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EARTHQUAKE LOCATION PROBLEM-SECOND ORDER DERIVATIVES FOR MULTILAYER EARTH AND THEIR IMPLICATIONS

by

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ABSTRACT

Obtaining accurate earthquake hypocenter locations has received considerable attention. Although the problem is non-linear yet the existing algorithms and computer programs primarily make use of linearisation. It is only recently that in some studies the nonlinearity has been incorporated in the computational schemes with a view to make improvements in the locations. A study incorporating the second order partial derivatives for half space and a layer over half space have been reported and in the present study the results have been extended for a multilayer one dimensional earth model. The existing formulation of the nonlinear problem alongwith the details of Geiger method has been included for ready reference as the required background for these derivations.

The expressions developed for second order partial derivatives have been incorporated in an existing program of locations using Geiger method. The results from both linear and nonlinear techniques for test earthquake data, synthetic earthquake data and 3 to 6 station real data have been obtained and compared. Numerous aspects require further examination before any generalised conclusions can be drawn. It has, however, been demonstrated that the incorporation of nonlinear terms in computations of hypocenter locations is easily feasible. Based on the results obtained and discussed in the paper, it is concluded that nonlinear inverse seems to provide better control in the estimation of focal depths, improves convergence particularly for events falling outside the array and provides better information of solution statistics.

Key words : Hypocenter, Nonlinear Inverse, Travel time derivatives, Computer Program, Focal depth, Convergence.

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INTRODUCTION

Earthquake hypocenter locations are computed from a set of observed arrival time data obtained from a seismological network of stations. The problem may be an overdetermined, even-determined and underdetermined depending upon the number of available independent recordings. When the number of independent recordings are more than the parameters to be estimated the problem is overdetermined and does not possess an exact mathematical solution. The estimation of hypocenter coordinates and origin time as linear least square problem was first obtained by Geiger (1910, 1912). The approach remained unutilised for a number of years because of tedious and lengthy computations involved in obtaining least square inverse. However, with the availability of digital computers a number of studies on hypocenter locations employing Geiger method have been reported (Bolt, 1960; Flinn, 1960; Nordquist, 1962; Bolt and Turcotte, 1964; Engdahl and Gunst, 1966). Many algorithms and programs have since been developed and some of these are commercially available for locating earthquakes (Eaton, 1969; Crampin, 1970; Lee and Lahr, 1975; Klein, 1978; Herrmann, 1979; Lahr, 1979). Numerous other studies leading to the understanding of the inherent drawbacks in these algorithms and possible methods of improvements have been reported (Smith, 1976; Buland, 1976; Aki and Richards, 1980; Lee and Stewart, 1981; Anderson, 1982).

Locations are obtained by solving a set of linear simultaneous equations i.e., equations of condition. The mathematical techniques are essentially based on computing hypocenter correction vector in an iterative manner for a given partial derivative matrix of travel time and residual vector. Although obtaining linear least square inverse from a set of linear equations looks straightforward yet numerical instability can arise in computing inverse of the matrix. This happens due to the presence of nearly dependent column vectors in the matrix to be inverted and is attributed to poorly constrained model parameters by the data. In order to overcome this, the ill constrained components (generally the focal depth) are taken out from the normal equations and their values fixed at each iterations step (Lee and Lahr, 1979).

Numerical stability can be better achieved if the matrix inversion is computed employing QR algorithm or singular value decomposition methods (generalized inverse) as these allow convenient detection of ill-conditioned matrices (Bolt, 1970; Buland, 1976; Klein, 1978). To allow deletion of the ill-conditioned vectors from the matrix while computing inverse, data from large number of stations is required. Studies have been carried out on the 'condition number' of the matrix which is the ratio of the largest to the smallest singular value. The condition number is large for events falling outside the array, may become very large (10[illegible]) for earthquakes located few array dimensions outside and reduces if the S and other phase arrivals are also used (Buland, 1976). The proper choice of model parameters leading to scaling of column of matix also reduces the condition number (Smith, 1976).

The earthquake location problem is nonlinear in nature yet the methode of locating hypocenters make use of linearisation employing Taylor series. Therefore, in addition to the use of normal equations and generalized inverse methods, a third approach is a modification of Geiger method (Gauss-Newton method) and incorporates the nonlinear behaviour (Newton method) of travel time as a function of source position (Lee and Stewart, 1981; Thurber, 1985). Advantages of nonlinear method as a means to provide stability during hypocenter locations in situations when first order partial derivatives vanish while the second order partial derivatives are maximized, especially, for short path lengths have been demonstrated (Thurber, 1985). The tendency for very shallow events to have unstable focal depths in standard location algorithms is a common experience. This is due to first order partial derivatives being very small positive numbers and a small perturbation in earthquake focal depth causes substantial upward shift resulting in an "airquake". Inclusion of second order terms would prevent this instability. Second order partial derivatives may also add stability in the locations of events falling outside the network. Fast convergence towards global minimum and reliable estimates of hypocenters have been demonstrated on the synthetic data (Thurber, 1985). However, detailed studies are required to fully assess the practical utility of this method.

Keeping in view the expected benefits of fast convergence and stability, the locations of events in present study have been attempted employing nonlinear least square method. Events have also been located by linear least square technique using a modification of

HYPOLAYR program. The superiority and validity of the nonlinear least square method has been studied earlier for synthetic data. This study has permitted a comparison of locations obtained employing linear and nonlinear methods on the real data which has an inherent component of noise.

FORMULATION OF EARTHQUAKE LOCATION PROBLEM

An earthquake is an independent event in space and time. Its location involves estimation of four dimensional vector in the Euclidean space. The model parameters in the earthquake location problem are the three space coordinates, namely, latitude, longitude, focal depth and the fourth coordinate is the origin time. These are the components of the four dimensional vector. The observables are the arrival times of events at the recording stations (Aki and Richards, 1980). It is adequate to make use of a Cartesian coordinate system provided the horizontal dimensions of the network are small as is the case for locating local earthquakes. In addition to the arrival time of various phases, two more data sets are needed to initiate the event location. These are the trial hypocenter and a representative velocity model of the region. The process of hypocenter location is accomplished in the following three steps :

(i) From the trial hypocenter the travel times to the various recording stations are computed based on the selected velocity model.

(ii) The observed travel times are compared with the computed travel times, and travel time residuals for each recording station are obtained.

(iii) These residuals are minimized using least square or other optimization techniques.

Geiger Method

The hypocenter vector, X, to be estimated in the four dimensional Euclidean space, may be expressed in a Cartesion coordinate system (x,y,z) as

$$X = (x, y, z, t)^T \qquad (1)$$

where t is the origin time of the event and the superscript T denotes the transpose. Let the trial hypocenter vector be given by

$$\bar{X} = (\bar{x}, \bar{y}, \bar{z}, \bar{t})^T \qquad (2)$$

Following notations are defined

- M - number of recording stations
- i - subscript to various variables for denoting station number 1 to M
- τ_i - observed arrival time
- t_i - computed travel time
- t_i - computed arrival time
- r_i - arrival time residual

The computed arrival time t_i from the $\overline{X}$ at the ith station is given by

$$t_i(\overline{X}) = T_i(\overline{X}) + \overline{t}$$

The arrival time residual at the ith station is

$$r_i(\overline{X}) = \tau_i - t_i(\overline{X})$$

$$r_i(\overline{X}) = \tau_i - T_i(\overline{X}) - \overline{t} \tag{3}$$

The objective function for the least square minimization is the sum of the squares of the residuals for all the stations and is

$$F(\overline{X}) = \sum_{i=1}^{M} [r_i(\overline{X})]^2 = r' r \tag{4}$$

where r is the vector in the M dimensional space having the components

$$r = [r_1(\overline{X}), r_2(\overline{X}) \ldots\ldots r_M(\overline{X})]^T \tag{5}$$

The computed arrival time (t_i) is a function of hypocenter and small change t_i due to small changes in (x, y, z, t) can be represented by Taylor series as

$$dt_i = \frac{\partial t_i}{\partial x} dx + \frac{\partial t_i}{\partial y} dy + \frac{\partial t_i}{\partial z} dz + \frac{\partial t_i}{\partial t} dt$$

$$+ \text{ higherorder terms} \tag{6}$$

Since $t = T_i(x,y,z) + t$, equation (3.6) is modified as

$$dt_i = \frac{\partial T_i}{\partial x} dx + \frac{\partial T_i}{\partial y} dy + \frac{\partial T_i}{\partial z} dz + dt \tag{7}$$

$$r_i(\overline{X}) = \frac{\partial T_i}{\partial x} dx + \frac{\partial T_i}{\partial y} dy + \frac{\partial T_i}{\partial z} dz + dt \tag{8}$$

For each station there will be one equation of the type (3.8). These equations are represented in the matrix form as

$$A\,\delta X = r \tag{9}$$

where

$$A = \begin{bmatrix} \frac{\partial T_1}{\partial x} & \frac{\partial T_1}{\partial y} & \frac{\partial T_1}{\partial z} & 1 \\ \frac{\partial T_2}{\partial x} & \frac{\partial T_2}{\partial y} & \frac{\partial T_2}{\partial z} & 1 \\ \frac{\partial T_3}{\partial x} & \frac{\partial T_3}{\partial y} & \frac{\partial T_3}{\partial z} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial T_M}{\partial x} & \frac{\partial T_M}{\partial y} & \frac{\partial T_M}{\partial z} & 1 \end{bmatrix} \tag{10}$$

and $\delta X\,(dx, dy, dz, dt)^T$ is the hypocenter adjustment vector in the four dimensional space and r is the M dimensional residual vector. The system of equations represented by (9) is overdetermined. By multiplying both the sides by A^T it reduces to even-determined system as

$$[A]_{M\times 4}\ [\delta X]_{4\times 1} = [r]_{M\times 1} \tag{11}$$

$$[A^T]_{4\times M}\ [A]_{M\times 4}\ [\delta X]_{4\times 1} = [A^T]_{4\times M}\ [r]_{M\times 1} \tag{12}$$

$$[A^T A]_{4\times4} \quad [\delta X]_{4\times1} = [A^T r]_{4\times1} \tag{13}$$

The expression (13) represents the normal equations which allow obtaining least square solutions.

