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ON CRITICAL EARTHQUAKE DIRECTION AND TREATMENT OF ACCIDENTAL TORSIONAL RESPONSE IN 3D FRAME ANALYSIS

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Abstract

The conventional approach of seismic analysis of multistorey buildings involves dynamic analysis under a pure translational excitation, usually applied in two orthogonal directions combined with an additional quasi-static analysis for torsional response. The choice of two orthogonal directions for applying seismic excitation, is easy to make when structure the is symmetrical, but for unsymmetrical structures the choice of seismic excitation direction cannot be made arbitrarily. The critical seismic excitation direction provides a way to identify the most unfavourable direction of seismic excitation for various modes but fails to do so in case of a symmetric structure having pure torsional modes. To this end mass perturbation procedure has been studied, primarily to account for accidental unsymmetry. By mass perturbation torsional modes can be excited in symmetric structures. Mass perturbation procedure together with critical direction alleviates the need for dual dynamic and static analysis to model the torsional response as all effects are accounted for within the realm of dynamic analysis only.

INTRODUCTION

With the computational capabilities now available it is not difficult to conduct a three dimensional (3D) frame analysis of buildings. While doing a 3D analysis one faces two problems (amongst others) *viz.* (a) the direction in which earthquake excitation is to be applied and (b) taking into account accidental torsional forces.

Generally the analysis is done by applying earthquake excitation in at least two orthogonal directions and to take into account accidental torsional response, an additional quasi-static analysis is conducted (Newmark and Rosenblueth, 1971).

The practice of analysis a structure by applying seismic excitation in two orthogonal directions poses problems when these directions have to be chosen. The choice of orthogonal directions is obvious when the structure is symmetrical. However, for unsymmetrical buildings - for example those that are L or Y shaped in plan; the choice of seismic excitation directions cannot be made arbitrarily.

In this paper the concept of applying the seismic excitation in a direction, in which the structure is weak or "critical direction" is studied. Critical directions of seismic excitation are the most unfavourable directions of seismic excitation and in this case the response of the structure will be the most severe.

The problem of torsional response is also very important and it has been addressed in the seismic codes of practice of many countries. The damage generally occurs when the so called centre of rigidity and centre of mass do not coincide or due to torsional component of ground motion. For the analysis of torsional response most codes resort to a static analysis wherein provisions are made for the increase in shear resulting from torsion about a vertical axis due to an eccentricity between centre of mass and centre of rigidity. The evaluation of the floor eccentricities requires some additional computational effort and the construction of an additional model for static analysis. It is, therefore, logical to search for the possibilities of obtaining the desired torsional forces within the framework of the dynamic analysis only.

CRITICAL EARTHQUAKE DIRECTION

Seismic design of structures invariably involves the use of the design spectra. The spectra may either be obtained from the relevent code of practice or may be specifically tailor-made for the site. This spectra may be used as it is for analysis in conjunction with the response spectrum method. In some cases, a spectra compatible acceleration time history is generated to be used for time-wise response analysis. In most cases, the analysis of primary structures involves the use of single spectra or a single time history. In other words identical spectra or time history is used to represent seismic loading in the so called "x and y horizontal directions" of the structures. Often when the horizontal excitation is combined with vertical excitation, a scaled down spectra or time history is used.

The use of identical or scaled response spectra or acceleration time history in the three directions implies that the ground motion in the three orthogonal directions are assumed to be in phase and perfectly correlated. Such perfect correlation automatically implies that the excitation is along a specific direction. For example if identical seismic excitation is applied in two orthogonal horizontal directions x and y then it is equivalent to applying a single seismic excitation at an angle of 45° from x or y horizontal axes. Clearly the magnitude of this excitaiton would be scaled up by a factor $\sqrt{2}$.

There is inherently nothing wrong in applying seismic excitation along a specific direction. It is, however, important that the chosen direction/directions of excitation be the ones that are the most unfavourable to the structure. This feature has been recognised by several authors (Bicanic *et al.*, 1986; Pankaj and Bicanic, 1988; Singh, 1991).

Critical directions of seismic excitation can be computed by maximisation of the modal amplitudes (Bicanic *et al.*, 1986, Singh, 1991) and lead to the most unfavourable directions of seismic excitation for each of the vibration modes. The familiar equations of motion of the discretized system undergoing base excitation are given by

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\ddot{\mathbf{y}} \tag{1}$$

where $\mathbf{x}$, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ denote relative displacement, relative velocity and relative acceleration vectors and $\ddot{\mathbf{y}}$ is the ground acceleration vector. If the components of the ground acceleration vector are denoted by $\ddot{Y}_x$, $\ddot{Y}_y$ and $\ddot{Y}_z$ then

$$\begin{aligned} \ddot{Y}_x &= \ddot{Y}_g \cos\psi_x \\ \ddot{Y}_y &= \ddot{Y}_g \cos\psi_y \\ \ddot{Y}_z &= \ddot{Y}_g \cos\psi_z \end{aligned} \tag{2}$$

where $\ddot{Y}_g(t)$ is the scalar function of ground acceleration in time. Thus the equations of motion can be simplified to read

