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SUGGESTED REQUIREMENTS ON MAXIMUM TENSION REINFORCEMENT FOR FLEXURAL MEMBERS IN IS:4326

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ABSTRACT

The provision of IS:4326-1976 on maximum tension steel ratio for reinforced concrete flexural members is reviewed. This provision intends to provide curvature ductility of 5, assuming unconfined concrete. However, a discrepancy exists in the method by which this provision is arrived at. A procedure is described herein to calculate the maximum tension steel ratio that ensures a specified value of curvature ductility. Simple empirical expressions are proposed for the maximum tension steel ratio, which ensure curvature ductility of 5.

INTRODUCTION

The philosophy of earthquake resistant design requires that a structure should be capable of surviving a severe earthquake without collapse. This requires the structure to have adequate ductility. Such structures are subjected to less earthquake force and are able to absorb and dissipate seismic energy by inelastic deformations. A measure of the ductility of a structure is the displacement ductility factor, μ, defined as $\mu = \Delta u/\Delta y$; where Δu is the lateral deflection at the ultimate condition and Δy is the lateral deflection at first yield. These displacements are measured at a suitable point, for instance at the roof level. In a structure which is responding inelastically to earthquake excitation, plastic hinges are formed at critically stressed sections of beams and columns. Here, these sections must also have adequate ductility. A measure of the ductility of a section subjected to flexure or combined flexure and axial load is the curvature ductility, which is defined as the ratio of ultimate curvature (ϕ_u) attainable without significant loss of strength, to the curvature corresponding to first yield of the tension reinforcement (ϕ_y). Typical values of the displacement ductility factor, μ, for reinforced concrete structures range from 3 to 5 (Ref. 9). This requires the sections at the plastic hinges in beams and column bases to have curvature ductility ranging from 12 to 20, i.e., $\phi_u/\phi_y \geq 4\mu$ (Ref. 6). However, reinforced concrete flexural members may be designed for curvature ductility of 5, assuming concrete as unconfined (Refs. 6, 7 and 10). Confined concrete has a higher value of crushing strain which increases curvature ductility significantly. Since concrete is always confined with transverse reinforcement near the column face where the plastic hings forms, the actual curvature ductility will be significantly greater than that calculated by assuming the concrete as unconfined.

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Curvature ductility increases with an increase in compression steel content, concrete compressive strength, and crushing strain of concrete. It decreases with an increase in tension steel content, yield strength of steel, and axial load on the member. Among these factors, the percentage of tension steel plays a dominant role in providing curvature ductility. Hence, most building codes impose restrictions on the maximum tension reinforcement in reinforced concrete flexural members.

In this paper, the provision of IS:4326-1976 (Ref. 3) on maximum tension steel ratio for reinforced concrete flexural members has been reviewed. The code defines a flexural member as one in which the average axial stress is less than 0.10 f_{ck}, where f_{ck} is the characteristic strength of concrete cube. The provision for maximum tension steel ratio intends to ensure curvature ductility of 5 (Ref. 10), assuming concrete as unconfined. However, a discrepancy exists in the method by which this provision is arrived at. A procedure is outlined herein to calculate the maximum tension steel ratio to ensure a required value of curvature ductility for reinforced concrete flexural members which are not subjected to any axial load. On the basis of a parametric study, simple empirical expressions are suggested for the maximum tension steel ratio, which ensure curvature ductility of 5.

PROVISION OF IS:4326-1976

IS:4326-1976 stipulates that the maximum tension steel ratio, ρ_{max}, on any face, at any section, must not exceed $\rho_c + 0.19\ f_{ck}/f_y$ for mild steel reinforcement; and $\rho_c + 0.15\ f_{ck}/f_y$ for cold worked deformed bars; where ρ_c is the steel ratio on the compression face, and f_y is the yield stress of steel.

