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ANTIPLANE DEFORMATIONS NEAR ARBITRARILY SHAPED ALLUVIAL VALLEYS

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ABSTRACT

The weighted residual method was applied to the problem of scattering and diffraction of plane elastic waves, in the form of antiplane waves, by a soft alluvial valley of arbitrary shape on the surface of a two-dimensional half-space. In order to demonstrate the versatility of the method, it was applied to shallow and deep circular, elliptical, and rectangular alluvial valleys. Results obtained are quite similar to those obtained using available closed form solutions. It was shown that significant ground motion amplitudes, with respect to the amplitude of incident waves, occurred on the ground surface of or near the alluvial valley. Amplitudes were dependent upon the shape and depth of the valley, the relative properties of the material in the valley and the surrounding medium, and the frequency and angle of incidence of incoming waves. Surface amplitude profiles for the lower frequency incident waves were simple near the discontinuity with peak amplitudes that did not vary much from 2 except on the surface of the alluvium. As the frequency of the incident waves was increased, the amplitude profiles near the valley became more complicated with peak values exceeding 5 on the surface of the half-space adjacent to the valley and exceeding 10 on the surface of the soft valley alluvium.

KEYWORDS: Weighted Residuals, SH-Wave, Diffraction, Alluvial Valley, Half-Space

INTRODUCTION

One of the many areas of earthquake engineering and seismological research has been the effect of local site conditions on ground motion. Among the local site topographies of interest are alluvial valleys located on the ground or half-space surface. In this paper, the problem of the scattering and diffraction of incident SH waves by an arbitrarily shaped alluvial valley on surface of a two-dimensional half-space is studied. A numerical solution for the problem is appropriate as the boundary between the material of the discontinuity and the half-space may be irregular, making it difficult to describe the solution in closed form. The method of weighted residuals is implemented in order to study possible amplifications and de-amplifications of displacements on the surface of the half-space adjacent to and within an alluvial valley.

Currently, with respect to sub-surface discontinuities, closed form solutions exist for incident SH-waves for a cavity in an infinite space (Pao and Mow, 1973), a circular cavity in a half-space (Lee, 1977) and an elastic tunnel and inclusion in a half-space (Lee and Trifunac, 1979). Among others, analytic solutions have been developed by Gregory (1967), Gregory (1970), Datta (1978), Dravinski (1983a). Currently, with respect to surface discontinuities, closed form solutions for incident SH-waves exist for a semi-cylindrical valley (Trifunac, 1971), a semi-elliptical valley (Wong and Trifunac, 1974a), a semi-cylindrical canyon (Trifunac, 1973), a semi-elliptical canyon (Wong and Trifunac, 1974b), a circular arc hill (Yuan and Men, 1992), a cylindrical canyon of circular arc cross-section (Yuan and Liao, 1994), a cylindrical alluvial valley of circular arc cross-section (Yuan and Liao, 1995), and a valley in a wedge shaped half-space (Sherif and Lee, 1997). Analytical solutions exist for a shallow circular alluvial valley subjected to incident SH-waves (Todorovska and Lee, 1991) and a shallow circular cylindrical canyon for P, SV, and SH-waves (Cao and Lee, 1989, 1990; Lee and Cao, 1989). Closed form solutions have been developed for the scattering and diffraction of P, SV, and SH-waves by a three-dimensional alluvial valley (Lee, 1984) and a three dimensional spherical cavity (Lee, 1988).

Several numerical approaches for the analysis and diffraction of SH-waves by an alluvial valley have been studied. A combined finite element and analytical technique was used for the scattering of plane SH-waves by arbitrarily shaped canyons (Shah et al., 1982), the three dimensional scattering of seismic waves by a cylindrical alluvial valley (Liu et al., 1991), the three dimensional scattering of seismic waves by cylindrical alluvial valleys of arbitrary cross-section (Khair et al., 1980), the scattering of SH-waves and Rayleigh waves by a cylindrical alluvial valley of arbitrary shape (Khair et al., 1991). Indirect and direct boundary element methods have been applied for the scattering of elastic waves by a dipping layer of alluvium (Wong et al., 1977), dipping layers of arbitrary shape (Dravinski, 1983b; Dravinski and Mossessian, 1987; Mossessian and Dravinski, 1990a, 1990b), extended to three dimensional problems by Sanchez-Sesma (1983), an infinitely long cylindrical canyon with arbitrary but uniform cross-section (Luco et al., 1990), an alluvial valley of arbitrary shape (Sanchez-Sesma et al., 1993), an irregular valley (Reinoso et al., 1997), multilayered media with irregular interfaces (Ding and Dravinski, 1996), the three dimensional response of a layered cylindrical valley in a layered half-space (Luco and deBarros, 1995), and the seismic response of three dimensional alluvial valleys for incident P, S, and Rayleigh waves (Sanchez-Sesma and Luzon, 1995). Other approaches include a generalized inverse method to model the diffraction of P, SV, and Rayleigh waves by a canyon of general shape (Wong, 1979, 1982), a wave expansion technique to address the scattering and diffraction of plane SH-waves by two dimensional homogeneities (Moeen-Vaziri and Trifunac, 1988a), and the scattering of elastic waves by three dimensional canyons (Esraghi and Dravinski, 1989), a least squares technique and series expansion to model the scattering and diffraction of plane P and SV-waves by two dimensional homogeneities (Moeen-Vaziri and Trifunac, 1988b), a method of matched asymptotic expansions to address the scattering of SH waves by surface irregularities (Sabina and Willis, 1975), a discrete wave number boundary element method to address the scattering of elastic waves by a hemispherical canyon (Kim and Papageorgiou, 1993) and the valley of Caracas, Venezuela (Papageorgiou and Kim, 1993), an approximate solution for surface ground motions of alluvial valleys subjected to incident SH-waves (Sanchez Sesma et al., 1988). Sanchez-Sesma (1987) reviews some methods available to study strong ground motion.

