Bull.Ind.Soc.Earth.Tech., Paper No.330, Vol.30, No.1, Mar.1993, pp.27-33

A NOTE ON MODAL ANALYSIS

V.K. Manicka Selvam and K. Jayasree
Professor Graduate Student

Department of Civil Engineering
Regional Engineering College
CALICUT - 673 601

ABSTRACT

For performing the modal analysis recommended by IS:1893-1984, modal periods and amplitudes of first three modes are necessary. For finding the same, use of a computer is inevitable. This note presents a simplified manual procedure for estimating the period and amplitudes of concrete shear building. The suggested procedure is best suited for use in preliminary design and for checking the computer solution.

INTRODUCTION

Design of tall buildings is an iterative process. During preliminary stages, use of a dependable and informative approximate method of analysis will be of assistance in making selection of member sizes and this in turn enhances the possibility of converging to an optimum design. Therefore, a good preliminary analysis and design will lead to a better final structure and reduce the number of cycles in the analysis - design iteration.

Approximate methods are also important as a means of checking computer results. Errors are often made, perhaps because of bugs in the program but more frequently because of incorrect input. A prudent analyst always checks computer results by some independent means. In many instances this can be best accomplished by a simple approximate hand computation methods.

Modern tall buildings are lighter in weight than those constructed in the past. Because of lesser weight, these buildings are highly sensitive to the effects of lateral loads caused by wind and earthquake motion. Therefore, lateral load analysis of tall buildings has acquired more importance in structural engineering practice.

RESEARCH SIGNIFICANCE

More than 99 percent of tall buildings constructed in India are less than ten storeys and this trend will continue in future also. IS:1893-1984[1] suggests either modal analysis using response spectrum method or seismic coefficient method for earthquake analysis for buildings less than 40 m in height in all the five zones of India. Of these two, the latter procedure is much simpler than the former which can be mechanically and manually implemented by a non specialist as every step is clearly outlined in the code. Simplicity makes seismic coefficient method very elegant and attractive. On the other hand, for accomplishing the modal analysis using the codal procedure, the modal periods and amplitudes for the first three modes are essential. This step necessitates the use of a computer. It is likely that computer program may not be readily available or the structural engineer may not be experienced in their proper use. Currently in the market, many soft wares are available wherein it is stated that the programmes were written according to IS:1893-1984 specifications. In such a situation it is necessary to verify which of the two methods and to what type of building the soft ware is written since there is option for the software manufacturer to choose between the two methods. Thirdly, during preliminary stages use of a computer may not be necessary. Finally, approximate methods serve the purpose of checking the computer output when the soft ware is used for the first time. Considering all these facts in this note a simplified approximate procedure is put forward for computing the periods and amplitudes manually for concrete shear buildings.

SIMPLIFIED EQUATIONS

(a) Modal Periods

Framed buildings are of two types, viz., moment resistant frames and shear frames. It is a fact that most concrete real buildings, lie in the shear zone or fall close to shear zone of frequency pattern. From the analysis of nearly one hundred and fifty buildings Radhakrishnan and Subramanian evolved the following formula for the fundamental period of shear building

$$T^2 = \frac{M}{K}\,[16\,n^2 + 16\,n + 7.3] \tag{1}$$

where

T = natural period in seconds
M = total mass of all floors in a frame
K = total stiffness of all columns in a frame
n = number of storeys.

For finding the periods of the second and third modes, the frequency ratios are furnished below.

n = 5	n = 10	n = 15
1.00	1.00	1.00
2.92	2.98	2.99
4.60	4.89	4.95

Hence from the fundamental period calculated using Eq. (1), the period of the second and third mode can be obtained to a very high degree of accuracy by making use of the frequency ratio. In this way, the periods of the first, second and third mode are reckoned.

(b) Modal Amplitudes

From a study of mode shapes of the shear building, the following expressions are proposed for finding the various amplitudes in the first three modes.

Fundamental mode : The amplitude of the fundamental mode of free vibration of a concrete shear building is given by the expression

$$a = [1 + (n'/150)^{0.5}\,]\,[n'/n]^{0.6} - [n'/150]^{0.5} \tag{2}$$

where

n = total number of storeys
n' = storey under consideration from bottom where the amplitude is needed
a = amplitude of nth storey.

The mode shape pattern is shown in Fig. 1(a).

Second mode : The second mode pattern is shown in Fig. 1(b). The profile of this mode consists of two parts AB and BC.

