms, FS is the maximum velocity of the undamped oscillator in the free vibrations following the earthquake, whereas SV is the maximum velocity during both the earthquake and the subsequent free vibrations.

The Fourier amplitude spectrum FS is the quantity used in the investigations of the earthquake mechanism, as it relates to the amplitudes of recorded waves (Trifunac, 1993, 1995a, 1995b). Similarly, the relative velocity spectrum SV characterizes the earthquake ground motion in terms of its influence on engineering structures. The above relation in SV and FS links the two measurements of the same physical phenomenon from different points of view.

3. Multi-Degree-of-Freedom Systems

An example of a fixed-base multi-degree-of freedom system used to study vibrations of tall buildings is shown in Figure A.2. In this model, masses m_i are lumped at the floor levels and are interconnected with massless columns, which have the equivalent spring constants k_i. Dashpots characterized by the damping constants c_i model the energy dissipation in this system. The shear force at each floor level is designated by V_i. V_n corresponds to the base shear acting between the structure and the rigid soil.

The system of equilibrium equations, one of which corresponds to each mass m_i, can be written in a compact matrix form as

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -\ddot{z}[m]\{I\} \quad \text{(A.20)}$$

Here, [·] designates a square $n \times n$ matrix, while $\{\cdot\}$ represents a column vector with n components. In general, the mass matrix [*m*], the damping matrix [*c*], and the stiffness matrix [*k*] may have all elements different from zero. For most structural models, however, like the one shown in Figure A.2, these matrices have only the diagonal and few off-diagonal terms that differ from zero.

In the special case in which the damping matrix [*c*] is a linear combination of the mass matrix and the stiffness matrix, a set of uncoupled coordinates $\{\xi\}$ and the corresponding transformation

$$\{x\} = [A] \{\xi\} \quad \text{(A.21)}$$

can be found in such a way that the classical normal mode solution is possible (Rayleigh, 1945; Caughey, 1959). Then, by substituting (A.21) into (A.20) and pre-multiplying (A.20) by the transpose of $[A]$, there follows:

$$[A]^T[m][A]\{\ddot{\xi}\}+[A]^T[c][A]\{\dot{\xi}\}+[A]^T[k][A]\{\xi]=\ddot{z}[A]^T[m]\{I\} \qquad \text{(A.22)}$$

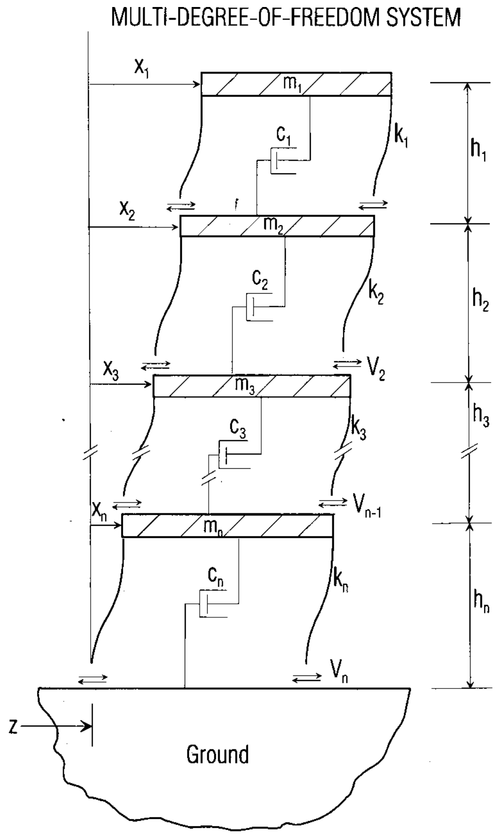

Fig. A.2 Multi-degree-of-freedom system

It is possible to normalize the coefficients of $\{\ddot{\xi}\}$ and $\{\xi\}$ in such a way that

$$[A]^T[m][A]=[I] \qquad \text{(A.23)}$$

where $[I]$ is the unit matrix, and

$$[A]^T[k][A]=[\omega^2] \qquad \text{(A.24)}$$

Denoting

$$[C]\equiv[A]^T[c][A] \qquad \text{(A.25)}$$

Equation (A.22) becomes

$$[I]\{\ddot{\xi}\}+[C]\{\dot{\xi}\}+[\omega^2]\{\xi\}=-\ddot{z}[A]^T[m]\{I\} \qquad \text{(A.26)}$$