This paper presents the application of the weighted residual approach to the scattering and diffraction of incident SH waves by arbitrarily shaped alluvial valleys on the surface of a half-space. This approach is used to evaluate boundary conditions and is a special case of the method of moments (Harrington 1967, 1968). This approach has been applied to electromagnetic wave fields (Harrington, 1967), acoustic radiation fields (Fenlon, 1969), elastic inclusions, canyons, cavities, alluvial valleys, canals, and tunnels of arbitrary shape (Manoogian, 1992, 1995, 1996, 1998a, 1998b); Lee and Wu, 1994a, 1994b; Manoogian and Lee, 1995, 1996; Lee and Manoogian, 1995). Use of this method results in a matrix equation from which the unknown coefficients are determined and used to develop a series solution for the scattering, diffraction, and transmission of waves by the alluvial valley. In each case, it is shown that the numerical solutions by this method can be reduced to the closed-form analytic solution when the topography is circular. This paper presents a new application of the weighted residual approach used to determine the scattering and diffraction of antiplane waves by discontinuities on an elastic half-space. The method is applied to alluvial valleys of many shapes, verified using closed form solutions for semi-circular and semi-elliptical alluvial valley solutions, and the character of amplitudes above and near the discontinuity are studied.

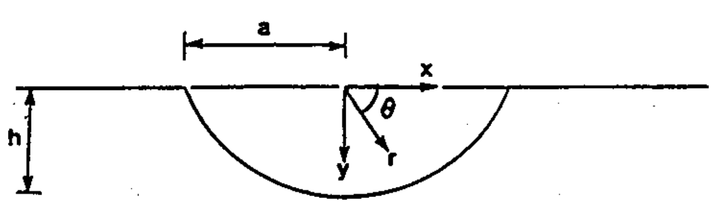

Fig. 1 Alluvial valley model

MODEL, EXCITATION, AND SOLUTION

The cross-section of the model to be studied is shown in Figure 1. It represents a shallow valley of circular shape situated on the surface of the half-space. Although the approach is derived using a shallow circular alluvial valley, the resulting equations may be used for a valley of any size or shape. The origin is on the surface of the half-space, centered with respect to the canyon edges. The half-space is assumed to consist of an elastic, homogeneous, isotropic material with rigidity μ and shear wave velocity c_β. The valley is assumed to consist of an elastic, homogeneous, isotropic material with rigidity μ_v and shear wave velocity $c_{\beta v}$. Two coordinate systems are required; a Cartesian coordinate system and a cylindrical coordinate system as shown in Figure 1. The z-axis may be assumed to be perpendicular to the plane defined by these coordinate systems.

Initially, define the excitation, $w^{(i)}$, as shown below:

$$w^{(i)} = \exp(-i\omega t)\exp\left(i\omega\left(\frac{x}{c_x}-\frac{y}{c_y}\right)\right) \tag{1}$$

This corresponds to a wave with incidence angle δ, amplitude of 1, excitation frequency ω, and wavelength $\lambda = 2\pi/k$, where $k=\omega/c_\beta$, c_x and c_y are the components of the phase velocity in the direction of the coordinate axes. In the absence of the valley, the incident SH-wave is reflected by the free surface (y = 0) and defined as shown below:

$$w^{(r)} = \exp(-i\omega t)\exp\left(i\omega\left(\frac{x}{c_x}+\frac{y}{c_y}\right)\right) \tag{2}$$

Omit $\exp(-i\omega t)$ from latter expressions. Add Equations (1) and (2) and define the result as shown below:

$$w^{(i+r)} = w^{(i)} + w^{(r)} \tag{3}$$

The following relations are applied to Equations (1) and (2) to convert from the Cartesian to the polar (r,θ) coordinate system:

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ c_x &= c_\beta/\cos\delta \\ c_y &= c_\beta/\sin\delta \\ k &= \omega/c_\beta \end{aligned} \tag{4}$$

The incident and reflected waves are combined into the expression shown below:

$$w^{(i+r)} = \exp(ikr\cos(\theta+\delta)) + \exp(ikr\cos(\theta-\delta)) \tag{5}$$

Due to the presence of the valley, the waves are scattered and diffracted within the half-space and transmitted into the valley. Within the half-space, the result is a sum of the incident, reflected, scattered, and diffracted waves. Within the valley, the result consists of the transmitted waves. These must satisfy the wave equation as defined below:

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} = \frac{1}{c_\beta^2}\frac{\partial^2 w}{\partial t^2} \tag{6}$$

Assume a scattered wave in the form shown below:

$$w^{(s)} = \sum_{n=0}^{\infty} A_n H_n^{(1)}(kr)\cos n\theta \tag{7}$$

The transmitted wave is defined as shown below:

$$w^{(v)} = \sum_{n=0}^{\infty} C_n J_n(k_v r)\cos n\theta \tag{8}$$

Here $k_v = \omega / c_{\beta v}$ is the wave number in the valley. Both $w^{(s)}$ and $w^{(v)}$ are Fourier series in θ. Only the cosine terms are used so that the zero stress boundary conditions at the surface of the half-space and valley are automatically satisfied (Cao and Lee, 1989a). In addition to the stress free boundary conditions at the free surface of the half-space, displacement and stress continuity conditions between the half-space and the valley must be used. These may be defined as follows:

$$0 = \frac{\mu}{r}\frac{\partial w}{\partial \theta} = \frac{\mu_v}{r}\frac{\partial w^{(v)}}{\partial \theta} \qquad \theta = 0, \pi \tag{9}$$

at the surface of the half-space, and continuity conditions

$$0 = w^{(i+r)} + w^{(s)} - w^{(v)} \tag{10}$$

$$0 = \mu\left(\frac{\partial w^{(i+r)}}{\partial r} n_r + \frac{1}{r}\frac{\partial w^{(i+r)}}{\partial \theta} n_\theta + \frac{\partial w^{(s)}}{\partial r} n_r + \frac{1}{r}\frac{\partial w^{(s)}}{\partial \theta} n_\theta\right) - \mu_v\left(\frac{\partial w^{(v)}}{\partial r} + \frac{1}{r}\frac{\partial w^{(v)}}{\partial \theta} n_\theta\right) \tag{11}$$