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\boldsymbol{\alpha}\ddot{Y}_g(t) \tag{3}$$

where $\boldsymbol{\alpha}$ is the vector of ground acceleration direction cosines. Thus if the three dimensional frame structure has n nodes and each node has 6 degrees of freedom (*dof*) and if the first n *dof* represent the translation in the x direction, the second n *dof* represent the translation in the y direction, the third n *dof* represent the translation in the z direction and the last $3n$ *dof* represent rotation about x, y and z axes respectively then

$$\begin{aligned} \boldsymbol{\alpha}^T = \ & (\cos\psi_x,\ \cos\psi_x,\ \ldots \quad n \text{ times} \\ & \cos\psi_y,\ \cos\psi_y,\ \ldots \quad n \text{ times} \\ & \cos\psi_z,\ \cos\psi_z,\ \ldots \quad n \text{ times} \\ & 0,\ 0, \ldots \qquad\qquad 3n \text{ times}) \end{aligned}$$

Clearly here the assumption that the ground motion in the three directions are perfectly correlated and in phase has been made. It has also been assumed that the torsional component of ground motion is absent.

Substituting displacement x as the product of eigen vector matrix $\boldsymbol{\Phi}$ and modal amplitude vector $\boldsymbol{\xi}$ *i.e.* $\mathbf{x} = \boldsymbol{\Phi}\,\boldsymbol{\xi}$ in equation 3 and using the principle of orthogonality one obtains the familiar decoupled equations of motion as

$$\ddot{\xi}_i + 2\varsigma_i\omega_i\dot{\xi}_i + \omega_i^2\xi_i = -C_i\ddot{Y}_g(t) \tag{4}$$

where

$$\omega_i^2 = \frac{\boldsymbol{\phi}_i^T\mathbf{K}\,\boldsymbol{\phi}_i}{\boldsymbol{\phi}_i^T\mathbf{M}\,\boldsymbol{\phi}_i} \tag{5}$$

and

$$C_i = \frac{\boldsymbol{\phi}_i^T\mathbf{K}\,\boldsymbol{\alpha}}{\boldsymbol{\phi}_i^T\mathbf{M}\,\boldsymbol{\phi}_i} \tag{6}$$

are the square of the i^{th} natural frequency and mode participation factor respectively, while $\boldsymbol{\phi}_i$ is the i^{th} eigenvector and ς_i is the damping ratio in the i^{th} mode.

If one is interested in time-wise response then the modal amplitude as a function of time $\xi_i(t)$ can be found using the Duhamel integral. On the other hand the maximum modal amplitude $\hat{\xi}_i$ can be found using the response spectrum. In either case the modal amplitude depends on the mode participation factor C_i which in turn depends on α the direction of ground acceleration vector. To maximise ξ_i or $\hat{\xi}_i$ it is necessary to find the maximum value of the mode participation factor or of the scalar quantity

$$q_i = \boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\alpha} \tag{7}$$

which can be written for a diagonal mass matrix as

$$q_i = \alpha_1 \sum_{j=1}^{n} \phi_{ji} m_j + \alpha_2 \sum_{j=n+1}^{2n} \phi_{ji} m_j + \alpha_3 \sum_{j=2n+1}^{3n} \phi_{ji} m_j \tag{8}$$

where ϕ_{ji} is the mode shape value of the j^{th} *dof* for the i^{th} eigenvector; m_j is the translational mass at node j; and α_1, α_2 and α_3 represent $\cos\phi_x$, $\cos\phi_y$ and $\cos\phi_z$ respectively. Equation 8 can be compactly written as

$$q_i = \sum_{l=1}^{3} \alpha_l b_l \tag{9}$$

The direction cosines are constrained by

$$\sum_{l=1}^{3} \alpha_l^2 = 1 \tag{10}$$

which leads to the maximum value of q_i given by

$$\begin{aligned} \frac{\partial q_i}{\partial \alpha_1} = 0 \;\Rightarrow\; b_1 - \frac{b_3 \alpha_1}{\sqrt{1 - \alpha_1^2 - \alpha_2^2}} = 0 \\ \frac{\partial q_i}{\partial \alpha_2} = 0 \;\Rightarrow\; b_2 - \frac{b_3 \alpha_2}{\sqrt{1 - \alpha_1^2 - \alpha_2^2}} = 0 \end{aligned} \tag{11}$$

Solving the above simultaneous equations and using equation 10 one gets

$$\alpha_i = \frac{b_i}{\sqrt{\sum_{l=1}^{3} b_l^2}} \qquad i = 1, 2, 3 \tag{12}$$

and

$$q_i(\max) = \sqrt{\sum_{l=1}^{3} b_l^2} \tag{13}$$

The above equation allows the evaluation of maximum of the maximum modal amplitude as

$$\hat{\xi}_i(\max) = \frac{q_i(\max)}{\boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\phi}_i} S_{d_i} \tag{14}$$

The after eigen analysis, it is easy to find the critical earthquake direction for each mode of vibration with the aid of equation 12.