If $[C]$ is a diagonal matrix,

$$[C]=\begin{bmatrix} 2\zeta_1\omega_1 & 0 & 0 & \cdots & 0 \\ 0 & 2\zeta_2\omega_2 & 0 & \cdots & \vdots \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & 2\zeta_n\omega_n \end{bmatrix} \qquad \text{(A.27)}$$

then, Equation (A.16) represents a set of *n* independent equations, each describing a single-degree-of-freedom system response in terms of a normal coordinate ξ_i:

$$\ddot{\xi}_i + 2\zeta_i\omega_i\dot{\xi}_i + \omega_i^2\xi_i = -\ddot{z}\alpha_i \tag{A.28}$$

where the constant

$$\alpha_i = \frac{\{A^i\}^T[m]\{I\}}{\{A^i\}^T[m]\{A^i\}}; \quad i = 1, 2, 3, \ldots, n \tag{A.29}$$

is the mode participation factor. It shows the extent to which the i^{th} mode is excited by the earthquake.

As may be seen, the i^{th} equation (Equation (A.28)) is the same as Equation (A.5) for a single-degree-of-freedom oscillator. Its solution therefore becomes

$$\xi_i(t) = \frac{-\alpha_i}{\omega_i\sqrt{1-\zeta_i^2}} \int_0^t \ddot{z}(\tau)e^{-\zeta_i\omega_i(t-\tau)} \sin\omega_i\sqrt{1-\zeta_i^2}(t-\tau)d\tau \tag{A.30}$$

After solving Equation (A.30) for all *n* coordinates, $\{x\}$ can be calculated from

$$\{x(t)\} = [A]\{\xi(t)\} \tag{A.31}$$

When the displacements $\{x(t)\}$ of masses are determined, the earthquake forces $F_i(t)$ acting on each mass, m_i are given by

$$\{F(t)\} = [k]\{x(t)\} \tag{A.32}$$

The shear forces V_i (see Figure (A.2)) are then

$$\{V(t)\} = [S]\{x(t)\} \tag{A.33}$$

where $[S]$ is the lower triangular matrix

$$[S] = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \tag{A.34}$$

Similarly, the moments at each level of the structure (Figure A.2) are

$$\{M(t)\} = [H][S][k]\{x(t)\} \tag{A.35}$$

where $[H]$ is the lower triangular matrix

$$[H] = \begin{bmatrix} h_1 & 0 & 0 & \cdots & \cdots & 0 \\ h_1 & h_2 & 0 & \cdots & \cdots & 0 \\ h_1 & h_2 & h_3 & & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ h_1 & h_2 & h_3 & \cdots & h_{n-1} & h_n \end{bmatrix} \tag{A.36}$$

The ultimate objective of this analysis, from the engineering point of view, is to calculate the envelope of maximum response $\{x\}_{\max}$, forces $\{F\}_{\max}$, shears $\{V\}_{\max}$, and moments $\{M\}_{\max}$, which are to be used for design purposes. Here, $\{\cdot\}_{\max}$ defines the following operation:

$$\{g\}_{\max} \equiv \begin{Bmatrix} g_{1_{\max}} \\ g_{2_{\max}} \\ \vdots \\ g_{n_{\max}} \end{Bmatrix} \tag{A.37}$$

It may be noted that the maximum response vector, $\{x\}_{\max}$, for example, does not define the response condition at any one time, but the maximum response experienced by the different levels of a structure during the entire time history of the analysis.

4. Response Spectrum Superposition

In the preceding sections, it was demonstrated that for a multi-degree-of-freedom system, the dynamic response of the r-th mode alone may be described by the equation that corresponds to a single-degree-of-freedom system (see Equations (A.4) and (A.28)), and that the total displacement response may be computed by adding the contributions of each individual mode (Equation (A.31)). For example, the contribution of the r-th mode to the total displacement of a multi-degree-of-freedom system would be :

$$\{x^{(r)}(t)\} = \frac{\{A^{(r)}\}\alpha_r}{\omega_i\sqrt{1-\zeta_r^2}} \int_0^t \ddot{z}(\tau) e^{-\zeta_r\omega_r(t-\tau)} \sin\omega_r\sqrt{1-\zeta_r^2}(t-\tau)\, d\tau \tag{A.38}$$

This contribution is seen to be directly proportional to $1/\omega_r\sqrt{1-\zeta_r^2}$ times the integral term and will depend on the integral's maximum absolute value. The latter was already defined in Equation (A.5), and its absolute value SD was given by Equation (A.9). Thus, in terms of displacement and velocity spectra (Equations (A.9) through (A.15)), Equation (A.38) can be written as

$$\{|x^{(r)}(t)|\}_{\max} = \{|A^{(r)}|\}\alpha_r SD^{(r)} = \{|A^{(r)}|\}\alpha_r \frac{T_r}{2\pi} SV^{(r)} \tag{A.39}$$

where the superscripts (r) on SD and SV indicate that these spectral values are computed for damping ζ_r and frequency $\omega_r = \frac{2\pi}{T_r}$, corresponding to the r-th mode of vibration. However, the different modal maxima do not occur at the same time, and therefore, the individual modes do not simultaneously contribute their peak values to the maximum total response.