Nonlinear Optimisation

The Newton's method of nonlinear optimization differs from Geiger method in that it also requires computations of second order partial derivatives to incorporate nonlinear behaviour of travel times. The objective function $F(\overline{X})$ to be minimized is

$$F(\overline{X}) = r^T r$$

where $\overline{X}$ and r are hypocenter vector and residual vector respectively as before. Expanding $F(\overline{X})$ in the Taylor series

$$F(\overline{X} + \delta X) = F(\overline{X}) + g^T \delta X + 1/2\, \delta X^T \overline{H}\, \delta X + \ldots \tag{14}$$

where g is the gradient vector, i.e., $g^T = \nabla F(\overline{X})$

It has been shown (Lee and Stewart, 1981; Thurber, 1985) that

$$g = -2A^T r \tag{15}$$

$$\overline{H} = 2\,[A^T A - (\nabla A^T) r] \tag{16}$$

where ∇ is the vector gradient operator and $\overline{H}$ is the Hessian matrix. Minimization of the F leads to

$$\delta X = -\overline{H}^{-1} g \tag{17}$$

$$= -1/2\,[A^T A - (\nabla A^T)\, r]^{-1}\,[-2A^T r]$$

$$= [A^T A - (\nabla A^T)\, r]^{-1}\; A^T r \tag{18}$$

The components of (4×4) matrix $\overline{N} = (\nabla A^T) r$ are represented as

$$
\begin{aligned}
\bar{N}_{xx} &= \sum_{i=1}^{M} \frac{\partial^2 T_i}{\partial x^2} r_i \\
\bar{N}_{xy} = \bar{N}_{yx} &= \sum_{i=1}^{M} \frac{\partial^2 T_i}{\partial x \partial y} r_i \\
\bar{N}_{xz} = \bar{N}_{zx} &= \sum_{i=1}^{M} \frac{\partial^2 T_i}{\partial x \partial z} r_i \\
\bar{N}_{yz} = \bar{N}_{zy} &= \sum_{i=1}^{M} \frac{\partial^2 T_i}{\partial y \partial z} r_i \\
\bar{N}_{yy} &= \sum_{i=1}^{M} \frac{\partial^2 T_i}{\partial y^2} r_i \\
\bar{N}_{zz} &= \sum_{i=1}^{M} \frac{\partial^2 T_i}{\partial z^2} r_i
\end{aligned} \tag{19}
$$

and the remaining components of N are zero.
In the matrix notation equation (19) can be written as

$$
(\nabla A^T)r = \begin{bmatrix} \bar{N}_{xx} & \bar{N}_{xy} & \bar{N}_{xz} \\ \bar{N}_{yx} & \bar{N}_{yy} & \bar{N}_{yz} \\ \bar{N}_{zx} & \bar{N}_{zy} & \bar{N}_{zz} \end{bmatrix} \tag{20}
$$

From equation (10) and (13) $A^T A$ is given by

$$\begin{bmatrix} \sum_{i=1}^{M} \frac{\partial T_i}{\partial x} \times \frac{\partial T_i}{\partial x}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial x} \times \frac{\partial T_i}{\partial y}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial x} \times \frac{\partial T_i}{\partial z}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial x} \\ \sum_{i=1}^{M} \frac{\partial T_i}{\partial y} \times \frac{\partial T_i}{\partial x}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial y} \times \frac{\partial T_i}{\partial y}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial y} \times \frac{\partial T_i}{\partial z}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial y} \\ \sum_{i=1}^{M} \frac{\partial T_i}{\partial z} \times \frac{\partial T}{\partial x}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial z} \times \frac{\partial T_i}{\partial y}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial z} \times \frac{\partial T_i}{\partial z}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial z} \\ \sum_{i=1}^{M} \frac{\partial T_i}{\partial x}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial y}, & \sum_{i=1}^{M} \frac{\partial T_i}{\partial z}, & M \end{bmatrix} \qquad (21)$$

The vector matrix equation 18 is now expressed as

$$[\delta X]_{4\times1} = [(A^T A)_{4\times4} - (\nabla A^T r)_{4\times4}]^{-1} (A^T r)_{4\times1}$$

$$[\delta X]_{4\times1} = [R]_{4\times4} \; [A^T r]_{4\times1}$$

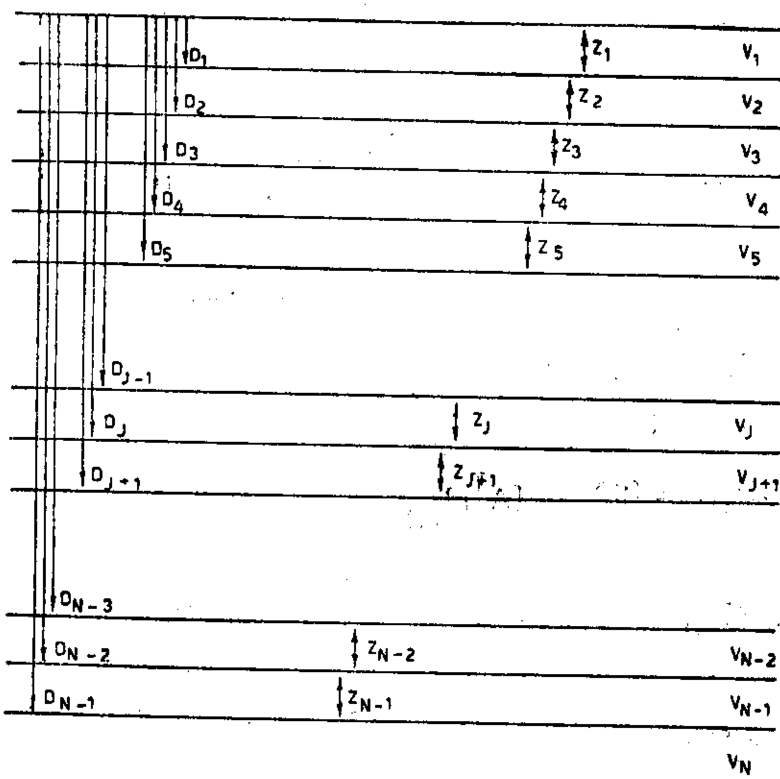
$$[\delta X]_{4\times1} = [R \; A^T r]_{4\times1} \qquad (22)$$

where $[\,(A^T A)_{4\times4} - (\nabla A^T r)_{4\times4}]^{-1} = [R]_{4\times4}$.

Equation (22) provides the desired solution for the hypocenter parameter correction vector (δX) whereas matrices A, ($\nabla A^T r$) and $A^T A$ are given by equations (10), (20) and (21) respectively. The expressions for the second order partial derivatives required in equation (20) have been derived in the following section for one dimensional multi-layer velocity model The expressions for first order derivatives are also obtained for convenience.

COMPUTATIONS OF TRAVEL TIMES AND DERIVATIVES

Computations of travel times of seismic rays and their first and second order partial derivatives are required in equations (20) and (21) for estimations of correction vector of hypocenter parameters. The earth model adopted in the present study consists of a sequence of horizontal layers of constant velocity which increase with depth and is shown in Figure 1. This is one dimensional multilayer velocity



$V_1 < V_2 < V_3 < V_4 < V_5 < \cdots V_J < V_{J+1} \cdots V_{N-2} < V_{N-1} < V_N$

FIG. 1-ONE DIMENSIONAL MULTILAYER VELOCITY MODEL- CONSTANT VELOCITY IN EACH LAYER BUT INCREASING WITH DEPTH

model and is most widely used in microearthquake studies. The analytical expressions required to compute travel times and their first order partial derivatives are well known (Eaton, 1969). Expressions for second order partial derivatives are already available for two earth models, half space and a layer over half space, and these are rederived here for ready reference (Thurber, 1985). The following notations are used in the derivation of these expressions.

V_J - propagation velocity of longitudinal wave in the Jth layer ($^J = 1, N$)

D_J - depth to the top of the ^{J}th layer $^J = 1, N$)

Z_J - thickness of the ^{J}th layer (^{J}=1, N)

$\triangle$ - epicentral distance

H - depth of the focus from the top of the Jth layer

V - velocify in the half space

θ_k^J - take off angle of the ray in the ^{J}th layer refracted from the top of Kth layer

During the computations of travel times and derivatives it has to be ascertained as to which ray (direct or refracted) is the first arrival at the recording station located at an epicentral distance ($\triangle$) from the hypocenter.

The steps involved in the computation are : determining which layer contains initial or trial hypocenter, determining which ray is the first arrival at epicentral distance ($\triangle$) and computing travel time and the first and second order partial derivatives for the appropriate ray. The three cases of ray paths for the seismic waves propagating from hypocenter to the recording station are shown in Figure 2. Expressions for travel time and their derivatives in each of these three cases are derived as follows.

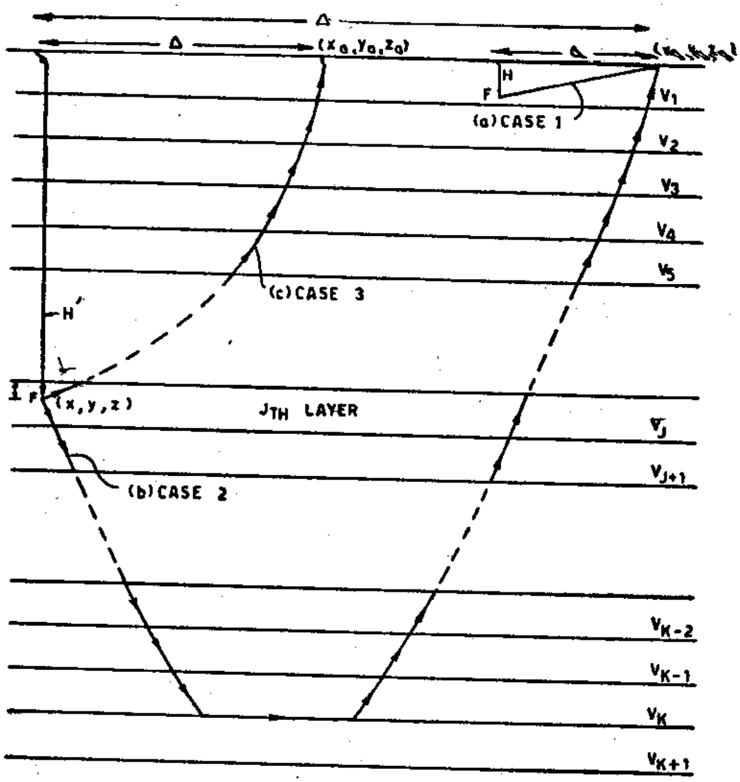


FIG. 2. DIRECT RAY FOR SOURCE IN FIRST LAYER (a), DOWNWARD RAY CRITICALLY REFRACTED FROM K_{TH} LAYER (b), AND DIRECT RAY (c) FOR SOURCE IN THE J_{TH} LAYER