Consider a doubly reinforced section at its ultimate strength (Fig. 1). The non-dimensional depth of the neutral axis at ultimate condition, Ku, is given by

$$Ku = \left\{ \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{su}} \right\} \tag{1}$$

where ε_{cu} is the crushing strain of concrete and ε_{su} is the strain in tension steel at the ultimate strength of the section. SP:22 (Ref. 10), the explanatory handbook to IS:4326-1976, states that to have curvature ductility of 5, strain in tension steel should be 5 times ε_y at the ultimate strength of the section, i.e., $\varepsilon_{su} = 5\ \varepsilon_y$, where ε_y is the yield strain of steel. This gives the following equation for the maximum tension steel ratio, ρ_{max} :

$$(\rho_{max} - \rho_c)\frac{f_y}{f_{cu}} \leq \left\{ \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 5\ \varepsilon_y} \right\} \tag{2}$$

where f_{cu} is the average stress in concrete at the ultimate strength of the section. SP:22 uses this equation to derive expressions for ρ_{max}. However, this does not ensure curvature ductility of 5. To explain this, consider a doubly reinforced section at yield (Fig. 2). The curvature at yield is given by $\phi_y = \varepsilon_y/d(1.0\text{-}K)$, where d = effective depth of member, and K = non-dimensional depth of the neutral axis at yield. The ultimate curvature is given by $\phi_u = \varepsilon_{su}/d(1.0\text{-}Ku)$ (Fig. 1). As per SP:22, $\varepsilon_{su} = 5\,\varepsilon_y$. Hence, curvature ductility will be

$$\frac{\phi_u}{\phi_y} = \frac{5\,(1.0-K)}{(1.0-Ku)} \qquad (3)$$

From equation (3), it is evident that curvature ductility will be 5 only if $K = Ku$, i.e., the depth of the neutral axis at yield ($K\,d$) and at ultimate ($Ku\,d$) is equal. However, for an under-reinforced section, Ku is always less than K. Hence, curvature ductility provided by the existing provision of IS:4326 will be less than 5.

Further, the values of ρ_{max}, specified in IS:4326-1976 are based on $\varepsilon_{cu} = 0.003$; $f_{cu} = 0.7\,f'_c$; and $f'_c = 0.786\,f_{ck}$; where f'_c is the compressive strength of concrete cylinder. ACI code (Ref. 1) also uses ε_{cu} equal to 0.003. However, research has indicated that use of $\varepsilon_{cu} = 0.003$ is too conservative for calculation of ultimate deformation (Refs. 6 and 7); instead a value of $\varepsilon_{cu} = 0.004$ is considered more reasonable. As IS:456-1978 (Ref. 4) uses $\varepsilon_{cu} = 0.0035$, this value may be used instead of 0.003. Also, IS:456-1978 takes the value of f_{cu} as 0.81 f'_c. The concrete cylinder strength; f'_c, varies from 0.77 to 0.92 times the cube strength for concrete grade varying from M15 (cube strength, f_{ck}, as 15 MPa) to M35, respectively (Ref. 5); and f'_c is usually taken equal to 0.8 f_{ck}. Hence, $\varepsilon_{cu} = 0.0035$, $f_{cu} = 0.81$ and f'_c and $f'_c = 0.8\,f_{ck}$ may be used to derive the maximum tension steel ratio.

ESTIMATION OF MAXIMUM TENSION STEEL RATIO

To calculate the maximum tension steel ratio for providing a specified curvature ductility, it is necessary to obtain curvature ductility as a function of the tension steel ratio, ρ, and the ratio of compression to tension steel, ρ_c/ρ. The idealized stress-strain properties of steel and concrete used herein are shown in Fig. 3. It may be noted that partial safety factors for material strength of steel (= 1.15) and concrete (= 1.5) are not being used herein. These partial safety factors are used in design to ensure adequate factor of safety against failure under service load conditions. During a severe earthquake, a structure is expected to undergo inelastic deformations, and hence damage. The idea herein is to provide adequate curvature ductility during such an event. Curvature ductility is defined as the ratio of curvature at ultimate strength of the section to the curvature at first yield to tension steel in the section. Hence, use of actual yield stress of steel and crushing strength of concrete is justified. This is consistent with other researchers (e.g. Ref. 6) who also do not use any material strength reduction factors while calculating curvature ductility.

