Bull. Ind. Soc. Earth. Tech., Paper No. 335, Vol. 30, No. 4, Dec. 1993, pp. 113-124

SIMPLIFIED SEISMIC ANALYSIS OF FRAMED BUILDINGS ON HILL SLOPE

D.K. Paul
Professor, Dept. of Earthquake Engineering,
University of Roorkee, Roorkee-247667

ABSTRACT

This paper looks into the various design problems and various precautions to be taken for framed buildings on hill slopes. A one dimensional (1D) simplified method of analysis of framed buildings on hill slopes is presented and the results of natural time periods are compared with those obtained by using three dimensional (3D) frame analysis. Method of evaluating the displacement and shear forces and its distribution within the interstorey resisting elements and the resisting elements resting on the hill slopes based on Indian Standard Code of Practice is worked out for an example.

INTRODUCTION

Most of the northern hilly region of India lies in the seismically active belt of the Himalayan range. Three major earthquakes (M>8), Kangra(1905), Bihar-Nepal (1934) and Assam(1950) have occurred in this hilly track during the last century. The hilly seismic regions of our country ranges from Jammu and Kashmir, Himachal Pradesh, North Uttar Pradesh, North Bihar, Sikkim, North Bengal, Assam, Meghalaya, Nagaland, Arunachal Pradesh, Manipur, Tripura and Mizoram. All these vast regions are undergoing rapid changes due to economic development and being the frontier states. It has vast populated area in the hilly regions and all kinds of construction practices (i.e. engineered and non-engineered, traditional and modern) are followed. All short of building materials i.e. adobe, burnt bricks, stone masonry, dressed stone masonry; bamboo, timber reinforced concrete etc. are used depending upon the locally available material.

The adobe, burnt brick, stone masonry and dressed stone masonry buildings are generally made over level ground in hilly regions. Since, the level land on hilly region is very limitted, there is a pressing demand to construct buildings on hill slopes in a hilly terrain as shown in Fig. 1. The height and gradient of hill slopes depend on strength and deformation characteristics of soil/rock mass. In most of the hilly region, the ground is subjected to freezing and thawing or permanent frozen condition. Improper selection and development of sites, drainage and variation of bearing capacity are some of the important factors which should be considered in planning and design of hill buildings. The bamboo, timber and reinforced concrete frame structures are the suitable building materials on hill slopes. Because of harsh weather conditions in most hilly regions, durability of reinforced concrete over bamboo and