STUDY OF TWO EXAMPLE BUILDINGS

In order to study the concept of critical directions of seismic excitation two two-storey buildings are considered. The first building is symmetrical with a square plan (Figure 1) and the second is a L-shaped unsymmetrical building (Figure 2).

In each case masses are assumed to be lumped at beam and column joints. A diagonal mass matrix with negligible mass moment of inertia is used. The seismic loading in the form of an acceleration spectra as given in IS-1893 (1984) was used.

Example 1: Symmetrical building

The first six frequencies and mode shapes were computed to illustrate the concept of critical earthquake directions. The mode shapes have been illustrated in Figure 3. In these mode shape plots only joint translational components have been plotted.

The first two modes are translational modes in x and y directions respectively (Figures 3(a) and (b)). The third mode (Figure 3(c)) is a pure torsional mode with both floors rotating the same way. The fourth mode (Figure 3(d)) is a mode in which diagonally opposite columns move towards/away from each other. Obviously this mode would be absent if floor stiffnesses were taken into account. The fifth mode (Figure 3(e)) is again a translational mode in x direction with both floors moving in opposite directions. The sixth mode (Figure 3(f)) is again a torsional mode with one floor rotating clockwise while the other rotates anticlockwise.

The critical earthquake directions were computed for the first six modes with the aid of equation 13. The direction cosines of these critical directions are tabulated in Table 1.

Table 1: Critical directions for Example 1

Mode	α_1	α_2	α_3
1	1	0	0
2	0	1	0
3	Indeterminate	Indeterminate	Indeterminate
4	Indeterminate	Indeterminate	Indeterminate
5	1	0	0
6	Indeterminate	Indeterminate	Indeterminate

For a purely symmetrical building, no translational earthquake will excite a purely torsional mode. As a result, critical directions corresponding to torsional modes are indeterminate. The critical directions corresponding to modes 1 and 2 are employed as seismic excitation directions to evaluate mode participation factors, C_i, in the first

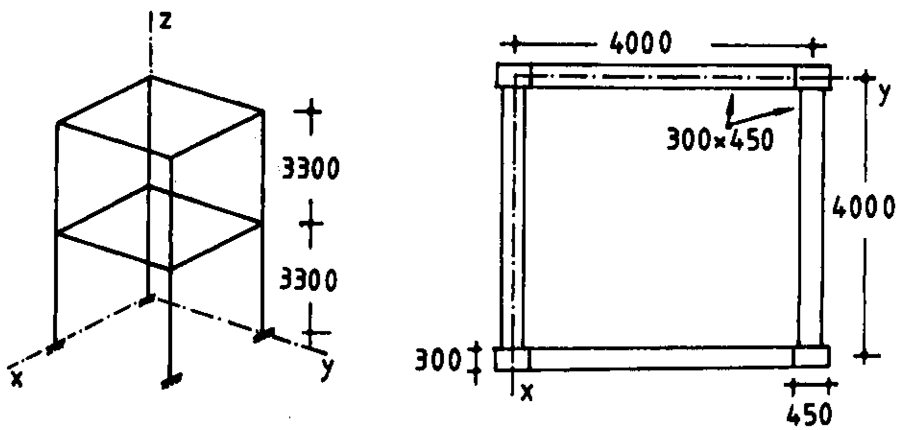


Figure 1: Symmetrical building square in plan

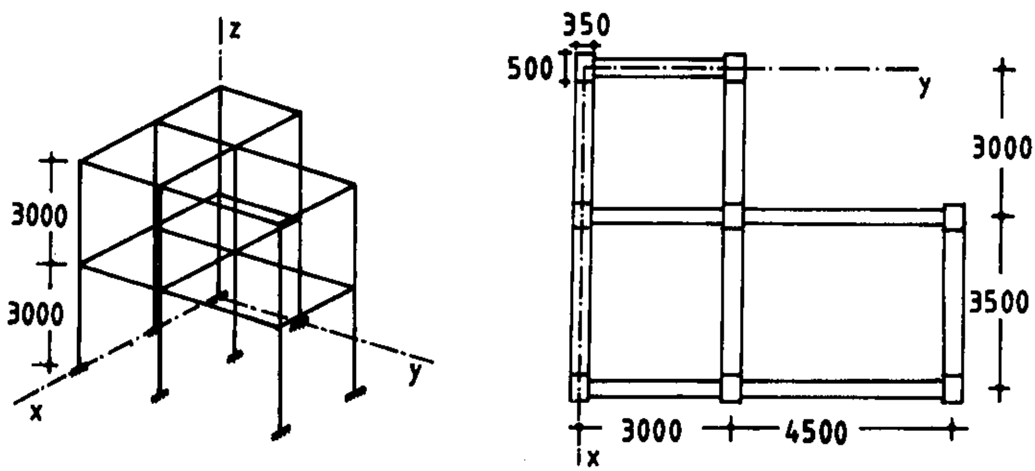


Figure 2: Unsymmetrical L-shaped building

six modes of vibration. In addition an arbitrary earthquake direction corresponding to $\alpha = (0.894,\ 0.447,\ 0.000)$ was also employed.