Case 1 : Direct Ray with Hypocenter in the First Layer

The earth model reduces to half space and is show in Figure 2 (a). The expressions for travel time and derivativas are

a) Travel time (T) $= (\Delta^2 + H^2)^{1/2}/v$ (23)

b) first order partial derivatives are

$$\frac{\partial T}{\partial x} = \frac{x - x_0}{V(\Delta^2 + H^2)^{1/2}} \tag{24}$$

$$\frac{\partial T}{\partial y} = \frac{y - y_0}{V(\Delta^2 + H^2)^{1/2}} \tag{25}$$

$$\frac{\partial T}{\partial z} = \frac{\partial T}{\partial H} = \frac{H}{V(\Delta^2 + H^2)^{1/2}} \tag{26}$$

where $\Delta = (x - x_0)^2 - (y - y_0)^2)^{1/2}$ (27)

c) Second order partial derivatives are

$$\frac{\partial^2 T}{\partial x^2} = \frac{(\Delta^2 + H^2) - (x - x_0)^2}{V(\Delta^2 + H^2)^{3/2}} \tag{28}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{(\Delta^2 + H^2) - (y - y_0)^2}{V(\Delta^2 + H^2)^{3/2}} \tag{29}$$

$$\frac{\partial^2 T}{\partial H^2} = \frac{\Delta^2}{V(\Delta^2 + H^2)^{3/2}} \tag{30}$$

$$\frac{\partial^2 T}{\partial x\, \partial y} = \frac{\partial^2 T}{\partial y\, \partial x} = - \frac{(x - x_0)\ (y - y_0)}{V(\Delta^2 + H^2)^{3/2}} \tag{31}$$

$$\frac{\partial^2 T}{\partial x\, \partial z} = \frac{\partial^2 T}{\partial z\, \partial x} = - \frac{(z - z_0)\ (x - x_0)}{V(\Delta^2 + H^2)^{3/2}} \tag{32}$$

$$\frac{\partial^2 T}{\partial y\, \partial z} = \frac{\partial^2 T}{\partial z\, \partial y} = - \frac{(y - y_0)(z - z_0)}{V(\Delta^2 + H^2)^{3/2}} \qquad (33)$$

In the expressions from (23) to (33), V is the half space velocity.

3.3.2 Case 2 : Refracted Ray with Hypocenter in Jth Layer

The ray path geometry for the refracted ray follows the laws of refraction optics as is shown in Figure 2 (b). The travel time (T_J), to epicentral distance (Δ) for a ray with hypocenter lying in the layer J (J = 1, N) and refracting from the top of the layer K (K = 1, N) (Eaton, 1969) is

$$T_J = T_K + \frac{\Delta}{V_K} \qquad (34)$$

where T_K is the intercept time and is expressed as

$$T_K = T_{JK} - \frac{H \cos \theta^J_K}{V_J} \qquad (35)$$

The time T_{JK} is the travel time for the ray with hypocenter placed of the layer J and refracted along the top of the layer K. This is expressed by successive summation of the two way times in each layer as

$$T_{JK} = \sum_{L=J}^{K-1} \frac{D_L \cos \theta^L_K}{V_L} + \sum_{L=1}^{K-1} \frac{D_L \cos \theta^L_K}{V_L} \qquad (36)$$

The expression for the travel time (T_J) is obtained by substituting expressions (36) and (35) in (34) and converting angles into velocities employing Snell's law.

$$T_J = \frac{\Delta}{V_K} + \sum_{L=J}^{K-1} \frac{D_L (V_K^2 - V_L^2)^{1/2}}{V_K V_L} + \sum_{L=1}^{K-1} \frac{D_L (V_K^2 - V_L^2)^{1/2}}{V_K V_L} - \frac{H (V_K^2 - V_J^2)^{1/2}}{V_K V_J} \qquad (37$$

The expressions for first and second order derivatives are obtained by differentiating expression (37) as

$$\frac{\partial T_J}{\partial x} = \frac{x - x_0}{V_K \Delta} \tag{38}$$

$$\frac{\partial T_J}{\partial y} = \frac{y - y_0}{V_K \Delta} \tag{39}$$

$$\frac{\partial T_J}{\partial H} = \frac{(V_K^2 - V_J^2)^{1/2}}{V_K V_J} \tag{40}$$

$$\frac{\partial^2 T_J}{\partial x^2} = \frac{1}{V_K}\left(\frac{1}{\Delta} - \frac{(x - x_0)^2}{\Delta^3}\right) \tag{41}$$

$$\frac{\partial^2 T_J}{\partial y^2} = \frac{1}{V_K}\left(\frac{1}{\Delta} - \frac{(y - y_0)^2}{\Delta^3}\right) \tag{42}$$

$$\frac{\partial^2 T_J}{\partial y \partial x} = \frac{\partial^2 T_J}{\partial x \partial y} = - \frac{(x - x_0)(y - y_0)}{V_K \Delta^3} \tag{43}$$

Remaining expressions of second order partial derivatives are zero.

Case 3 : Direct Ray with Hypocenter in the Jth Layer

The travel time (T), for the direct ray with hypocenter in the Jth layer, as shown in Figure 2 (c), can not be conveniently expressed in terms of Δ. However, both T and Δ are separately expressed (Eaton, 1969) in terms of H, Sinθ, velocities and layer thicknesses as

$$T = \frac{H}{V_J(1-\mathrm{Sin}^2\theta)^{1/2}} + \sum_{L=1}^{J-1} \frac{Z_L V_J}{V_L^2 \left(\frac{V_J^2}{V_L^2} - \mathrm{Sin}^2\theta\right)^{1/2}} \tag{44}$$

$$\Delta = \frac{H \,\mathrm{Sin}\,\theta}{(1-\mathrm{Sin}^2\theta)^{1/2}} + \sum_{L=1}^{J-1} \frac{Z_L \,\mathrm{Sin}\,\theta}{\left(\frac{V_J^2}{V_L^2} - \mathrm{Sin}^2\theta\right)^{1/2}} \tag{45}$$

and $\frac{\partial T}{\partial \Delta}$ is expressed as

$$\left.\frac{\partial T}{\partial \Delta}\right|_H = \frac{\left.\frac{\partial T}{\partial \operatorname{Sin}\theta}\right|_H}{\left.\frac{\partial \Delta}{\partial \operatorname{Sin}\theta}\right|_H} \tag{46}$$

Differentiation of expressions (44) and (45) with respect to Sinθ gives

$$\left.\frac{\partial T}{\partial \Delta}\right|_H = \frac{\frac{H \operatorname{Sin}\theta\,(1-\operatorname{Sin}^2\theta)^{-3/2}}{V_J} + \sum_{L=1}^{J-1} \frac{Z_L\, V_J\, \operatorname{Sin}\theta}{V_L^2}\left(\frac{V_J^2}{V_L^2} - \operatorname{Sin}^2\theta\right)^{-3/2}}{H(1-\operatorname{Sin}^2\theta)^{-3/2} + \sum_{L=1}^{J-1} \frac{Z_L\, V_J^2}{V_L^2}\left(\frac{V_J^2}{V_L^2} - \operatorname{Sin}^2\theta\right)^{-3/2}} \tag{47}$$

The first order partial derivatives are given by

$$\left.\frac{\partial T}{\partial x}\right|_H = \left.\frac{\partial T}{\partial \Delta}\right|_H \frac{\partial \Delta}{\partial x} = \left.\frac{\partial T}{\partial \Delta}\right|_H \left(\frac{x - x_0}{\Delta}\right) \tag{48}$$

$$\left.\frac{\partial T}{\partial y}\right|_H = \left.\frac{\partial T}{\partial \Delta}\right|_H \frac{\partial \Delta}{\partial X} = \left.\frac{\partial T}{\partial \Delta}\right|_H \left(\frac{y - y_0}{\Delta}\right) \tag{49}$$

Travel time (T) is a function of (H, Sin θ) and epicentral distances, △, is also a function of (H, Sin θ). Therefore, $\left.\frac{\partial T}{\partial H}\right|_{\operatorname{Sin}\theta}$ can be expressed as

$$\left.\frac{\partial T}{\partial H}\right|_{\operatorname{Sin}\theta} = \left.\frac{\partial T}{\partial H}\right|_{\Delta} + \left.\frac{\partial T}{\partial \Delta}\right|_H \left.\frac{\partial \Delta}{\partial H}\right|_{\operatorname{Sin}\theta}$$

$$\left.\frac{\partial T}{\partial H}\right|_{\Delta} = \left.\frac{\partial T}{\partial H}\right|_{\operatorname{Sin}\theta} - \left.\frac{\partial \Delta}{\partial H}\right|_{\operatorname{Sin}\theta} \left.\frac{\partial T}{\partial \Delta}\right|_H \tag{50}$$

where

$$\left.\frac{\partial T}{\partial H}\right|_{\operatorname{Sin}\theta} = \frac{1}{V_J\,(1-\operatorname{Sin}^2\theta)^{1/2}} \tag{51}$$

and

$$\frac{\partial \Delta}{\partial H}\bigg|_{\mathrm{Sin}\theta} = \frac{\mathrm{Sin}\theta}{(1-\mathrm{Sin}^2\theta)^{1/2}} \tag{52}$$

Hence, $$\frac{\partial T}{\partial H}\bigg|_{\Delta} = \frac{1}{V_J(1-\mathrm{Sin}^2\theta)^{1/2}} - \frac{\mathrm{Sin}\theta}{(1-\mathrm{Sin}^2\theta)^{1/2}} \frac{\partial T}{\partial \Delta}\bigg|_{H} \tag{53}$$