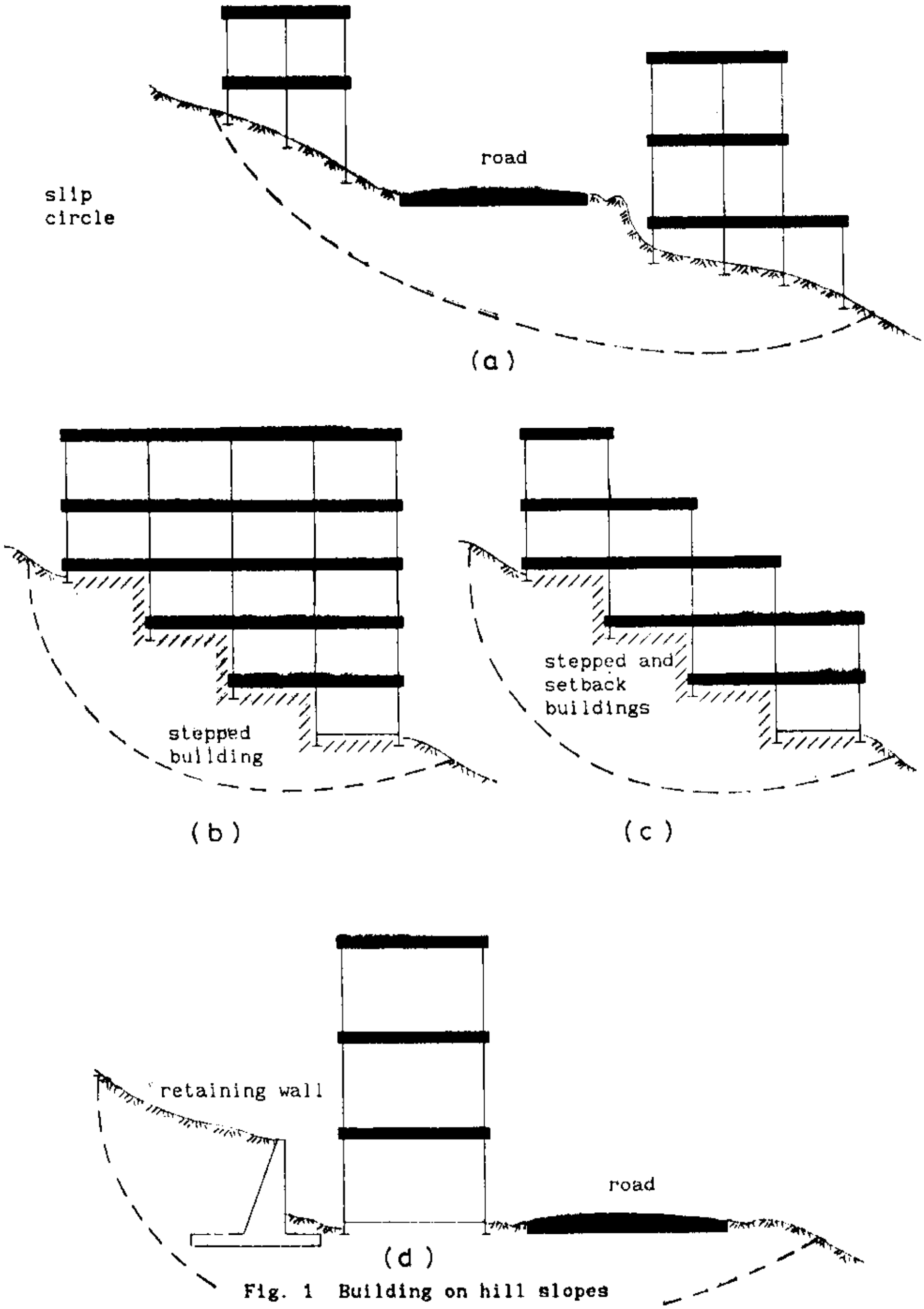

Fig. 1 Building on hill slopes

timber, and improved present day transport conditions, the reinforced concrete construction is becoming more popular in hilly regions. In this paper, a simplified analysis procedure of a framed building on hill slope is presented.

HILL BUILDINGS

The buildings situated on hill-slopes are generally irregular, torsionally coupled and hence susceptible to severe damage during an earthquake. Such buildings have mass and stiffness varying along the horizontal and vertical planes. The centre of mass and the centre of rigidity do not coincide on various floors.

Typical buildings on either side of road are constructed as shown in Fig. 1. Seismic analysis for lateral vibration of such buildings are some what different than the buildings on level ground since the columns of the building rest at different levels on the slope. Such buildings are also unsymmetrical in elevation and plan which demand torsional analysis as well.

Simplified methods are generally preferred in design offices over sophisticated 2D or 3D frame analysis. For buildings having irregular shape and/or irregular distribution of mass and stiffness in horizontal and vertical planes, IS:1893-1984 recommends Modal analysis using Response Spectrum method. It also states that provision shall be made for the increase in shear resulting from the horizontal torsion due to an eccentricity between the centre of mass and the centre of rigidity. The IS Code 1893-1984 does not give ample guidance in the analysis of buildings on hill slopes. An analysis of such a building is described through an example enumerating the steps in arriving a mathematical model, and distribution of shear forces.

PROBLEMS REGARDING BUILDINGS ON HILL-SLOPES

Very few buildings receive the careful planning of architect and engineer's rigorous analysis and design. People construct their houses any way they like without thoughts about the structural safety aspects. Even though most of the hilly regions of India are highly seismic, buildings are not designed for earthquake forces except for a very few important government buildings. Apart from these problems which arises due to human ignorance, the more technical problems faced by buildings on hill-slopes regarding seismic design are discussed below.