The mode participation factors for seismic excitation in critical directions computed from the first two modes are given in Table 2. The table also gives mode participation factors for the above mentioned arbitrary excitation direction.

Table 2: Mode participation factors for different earthquake directions for Example 1

Mode	$\alpha_1 = 1.000$ $\alpha_2 = 0.000$ $\alpha_3 = 0.000$	$\alpha_1 = 0.000$ $\alpha_2 = 1.000$ $\alpha_3 = 0.000$	$\alpha_1 = 0.894$ $\alpha_2 = 0.447$ $\alpha_3 = 0.000$
1	165.3	0.0	0.0
2	0.0	163.4	73.1
3	0.0	0.0	0.0
4	0.0	0.0	0.0
5	50.1	0.0	45.4
6	0.0	0.0	0.0

While the x and y direction earthquake (critical directions corresponding to modes 1 and 2) excite only corresponding translational modes, an earthquake applied at an inclination excites both x and y direction modes. Table 2 also illustrates that for a symmetrical building no translational earthquake is able to excite a torsional mode of vibration. Thus mode participation factors for modes 3 and 6 would be zero for any translational earthquake direction. As a result when seismic excitation is applied in x direction (critial direction obtained from mode 1) member forces will be contributed by modes 1 and 5 only (out of first six modes). When seismic excitation is applied in the y direction ($\alpha = (0,\ 1,\ 0)$), forces are contributed by mode 2 only out of the first six modes. When seismic excitation is applied in an inclined direction corresponding to $\alpha = (0.894,\ 0.447,\ 0.000)$ forces are contributed by modes 1,2 and 5.

Example 2: Unsymmetrical building

The first six frequencies and mode shapes were computed in order to illustrate the concept of critical earthquake directions. The mode shapes along with corresponding frequencies are shown in Figure 4. In these mode shape plots only translations have been plotted.

The first mode (Figure 4(a)) is predominantly a translational mode in y direction. The second and third modes (Figures 4(b) and (c)) are a combination of translation and rotation with translation being predominantly in y and x directions respectively. The modes 4 and 5 (Figures 4(d) and (e)) would perhaps be absent if floor rigidities were taken into account. Mode 6 (Figure 4(f)) is again a translational mode in y direction with both floors moving the opposite way.