I. Curvature at Yield (ϕ_y)

Consider a doubly reinforced section at yield (Fig. 2). Let ρ be the tension steel ratio such that compressive strain in the extreme concrete fiber is ε_c when the tension steel commences to yield. Assume that the ratio of compression to tension steel, ρ_c / ρ, is fixed initially at some value. From the strain diagram (Fig. 2), the non-dimensional depth of the neutral axis at yield, K, is

$$K = \{ \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_y} \} \tag{4}$$

Thus, curvature at yield is

$$\phi_y = \frac{\varepsilon_y}{d\,(1.0-K)} \tag{5}$$

The compressive force, C (Fig. 2), is given by

$$C = \bar{\alpha}\, f'_c\, K\, b\, d + \rho_c\, f_{sc}\, b\, d \tag{6}$$

where b = breadth of member; and $\bar{\alpha}$ = average stress factor for the compressive stress block, which is equal to

$$\bar{\alpha} = \{ \frac{\varepsilon_c}{\varepsilon_{c1}} - \frac{1}{3} [\frac{\varepsilon_c}{\varepsilon_{c1}}]^2 \} \quad \text{when} \quad \varepsilon_c \leq \varepsilon_{c1}$$

$$\bar{\alpha} = \{ 1.0 - \frac{1}{3} \frac{\varepsilon_{c1}}{\varepsilon_c} \} \quad \text{when} \quad \varepsilon_c > \varepsilon_{c1} \qquad \text{....(7)}$$

ε_{c1} (= 0.0020) is the strain at which the stress-strain curve of concrete changes from parabolic to linear (Fig. 3). f_{sc} is the stress in the compression steel, which is calculated as

$$f_{sc} = \{ \frac{K - K'}{K} \} \; \varepsilon_c\, E_s \tag{8}$$

Here, $K' = d'/d$; and d' = effective cover to compression steel. If f_{sc} calculated from equation (8) exceeds f_y, then it must be taken as f_y itself. The tensile force, T, on the cross section is given by

$$T = \rho\, f_y\, b\, d \tag{9}$$

Considering equilibrium of the section (C = T) equations (6) and (9) give the tension steel ratio, , as

$$\rho = \{ \frac{\bar{\alpha}\, f'_c\, K}{f_y - (\rho_c/\rho)\, f_{sc}} \} \tag{10}$$

This is the tension steel ratio such that compressive strain in the extreme fiber of concrete is ε_c, when the tension steel commences to yield.

II Ultimate Curvature (ϕ_u)

Let the same section (i.e., with tension steel ratio, ρ, as calculated by equation 10) be loaded to its ultimate strength. Equilibrium of the cross section (Fig. 1) gives

$$f_{cu}\ Ku\ b\ d + \rho_c\ f_{sc}\ b\ d = \rho\ f_y\ b\ d \qquad (11)$$

The value of f_{cu} is taken as 0.81 f'_c. Stress in compression steel, f_{sc}, is given by

$$f_{sc} = \left\{\frac{Ku - K'}{Ku}\right\} \varepsilon_{cu}\ E_s \qquad (12)$$

Initially assume that f_{sc} is less than f_y. Equations (11) and (12) give the following quadratic equation for Ku

$$\alpha\ (Ku)^2 + \beta\ Ku - \delta = 0 \qquad (13)$$

where $\alpha = 0.81\ f'_c$; $\beta = \rho_c\ \varepsilon_{cu}\ E_s - f_y\ \rho$; and $\delta = K'\rho_c\ \varepsilon_{cu}\ E_s$. The value of Ku obtained from this equation is used to calculate stress in the compression steel, f_{sc}, from equation (12). If f_{sc} obtained from equation (12) is less than f_y, the calculated value of Ku (from equation 13) is correct, whereas, if f_{sc} is greater than f_y, it must be taken as f_y itself and the value of Ku must then be found from the equation