Ground motion

In seismic analysis, it is usually assumed that the motion which takes place during an earthquake has one principal direction. Therefore, a well designed structure should be capable of resisting destructive earthquake energy reaching it from any possible direction. The usual practice in earthquake analysis involves two different analyses at two perpendicular directions. For structures which are symmetrical having well defined principal axes, this approach yields satisfactory results. For complex three dimensional structures such as the typical buildings on hill-slopes, there may be a critical direction of earthquake which produces the maximum force in a particular member. Such an analysis is likely to be very costly.

When the response spectra is taken as the input excitation the problem could be even more complex. Combining modal contributions to response under a single input direction is not straight forward since the maximum response from different modes occur at different times.

Effects of soil profile

Damage to buildings and structures on hill slopes as observed in the past earthquakes seem to indicate that to a certain extent, non-uniform soil profile may be responsible for it. At the time of the 1968 Tokachi-oki earthquake in north-eastern region of Japan, low rigid reinforced concrete buildings located near the edge of a stretch of hills in the city of Hachinohe suffered serious structural damages. Also, in the 1975 Oita earthquake in the southern region of Japan, the ground floor column of a 4-storeyed reinforced concrete hotel building of modern design built on a hill-slope, were severely damaged.

On the other hand, residences of wooden construction on sedimentary soil near exposed hill sides at the time of the 1948 Fukui earthquake suffered large heaving due to violent ground movements induced, in part if not entirely, due to the soil profile. In the Nov.23,1980 earthquake of Italy, it was observed that most of the damage occurred to towns located atop hill sides, some of which were completely razed to the ground. This implies that there could be some amplification of vibrations or motions at these higher elevations.

Sakurai and Minami (1969) investigated the effects of soil profiles on the response of buildings by carrying out simulated model study of buildings located on different soil profiles and found that the seismic response of buildings located on such soil profiles differs markedly from the case of buildings supported on horizontally uniform soil layers. The series of studies that they made relating to the seismic response of buildings on the soil-foundation-building interaction system has prompted Sakurai and Minami to suggest the inclusion of "soil profile factor" for the determination of design seismic coefficient.

Variation in column stiffness

Due to site conditions, buildings on hill-slopes are characterised by unequal column heights which result in variation of column stiffnesses. It is obvious that a short column would be stiffer than a longer one of the same cross-sectional area. For vertical loads, it may not be susceptible to buckling, and hence capable of receiving higher loads. The short column is also stiffer under lateral loading. Since the loads are distributed according to stiffness of resisting elements, the short, stiff column will "attract" forces that may be quite out of proportion to its strength. The "short column effect" thus results in greater load and greater damage to the shorter columns.

If the condition of short column cannot be avoided, one solution is to equalize the stiffness of the columns by introducing struts that increase the stiffness of the longer columns.

Slope stability

A building on a hill slope may lead to slope stability problem specially during the rainy season and earthquakes. A detailed slope stability analysis should be carried out considering the loads due to buildings.

Torsional coupling

Buildings with eccentric centers of mass and resistance respond in coupled translational and rotational motions when subjected to ground motion. As a result of coupled lateral-torsional motions, the lateral forces experienced by various resisting elements such as floor slabs, frames and shear walls would differ from those experienced by the same elements if the building had symmetric plan and hence responded only plane translational vibrations.

It is customary to simplify the dynamic analysis of buildings by assuming single degree of freedom per floor mass corresponding to translation in the chosen direction of ground motion. The modal response is then studied for ground motion along which the resisting elements are located. The design seismic forces so obtained may be satisfactory for a purely symmetrical building. However, a little change in the distribution of mass and/or stiffness will also produce floor rotation along with translations. Such response is the characteristic of a generally unsymmetrical building. Therefore, analysis of torsionally coupled buildings requires that in addition to the usually considered translational degrees of freedom, the torsional degree of freedom be included for each floor.

Buildings on hill-slopes, those shown in Fig.1 will invariably undergo coupled translational and rotational motions even when the ground motion is uniform over the base and contains no rotational component. The plan of a particular floor may be symmetrical along both axes, but due to the change in floor area as building height increases, and variation of column stiffnesses, the center of mass and center of rigidity will most likely not coincide.