The sum of maximum modal responses (Biot, 1942)

$$\sum_{r=1}^{n} \{|x^{(r)}(t)|\}_{\max} = [|A|] \begin{Bmatrix} SD^{(1)} \\ SD^{(2)} \\ \vdots \\ SD^{(n)} \end{Bmatrix} \tag{A.40}$$

would clearly give an upper bound to the total system response, but at the same time may be too conservative. An alternative approach, based on statistical considerations (Goodman et al., 1958), is to take the square root of the sum of the squares of the individual modal maxima. This method (RMS) has been shown to give reasonable results (Jennings and Newmark, 1960) for structures in which the main contributions come from the lowest few modes. The question of mode superposition has been studied extensively (Amini and Trifunac, 1985; Gupta and Trifunac, 1987, 1989; Merchant and Hudson, 1962), and the conditions under which a meaningful degree of conservatism can be achieved, have been determined. When the maxima of each response quantity have been determined from equations analogous to Equation (A.39), the RMS approximation is given by

$$\{x\}_{\max} \approx \begin{Bmatrix} \left[\sum_{i=1}^{n} \left(x_{1_{\max}}^{i} \right)^2 \right]^{1/2} \\ \left[\sum_{i=1}^{n} \left(x_{2_{\max}}^{i} \right)^2 \right]^{1/2} \\ \vdots \\ \left[\sum_{i=1}^{n} \left(x_{n_{\max}}^{i} \right)^2 \right]^{1/2} \end{Bmatrix} \tag{A.41}$$

Analogous expressions for $\{V\}_{\max}$ and $\{M\}_{\max}$ follow from Equation (A.41).

The above-described response spectrum superposition method provides only an approximate indication of the maximum response in the multi-degree-of-freedom systems. The advantage of this method is that it avoids lengthy computations associated with the exact method, and at the same time, takes into account the dynamic nature of the problem. It can often provide reasonable results for design purposes.

APPENDIX B

For the standard corrected accelerograms (Trifunac et al., 1971), which are available at equally spaced time intervals Δt, an approach based on the exact analytical solution of the Duhamel integral for the successive linear segments of excitation appears to be most practical. This approach is described in Nigam and Jennings (1968). For completeness of this paper, the important features of this method are briefly summarized here.

The differential equation for the relative motion $x(t)$ of a single-degree-of-freedom oscillator subjected to base acceleration $a(t)$ is given by

$$\ddot{x} + 2\omega\zeta\,\dot{x} + \omega^2 x = -a(t) \tag{B.1}$$

where, ζ = fraction of critical damping and ω = the natural frequency of vibration of the oscillator. For $a(t)$ given by a segmentally linear function for $t_i \le t \le t_{i+1}$, Equation (B.1) becomes

$$\ddot{x} + 2\omega\zeta\,\dot{x} + \omega^2 x = -a_i + \frac{\Delta a_i}{\Delta t}(t - t_i)\,, \tag{B.2}$$

where

$$\Delta t = t_{i+1} - t_i = \text{const.} \tag{B.3}$$

and

$$\Delta a_i = a_{i+1} - a_i \tag{B.4}$$

The solution of Equation (B.2), for $t_i \le t \le t_{i+1}$, then becomes

$$x(t) = e^{-\zeta\omega(t-t_i)}\left[C_1 \sin\omega_d(t-t_i) + C_2 \cos\omega_d(t-t_i)\right] - \frac{a_i}{\omega^2} + \frac{2\zeta}{\omega^3}\frac{\Delta a_i}{\Delta t} - \frac{1}{\omega^2}\frac{\Delta a_i}{\Delta t}(t-t_i) \tag{B.5}$$

where

$$\omega_d \equiv \omega\sqrt{1-\zeta^2} \tag{B.6}$$

Setting $x = x_i$ and $\dot{x} = \dot{x}_i$ at $t = t_i$, C_1 and C_2 become

$$C_1 = \frac{1}{\omega_d}\left(\zeta\omega x_i + \dot{x}_i - \frac{2\zeta^2 - 1}{\omega^2} + \frac{\zeta}{\omega} a_i\right) \tag{B.7}$$

$$C_2 = x_i - \frac{2\zeta}{\omega^3}\frac{\Delta a_i}{\Delta t} + \frac{a_i}{\omega^2} \tag{B.8}$$