$$Ku = \frac{(\rho - \rho_c)\ f_y}{0.91\ f'_c} \qquad (14)$$

The appropriate value of Ku is then used to calculate the ultimate curvature

$$\phi_u = \frac{\varepsilon_{cu}}{Ku\ d} \qquad (15)$$

Curvature ductility (ϕ_u/ϕ_y) for a tension steel ratio of ρ is obtained by using equations (5) and (15). In order to generate the graph of curvature ductility versus tension steel ratio, the value of ε_c is initially taken as a low value, say 0.0001; it is gradually increased and for each of its value the tension steel ratio, ρ (equation 10), and the corresponding curvature ductility, ϕ_u/ϕ_y, are found. The value of ratio of compression to tension steel, ρ_c/ρ, can be changed and the process repeated. These graphs are shown in Figs. 4, 5 and 6 for concrete of grade M20 and M25 and steel of grade Fe 250, Fe 415 and Fe 500. The maximum permissible tension steel ratio, ρ_{max}, for a desired value of curvature ductility (5 in the present case) and compression to tension steel ratio, ρ_c/ρ, can be read from these graphs. Thus ρ_{max} and the corresponding value of ρ_c/ρ are obtained. It is observed that ρ_{max} increases almost linearly with ρ_c. Hence, ρ_{max} may be expressed as $A\rho_c + B$, where A and B are constants which are to be found by linear regression analysis. Such an analysis was carried out on data obtained for concrete of grade M15, M20 and M25 with steel of grade Fe 250, Fe 415 and Fe 500. The correlation constant in the linear regression analysis works out to be 0.999, which means that the assumed linear relation between ρ_{max} and ρ_c is valid. The following expressions were obtained for ρ_{max} :

$$\rho_{max} \leq 0.965\,\rho_c + 0.00074\,f_{ck} \qquad (\text{for } f_y = 250 \text{ MPa})$$
$$\rho_{max} \leq 0.759\,\rho_c + 0.00034\,f_{ck} \qquad (\text{for } f_y = 415 \text{ MPa})$$
$$\rho_{max} \leq 0.577\,\rho_c + 0.00025\,f_{ck} \qquad (\text{for } f_y = 500 \text{ MPa}) \qquad .....(16)$$

However, for design convenience the coefficient A was rounded off and the constant term B was calculated again. This gives the following equations for ρ_{max} :

$$\rho_{max} \leq \rho_c + 0.00072\,f_{ck} \qquad (\text{for } f_y = 250 \text{ MPa})$$
$$\rho_{max} \leq 0.75\,\rho_c + 0.00034\,f_{ck} \qquad (\text{for } f_y = 415 \text{ MPa})$$
$$\rho_{max} \leq 0.55\,\rho_c + 0.00025\,f_{ck}. \qquad (\text{for } f_y = 500 \text{ MPa}) \qquad .....(17)$$

These expressions are slightly different from those suggested in Ref. 8 as the present analysis considers values of ρ_{max} up to a maximum of 4 percent while that in Ref. 8 considers values of ρ_{max} up to a significantly higher value. It is observed that for mild steel reinforcement, IS:4326-1976 and equation (17) recommend about the same value of ρ_{max}. For cold worked deformed bars, IS:4326-1976 recommends ρ_{max} which is considerably higher than that recommended by equation (17). As curvature ductility reduces appreciably with an increase in the ratio of tension steel (Figs. 5 and 6), curvature ductility obtained by adopting ρ_{max} values as per the existing provision of IS:4326-1976 will be less than 5 for cold worked deformed bars.

It is evident from Figs. 4, 5 and 6 that curvature ductility decreases as the grade of steel is increased. In an extreme case, if the ratio ρ_c/ρ is taken as 1.0, equation (17) gives ρ_{max} equal to 0.83%, 1.11%, and 1.39% for steel of grade Fe 500 and concrete of grade M15, M20 and M25, respectively. This is quite a low percentage of steel to provide. Hence, use of steel of grade Fe 500 should not be allowed in seismic zones IV and V. Grade of steel above Fe 415 is not permitted in some earthquake areas, e.g., California and New Zealand (Refs. 1, 2 & 9).