PRECAUTIONS FOR BUILDINGS ON HILL-SLOPES

In hilly regions where landslides are frequent and hazardous, the chances of earthquake triggered landslides and their consequent disaster must be minimised as far as possible. For this reason, slope stability analysis of the chosen site should be carried out taking into consideration the forces resulting from earthquakes, seepage water pressure and the estimated weight of the proposed structure.

Improper selection and development of sites often causes landslides which may damage the building and result in loss of life and property. A suitable breast-wall may be constructed when a soil mixed boulder rests over rocks. The use of random rubble dry stone masonry retaining walls as a protection measure is a common practice, and their performance in the past earthquakes have been quite satisfactory for a maximum height of 4m [Arya and Gupta(1983)]. The resistance and behaviour of random rubble dry stone masonry retaining walls of height even more than 10m are found to be satisfactory and safe in seismic regions [Arya and Gupta(1983)].

On hill sites, drainage requires special consideration. The natural flow of storm water should be diverted away from the foundations. To minimise seepage, a minimum of 1 meter wide apron should be provided all around the building to prevent entry of water into the foundation. Since the inner side of cut-slope may have higher bearing capacity, the building should be oriented in such a way that higher load comes on the inner side of the cut-slope. Construction of buildings on filled up ground should be avoided.

When adequate provision for lateral support, such as retaining walls, the higher footings are not provided, the IS:1904-1986 on Design and Construction of Foundations, has specified some guidelines as follows:

(i) When ground surface slopes downwards adjacent to a footing, the sloping surface shall not intersect a frustum of bearing material under the footing having sides which makes an angle of 30 degrees with the horizontal for soil and the horizontal distance from the lower edge of the footing to the sloping surface shall be at least 60cms for rock and 90cms for soil.

(ii) In the case of footings in granular and clayey soils, a line drawn between the lower adjacent edge of the upper footing and the upper edge of the lower footings shall not have a steeper slope than one vertical to two horizontal.

MATHEMATICAL MODEL FOR FRAMED BUILDINGS ON HILL-SLOPES

Lateral and torsional vibrations will be predominant in a building on a hill slope when subjected to seismic forces. Uncoupled analyses for two lateral (one along the hill slope and another across the hill slope) and torsional vibrations are proposed to be carried out. Therefore, separate mathematical models are required for two lateral and torsional analysis.

Lateral Vibration Analysis

The floor masses are lumped at the floor level together with the appropriate stiffness contribution of columns and walls. Two uncoupled lateral motions in two principal directions are considered. The inter storey stiffnesses are evaluated as usual by assuming the floors to be rigid in plane as compared to the columns and is given as:

$$k_i = \sum_{i=1}^{n} \frac{12E_i I_i}{h_i^3} \qquad \text{......(1)}$$

where E_i, I_i, k_i and h_i are the modulus of elasticity, moment of inertia and clear height of a column corresponding to ith storey, n is the total number of columns between the floors considered.

If the floors are supported by two types of column such as columns resting on lower floors and columns resting on the ground directly, then the column stiffness should be calculated as follows:

(i) The columns supporting the floor and resting on lower floor is calculated as above and contribute to the inter storey stiffness only.

(ii) The columns supporting the floor and resting on sloping ground is calculated as above and contribute to the stiffness between the floor and the ground only.

EVALUATION OF RESPONSE

The evaluation of response of a building resting on sloping ground is carried out with the help of a numerical example. Figure 2 shows the