To obtain expressions for second order derivatives, differentiating expression (50) with respect to H as

$$\frac{\partial}{\partial H}\left[\frac{\partial T}{\partial H}\bigg|_{\Delta}\right]_{\Delta} = \frac{\partial}{\partial H}\left[\frac{\partial T}{\partial H}\bigg|_{\Delta}\right]_{\mathrm{Sin}\theta} - \frac{\partial \Delta}{\partial H}\bigg|_{\mathrm{Sin}\theta} \frac{\partial}{\partial \Delta}\left[\frac{\partial T}{\partial H}\bigg|_{\Delta}\right]_{H} \tag{54}$$

Let $\frac{\partial T}{\partial H}\big|_{\Delta} = P$

$$\frac{\partial P}{\partial H}\bigg|_{\Delta} = \frac{\partial P}{\partial H}\bigg|_{\mathrm{Sin}\theta} - \frac{\partial \Delta}{\partial H}\bigg|_{\mathrm{Sin}\theta} \frac{\partial P}{\partial \Delta}\bigg|_{H} \tag{55}$$

Where operator $$\frac{\partial}{\partial \Delta}\bigg|_{H} = \frac{\dfrac{\partial}{\partial\, \mathrm{Sin}\theta}}{\dfrac{\partial \Delta}{\partial\, \mathrm{Sin}\theta}}\Bigg|_{H} \tag{56}$$

Equatian (55) provides the expression for $\frac{\partial^2 T}{\partial H^2}\big|_{\Delta}$

wherein the quantities

$$\frac{\partial P}{\partial H}\bigg|_{\mathrm{Sin}\theta}, \quad \frac{\partial P}{\partial\, \mathrm{Sin}\theta}\bigg|_{H} \quad \text{and} \quad \frac{\partial \Delta}{\partial\, \mathrm{Sin}\theta}\bigg|_{H}$$

have been computed as

$$\left.\frac{\partial P}{\partial H}\right|_{Sin\theta} = -\frac{Sin\theta}{(1-Sin^2\theta)^{1/2}} \frac{\partial}{\partial H}\left(\left.\frac{\partial T}{\partial \Delta}\right|_H\right)_{Sin\theta}$$

$$\left.\frac{\partial P}{\partial H}\right|_{Sin\theta} = \frac{\partial}{\partial H}\left(\frac{1}{V_J(1-Sin^2\theta)^{1/2}} - \frac{Sin\theta}{(1-Sin^2\theta)^{1/2}}\left.\frac{\partial T}{\partial \Delta}\right|_H\right)_{Sin\theta} \tag{57}$$

(The value of $\left.\frac{\partial T}{\partial \Delta}\right|_H$ is given by equation (47)

$$\left.\frac{\partial P}{\partial Sin\theta}\right|_H = \frac{\partial}{\partial Sin\theta}\left(\frac{1}{V_J(1-Sin^2\theta)^{1/2}} - \frac{Sin\theta}{(1-Sin^2\theta)^{1/2}}\left.\frac{\partial T}{\partial \Delta}\right|_H\right)_H$$

$$\left.\frac{\partial P}{\partial Sin\theta}\right|_H = \frac{Sin\theta\,(1-Sin^2\theta)^{-3/2}}{V_J} - \left(\left.\frac{\partial T}{\partial \Delta}\right|_H\right)(1-Sin^2\theta)^{-3/2}$$

$$Sin\theta\,(1-Sin^2\theta)^{-1/2}\frac{\partial}{\partial Sin\theta}\left(\left(\frac{\partial T}{\partial \Delta}\right)_H\right) \tag{58}$$

To complete the expressions (57) and (58) the partial derivatives to be computed are

$$\frac{\partial}{\partial H}\left[\left.\frac{\partial T}{\partial \Delta}\right|_H\right]_{Sin\theta} \quad \text{and} \quad \frac{\partial}{\partial Sin\theta}\left[\left.\frac{\partial T}{\partial \Delta}\right|_H\right]$$

Differentiation of expression (47) leads to

$$\frac{\partial}{\partial H}\left[\left.\frac{\partial T}{\partial \Delta}\right|_H\right]_{Sin\theta} = (A_1B_2 - B_1A_2)(A_2H + B_2)^{-2} \tag{59}$$

$$\text{where } A_1 = \frac{Sin\theta\,(1-Sin^2\theta)^{-3/2}}{V_J} \tag{60}$$

$$A_2 = (1 - \sin^2\theta)^{-3/2} \tag{61}$$

$$B_1 = \sum_{L=1}^{J-1} \frac{Z_L V_J \sin\theta}{V_L^2} \left(\frac{V_J^2}{V_L^2} - \sin^2\theta \right)^{-3/2} \tag{62}$$

$$B_2 = \sum_{L=1}^{J-1} \frac{Z_L V_J^2}{V_L^2} \left(\frac{V_J^2}{V_L^2} - \sin^2\theta \right)^{-3/2} \tag{63}$$

$$\frac{\partial}{\partial \sin\theta} \left(\left. \frac{\partial T}{\partial \Delta} \right|_H \right) = \frac{AA1\,(AA2 + SUM1) - BB1\,(BB2 + SUM2)}{(AA1)^2} \tag{64}$$

where

$AA1 = b_1 (1 - \sin^2\theta)^{-3/2} + B_2$
$BB1 = a_1 \sin\theta\, (1 - \sin^2\theta)^{-3/2} + B_1$
$a_1 = H/V_J$
$b_1 = H$
$AA2 = a_1 (2 \sin^2\theta + 1 (1 - \sin^2\theta^{-5/2}$
$BB2 = 3b_1 \sin\theta\, (1 - \sin^2\theta)^{-5/2}$

$$SUM1 = \sum_{L=1}^{J-1} \frac{Z_J V_J}{V_L^2} \left(\frac{V_J^2}{V_L^2} - \sin^2\theta \right)^{-5/2} \left(2 \sin^2\theta + \frac{V_J^2}{V_L^2} \right)$$

$$SUM2 = \sum_{L=1}^{J-1} \frac{3\sin\theta\, Z_L V_J^2}{V_L^2} \left(\frac{V_J^2}{V_L^2} - \sin^2\theta \right)^{-5/2}$$

Substitution of expressions (59) and (64) in equations (57) and (58) yields

$$\left. \frac{\partial P}{\partial H} \right|_{\sin\theta} = DPHU = - \frac{\sin\theta}{(1-\sin^2\theta)^{1/2}} (A_1 B_2 - B_1 A_2)(A_2 H + B_2)^{-2} \tag{65}$$

and

$$\left. \frac{\partial P}{\partial \sin\theta} \right|_H = DPUH = \left(\frac{\sin\theta}{V_J} (1-\sin^2\theta)^{-3/2} \right) - [DTDH\, (1-\sin^2\theta)^{-3/2} + \sin\theta\, (1-\sin^2\theta)^{-1/2}\, DTDD] \tag{66}$$

where DTDH is given by equation (47)

and DTDD $= \frac{\partial}{\partial \text{Sin}\theta} \left(\frac{\partial T}{\partial \Delta}\Big|_H \right)$ (67)

From equations (52) and (45) we obtain

$$\frac{\partial \Delta}{\partial H}\Big|_{\text{Sin}\theta} = \text{DDHU} = \frac{\text{Sin}\theta}{(1-\text{Sin}^2\theta)^{1/2}}, \quad (68)$$

$$\frac{\partial \Delta}{\partial \text{Sin}\theta}\Big|_H = \text{DDUH} = H\,(1-\text{Sin}^2\theta)^{-3/2} + B_2 \quad (69)$$

Substituting expressions (65), (66), (68) and (69) in (55)

$$\frac{\partial^2 T}{\partial H^2}\Big|_\Delta = \frac{\partial P}{\partial H}\Big|_\Delta = (\text{DPHU}) - \frac{(\text{DDHU})\,(\text{DPUH})}{(\text{DDUH})} \quad (70)$$

$\frac{\partial^2 T}{\partial x^2}$ can be expressed as

$$\frac{\partial^2 T}{\partial x^2}\Big|_H = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial \Delta}\cdot\frac{\partial \Delta}{\partial x}\right) \quad (71)$$

$$= \frac{\partial T}{\partial \Delta}\cdot\frac{\partial^2 \Delta}{\partial x^2} + \frac{\partial \Delta}{\partial x}\cdot\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial \Delta}\right)$$

$$= \frac{E}{F}\left(\frac{\Delta^2 - (x-x_0)^2}{\Delta^3}\right) + \left(\frac{\frac{\partial}{\partial \text{Sin}\theta}\left(\frac{\partial T}{\partial \Delta}\Big|_H\right)}{\text{DDUH}}\;\frac{(x-x_0)^2}{\Delta^2}\right) \quad (72)$$

where

$$E = A + B_1 \text{ and } F = C + B_2 \quad (73)$$

$$A = \frac{H \,\mathrm{Sin}\theta}{V_3} (1-\mathrm{Sin}^2\theta)^{-3/2}, \quad C = H\,(1-\mathrm{Sin}^2\theta)^{-3/2} \tag{74}$$

Similarly $(\partial^2 T/\partial y^2)|_H$ is expressed as

$$\left.\frac{\partial^2 T}{\partial y^2}\right|_H = \frac{E}{F}\left(\frac{\Delta^2 - (y-y_o)^2}{\Delta^3}\right) + \left(\frac{\frac{\partial}{\partial \mathrm{Sin}\theta}\left(\left.\frac{\partial T}{\partial \Delta}\right|_H\right)}{DDUH} \;\frac{(y-y_o)^2}{\Delta^2}\right) \tag{75}$$

express $\frac{\partial^2 T}{\partial x \partial y}$ as

$$(\partial^2 T/\partial x\,\partial y) = (\partial/\partial x)(\partial T/\partial y)$$

$$= [\partial/\partial x][(\partial T/\partial \Delta)(\partial \Delta/\partial y)]$$

$$= [\partial T/\partial \Delta][(\partial/\partial x)(\partial \Delta/\partial y)] + [\partial \Delta/\partial y][(\partial/\partial x)(\partial T/\partial \Delta)]$$

$$- \frac{E}{F}\left(\frac{(x-x_o)(y-y_o)}{\Delta^3}\right) + \left(\frac{\frac{\partial}{\partial \mathrm{Sin}\theta}\left(\left.\frac{\partial T}{\partial H}\right|_H\right)}{DDUH}\;\frac{(x-x_o)(y-y_o)}{\Delta^2}\right) \tag{76}$$

and identically obtain the expressions for

$$\frac{\partial^2 T}{\partial y\,\partial x} = -\frac{E}{F}\left(\frac{(x-x_o)(y-y_o)}{\Delta^3}\right) + \frac{\frac{\partial}{\partial \mathrm{Sin}\theta}\left(\left.\frac{\partial T}{\partial H}\right|_H\right)}{DDHU}\;\frac{(x-x_o)(y-y_o)}{\Delta^2} \tag{77}$$