SUMMARY AND CONCLUSIONS

The provision of IS:4326-1976 on the maximum tension steel ratio for reinforced concrete flexural members endeavours to provide curvature ductility of 5, assuming concrete as unconfined. However, the method used by the code of ensure curvature ductility of 5 is incorrect. A procedure is described herein to calculate curvature ductility as a function of the tension steel ratio. Using this procedure, simple empirical expressions are derived for the maximum tension steel ratio, ρ_{max}, which ensure curvature ductility of 5. IS:4326-1976 recommends about the same value of ρ_{max} for mild steel reinforcement. However, the code recommends higher values of ρ_{max} for cold worked deformed bars, which results in curvature ductility less than 5. It is also recommended that use of steel reinforcement of grade Fe 500 be not allowed in seismic zones IV and V due to poor curvature ductility.

APPENDIX I : REFERENCES

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APPENDIX II : LIST OF SYMBOLS

A, B	=	constants in linear regression analysis ;
b	=	breadth of member ;
C	=	total compressive force on section ;
d	=	effective depth of member ;
d'	=	effective cover to compression steel ;
E_s	=	elastic modulus of steel ;
f_{ck}	=	characteristic strength of concrete cube ;
f_{cu}	=	average stress in concrete at ultimate strength of section ;
f'_c	=	compressive strength of concrete cylinder ;

f_{sc}	=	stress in compression steel ;
f_y	=	yield stress of steel ;
K	=	non-dimensional depth of neutral axis at yield ;
K'	=	ratio d'/d ;
Ku	=	non-dimensional depth of neutral axis at ultimate ;
T	=	tensile force on section ;
α	=	coefficient of quadratic ;
$\bar{\alpha}$	=	average stress factor for compressive stress block ;
β	=	coefficient of quadratic ;
Δu	=	lateral deflection of structure at ultimate condition ;
Δy	=	lateral deflection of structure at first yield ;
δ	=	coefficient of quadratic ;
ε_c	=	strain in extreme compression fiber of concrete ;
ε_{cu}	=	crushing strain of concrete (= 0.0035) ;
ε_{cl}	=	strain in concrete when its stress strain curve changes from parabolic to linear (= 0.0020) ;
ε_{su}	=	strain in tension steel at ultimate strength of section ;
ε_y	=	yield strain of steel (= f_y/E_s) ;
ϕ_u	=	curvature at ultimate strength of section ;
ϕ_y	=	curvature of section at yield of tension steel ;
μ	=	displacement ductility factor ;
ρ	=	tension steel ratio ;
ρ_c	=	compression steel ratio ;
ρ_{max}	=	maximum permissible tension steel ratio for curvature ductility of 5.