Here, at the interface between the valley and the half-space, n_r and n_θ are the unit normals in the r and θ directions. The free surface boundary condition is automatically satisfied by $w^{(s)}$ and $w^{(v)}$. The displacement continuity condition, Equation (10), is satisfied by substituting Equations (5), (7), and (8). The stress continuity condition, Equation (11), is satisfied by substituting Equations (5), (7), and (8). Assemble the resulting equations into the weighted residual forms shown below:

$$0 = \int_{\theta_1}^{\theta_N} W_m(r(\theta), \theta)\left(w^{(i+r)} + w^{(s)} - w^{(v)}\right) d\theta \qquad m = 0, 1, 2, \ldots \tag{12}$$

$$0 = \int_{\theta_1}^{\theta_N} W_m(r(\theta), \theta)\left(\tau^{(i+r)} + \tau^{(s)} - \tau^{(v)}\right) d\theta \qquad m = 0, 1, 2, \ldots \tag{13}$$

Define $W_m(r(\theta), \theta)$ as the weight function and $\tau^{(i+r)}, \tau^{(s)}$, and $\tau^{(v)}$ as stresses due to the incident and reflected waves, scattered waves, and transmitted waves. The weight function used in this case was $\cos m\theta$. Since convergence was achieved and solutions matched closed form solutions, others were not tested. The weighted residual forms are assembled into the matrix form shown below:

$$[C_{mn}]\begin{bmatrix} A_n \\ C_n \end{bmatrix} = [b_m] \tag{14}$$

Denote the matrix $[C_{mn}]$ as the matrix of coefficients from the scattered and transmitted wave weighted residual expressions, Equations (12) and (13). The vector $[b_m]$ consists of the coefficients from incident and reflected wave weighted residual expressions. The matrix $[C_{mn}]$ and the vector $[b_m]$ may be partitioned as shown below:

$$\begin{bmatrix} C1 & C2 \\ C3 & C4 \end{bmatrix}\begin{bmatrix} A_n \\ C_n \end{bmatrix} = \begin{bmatrix} b_{m1} \\ b_{m2} \end{bmatrix} \tag{15}$$

where $[C1]$ and $[C2]$ are the scattered and transmitted wave terms from the displacement continuity condition, $[C3]$ and $[C4]$ are the scattered and transmitted wave terms from the stress continuity condition, and $[b_{m1}]$ and $[b_{m2}]$ are the excitation terms from the displacement and stress continuity conditions. Solve for C_n in order to obtain the following expression:

$$[C_n] = [C4]^{-1}\left[[b_{m2}] - [C3][A_n]\right] \tag{16}$$

Solve for A_n in order to obtain the following expression:

$$[A_n]=\left[[C1]-[C2][C4]^{-1}[C3]\right]^{-1}\left[[b_{m1}]-[C2][C4]^{-1}[b_{m2}]\right] \tag{17}$$

Constants A_n and C_n are determined and substituted into Equations (7) and (8). The transmitted wave amplitudes are defined by Equation (8). The scattered wave amplitudes are added to Equation (5) to obtain amplitudes in the half-space.

SPECIAL CASE--SEMI-CIRCULAR ALLUVIAL VALLEY

The case of a semi-circular alluvial valley on the surface of an elastic half-space is worthy of special consideration as the closed form analytic solution for incident plane SH waves has been available for many years (Trifunac, 1971). Using the (moment) method of weighted residues introduced above, the same continuity equations of displacement and stress can be applied at the interface of the half-space and alluvial valley materials. Then the continuity conditions (Equations (10) and (11)) take the following form. At $r=a$, the displacement continuity Equation (10) stays the same.

$$w^{(i)}+w^{(r)}+w^{(s)}=w^{(v)} \tag{18}$$

The stress continuity equation at $r=a$ becomes

$$\mu\frac{\partial}{\partial r}\left(w^{(i)}+w^{(r)}+w^{(s)}\right)=\mu_v\frac{\partial w^{(v)}}{\partial r} \tag{19}$$

with μ and μ_v respectively the shear modulus of the half-space and the alluvium. The surface boundary conditions remain the same. In applying the method of weighted residues, Equations (12), (13) may be further simplified using the orthogonality of the cosine functions. Expressions for the incident and reflected plane waves, $w^{(i)}$ and $w^{(r)}$ in Equation (5) may be expanded in terms of Bessel functions in the polar coordinate system with the origin at the center of the surface of the valley:

$$w^{(i)}+w^{(r)}=\sum_{m=0}^{\infty}\left(\varepsilon_m i^m J_m(ka)+\varepsilon_m(-i)^m J_m(ka)\right)\cos m\theta \ \cos m\delta \tag{20}$$

where, as before, $\cos m\theta$ are the weight functions, the orthogonality of these trigonometric functions may be applied, using the well known identities (Abramowitz and Stegun, 1964)

$$\int_0^{\pi}\cos m\theta \ \cos n\theta \ d\theta=\begin{cases}0 & \text{for } m\neq n\\ \pi/2 & \text{for } m=n\end{cases} \tag{21}$$

The weighted residues integral for the displacement continuity at $r=a$ takes the form for $m=0,1,2\ldots$

$$\int_0^{2\pi}\cos \ m\theta \ \left(w^{(i)}+w_I^{(i)}+w^{(s)}+w_I^{(s)}-w^{(v)}\right)_{r=a} d\theta=0 \tag{22}$$