Expression for $\frac{\partial^2 T}{\partial x\,\partial H}$ is given as

$$\frac{\partial^2 T}{\partial x\,\partial H} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial H}\right) = \frac{\partial}{\partial \Delta}\left(\left.\frac{\partial T}{\partial H}\right|_\Delta\right)\frac{\partial \Delta}{\partial x}$$

$$\frac{\partial^2 T}{\partial x\,\partial H} = \frac{\frac{\partial}{\partial \mathrm{Sin}\theta}\left(\left.\frac{\partial T}{\partial H}\right|_\Delta\right)\left(\frac{\partial \Delta}{\partial x}\right)}{\left.\frac{\partial \Delta}{\partial \mathrm{Sin}\theta}\right|_H}$$

Using various derived expressions, we can write

$$(\partial^2 T/\partial x \ \partial H) = [DPUH/DDUH]\,[x - x_0)/\Delta\,] \qquad (78)$$

and

$$(\partial^2 T/\partial y \ \partial H) = [DPUH/DDUH]\,[(y - y_0)/\Delta\,] \qquad (79)$$

As the partial differential operators commute, we have

$$(\partial^2 T/\partial x \ \partial H) = [DPUH/DDUH]\,[x - x_0)/\Delta\,] \qquad (80)$$

$$(\partial^2 T/\partial H \ \partial y) = (\partial^2 T/\partial y \ \partial H) = [DPUH/DDUH]\,[(y - y_0)/\Delta\,] \qquad (81)$$

Equations (70, (72) and (75 to 81) provide the expressions of the second order Partial derivatives for the direct ray arriving at a station when the focus is not in the first layer.

ERROR ANALYSIS

The arrival time data of the seismic phases used in locating earthquakes is contaminated by noise from a variety of sources. The common sources of noise are errors in reading time clock synchronisation errors and minor variation in the rate of drift of chronometers at individual recording stations. Diurnal temperature variations may also affect the chronometer drift rate. Small variations in the drum speed and unknown station corrections due to the local site geology below the recording stations may also introduce errors in the arrival time data. Furthermore, errors also creep in due to phase identification, namely, on account of inaccurate reading of P-time when its onset is emergent and the misidentification of S–phase whose onset is normally submerged in P–coda or due to the presence of supplimentary phases. The a–priori knowledge about the statistical characteristics of this complex noise pattern are not available. However, it is normally assumed that all components of the data vectors are statistically independent, share the same variance and follow Gaussian distribution.

The objective function to be minimized in the earthquake location problem follows χ^2 distribution with M degrees of freedom. However, for each set of data there are four linear constraints due to the estimation of four unknown hyocenter parameters. This imposes a constraint on the system and reduces its degrees of freedom to M-4. The standard statistical tables on χ^2 distribution can be consulted to determine the quality of the obtained hypocenter parameters. If the solution quality is poor, it can be improved by re-evaluating the arrival time data or by assuming different velocity models.

To estimate the reliability of hypocenter solutions, the following statistical parameters are normally computed in terms of arrival time residual (RT).

$$\text{Mean deviation (MD)} = \sum_{i=1}^{M} |RT_i| / M \tag{82}$$

$$\text{Average residual (AVR)} = \sum_{i=1}^{M} RT_i / M \tag{83}$$

$$\text{Standard deviation of } RT_i \text{ (SDAT)} = \sum_{i=1}^{M} [(RT_i^2) / (M-\bar{P}-1)]^{1/2} \tag{84}$$

$$\text{Variance } (\sigma^2) = \sum_{i=1}^{M} (RT_i)^2 / (M-\bar{P}-1) \tag{85}$$

$$\text{RMS residuals} = \sum_{i=1}^{M} [(RT_i)^2/M]^{1/2}$$

Standard error in origin time ($SDAT_0$) = SDAT $(C_{44})^{1/2}$ (86)

Standard error in longitude (SDAX) = SDAT $(C_{11})^{1/2}$ (87)

Standard error in latitude (SDAY) = SDAT $(C_{22})^{1/2}$ (88)

Standard error in focal depth (STAZ) = SDAT $(C_{33})^{1/2}$ (89)

In these expressions M stands for the number of observations available and P for the number of parameters to be adjusted. C_{11}, C_{22}, C_{33} and C_{44} are the elements of the principal diagonal of the inverse matrix

$(A^T A)^{-1}$ or $(A^T A)\text{-}\nabla A^T{}_r)^{-1}$

Matrix of Resolution

The matrix of resolution (R) is a useful parameter which provides insight into how well the components of the hypocenter vector are resolved. In the overdetermined least square problem expressed by equation (9),i.e., $A\delta X = r$, the least square operator gives the estimated solution vector ($\delta \bar{X}$) as

$$\delta \bar{X} = (A^T A)^{-1} \quad A^T r$$

$$\delta \bar{X} = (A^T A)^{-1} A^T A \, \delta X$$

$$\delta \bar{X} = R \delta X$$

where $R = (A^T A)^{-1} A^T A = 1$ (90)

In the normal equation method, the resolution matrix for the overdetermined least square problem equals identity matrix and the resolution is perfect. However, a perfect resolution may not be achievable in practice for the overdetermined least square problem due to inherent errors in the data. The estimated components of the hypocenter correction vector $\delta \bar{X}$ are the weighted sum of the components of the vectors δX with the weights given by the resolution matrix. Therefore, row vectors of the matrix provide a view about the resolution in the various components of the estimated hypocenter vector. The resolution is perfect when $R = I$. The solution is better if its computed diagonal and the off diagonal elements approach unity and zero respectively.

The matrix of resolution for the nonlinear least square problem (NLLS) is derived from the equation (.18) as

$$\bar{H} \delta X = 2A^T r$$

$$(A^T A - (\nabla A^T) \; r) \, \delta X = A^T r$$

$$\delta \bar{X} = (A^T A) - (\nabla A^T) \; r)^{-1} \quad A^T r \text{ (using } A\delta X = r)$$

$$\delta \bar{X} = (A^T A - (\nabla A^T) \; r)^{-1} \, A^T A \delta X$$

This gives the matrix of resolution as

$$R_n = (A^T A - \nabla A^T r)^{-1} A^T A \quad (91)$$

Similar expression has been reported in literature earlier (Thurber, 1985). Study of resolution matrix for hypocenter locations obtained employing linear and nonlinear methods will be useful to find out improvement, if any, in resolution of hypocenter parameters.

Covariance Matrix

The covariance matrix (C) is useful in investigating correlations or near dependencies between various pairs of components of solution vector. In the least square problem the symmetric positive definite matrix $C = (A^T A)^{-1}$ is the covariance matrix. The covariance matrix may also be multiplied by scalars like variance for statistical interpretation. The value of elements C_{ij} $(i \neq j)$ being close to 1 or -1 will mean that these components of the solution vector are highly correlated. Thus, the following 2×2 principal submatrix of the covariance matrix is nearly singular

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \quad (92)$$

This type of analysis has an inherent weakness that only dependence between pairs of variables can be detected. There can be situations where a set of three variables possess mutual near dependence without any two variables being nearly dependent (Lawson and Hanson, 1974). In the case of earthquake location problem the errors are assumed to be uncorrelated, and the diagonal elements of scaled matrix $\sigma^2 C$ give the variance of the individual solution components and off diagonal elements provide the information about the covariance in the solution components.

The covariance matrix for the nonlinear least square problem (Thurber, 1985) is :

$$C_n = \sigma^2 \ (A^T A - (\nabla A \ \ T)r)\text{-}1 R_n \quad (93)$$

where R_n is already defined in equation (91)

Solution to the above equations provides the length of the principal axes in terms of $\frac{1}{\sqrt{\lambda_1}}$ and $\frac{1}{\sqrt{\lambda_2}}$. To obtain joint confidence region for two variables, the equation of error ellipsoid is simplified in two dimensions as

$$\left[x-\bar{x},\ y-\bar{y}\right]\begin{bmatrix} C'_{11} & C'_{12} \\ C'_{21} & C'_{22} \end{bmatrix}\begin{bmatrix} x-\bar{x} \\ y-\bar{y} \end{bmatrix} \leqslant \sigma^2 \bar{P} F_{0.5} \qquad [95]$$

where $\bar{P}$ is the number of regressors, σ^2 the estimated variance and $F_{0.5}$ is the F distribution with $\bar{P}$ and $[M-\bar{P}]$ degrees of freedom at 95% confidence level. In this equation all quantities are known except X. The value of X that satisfies this inequality forms an ellipse. This error analysis is based on the assumption that the errors in the arrival times are normally distributed. However, in the recently reported study attention has been focussed on the mechanics of error propogation in the hypocenter location *(*Pavlis, 1986*)*, and it is demonstrated the conventional error ellipsoid [Flinn, 1965] will include hypocenter if the errors in velocity model also follow normal distribution.

DEVELOPMENT OF THE COMPUTER PROGRAM AND ITS TESTING

The program used has been basically evolved from linear least square program HYPOLAYR (Eaton, 1969*)*. The same program allows both linear least square and nonlinear computations. The additional features incorporated in the HYPOLAYR for linear least square computations relate to obtaining resolution matrix, covariance matrix and axes of error ellipse. For the purpose of nonlinear computations the mathematical expressions for the second order derivatives developed for multilayer one dimensional earth model have been incorporated after suitable translations into Fortran language. The program has been installed and tested on DEC 2050 computer system. To reduce the round off errors in computing small quantities the program was run using double precision. For ready reference features of this program allowing both linear and nonlinear computations are described in the following section.