Profile AB : (origin at A)

$$a = \sin \frac{\pi n'}{n_1} \qquad (3)$$

where

a = amplitude of n'th storey
n = storey number counted from bottom A
n_1 = approximately $\frac{2}{3}n$ (an even number)
n - total number of floors.

NOTE : The number n_1 is determined as follows.
As an example, a building with ten storeys is considered.

$n_1 = \frac{2}{3} \times 10 = 6.66$. This can be taken as 6. A as second example, a building with 7 storeys is considered.

$n_1 = \frac{2}{3} \times 7 = 4.66$. This can be taken as 4. The number n_1 will have to be an even number.

Profile BC : (origin at B)

The profile is described by the equation

$$a = [q'/q]^{0.8} \qquad (4)$$

where

q' = storey under consideration counted from B
q = total number of storeys above B
a = amplitude of q'th storey

Third mode : The third mode pattern is shown in Fig. 1(c). The profile consists of two portions AB and BC.

Profile AB : (origin at A)

The amplitude profile is described by the function

$$a = \sin 2\pi \frac{n'}{n_1} \qquad (5)$$

where a, n' and n_1 have the same meaning described earlier.

Profile BC : (origin at B)

$$a = [s'/s] \qquad (6)$$

where

s' = storey under consideration counted from B
s = total number of storeys above B.

In Fig. 1(b) and 1(c) the signs are established from a study of computer solution. The use of above manual procedure is illustrated below in an example.

ILLUSTRATIVE EXAMPLE

Number of storeys (n)	=	10
Bay width	=	5 m
Height of floor (h)	=	3.2 m
Column size	=	0.4m x 0.6m
Beam size	=	0.3m x 0.65m
Weight of each floor in a frame	=	386.5 kN
Spacing	=	7.5 m
g	=	9.81 m/sec^2
number of bays	=	2
Young's modulus (E)	=	2.5×10^7 kN/m^2

$$M = \frac{10 \times 386.5}{9.81} = 393.985 \ \frac{\text{kN - sec}^2}{\text{m}}$$

$$K = m \ 12 \ EI/h^3$$

where m equal to number of columns in a frame.

$$\therefore \quad K = (10\text{x}3) \times 12 \times 2.5 \times 10^7 \times \frac{1}{12} \times 0.4 \times 0.6^3/3.2^3$$

$$= 1977539.06 \ \text{kN/m}$$

Substituting M, K and n in Eq. (1)

T_1 = 0.5934 sec (computer solution T_1 = 0.5894 sec)

$$P_1 = \frac{2\pi}{T_1} = 10.588 \ \text{rad/sec}$$

Second mode frequency = P_2 = 2.98 x 10.588

= 31.552 rad/sec

$$T_2 = \frac{2\pi}{31.552} = 0.1991 \text{ sec (computer solution } T_2 = 0.1979 \text{ sec)}$$

Third mode frequency = P_3 = 4.89 x 10.588

= 51.775 rad/sec.

$$T_3 = \frac{2\pi}{51.775} = 0.1213 \text{ sec. (computer solution } T_3 = 0.1206 \text{ sec.)}$$

The building is situated in zone V for which

β = 1
I = 1
F_o = 0.4

The amplitudes of the first, second the third modes computed using the proposed equations are compared with the exact solution in Table 1. Using the above data, the modal analysis was performed exactly along the lines given reference [3]. The example given in reference [3] was also reworked using the proposed method. In Table 2 the shear values of the proposed method and computer solution for the illustration problem are furnished. In Table 3 the shear values obtained by the proposed method and using the computer solution for the problem of reference [3] are given. A perusal of all these results shows that the simplified procedure proposed in the paper is quite efficacious for use in preliminary design.

SUMMARY

In reality tall concrete buildings behave like a shear building. For computing the modal periods and amplitudes of such buildings a simplified method is proposed. The prediction of the proposed method compares favourably well with the computer solution. Because of its simplicity, the method is suitable for use in preliminary design and for checking the computer solution.

REFERENCES

1. ______ IS:1893-1984, "Criteria for earthquake resistant design of structures", Indian Standards Institution, New Delhi, 1986.

2. Radhakrishnan, R. and Subramanian, N., "Dynamic analysis of multi-storey frames", Paper No. 183, Bulletin ISET, Vol. 15, No. 3, Sept. 1978, pp. 39-49.

3. "Explanatory handbook on codes for earthquake engineering", IS:1893-1975 and IS:4326-1976, Indian Standards Institution, New Delhi, 1983.