Similarly, the orthogonality property for the stress continuity equation gives at $r=a$, for $m=0,1,2\ldots$

$$\int_0^{\pi}\cos m\theta\frac{\partial}{\partial r}\left(w^{(i)}+w_I^{(i)}+w^{(s)}+w_I^{(s)}-\frac{\mu_v}{\mu}w^{(v)}\right)\Big|_{r=a} d\theta=0 \tag{23}$$

The following equation results

$$\begin{aligned}&\varepsilon_m i^m\frac{\partial}{\partial r}J_m(ka)\cos m\delta+\varepsilon_m(-i)^m\frac{\partial}{\partial r}J_m(ka)\cos m\delta\\&+A_m\frac{\partial}{\partial r}H_m^{(1)}(ka)\cos m\delta-\frac{\mu_v}{\mu}C_m\frac{\partial}{\partial r}J_m(k_v a)\cos m\delta=0\end{aligned} \tag{24}$$

The above expressions for the coefficients for $m=0,1,2\ldots$ in the above are the same form as the analytic closed form expressions developed by substituting Equations (5), (7), and (8) into the continuity conditions (10) and (11) in solving the diffraction problem of a semi-circular alluvial valley as a boundary value problem using wave propagation theory. By comparing the resulting displacement amplitudes calculated using the exact analytical approach (Trifunac, 1971) and the weighted residue approach used here as a numerical method, the validity of the approximate numerical method can be tested and improved upon. The weighted residual method can be applied with confidence to problems involving other arbitrarily shaped valleys when closed form solutions are not available.

SURFACE DISPLACEMENT AMPLITUDES

Of particular interest are the displacement amplitudes on the surface of the half-space near the valley and on the valley surface. If the amplitude of the incident plane SH-waves is 1, the responses shown define amplitude and de-amplitude factors. The resultant motion is defined by the modulus as shown below:

$$amplitude = \left(\mathrm{Re}^2(w) + \mathrm{Im}^2(w)\right)^{1/2} \tag{25}$$

In the absence of the valley, for a uniform half-space, the modulus of the ground displacement is 2. Due to the existence of the valley, incident waves are scattered and diffracted into the half-space and transmitted into the valley. As a result, the moduli may differ significantly from 2. Displacements were calculated for a discrete set of dimensionless frequencies, η, at intervals of 0.25 ranging from 0.25 to 2. The dimensionless frequency, η, is defined as shown below:

$$\eta = \frac{2a}{\lambda} = \frac{ka}{\pi} = \frac{\omega a}{\pi\beta} \tag{26}$$

Let a be the half-width of the valley, the distance between the edges of the valley at the surface.

Figures that follow show the displacement amplitudes on the surface of the half-space and the valley for an incident SH-wave of amplitude 1. All displacements are plotted with respect to the dimensionless distance x/a. The most interesting cases are those with alluvial fill that is softer and less dense than the surrounding material. The plots of surface amplitudes that follow include semi-circular and semi-elliptical alluvial valleys with configurations similar to those found in Trifunac (1971) and Wong and Trifunac (1974a) in order to verify the weighted residual formulation. The method is also applied to shallow circular $(h/a = 0.5)$ and rectangular shaped alluvial valleys to demonstrate the versatility of the weighted residual approach. Once formulated and programmed, the solution may be used to study the surface displacements on and near alluvial valleys of a variety of shapes. Computations were not attempted for alluvial valleys with h/a less than 0.5. Convergence becomes a problem for the shallowest valleys as the Hankel functions for the points closest to the origin, located on the surface and midpoint of the alluvium, become nearly singular. Manoogian (1992) includes results for a shallow elliptical canyon with $h/a = 0.3$. Attempts to obtain surface displacements for a shallow canyon with $h/a = 0.05$ were not successful. Yuan and Liao (1995) have addressed this issue, in part, by moving the origin above the half-space for the case of a shallow cylindrical alluvial valley. Use of the Yuan and Liao (1995) approach in combination with the methods used in this article could allow for solutions for very shallow alluvial valleys. The number of terms needed for the convergence of the solution was determined by experimentation. The number of terms used was actually larger than needed and was defined as the maximum number of terms that could be used without causing a numerical overflow.

Figure 2 shows the surface displacements on and near a soft semi-circular valley $(h/a = 1)$ for a dimensionless frequency range from 0.25 to 2 for an (horizontal) incident wave with an angle of incidence of 0°, $\mu_v/\mu = 1/6$ and $\rho_v/\rho = 2/3$. Figure 3 shows the amplitudes for $\eta = 2$. These match the results of the closed form solution (Trifunac, 1971). Surface responses are symmetrical for a vertically incident wave, $\delta = 90°$, with peak amplitudes of about 4 on the half-space near the valley and exceeding 10 on the surface of the valley for $\eta = 1.5$. As the angle of incidence decreases toward 0°, the responses become larger and more complex on the half-space for $x/a < -1$. This is due to the interference of incident and reflected waves, waves scattered by the valley and waves transmitted back into the half-space from the valley. On the other side of the valley, $x/a > 1$, a shadow zone exists in which the amplitudes are smoother and tend toward 2. On the surface of the valley, significant amplification occurs with respect to half-space amplifications. Since the wave number, k_v, in the valley is twice that of the half-space, the amplitude pattern is more complex than that found on the half-space.

Figure 4 shows the surface displacement amplitudes on and near a shallow circular valley for a frequency range from 0.25 to 2 for an incident wave with an angle of incidence of 30°, $\mu_v/\mu = 1/6$ and $\rho_v/\rho = 2/3$. Figure 5 shows the surface amplitudes for $\eta = 2$. As in the case of the circular valley discussed previously, amplitudes are symmetrical for an incident wave with vertical orientation. Amplitude patterns on the half-space on this shallower valley are similar to those found for the circular valley, but less complex and of smaller amplitude.