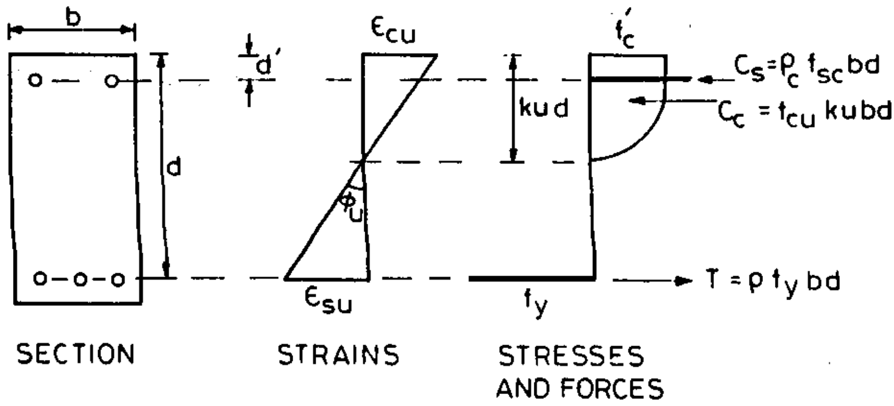

FIG.1 A DOUBLY REINFORCED SECTION AT ITS ULTIMATE STRENGTH

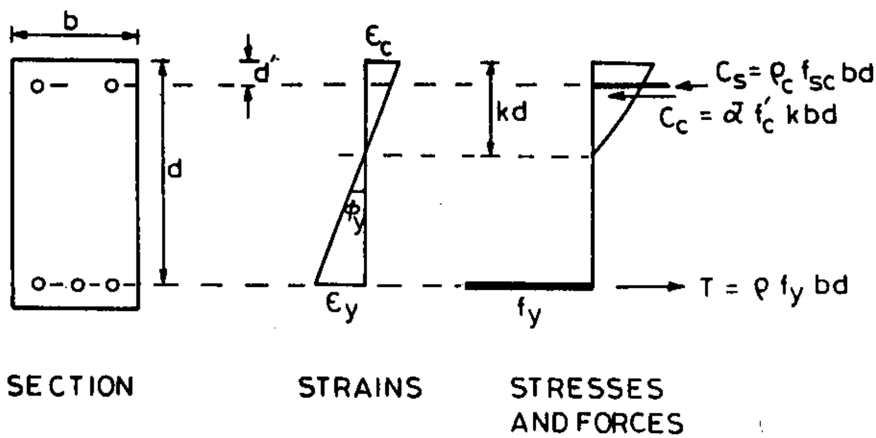

FIG.2 A DOUBLY REINFORCED SECTION AT YIELD

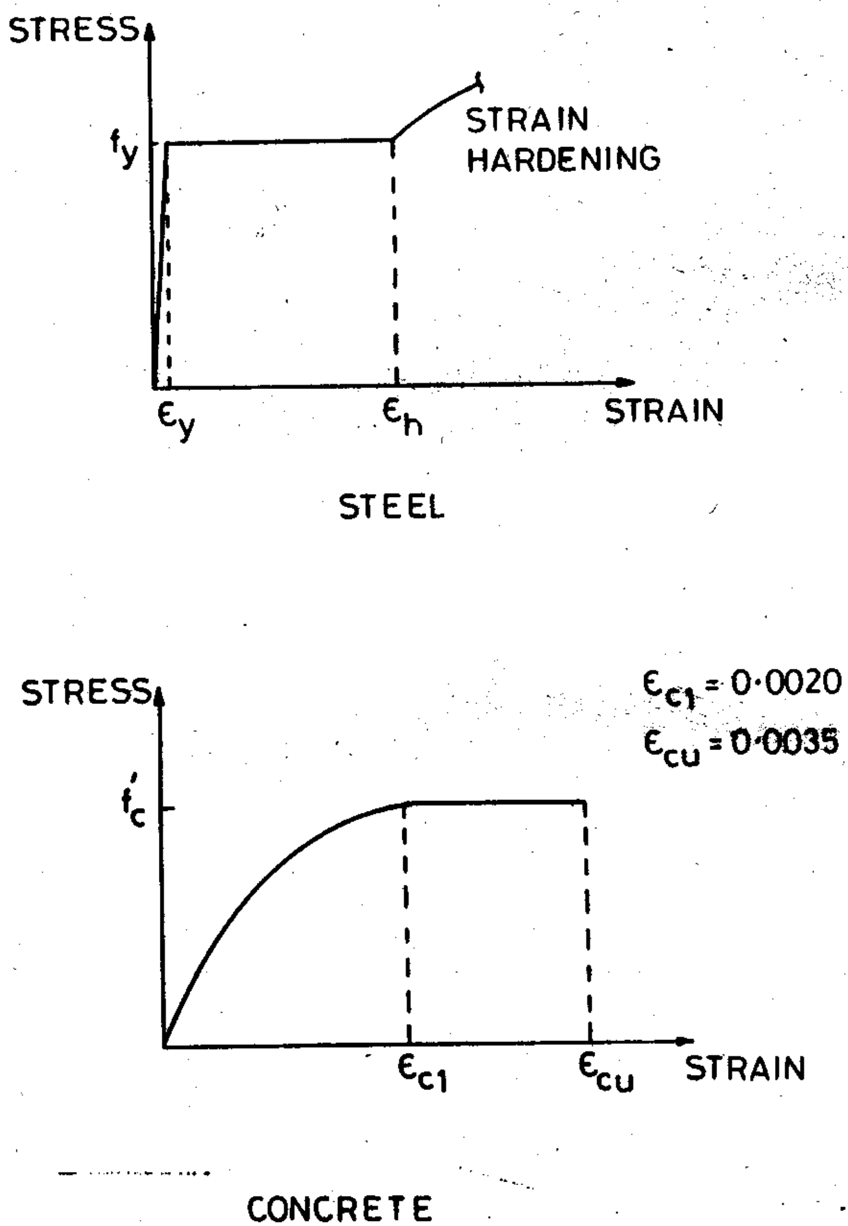