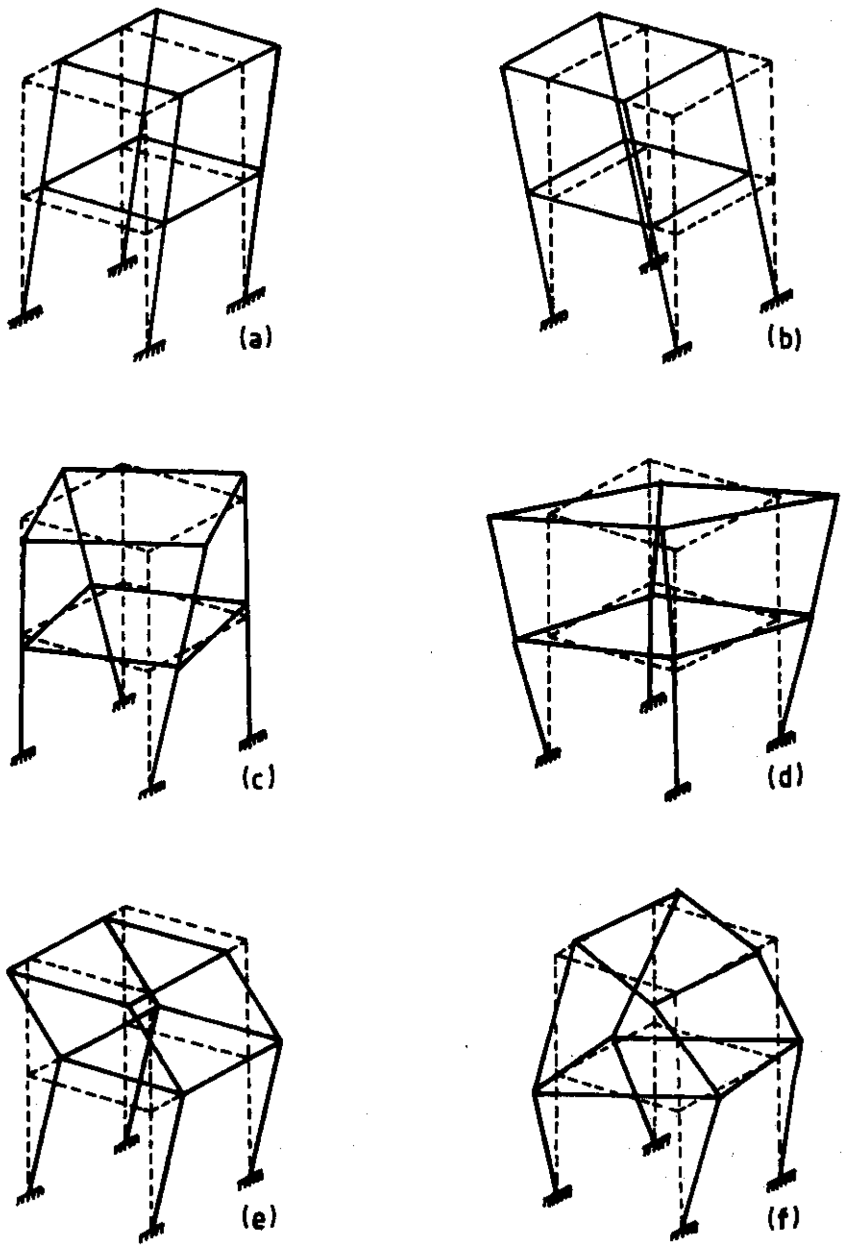


Figure 3: Mode shapes (translations only) for Example 1. (a) Mode 1 (3.31 Hz) (b) Mode 2 (4.25 Hz) (c) Mode 3 (4.27 Hz) (d) Mode 4 (8.63 Hz) (e) Mode 5 (9.51 Hz) (f) Mode 6 (11.07 Hz)

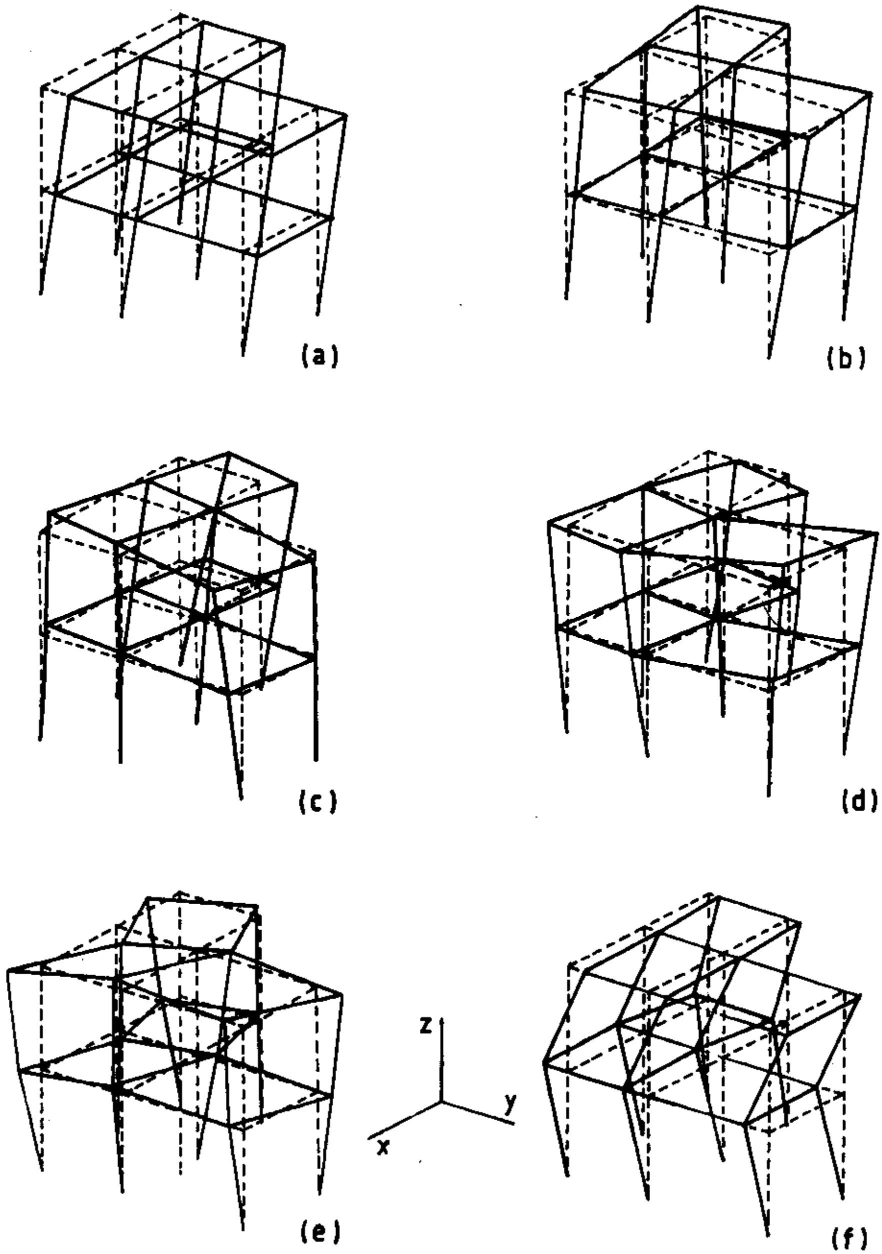


Figure 4: Mode shapes (translations only) for Example 2. (a) Mode 1 (4.07 Hz) (b) Mode 2 (5.07 Hz) (c) Mode 3 (5.42 Hz) (d) Mode 4 (8.39 Hz) (e) Mode 5 (11.46 Hz) (f) Mode 6 (11.61 Hz)

The critical earthquake directions were computed for the first six modes with the aid of equation 14. The direction cosines of these critical directions are tabulated in Table 3.