Figure 6 shows the surface displacement amplitudes for a semi-elliptical valley with $h/a = 0.7$, a frequency range from 0.25 to 2, an incident wave with an angle of incidence of 60°, $\mu_v/\mu = 1/6$ and $\rho_v/\rho = 2/3$. Figure 7 shows the surface amplitudes on and near the valley for $\eta = 2$. These solutions match those of the closed form solution (Wong and Trifunac, 1974a). As in the case of the circular valleys, amplitudes are symmetrical for an incident wave with a vertical orientation. As the angle of incidence decreases towards 0°, amplitude patterns on the half-space become more prominent and complex for $x/a < -1$. On the other side of the valley, $x/a > 1$, the amplitudes are smoother and tend towards 2. Within the soft valley, responses are significantly amplified and have greater complexity with respect to the half-space with peaks exceeding 10 for $\eta = 1.5$ and an angle of incidence of 0°.

Figure 8 shows the surface amplitudes for a rectangular shaped valley for a dimensionless frequency range from 0.25 to 2, incident SH-waves with an angle of incidence of 0°, $h/a = 1$, $\mu_v/\mu = 1/6$ and $\rho_v/\rho = 2/3$. Figure 9 shows the amplitudes for $\eta = 2$. As in the cases of the other soft valleys, amplitudes are symmetric for the case in which the angle of incidence is vertical. For other angles of incidence, amplitudes are more pronounced and patterns more complex for $x/a < -1$. On the other side of the valley, amplitudes are less complex and tend toward 2. Within the valley, amplitudes are significantly larger with respect to those on the half-space with some cases exceeding 10. It should be noted that amplitudes for the rectangular valleys are not plotted on the sides of the valleys. Jumps in the amplitude plots at $x = \pm a$ represent the range of the amplitudes on the sides of the valleys.

CONCLUSIONS

A number of observations result from the work presented:

1. The weighted residual approach approximation for the scattering, diffraction, and transmission of SH-waves by an alluvial valley yields solutions which will match the known closed form solutions.
2. The weighted residual approach is suitable for valleys of arbitrary shape though convergence for the shallowest valleys is more difficult to obtain.
3. Ground surface amplitudes on the half-space outside the valleys may be nearly 6, significantly larger than the expected value of 2 for a smooth homogeneous half-space and depend on the angle of incidence, dimensionless frequency, shape of the valley, and the relative properties of the valley.
4. Ground surface amplitudes on the half-space within the valleys may be as much as 10 to 15 times the amplitude of the incident wave for a softer valley.

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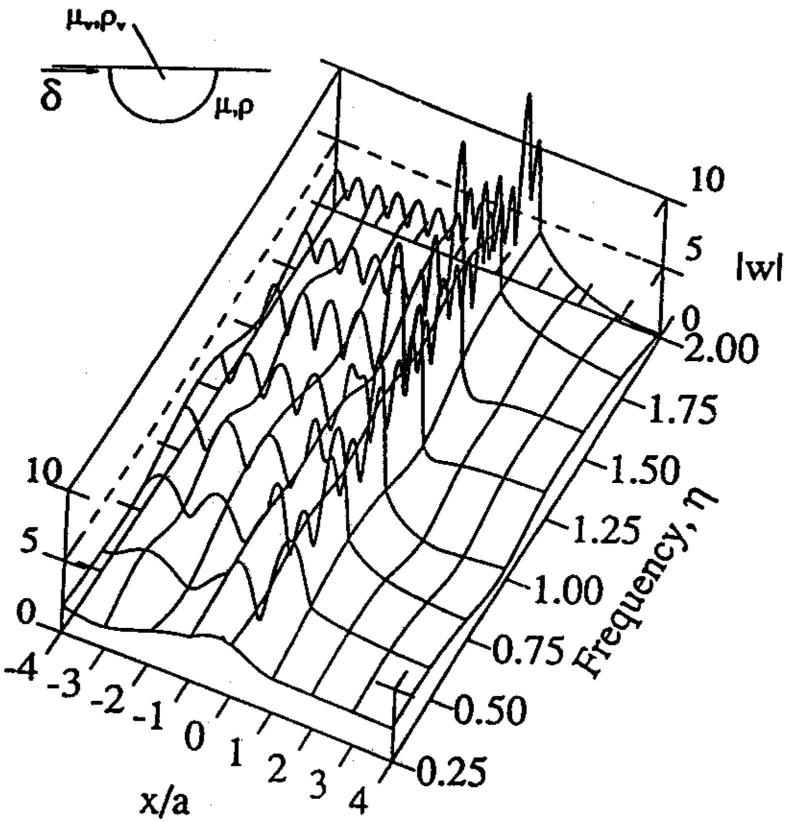

Fig. 2 Surface amplitudes on and near a semi-circular alluvial valley, $h/a=1$, $\delta=90^\circ$, $\mu_v/\mu=1/6$, $\rho_v/\rho=2/3$