Substituting C_1 and C_2 into Equation (B.5) and setting $t = t_{i+1}$ leads to the recurrence relationship for x_i and $\dot{x}_i$, given by

$$\begin{Bmatrix} x_{i+1} \\ \dot{x}_{i+1} \end{Bmatrix} = \left[A(\zeta,\omega,\Delta t)\right]\begin{Bmatrix} x_i \\ \dot{x}_i \end{Bmatrix} + \left[B(\zeta,\omega,\Delta t)\right]\begin{Bmatrix} a_i \\ a_{i+1} \end{Bmatrix} \tag{B.9}$$

The elements of matrices A and B are

$$\left.\begin{aligned}
a_{11} &= e^{-\zeta\omega\Delta t}\left(\frac{1}{\sqrt{1-\zeta^2}}\sin\omega_d\Delta t + \cos\omega_d\Delta t\right) \\
a_{12} &= \frac{e^{-\zeta\omega\Delta t}}{\omega_d}\sin\omega_d\Delta t \\
a_{21} &= -\frac{\omega}{\sqrt{1-\zeta^2}}e^{-\zeta\omega\Delta t}\sin\omega_d\Delta t \\
a_{22} &= e^{-\zeta\omega\Delta t}\left(\cos\omega_d\Delta t - \frac{1}{\sqrt{1-\zeta^2}}\sin\omega_d\Delta t\right)
\end{aligned}\right\} \tag{B.10}$$

$$\left.\begin{aligned}
b_{11} &= e^{-\zeta\omega\Delta t}\left[\left(\frac{2\zeta^2-1}{\omega^2\Delta t} + \frac{\zeta}{\omega}\right)\frac{\sin\omega_d\Delta t}{\omega_d} + \left(\frac{2\zeta}{\omega^3\Delta t} + \frac{1}{\omega^2}\right)\cos\omega_d\Delta t\right] - \frac{2\zeta}{\omega^3\Delta t} \\
b_{12} &= e^{-\zeta\omega\Delta t}\left[\left(\frac{2\zeta^2-1}{\omega^2\Delta t}\right)\frac{\sin\omega_d\Delta t}{\omega_d} + \frac{2\zeta}{\omega^3\Delta t}\cos\omega_d\Delta t\right] - \frac{1}{\omega^2}\frac{2\zeta}{\omega^3\Delta t} \\
b_{21} &= e^{-\zeta\omega\Delta t}\left[\left(\frac{2\zeta^2-1}{\omega^2\Delta t} + \frac{\zeta}{\omega^2}\right)\left(\cos\omega_d\Delta t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d\Delta t\right)\right. \\
&\quad \left. -\left(\frac{2\zeta^2}{\omega^2\Delta t} + \frac{1}{\omega^2}\right)\left(\omega_d\sin\omega_d\Delta t + \zeta\omega\cos\omega_d\Delta t\right)\right] + \frac{1}{\omega^2\Delta t} \\
b_{22} &= -e^{-\zeta\omega\Delta t}\left[\frac{2\zeta^2-1}{\omega^2\Delta t}\left(\cos\omega_d\Delta t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d\Delta t\right)\right. \\
&\quad \left. -\frac{2\zeta}{\omega^2\Delta t}\left(\omega_d\sin\omega_d\Delta t + \zeta\omega\cos\omega_d\Delta t\right)\right] - \frac{1}{\omega^2\Delta t}
\end{aligned}\right\} \tag{B.11}$$

Therefore, if the displacement and velocity of the oscillator are known at t_i, the complete response can be computed by a step-by-step application of Equation (B.9). The advantage of this method lies in the fact that for a constant time interval Δt, matrices A and B depend only upon ζ and ω, and are constant during the calculation of the response.

To calculate and plot complete response spectra, maximum values of displacement $SD = |x(t)|_{\max}$, velocity $SV = |\dot{x}(t)|_{\max}$ and absolute acceleration $SA = |\ddot{x}(t) + a(t)|_{\max}$ are stored for each period $T = \frac{2\pi}{\omega}$ and a fraction of critical damping ζ. The calculation of these maxima is approximate, because the displacement $x(t)$, velocity $\dot{x}(t)$, and acceleration $\ddot{x}(t)$ are found only at discrete points, where the values are x_i, $\dot{x}_i$, and $\ddot{x}_i + a_i$ for i = 1,2,…, N (where N is the total number of discrete, equally spaced points at which the input accelerogram is given). For standard spectrum calculations, the choice of the interval of integration Δt is selected to be

$$\Delta t \leq \frac{T}{10} \tag{B.12}$$

but it is always less than or equal to Δt = 0.02 s. Here, T is the period of the oscillator for which the spectrum point is calculated. For such a choice of integration interval, the discretization error is always less than 5%.

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