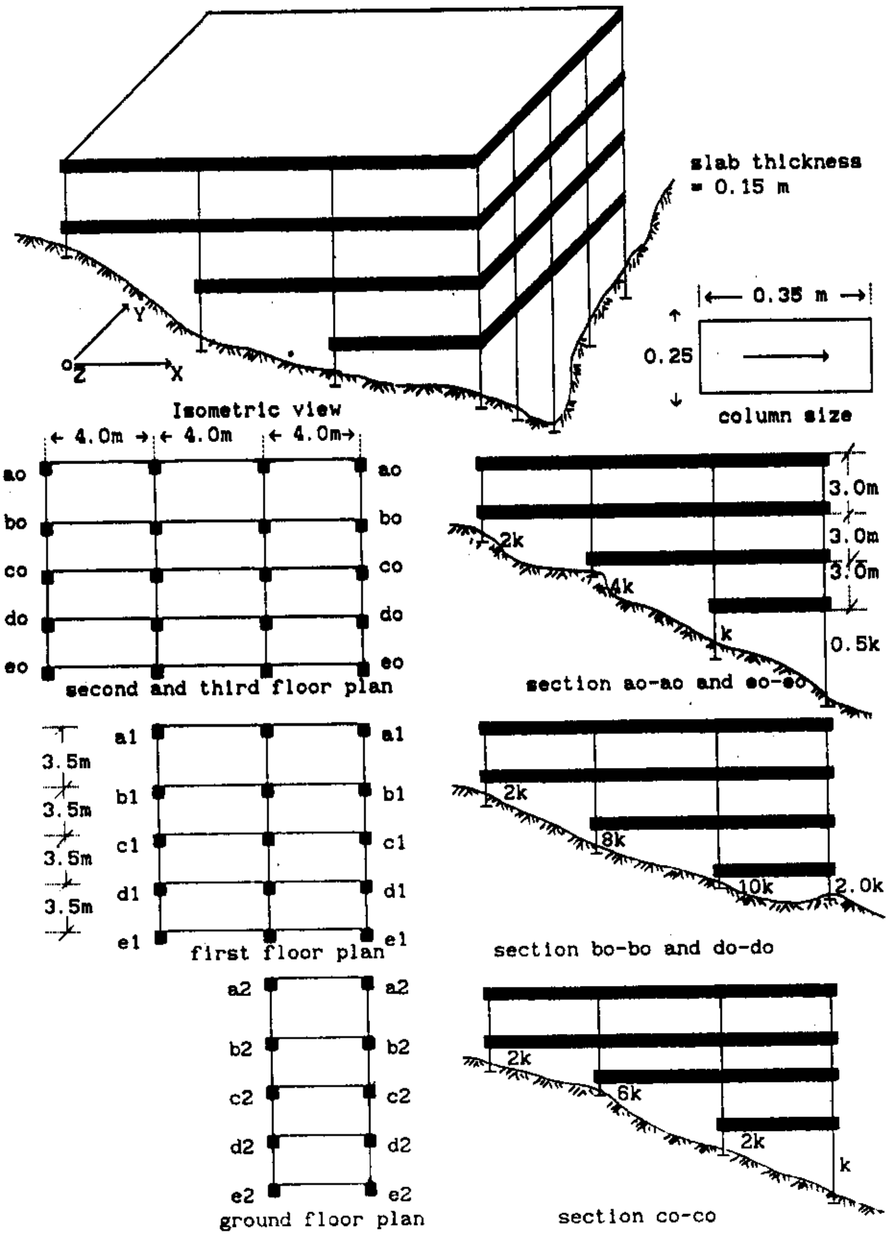

Fig. 2 Plans and sections

plans at ground, first and second floors of a three storey building on hill slope. The mathematical model obtained as described above is shown in the Fig. 3 and the properties are given in Table 1.

Table 1 Material and sectional properties

direction of vibration	$E_i (t/m^2)$	$\gamma (t/m^3)$	$h_i (m)$	$I_i (m^4)$	$k_i (t/m)$	$m (t\text{-}s^2/m^4)$
across slope	0.20e+07	2.4	3.0	0.4557e-03	405.1	0.514
along slope	0.20e+07	2.4	3.0	0.8930e-03	793.8	0.514

γ is the unit weight of concrete and m is the mass of floor size 3.5mx4.0m.