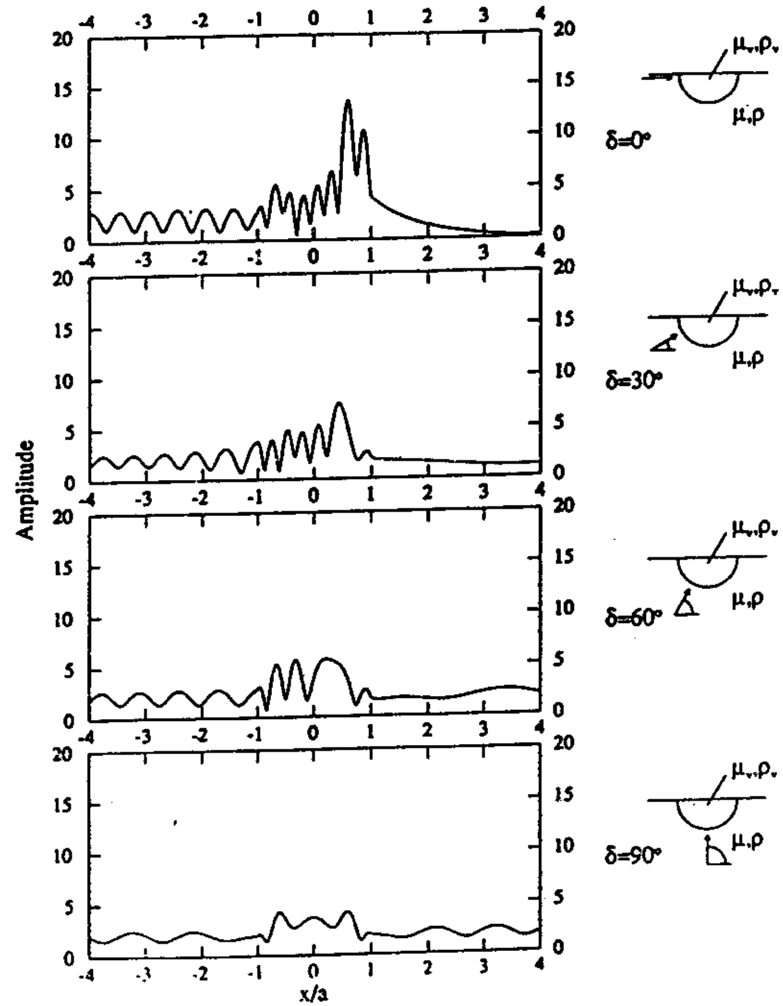

Fig. 3 Surface amplitudes on and near a semi-circular alluvial valley, $\eta=2$, $h/a=1$, $\mu_v/\mu=1/6$, $\rho_v/\rho=2/3$

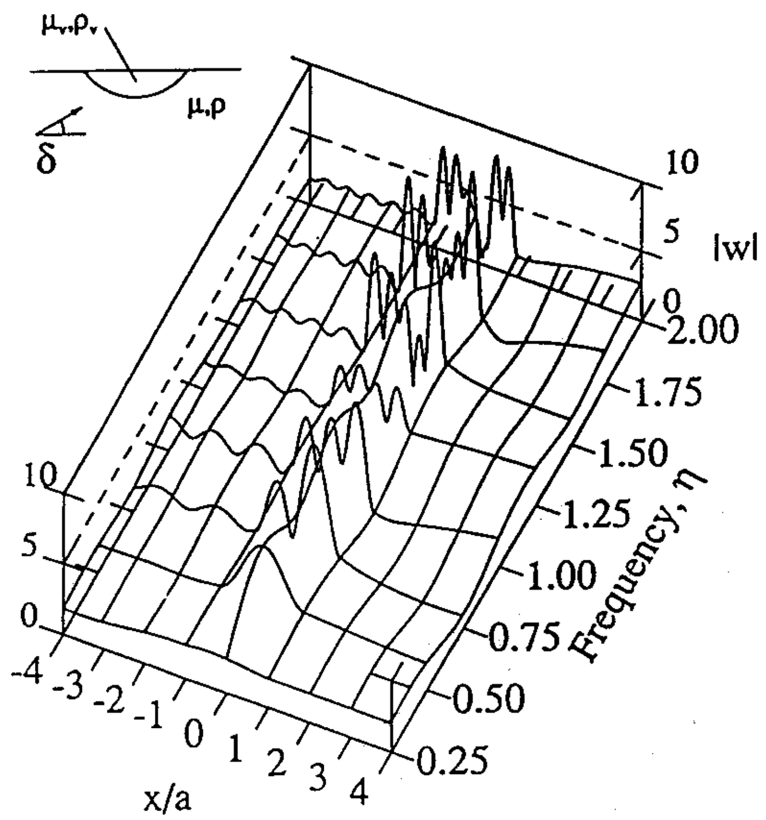

Fig. 4 Surface amplitudes on and near a shallow circular alluvial valley, $h/a = 1/2$, $\delta = 30^0$, $\mu_v/\mu = 1/6$, $\rho_v/\rho = 2/3$