Brief Features of the Linear Program

The modified HYPOLAYR program has been called as Linear Least Square program *(*LLS*)* in this study. The hypocenter correction vector is computed from the initially assumed hypocenter employing linear least square. This correction vector is incorporated in the initial hypocenter vector in an interative manner to achieve the desired refinement. The adjustments are continued until the average residuals, mean deviation of residuals and change in the mean deviation of residuals, all become smaller than prescribed limits or the number of iterations exceeds the preset value. Adjustment is not terminated prior to fifth iteration. On the fourth iteration the focal depth is restricted to the trial focal depth in case the effective depth control is lost [$\partial T/\partial z < 0.02$]. Focal depth is not adjusted during the first interation and also in case previous adjustment of epicenter is greater than 10 km. During the process of adjustment if the hypocenter moves above suface its parameters are scaled down to restrict the hyocenter below the earth's surface. The focal depth is scaled to a value 0.6 times the distance from the surface, whereas the adjustments in epicenter and origin time are also scaled down to 0.4 times of the original computed values. The individual arrival times are weignted during the adjustment process on the basis of the quality of P-arrival time and epicentral distance from the station. The program is designed to permit locations for events recorded on any number of stations from 3 to 25. For three station data the depth or origin time must be specified. Several solution modes can be computed for a single event. Use of S–phase data is made to compute origin time restricted solutions only and in this case the origin time is set to that computed from [S-P] time data. S-phase data is not used with P-phase data in adjusting the hypocenter. Following are the termination conditions for the hypocenter adjustment.

a) Adjustment is terminated after five iterations in case average residual < 0.002, mea deviation < 0.10 and change in mean deviation < 0.005 are obtained.

b) Adjustment is terminated after eight iterations when average residual < 0.002, mean deviation < 0.30 and change in mean deviation < 0.003 are obtained.

c) Adjustment is terminated when average residual ≥ 0.002 but change in tse mean deviation < 0.002 are obtained. In this case the iteration limit is set to 14 and average residual is added to the origin time and arrival time residuals and solution statistics are recomputed.

d) Adjustment is continued upto 12 iterations and in case adequacy criteria set for termination are not achieved, iteration limit is set at

13, average residual is added to the origin time and arrival time residuals and solution statistics are recomputed.

e) When $\frac{\partial T}{\partial z} < 0.02$ focal depth adjustment is blocked

f) Solutions can not be obtained if there are gross errors in arrival time data or if the epicenter falls much outside the network. When all the four parameters are adjusted atleast six observations are required to estimate the standard errors in hypocenter parameters.

Brief Features of Nonlinear Program

The part of the program which is allowing the nonlinear computations has been referred to as NLLS. The adequacy criteria for termination of solution have been set as explained in the linear program The computations of second order partial derivative matrix [Eq. 19] are carried out in the main program and two control cards KKLL and KKMM are provided in the main program with the following objectives.

(1) KKLL switch controls the execution of travel time derivative subroutines. KKLL = 0 passes control to subroutine TRVDR1 for computations of first order partial derivatives of travel time and KKLL = 1 facilitate computations of first and second order partial derivatives of travel time employing TRVDRV subroutine as given in the flow diagram in Figure 3.

(2) KKMM switch controls the weighting scheme of data. In case KKMM = 0 normal equations are computed employing weight factors. When KKMM is set = 1, normal equations are computed in the main program without weights and the control is passed to compute second order partial derivative matrix without weights.

The focal depth adjustment is blocked in case $\frac{\partial^2 T}{\partial Z^2} < 0.00001$. Main program also provides resolution matrix, covariance matrix and length of the axes of error ellipse for both linear and nonlinear methods to allow desired comparison of two techniques.

Results From Standard Test Data

The earthquake data given in the program HYPO71 (Lee and Lahr, 1975) has been used for the purpose of testing the nonlinear program

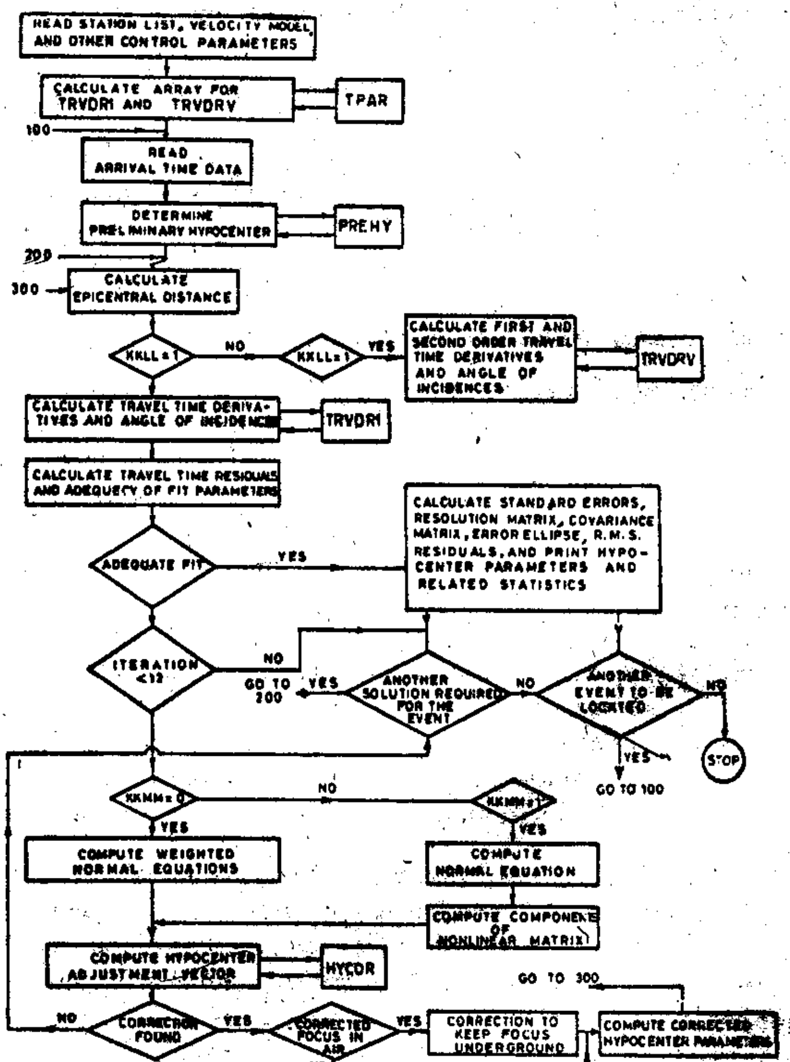


FIG. 3. FLOW DIAGRAM FOR THE COMPUTER PROGRAM FACILITATING COMPUTATIONS OF HYPOCENTER PARAMETERS EMPLOYING BOTH LLS AND NLLS TECHNIQUES. SUBROUTINES TPAR, PREHY, TRVDR1 AND HYCOR ARE FROM HYPOLAYR PROGRAM (EATON, 1969.)

(NLLS) and the results obtained ard presented in Table 1. Locations for this test earthquake obtained employing both LLS and HYPO71 programs are also included in the Table 1 to allow easy comparison. The hypocenter location remained almost same for both linear and nonlinear methods. The use of nonlinear technique has resulted in reduction of the standard errors. However, these standard errors are higher as compared to the standard errors obtained employing program HYPO71 which uses a different regression technique and weighting schemes for hypocenter locations. Iterations have decreased in nonlinear program from 14 to 10 which demonstrate that the introduction of second order terms have improved the convergence rate as compared to the linear method. The three programs have shown remarkable stability and the solutions converge to almost same hypocenter location except for focal depth in which relatively higher deviations have been obtained. The stability of solutions can be further seen in Table 2 which has listed the travel time residuals at each station and shows that variation of travel time residuals for all the three programs remains by and large same.

TABLE I Test Results for Nonlinear Program (NLLS) with other Standard Programs

Program Name	Origin Time (sec)	Latitude (min)	Longitude (min)	Focal Depth (km)	RMS* Residuals (sec)	Standard Errors: Longitude (SDAX) (km)	Standard Errors: Latitude (SDAY) (km)	Standard Errors: Focal Depth (SDAZ) (km)	Standard Errors: Origin Time (SDATo) (sec)	No. of stations	No. of Iterations
NLLS	52.69	28.44	42.00	9.24	0.18	0.56	0.49	1.28	0.11	17	10
LLS	52.68	28.55	42.02	9.32	0.18	0.68	0.63	1.64	0.15	17	14
HYPO71	52.83	28.59	41.94	8.41	0.16	0.50+		1.1		17	

$^{+}[(SDAX)^2 + (SDAY)^2]^{1/2} = 0.50$

*Residuals at individual stations are given in Table 3.2.

TABLE 2 Travel Time Residuals at Various Stations.

Station Name	Travel Time Residuals (sec) Employing		
	NLLS Program	LLS Program	HYPO71 Program
SR01	0.01	0.02	0.20
SR02	0.21	0.22	0.06
SR03	0.24	0.23	0.40
SR04	0.11	0.11	0.04
SR05	0.21	0.20	0.10
SR06	0.20	0.19	0.11
SR07	0.18	0.18	0.18
SR08	0.10	0.10	0.02
SR09	0.33	0.32	0.04
SR10	0.16	0.11	0.17
SR11	0.06	0.07	0.08
SR12	0.02	0.02	0.06
SR13	0.35	0.34	0.38
SR15	0.05	0.06	0.09
SR16	0.29	0.30	0.17
SR18	0.08	0.08	0.01
SR19	0.02	0.02	0.03

Results from Synthetic Test Data

A nine station fictitious array was designed as shown in Figure 4. The travel times for longitudinal waves were computed at these nine stations for half space overlain by a layer for a hypothetical event. Computed arrival times were inverted to obtain the hypocenter location of the hypothetical event using both NLLS and LLS and the results compared. This has been done for two events one falling within the array and other falling outside the array but both lying in the top layer. In addition an event below the top layer but outside the array was also taken. The outcome of the various test cases has been described here.