Table 3: Critical directions for Example 2 (truncated after three decimal places)

Mode	α_1	α_2	α_3
1	−0.018	0.999	−0.000
2	−0.292	−0.956	0.016
3	−0.993	0.113	0.002
4	−0.382	0.923	−0.023
5	0.620	−0.784	0.000
6	0.011	−0.999	−0.005

The critical directions corresponding to modes 1,2 and 3 were employed as seismic excitation directions to evaluate mode participation factors in the first six modes of vibration. It is important to observe that in a unsymmetrical building, critical directions of seismic excitation are inclined directions. This is due to the fact that all the six modes of vibration are mixed types of modes (translational as well as torsional). Table 3 indicates that although modes 1 and 6 are predominantly translational modes in y direction they are not completely so.

The mode participation factors for seismic excitation in critical directions computed from the first three modes are given in Table 4. Table 4 shows that in an unsymmetrical building in which pure translational or pure torsional modes are absent all modes contribute to the response.

Table 4: Mode participation factors for different earthquake directions for Example 2

Mode	$\alpha_1 = -0.018$ $\alpha_2 = 0.999$ $\alpha_3 = -0.000$	$\alpha_1 = -0.292$ $\alpha_2 = -0.956$ $\alpha_3 = 0.016$	$\alpha_1 = -0.993$ $\alpha_2 = 0.113$ $\alpha_3 = 0.002$
1	245.3	−233.4	31.7
2	−10.9	74.2	202.8
3	16.9	24.1	130.8
4	6.1	−5.1	3.2
5	−8.1	5.8	−7.1
6	−75.6	72.1	−9.2

ACCIDENTAL TORSIONAL RESPONSE

The torsional response in structures can arise due to unsymmetry (planned or accidental) or due to torsional component of ground motion. With regard to the latter, torsional spectra are still not available. In this study, consideration will be confined to torsional response caused by the non-coincidence of centre of mass and centre of rigidity which may arise due to accidental or planned unsymmetry in buildings.

Computation of torsional response — recommended procedures

Most codes of practice recommend a static method of analysis for the evaluation of torsional response. The method requires the evaluation of the locations of centre of mass and centre of rigidity to evaluate eccentricities. While it is easy to evaluate the location of centre of mass, both at each floor level as well as its position in space, the evaluation of rigidity centre presents a number of difficulties. Firstly, the resisting elements span from floor to floor and it is not easy to specify the centre of rigidity to a particular floor. Secondly, the changing centre of rigidity in each storey adds to the complications in static analysis. It is, perhaps, with this in mind ATC (Applied Technological Council) [1] recommends using the torsional moment on the basis of a displacement which depends upon the dimension of the building perpendicular to the direction of applied force. As per the 1992 world list of Earthquake Resistant Regulations [2] the codes of Argentina, Canada, Chile, Ethiopia, Indonesia, Israel, Mexico, Peru, Turky, U.S.A. and Venezuela recommend procedures which involve application of torsional moments caused by eccentricity between centre of mass and centre of rigidity as well as fraction of building plan dimensions. Thus torsion caused due to torsional irregularity as well as due to accidental causes is incorporated in design. The codes further recommend amplification of torsional irregularity (if it exists) for design purposes. Accidental eccentricity, which may arise due to uncertainties in location of loads, is envisaged, for design purposes, to be of the order of 5% to 10% of plan dimension perpendicular to the earthquake loading direction. On the other hand IS 1893-1984 relies completely on the eccentricity calculations, thus accidental sources of torsion are not taken into account. The National Building Code of Canada puts a condition for the applicability of static torsional analysis which states that the centre of mass and the centre of rigidity should fall approximately on two vertical lines. No such condition exists in IS 1893-1984. Some codes recommend the use of three dimensional dynamic modal analysis in many cases, with mass configuration perturbed. However, methodologies for such perturbations have not been outlined.

To summarise either a static torsional analysis is used for the evaluation of torsional response or a three dimensional dynamic analysis is recommended.

Mass perturbation

Critical seismic excitation direction fails in the case of an ideally symmetric structure having pure torsional modes. Mass perturbation provides a method to avoid ideal symmetry. It can also be used to increase torsional response in unsymmetrical buildings.