TABLE 1 - Comparison of modal amplitudes by computer solution and the proposed method

	Fundamental Mode		Second Mode		Third Mode	
Floor No.	Computer Solution	Proposed Method	Computer Solution	Proposed Method	Computer Solution	Proposed Method
1.	0.01047	0.01323	-0.03045	-0.03421	-0.04764	-0.05716
2.	0.02070	0.02153	-0.05478	-0.05925	-0.06954	-0.05716
3.	0.03046	0.02874	-0.06812	-0.06842	-0.05386	0.00000
4.	0.03953	0.03537	-0.06780	-0.05925	-0.00907	0.05716
5.	0.04771	0.04161	-0.05387	-0.03421	0.04061	0.05716
6.	0.05481	0.04757	-0.02912	0.00000	0.06835	0.00000
7.	0.06066	0.05331	0.01461	0.02257	0.05916	-0.01650
8.	0.06515	0.05890	0.03175	0.03929	0.01800	-0.03300
9.	0.06815	0.06431	0.05567	0.05435	-0.03289	-0.04950
10.	0.06962	0.06962	0.06842	0.06842	-0.06600	-0.06600

TABLE - 2 Comparison of the proposed method solution with the computer results

	Shear force V_1 using modal analysis in KN	
	Computer solution	Proposed method
1.	52.06	53.52
2.	49.73	49.87
3.	46.34	47.07
4.	42.94	43.05
5.	40.30	41.52
6.	36.00	39.20
7.	30.54	34.00
8.	24.68	27.30
9.	17.48	19.20
10.	8.69	9.71

TABLE - 3 Shear values of the problem given in reference [3]

Shear force V_1 using modal analysis, in tons		
Floor No.	Computer solution	Proposed method
1.	293.8	308.5
2.	287.3	295.5
3.	273.8	279.3
4.	259.6	257.8
5.	246.0	239.9
6.	233.8	224.9
7.	224.3	216.9
8.	210.5	213.3
9.	192.5	207.7
10.	173.0	196.4
11.	154.8	175.9
12.	131.7	149.4
13.	103.0	117.3
14.	69.0	79.8
15.	30.7	36.8

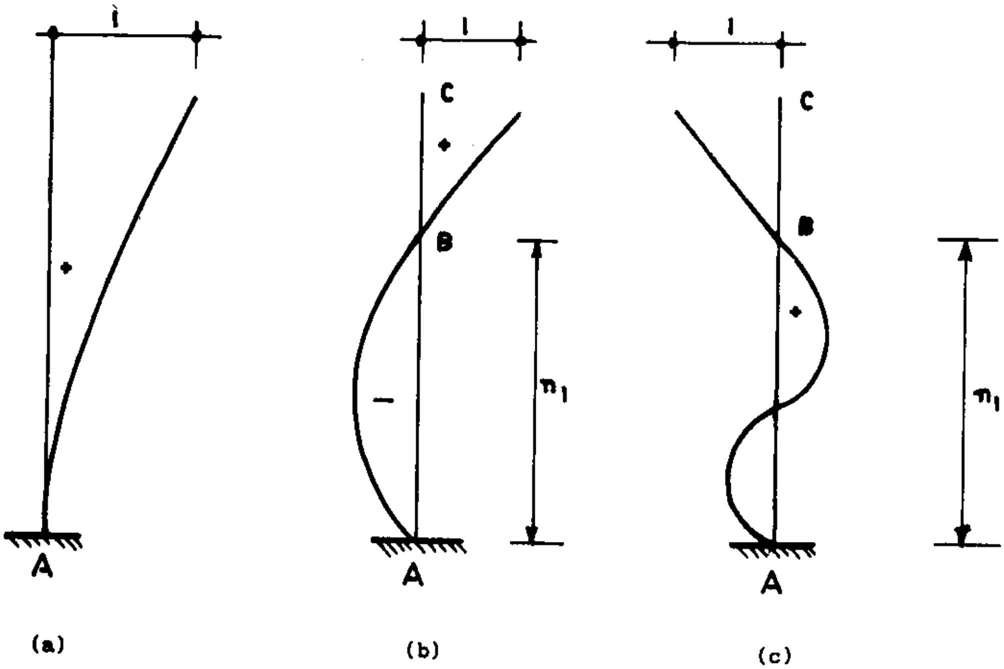

Fig. 1: Different mode patterns. (a) Mode 1 (b) Mode 2 (c) Mode 3