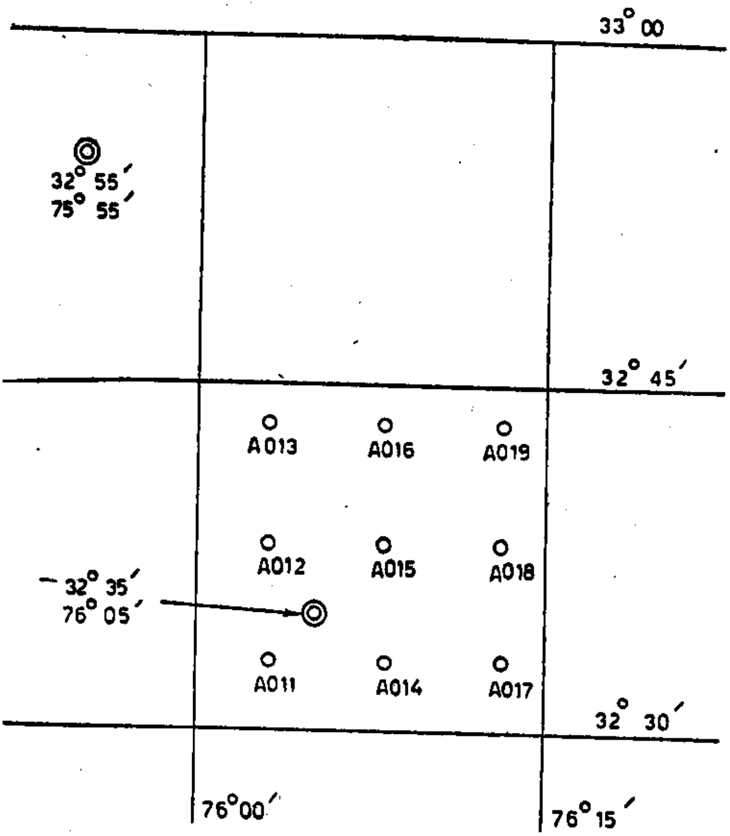


FIG. 4–LOCATIONS OF STATIONS IN THE FICTICIOUS ARRAY AND THE HYPOTHETICAL EPICENTERS WITHIN AND OUTSIDE THE ARRAY

An Event Falling within the Array and in the First Layer

Table 3 gives the coordinates of hypothetical stations, the hypocenter, and the assumed velocity model which are used for generating synthetic set of travel time data. The synthetic data set so generated is given in Table 4. The travel times were rounded off to second decimal place and converted to arrival time data by adding an arbitrary origin time of 10 sec. Both linear and nonlinear methods have given almost identical locations which differ from the original location by .01 min. of longitude [location No. 1 and 2 in Table 5]. The hypocenter converges to the

TABLE 3 Coordinates for Hypothetical Stations and an Event, and the Velocity Model

Station Name	Latitude deg	Latitude min	Longitude deg	Longitude min
AO11	32.0	33.00	76.0	03.00
AO12	32.0	38.00	76.0	03.00
AO13	32.0	43.00	76.0	03.00
AO14	32.0	33.00	76.0	08.00
AO15	32.0	38.00	76.0	08.00
AO16	32.0	43.00	76.0	08.00
AO17	32.0	33.00	76.0	13.00
AO18	32.0	38.00	76.0	13.00
AO19	32.0	43.00	76.0	13.00

Velocity Km/sec	Depth to the Top of Layer-Km	Coordinates of Hypothetical Event: Latitude deg	Latitude min	Longitude deg	Longitude min	Focal Depth km
5.0	00.0	32.0	35.0	76.0	05.0	02.0
6.0	10.0					
8.0	30.0					

original location after five iterations. The length of the axes which govern the joint confidence region are less in the case of nonlinear method.

TABLE 4 Synthetic P-Travel Time and Arrival Time Data

Station Name	Travel Time sec	Arrival Time sec
AO11	1.0467	11.05
AO12	1.3339	11.34
AO13	3.0503	13.05
AO14	1.2573	11.26
AO15	1.5073	11.50
AO16	3.1287	13.13
AO17	2.6303	12.63
AO18	2.7562	12.76
AO19	3.8880	13.89

An Event Falling Outside the Array but in the Top Layer

The same set of data on station coordinates and velocity as given in Table 1 were taken. A hyothetical event is taken with coordinates as

Latitude		Longitude		Focal Depth
deg	min	deg	min	km
32.0	55.0	75.0	55.0	08.0

This event was falling about one dimension outside the array with focus in the first layer. The computed P-travel and arrival times are given in Table 6. The test location obtained employing both NLLS and LLS techniques are given in Table 7. Using the P-times the LLS gives exact inversion [location No. 2] whereas the NLLS introduced 50m and 30m errors in latitude and longitude [location No. 1] respectively which are fairly small.

In general locations of events outside the array will require better data control. Thus, origin time was restricted by using [S-P] time and

TABLE 5 Test Locations for an Event within the Array and in the Top Layer Employing NLLS (No.1) and LLS (No.2)

No.	Origin Time sec	Latitude min	Longitude min	Focal Depth km	No. of Iterations	RMS Residuals sec	AX+	BX+
1	10.00	35.00	5.01	02.0	5	0.26×10^{-2}	2.21	2.72
2	10.00	35.00	5.01	02.0	5	0.26×10^{-2}	2.62	3.19

+AX and BX are the unscaled lengths of the axes of error ellipse.

TABLE 6 Synthetic P-Travel Time and Arrival Time Data

Station Name	Travel Time sec	Arrival Time sec
AO11	8.4179	18.42
AO12	6.9487	16.95
AO13	5.3329	15.33
AO14	8.8987	18.90
AO15	7.5554	17.56
AO16	6.2107	16.21
AO17	9.5579	19.56
AO18	8.3406	18.34
AO19	7.2762	17.28

locations obtained both using NLLS [No.3] and LLS [No.4]. Another set of locations were then obtained using these as trial locations and the P-times only [No. 5 and 6]. In both these exercises the NLLS has given superior results. In fact use of LLS did not allow the focal depth estimates.

An Event Falling Outside the Array but Below the Top Layer

The same hypocenter coordinates as above were assumed and in order to bring focus below first layer, the top of the 6 km/sec velocity layer in the velocity model in Table 3 has been moved up by 5 km, Table 8 gives the computed P-travel and arrival times. Hypocenter locations obtained using both nonlinear and linear methods are presented in Table 9. Locations No. 1 and 3 are obtained restricting origin time from [S-P] time and locations No. 2 and 4 have been computed using only P-time taking locations 1 and 3 as trial hypocenters. During the process of convergence employing linear method focal depth of 8.9 km has been estimated. The component of resolution matrix, which is 0.0, did show that focal depth could not converge further. This is due to $\frac{\partial T}{\partial z} < 0.02$ and hence focal depth was restricted to 8.9 km. The focal depth converged to the value of 8.5 km employing nonlinear method. Using P-times

TABLE 7 Test Locations for an Event Falling Outside the Array and in the Top Layer Employing NLLS (No. 1,3,5) and LLS (No. 2,4,6)

No.	Origin Time sec	Latitude min	Longitude min	Focal Depth km	No. of Iterations	RMS Residuals sec	AX‡	BX‡
1	10.01	54.97	55.02	8.0	11	0.19×10^{-2}	1.92	11.19
2	10.00	55.00	55.00	8 0	7	0.17×10^{-2}	2.49	13.98
3	9.62+	55.94	54 24	7.6	7	0.57×19^{-2}	1.92	11.40
4	9.62×	56.53	53.79	*	5	0.83×10^{-1}	2.34	14.19
5	10.00	55.00	55.00	8.0	5	0.16×10^{-2}	1.93	10.81
6	11.33	51.84	57.55	*	14	0.82×10^{-1}	2.63	14.48

+ Origin time from (S-P) time. * Focal depth could not converge.

‡ AX and BX are thk unscaled lengths of the axes of error elipse.

TABLE 8 Synthetic P-Travel Time and Arrival Time Data

Station Name	Travel Time sec	Arrival Time sec
AO11	7.6655	17.67
AO12	6.2166	16.62
AO13	4.8348	14.87
AO14	8.1447	18.14
AO15	6.8068	16.81
AO16	5.5875	15.59
AO17	8.8014	18.80
AO18	7.5885	17.59
AO19	6.5291	16.53

and locations 1 and 3 as trial hypocenters the focal depth did not show any improvement. Perhaps due to small variations in $\frac{\partial T}{\partial z}$ and $\frac{\partial^2 T}{\partial z^2}$, the convergence of focal depth failed when its trial value was 8.5 and 8.9 km and only three parameters were then estimated. These depths have still been retained in the results because they are with in about 10% of the assumed value.

It is thus concluded that for events falling outside the array [S-P] time will be useful for estimating focal depths and the nonlinear method provides relatively better estimates of focal depths as compared to LLS. If consideration is given to the shift in the hypocenter, i.e., $\sqrt{(\Delta x^2 + \Delta y^2 + \Delta z^2)}$, from the assumed value the nonlinear method which has given smaller shift has proved better. Further, the RMS residuals and length of the axis of ellipse [unscaled] governing the joint confidence region are also less in the case of nonlinear method (Table 9).

Other Test Results

Microearthquake locations for events recorded at 3 to 6 station networks operated in regions around Chamera, Jamrani and Navagam

TABLE 9 Test Locations for an Event Outside the Array with Focus Below the First Layer Employing NLLS (No.1 to 2) and LLS (No.3 to 4)

No.	Origin Time sec	Lati-tude min	Longi-tude min	Focal Depth km	No. of Iter-ations	RMS Residuals sec	AX*	BX*
1	09.12+	57.40	5[illegible].07	8.5	5	0.15×10^{-1}	2.04	12.91
2	10.04	54.87	55.11	8.5	5	0.43×10^{-2}	2.05	11.27
3	09.12+	57.31	53.09	8.9	7	0.15×10^{-1}	2.70	17.09
4	09.99	59.08	55.03	8.9	5	0.62×19^{-2}	2.66	14.68

+ Origin time restricted from (S-P) time.

* AX and BX are the unscaled lengths of the axes of error ellipse.

dam sites were obtained employing the LLS and NLLS techniques. The comparison of the results for two regions, viz., Chamera and Jamrani, employing both techniques will be reported shortly (Paper in preparation). However, salient features of the results are as follows :

The relative effectiveness of two techniques seems to depend on the position of hypocenter to be located with respect to recording stations. The results show by and large similar convergence history for events located within or close to the arrays. A special feature of the results around Chamera region is the convergence of focal depths for about 14% more events by NLLS as compared to LLS technique. For locations of events falling outside the array in the region of Jamrani dam site faster convergence has been observed employing NLLS technique. Comparison of few test locations obtained using 3 to 6 stations data are listed in Table 10. Epicenter parameters show insignificant variations but in few cases focal depths could converge employing NLLS technique.