FIG. 3 IDEALIZED STRESS-STRAIN CURVES

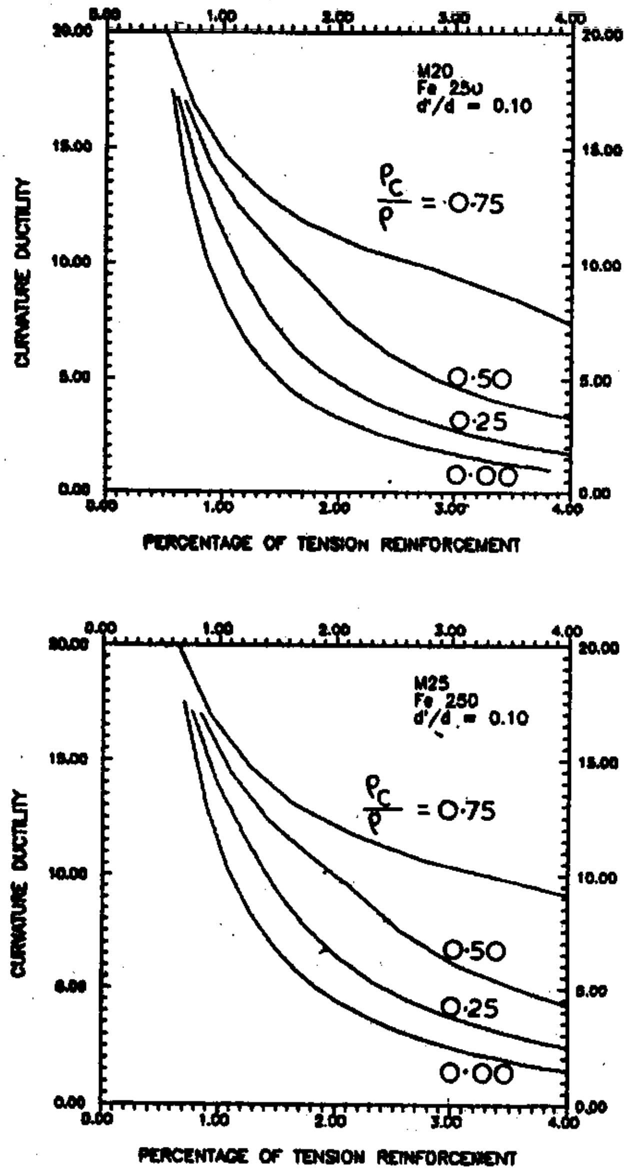

FIG. 4 GRAPH OF CURVATURE DUCTILITY VS. PERCENTAGE TENSION STEEL RATIO FOR Fe 250 STEEL

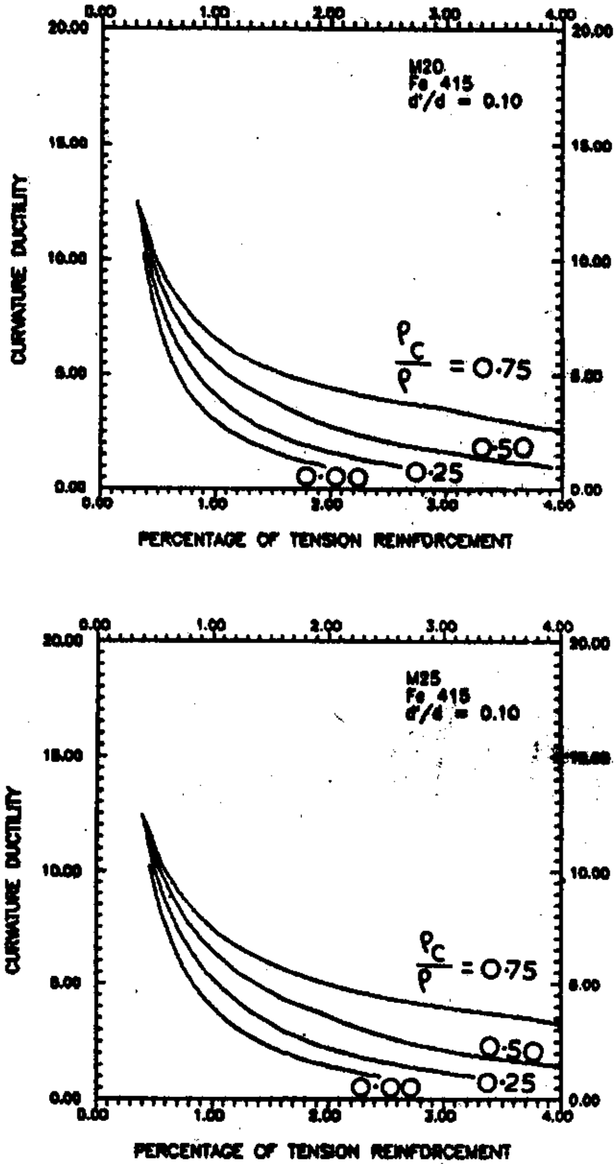