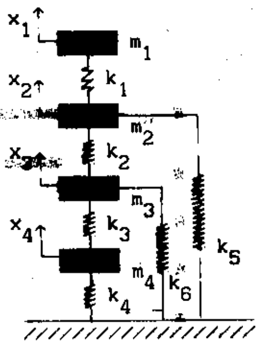

Fig. 3 Mathematical model for hill building

Equation of Motion: The equation of motion for undamped free vibration of the above system can be expressed as

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & (k_1+k_2+k_5) & -k_2 & 0 \\ 0 & -k_2 & (k_2+k_3+k_6) & -k_3 \\ 0 & 0 & -k_3 & k_3+k_4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = 0$$

$$\text{or} \quad \underline{M}\,\ddot{\underline{X}} + \underline{K}\,\underline{X} = 0 \qquad \ldots\ldots\ldots(2)$$

The modal analysis is carried out as per IS Code 1893. Let us assume that each column between any two successive floors are of same size and same height having stiffness k, then the interstorey stiffnesses would be obtained as follows

$$k_1 = 20k \quad k_2 = 15k \quad k_3 = 10k.$$

The stiffness of the columns resting on ground are of different height because of irregular hill slope but have the same size. The stiffness of each column is shown in the figure. The stiffness of columns connected to ground is worked out as

$$k_4 = 30k \quad k_5 = 10k \quad k_6 = 30k.$$

The floor mass in term of m is worked out as

$$m_1 = 12m \quad m_2 = 12m \quad m_3 = 8m \quad m_4 = 4m.$$

The equation (2) yields, the time periods and mode shapes in two modes of vibration along the slope as

$$T_1 = 0.2006 \text{ s}$$

$$\phi^{(1)^T} = [1.0,\ 0.6190,\ 0.2091,\ 0.05266]$$

$$T_2 = 0.0900 \text{ s}$$

$$\phi^{(2)^T} = [\ 1.0,\ -2.188,\ -2.2260,\ -0.619\ ] \qquad \ldots\ldots(3)$$

Table 2 shows the comparison of the free vibration time periods of 1D simplified analysis as compared with the 3D frame analysis where the floors and the roof are considered rigid in their plane. Three degrees of freedom per floor [illegible] lateral translations and one rotation about vertical axis) is considered in 3D frame analysis while 1D analysis considers only the translation in principal directions separately. The time periods obtained from 1D analysis are comparable and lower than the time periods as obtained from the 3D analysis.

Modal Analysis as per Is Code 1893-1984

The seismic coefficient is expressed as

$$\alpha_h^{(r)} = \beta\, I\, F_o\, S_a/g \qquad \ldots\ldots(4)$$

Table-2 Comparison of 1D and 3D analyses time periods (sec)

mode	3D analysis	1D analysis	remarks
1	0.3122	0.2810	vib. across the slope
2	0.2228	0.2006	vib. along the slope
3	0.1255	0.1260	vib.across slope & torsion
4	0.0958	-	vib.across slope & torsion
5	0.0895	0.0900	vib.along the hill slope

Assuming, β=1.2 (isolated r.c. footing without tie beams or unreinforced strip foundation), I=1.5 (hospital building), F=0.4 and 5% damping. The S_a/g works out to 0.20, $\alpha_h^{(1)}$=0.144 and $\alpha_h^{(2)}$=0.144. The mode shape coefficients obtained as C_1 = 1.2566 and C_2 = 0.17622.

The lateral and shear force ($Q_i^{(r)}$ and V_i) are obtained as:

$$Q_i^{(r)} = K\, W_i\, \phi_i^{(r)} C\ \alpha_h^{(r)}$$

$$V_i = (1-\gamma)\sum_{r=1}^{n} V_i^{(r)} + \gamma \sqrt{\Sigma\, (V_i^{(r)})^2} \qquad \ldots\ldots(5)$$

where, K is a constant given in code, W_i is the lumped weight, $\phi_i^{(r)}$ is the mode shape coefficient at ith level and rth mode, C_r is the mode participation factor in rth mode, r is a constant. Table 3 gives the modal displacement and Table 4 gives the shear forces at various floors of the building. Figure 4 shows the mode shapes and the distribution of shear forces.