Mass perturbation is performed by changing the mass distribution in the structural model, such that the new centre of mass is moved by a fraction of mass inertia gyration radius along a perturbation axis and hence does not coincide with the centre of stiffness. Using this procedure the need to evaluate the centre of rigidity is avoided. Here discussion is confined to mass perturbation for the planar case *i.e.* redistribution of floor masses, though this procedure can be generalized for a 3D situation (Bicanic *et al.*, 1986; Singh, 1991). Consider the mass distribution as shown in the plan of Figure 5 with centre of mass at CM. An axis of mass perturbation w at any angle is selected. For best effect it is suggested that this axis be chosen perpendicular to critical earthquake direction. The centre of mass is now moved to its new position $\overline{CM}$ while preserving total mass on axis $\bar{p}$. Both p and $\bar{p}$ are axes perpendicular to w. This shift is equal to a chosen percentage of gyration radius as shown in Figure 5. The two conditions — (a) preservation of total mass and (b) shift of mass centre lead to two scaling factors on two sides (L and R) of axis p (Figure 5). From the first condition one obtains

$$\sum m_i = M = (1-\beta_1)\sum m_i^L + (1+\beta_2)\sum m_i^R \tag{15}$$

while from the second one gets

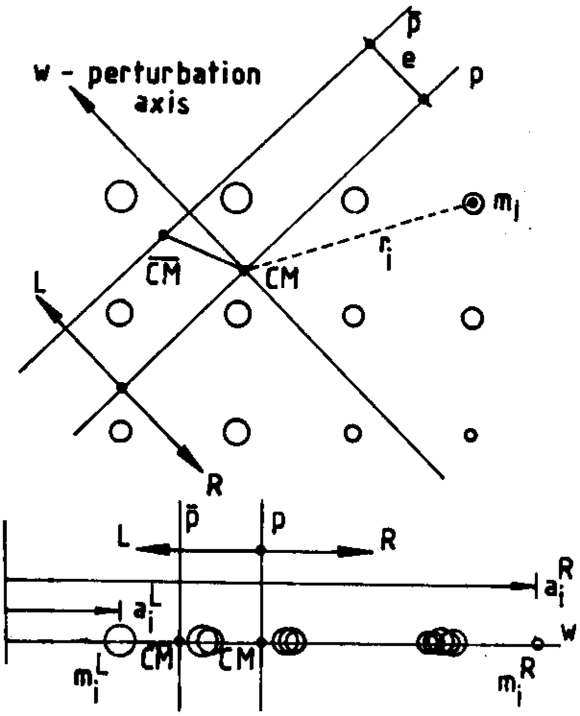
$$M(a_0+e) = (1-\beta_1)\sum m_i^L a_i^L + (1+\beta_2)\sum m_i^R a_i^R \tag{16}$$

Solving the above two equations for β_1 and β_2 one gets

$$\begin{aligned} \beta_1 &= \frac{Me\sum m_i^R}{\sum m_i^L \sum m_i^R a_i^R - \sum m_i^R \sum m_i^L a_i^L} \\ \beta_2 &= \frac{Me\sum m_i^L}{\sum m_i^L \sum m_i^R a_i^R - \sum m_i^R \sum m_i^L a_i^L} \end{aligned} \tag{17}$$

The two simultaneous conditions in practice may lead to movement of $\overline{CM}$ along $\bar{p}$ away from the perturbation axis thus resulting in an eccentricity which is more than e.

The above procedure eliminates the need to evaluate the centre of rigidities and takes into account accidental eccentricities. Moreover, the analysis does not require "static" torsional analysis as the complete analysis can take place in a dynamic domain. This procedure can be combined with the critical earthquake direction concept and the perturbation can be performed along a perturbation axis that is perpendicular to the critical earthquake direction. It should be obvious that mass perturbation would change the critical earthquake directions. It is proposed that the critical directions corresponding to the unperturbed structure be used for the dynamic torsional analysis.



$$\text{Gyration radius} = \sqrt{\frac{\sum m_i r_i^2}{\sum m_i}}$$

$$e = \text{fraction} \times \text{Gyration radius}$$

Figure 5: Mass perturbation in a plane

Example 3: Mass perturbation for the symmetric building

In order to study the mass perturbation procedure the symmetric building analysed earlier was considered. Planar mass perturbation was employed for each floor level. The radius of gyration was evaluated to be 2.83 m. An eccentricity of 20% of radius of gyration was assumed to illustrate the procedure. Perturbation was performed along a perturbation axis being taken parallel to the x axis. This corresponds to the second mode critical earthquake direction evaluated earlier. The factors β_1 and β_2 for this case came out to be 0.2 and 0.2.

Comparison of frequencies (Figures 3 and 6) shows that the difference in frequencies is small although considerable perturbation (20% of radius of gyration) has been employed. This supports the mass perturbation procedure employed. As the frequency response is not altered considerably, thereby, if response spectrum analysis is performed, the spectral values corresponding to different modes would not be significantly altered.