FIG. 5 GRAPH OF CURVATURE DUCTILITY VS. PERCENTAGE TENSION STEEL RATIO FOR Fe 415 STEEL

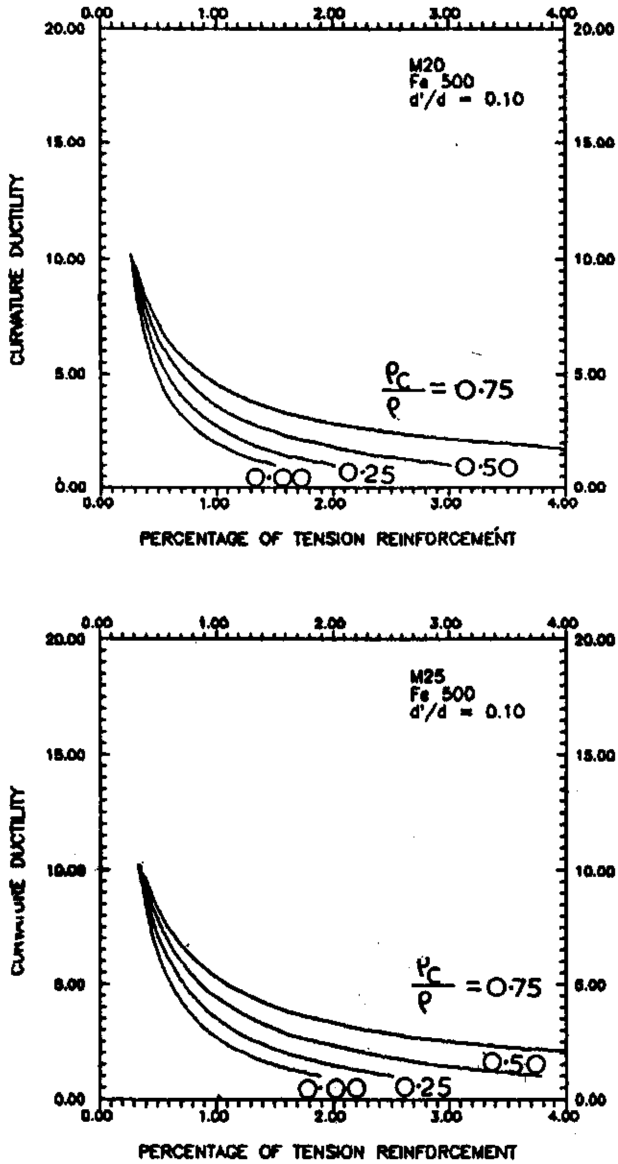

FIG. 6 GRAPH OF CURVATURE DUCTILITY VS. PERCENTAGE TENSION STEEL RATIO FOR Fe 500 STEEL