VARIATION OF SECOND ORDER DERIVATIVES WITH EPICENTRAL DISTANCE

The variation of second order partial derivatives with epicentral distance is studied for the two hypothetical events whose locations are showing in Figure 4. Both these events lie in the layer over a half space. Graph in Figure 5 gives the variation of the partial derivatives with epicentral distance [Δ] for an event falling within the array. The Graph shows that, by and large, variation of the $\frac{\partial^2 T}{\partial x^2}$ and $\frac{\partial^2 T}{\partial y^2}$ follow similar trend whereas the trend of $\frac{\partial^2 T}{\partial x \partial y}$ is different. The Graph in Figure 6 shows the variation of the second order derivatives for an event falling outside the array and shows that the second order terms viz., $\frac{\partial^2 T}{\partial x^2}$ and $\frac{\partial^2 T}{\partial y^2}$ follow opposite trends. This results in the improvements of nearly linearly dependent vectors comprising the linear matrix and hence, improves the convergence. It is interpreted that for locations of events falling outside the array the incorporation of second order derivatives play significant role in imparting stability to the matrix during inversion and resulting improvement in its convergence. The variation of $\frac{\partial^2 T}{\partial x \partial y}$ perhaps fluctuates more for an event falling within the network as compared to an event falling outside the network.

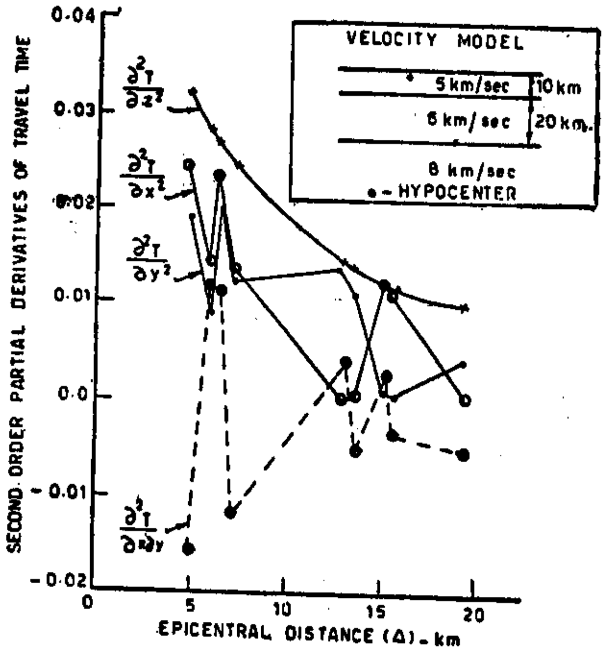


Fig. 5 Variation of Second Order Partial Derivatives With Epicentral Distance (Δ) For An Event Falling Inside The Array.

DISCUSSION

The problem of earthquake locations has been very actively persued for about three decades, yet the techniqes used are based on linear least square (LLS) optimisation. These teachniques differ on account of the use of alternate algorithms for computations of inverse and use first order partial derivatives of travel time. It is only recently that the nonlinear least square technique has been considered as a possible improvement over LLS technique, but has yet to find practical applications.

To study the role of second order derivatives in earthquake location problem, the expressions for the same are developed for one dimensional multilayer earth model. Further, it is dimonstrated by incorporating these expressions in an existing linear least square (LLS) program that these can be used in earthquake locations. The modified program

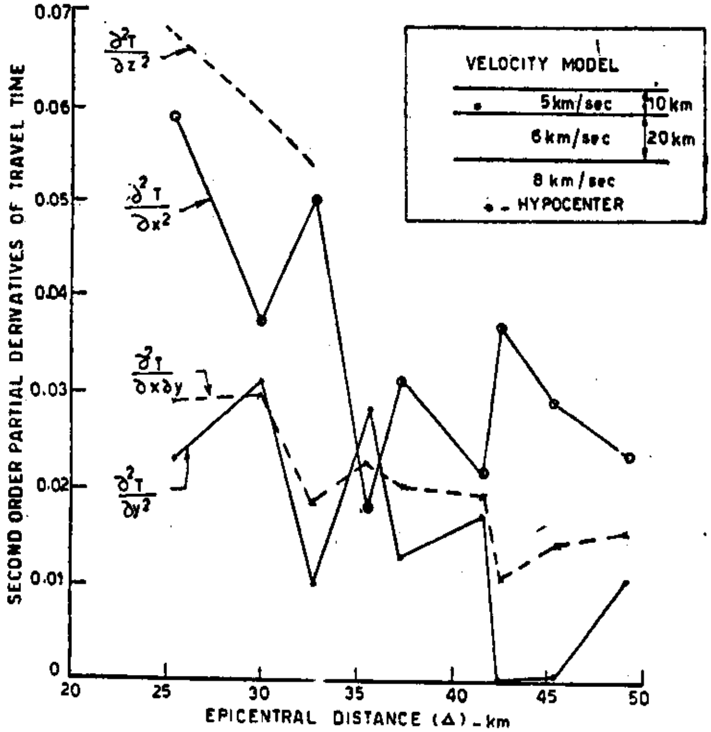


FIG. 6 - VARIATION OF SECOND ORDER PARTIAL DERIVATIVES WITH EPICENTRAL DISTANCE (Δ) FOR AN EVENT FALLING OUTSIDE THE ARRAY

termed as NLLS is tested with 17 station standard earthquake data, 9 station synthetic data and 3 to 6 station real earthquak data. Locations are also obtained using the LLS technique and the results are compared with the NLLS technique.

Testing with 17 station standard data set revealed that hypocenter converges to almost same solution employing both the NLLS and the LLS techniques. The hypocenter location by these programs was found not much different than that obtained employing HYPO 71 of Lee and Lahr (1975). There is a minor increase in the value of focal depth estimated with NLLS technique as compared to that obtained with HYPO 71 program. However, there has been reduction in the standard errors of hypocenter parameters as well as improvement in the convergence rate with NLLS technique as compared to LLS technique.

TABLE 10 Comparison of Test Locations Obtained Employing 3 to 6 Station Data by NLLS and LLS Techniques.

	Hypocenter Parameters Employing										
	NLLS Technique					LLS Technique					
No.	Origin Time sec	Lat-tude min	Longi-tude min	Focal Depth km	RMS Residuals sec	Origin Time sec	Lati-tude min	Longi-tude min	Focal Depth km	RMS Resi-duals sec	M‡
1	53.18	49.20	40.10	11.0	0.109	53.18	49.20	40.10	11.10	0.109	6
2	31.68	46.71	49.39	12.5	0.051	31.68	46.71	49.39	12.5	0.051	6
3	03.67	46.56	38.49	06.81	0.072	03.77	46.76	38.64	06.3	0.072	6
4	10.18	49.76	41.36	17.94	0.31	10.13	49.77	41.34	18.3	0.31	6
5	59.83	35.01	03.68	11.6	0.149	58.83	34.31	02.28	*	0.152	4
6	58.07	27.78	06.85	08.0	0.299	58.07	26.98	05.10	*	0.301	4
7	10.73	26.71	37.77	15.2	+	10.73	26.71	37.77	15.2	+	3
8	40.26	9.84	3.62	9.9	+	40.26	9.84	3.63	9.9	+	3
9	13.77	41.02	24.77	2.2	+	13.77	41.00	24.56	*	0.015	3
10	5.35	31.42	23.51	1.3	+	5.35	31.51	23.37	*	0.011	3

‡Nunber of recording stations.

* Focal depth convergence failed.

+RMS residuals are less than 10^{-6} sec.

Both the techniques gave identical hypocenter solution employing 9 station synthetic data for an event falling within the array. For an event falling outside the array in a layer over a half space identical solutions were obtained employing P-times only. Restricting origin time from (S-P) time and using P-times NLLS provided improved results. Almost identical epicenter locations were obtained for an event falling outside the array in a half space underlying a layer, whereas focal depth could be correctly obtained to about 10% of the assumed value. In all these test cases reduction in the lenght of the axes governing the cocfidence ellipse are brought out employing NLLS method as compared to the LLS method.

The locations obatained employing 3 to 6 station data show that hypocenter parameters converge to same solution employing both NLLS and LLS techniques. However, in certain cases focal depths could only converge employing NLLS technique. Thus, introduction of second order derivatives impart stability to the matrix during inversion.

Convegence of hypocenter to same solution employing both the techniques is due to the fact that there is only one point in space [i.e., hypocenter] towards which the solution should converge in case the location is well constrained. It has been demonstrated [Pavlis, 1986] that most of the existing location methods can be equated to the weighted least square problem. In case the stable solutions in the least square sense is obtainable from these weighted equations of condition, then various methods should converge to the same solution. The purpose of the teachniques like damped least square [Harrmann, 1979] and Newton's method [Thurber, 1985] is to promote convergence. As long as the location is well censtrained, various methods will converge to the same solution. This is the reason for the convergence of majority of locations to the same solution employing NLLS and LLS techniques.

The variations of second order derivatives [Figures 5 and 6] with epicentral distance shows that the dervatives $\frac{\partial^2 T}{\partial x^2}$ and $\frac{\partial^2 T}{\partial y^2}$ fluctuate with opposit trends for an event falling outside the array. For an event falling within the array variation of $\frac{\partial^2 T}{\partial x^2}$ and $\frac{\partial^2 T}{\partial y^2}$ have almost similar trends for short epicentral distances. Hence, the introduction of nonlinear terms promotes convergence of locations for events falling outside the array.

Both methods provide almost similar RMS residuals employing standard data set. Same RMS residuals are obtained for an event location within the array with synthetic data. For event falling outside the array the minimum RMS residual is obtained for the location which is obtained employing origin time restricted solution as the trial location . For location of an event falling in half space below a layer the RMS residuals are less employing LLS method. In brief, there are no systematic or definite patterns of RMS residuals revealed employing both the techniques.

The diagonal components of resolution matrix provide perfect resolution employing both the techniques, when the hypocenter solutions are well constrained, i.e., identical locations obtained employing both the methods. However, for unstable hypocenter solutions the LLS method still gave perfect resolution, whereas the diagonal components of resolution matrix employing NLLS deviate from unity under such conditions. It appears that more realistic estimation from the components of resolution matrix regarding the solution quality is possible employing NLLS method.

Employing covariance matrix, lengths of only two axes of the error ellipse are computed. These lengths are smaller with NLLS technique as compared to LLS technique. Thus, there is a reduction in the area of the ellipse and hence it seems NLLS technique provides better information of solution statistics in terms of joint confidence region. In view of overall considerations NLLS technique seems to provide improved estimation of hypocenter parameters.

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