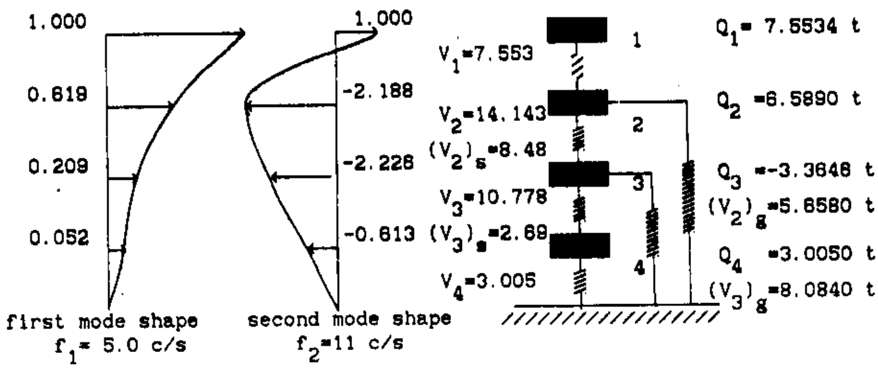

Fig.4 Mode shape and shear forces

Analysis for Torsion

Such buildings are unsymmetrical in elevation and therefore adequate provisions should be made according to IS: Code. When the centre of rigidity of various vertical resisting elements like (columns and shear walls) in a storey does not coincide with the centre of mass, torsion will occur. The distance between the centroid of rigidity and the centre of mass is usually called the eccentricity e. The eccentricity multiplied by the lateral force is the torsional moment and this moment must be resisted by the structure in addition to the normal design lateral force.

Codal Provisions for Torsion

The SEAOC Code requirement states that "Provision shall be made for the increase in shear resulting from the horizontal torsion due to an eccentricity between the center of mass and the center of rigidity. Negative torsional shears shall be neglected. In addition, where the vertical resisting elements depend on diaphragm action for shear distribution at any level, the shear resisting elements shall be capable of resisting a torsional moment assumed to be equivalent to the storey shear acting with an eccentricity of not less than five percent of the maximum building dimension at that level. The practice in USA is that

Table-3 Relative displacements

sno.	first mode Sd_1C_1=1.8121		second mode Sd_2C_2=-0.0508		z (mm)
	$\phi_1^{(1)}$	z_1(mm)	$\phi_1^{(2)}$	z_1(mm)	
1	1.000	1.8121	1.000	-0.0508	1.8128
2	0.619	1.1217	-2.188	0.1112	1.1272
3	0.209	0.3790	-2.226	0.1131	0.3955
4	0.053	0.0954	-0.613	0.0311	0.1004

z = relative displacement, Sd = spectral displacement, C = mode participation factor

Table - 4 Evaluation of shear force

sño.	first mode			second mode			shear	
	$V_1^{(1)}$	$(V_1^{(1)})_s$	$(V_1^{(1)})_g$	$V_1^{(2)}$	$(V_1^{(2)})_s$	$(V_1^{(2)})_g$	$(V_1)_s$	$(V_1)_g$
1	8.212*	8.2120	0.0000	-1.1516	-1.1516	0.0000	7.5534	0.000
2	13.295	7.9773	5.3182	1.3680	0.8208	0.5472	8.4846	5.658
3	9.122	2.2800	6.8420	2.5298	0.6325	1.8974	2.6940	8.084
4	2.424	-	2.4240	0.8678	-	0.8678	-	3.005

* The total shear V_i at a level is distributed in two components in the ratio of stiffness (i) to the interstorey stiffness $(V_i)_s$ and (ii) shear transferred to the ground $(V_i)_g$

even though there is no computed eccentricity still taking 5% of maximum building dimension as the eccentricity, the torsional moment shall be calculated and provision made for it.

The Indian Code of practice (Clause 4.2.4-IS 1893) states that provision shall be made for the increase in shear resulting from the horizontal torsion due to an eccentricity between the centre of mass and centre of rigidity. The design eccentricity shall be taken 1.5 times the computed eccentricity between the centre of mass and the centre of rigidity. Negative torsional shears shall be neglected. The significant point in the IS: Code is that 50% increase in the value of eccentricity for the calculation of torsional moments to be used in calculation of shears due to torsion.

CONCLUSIONS

A procedure is suggested to evaluate the response of a building on hill slope. The procedure is very simple and can be adopted in design offices easily. The 1D simplified model gives fairly good results as compared to the three-dimensional results and can be adopted for preliminary analysis and design based on codal methods.

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