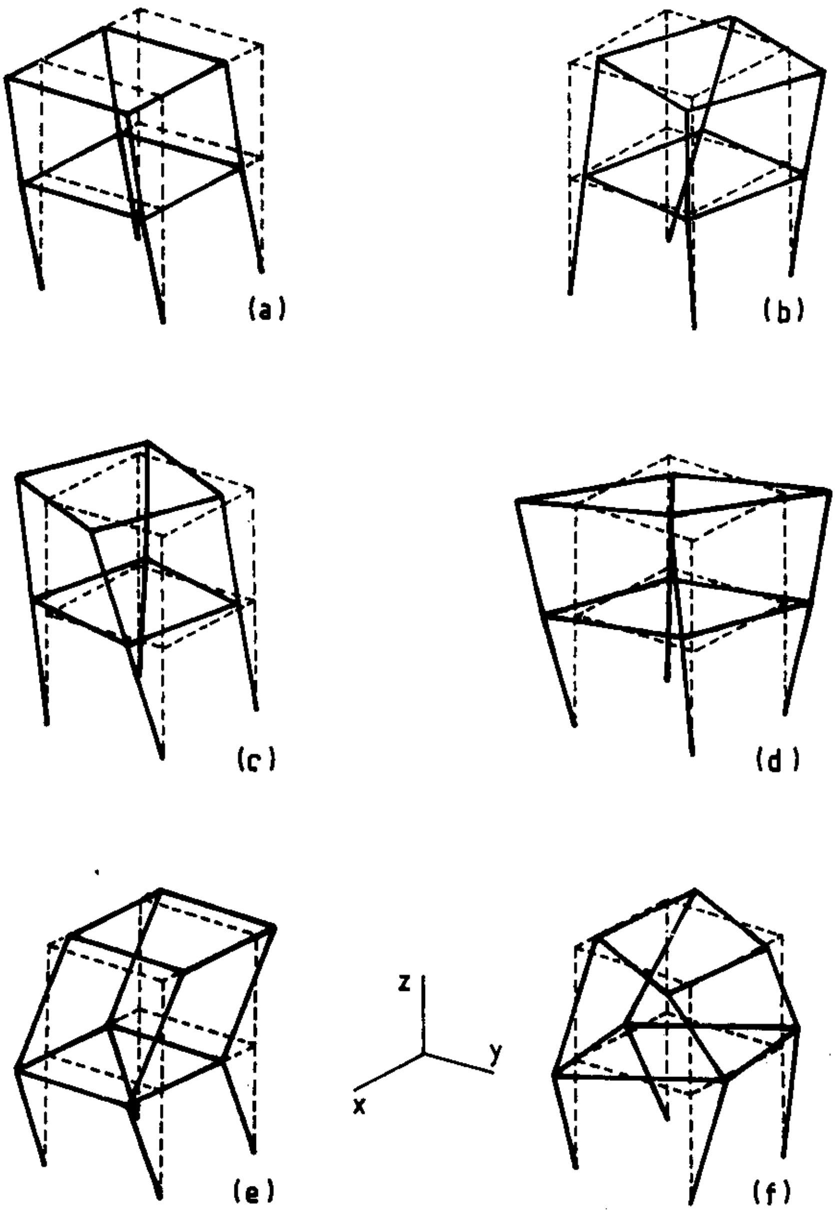


Figure 6: Mode shapes (translations only) after mass perturbation (Example 3). (a) Mode 1 (3.31 Hz) (b) Mode 2 (4.00 Hz) (c) Mode 3 (4.56 Hz) (d) Mode 4 (8.77 Hz) (e) Mode 5 (9.51 Hz) (f) Mode 6 (11.05 Hz)

With regard to mode shapes shown in Figure 6, it can be seen that for the perturbation axis parallel to the x direction, except for modes 1 and 5 all other modes (amongst the first six) are a combination of translational and torsional components. Modes 1 and 5 are pure translational modes in the x direction as the mass and stiffness components on both sides of this axis (passing through centre of mass) have identical mass and stiffness components.

A comparison of mode participation factors for the perturbed and unperturbed cases is given in Table 5. It can be seen that for an earthquake in y direction and perturbation axis along x direction, the contribution of first and fifth modes is zero. However, the most important observation is that modes 3 and 6 which were torsional modes for the symmetric case, have non-zero participation factors in the perturbed case.

Table 5: Comparison of Mode Participation Factors (MPF) in symmetrical building prior to and after mass perturbation with earthquake applied in the y–direction.

	MPF before	MPF after
1	0.0	0.0
2	163.4	125.7
3	0.0	104.2
4	0.0	7.6
5	0.0	0.0
6	0.0	7.5

Mass perturbation induces torsional effects if they are absent and if the torsional effects do exist in the original structure, they would be taken care of by an unperturbed analysis. It is proposed that a three dimensional unperturbed analysis be first carried out followed by an analysis subsequent to mass perturbation. The increased forces may be taken into consideration for design while the decreased member forces be neglected. Futher, perturbation be carried out on both sides of the perturbation axis.

CONCLUSIONS

Earthquake forces may be applied along critical directions rather than using arbitrarily chosen orthogonal directions and then combined using some statistical technique to give design forces. Thus most unfavourable response of the structure will be accounted for.

Using mass perturbation torsional modes in symmetrical structures can be excited. The natural periods of vibration, of the simple building studied, remain virtually unchanged even after considerable perturbation, using the procedure discussed, was employed. For torsionally irregular structures this procedure can be used to accentuate torsional response which may arise due to accidental causes. The process avoids the need of dual analysis (dynamic and static) to model torsional response. Thus mass perturbation procedure, together with critical direction concept provides a simple and efficient way of three dimensional analysis of multistorey buildings.

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