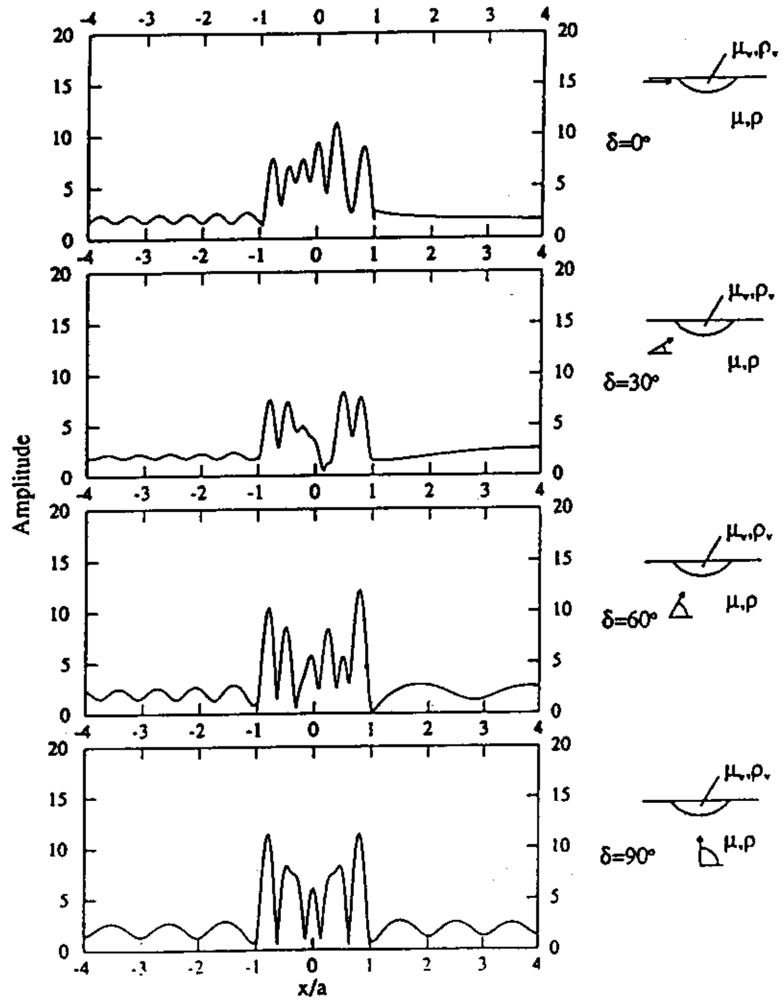

Fig. 5 Surface amplitudes on and near a shallow circular alluvial valley, $\eta = 2$, $h/a = 1/2$, $\mu_v/\mu = 1/6$, $\rho_v/\rho = 2/3$

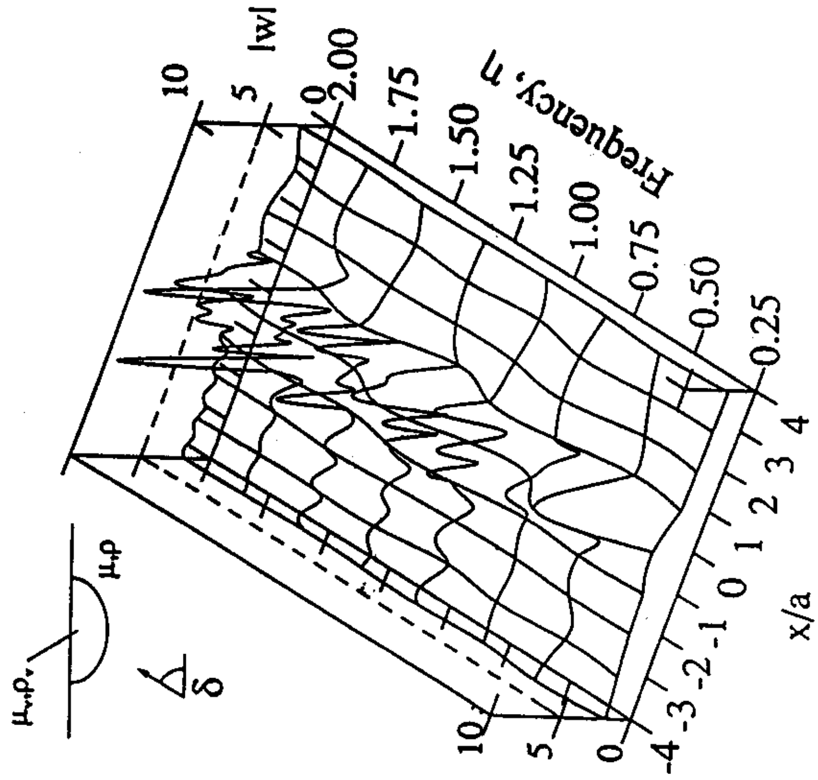

Fig. 6 Surface amplitudes on and near a semi-elliptical alluvial valley, $h/a = 0.7$, $\delta = 60^0$, $\mu_v/\mu = 1/6$, $\rho_v/\rho = 2/3$

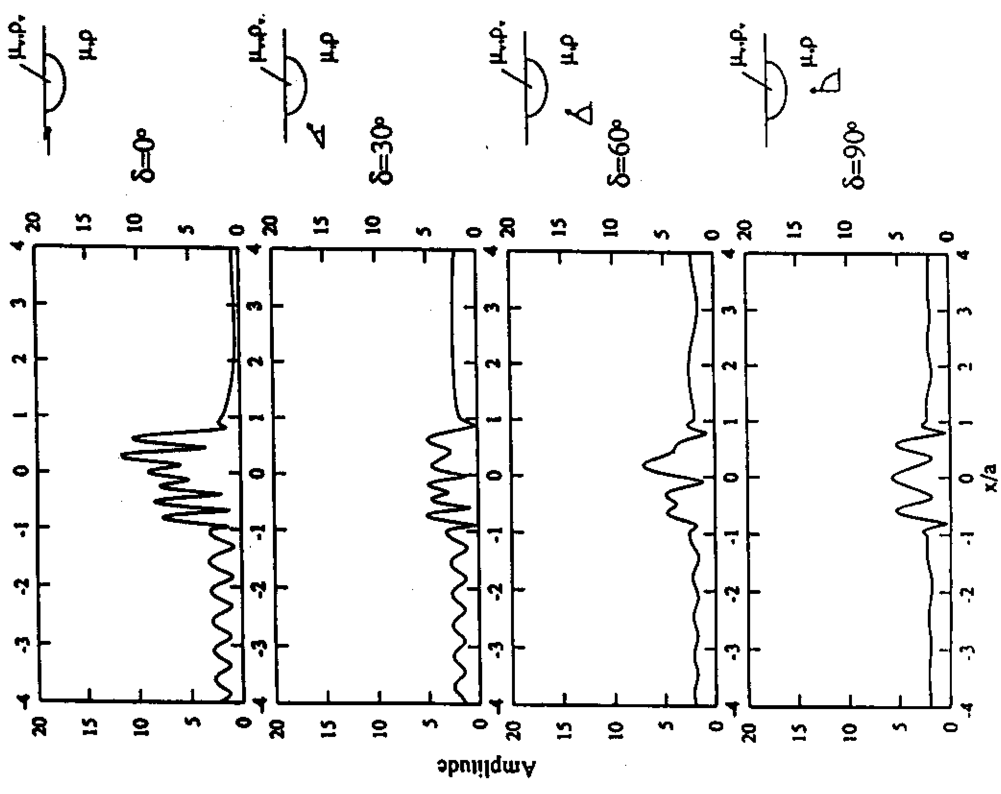

Fig. 7 Surface amplitudes on and near a semi-elliptical alluvial valley, $\eta = 2$, $h/a = 0.7$, $\mu_v/\mu = 1/6$, $\rho_v/\rho = 2/3$

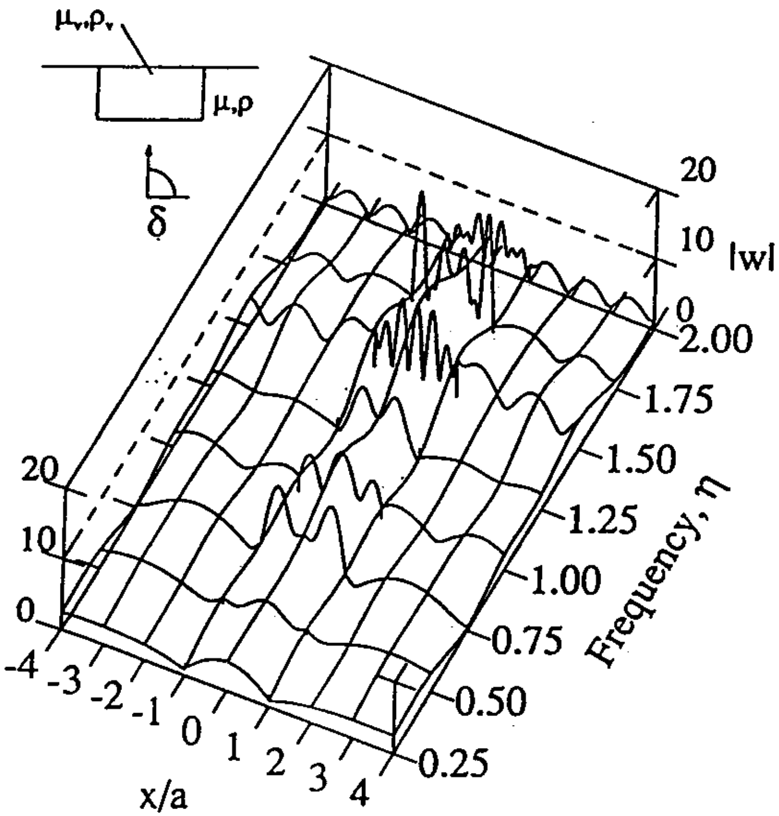

Fig. 8 Surface amplitudes on and near a rectangular alluvial valley, $h/a = 1$, $\delta = 90^0$, $\mu_v/\mu = 1/6$, $\rho_v/\rho = 2/3$

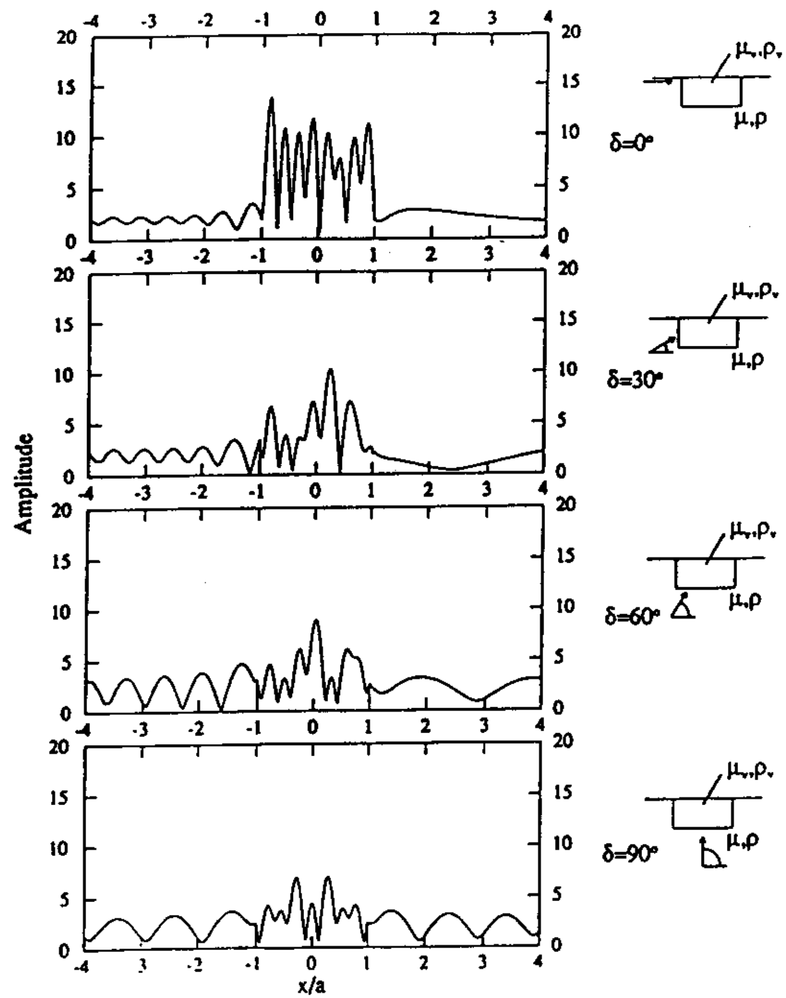

Fig. 9 Surface amplitudes on and near a rectangular alluvial valley, $\eta = 2$, $h/a = 1$, $\mu_v/\mu = 1/6$, $\rho_v/\rho = 2/3$

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