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RESPONSE OF BUILDING-FOUNDATION SYSTEM TO AN EARTHQUAKE MOTION

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INTRODUCTION

In all the cases of Earthquake excitation, it has been assumed that the earthquake motions are introduced as specified quantities at the structural support points. In effect, these important displacements are assumed to depend only on the earthquake generation and wave mechanism and are not influenced by the response of the structure. In actual fact, the structure and the soil on which it is founded form a combined dynamic response mechanism, and there may be significant feed back from the structure into the soil layer. Hence, the extent to which the structural response may alter the characteristics of earthquake motions observed at the foundation level depends on the relative mass and stiffness properties of soil as well as the structure. Thus the physical properties of the foundation medium forms an important factor in the intensity of the earthquake response of structures supported on it. Because of these reasons the soil layer will have influence on the structural response to earthquakes. The effects of soil condition in the response calculation are taken as follows :

(i) The soil is considered without the structure and its effect on the characterisitcs of the vibratory waves propogated upwards from the base ment rock to the surface is evaluated. The resulting soil surface accelerations-without the structure are termed as the free field motions.

(ii) The structural response to the free field motions is computed by taking into account the interaction of the soil and strucuture, if the interaction is considered to be significant.

In the soil structure interaction analysis, the influence of the foundation must be represented in the analysis by an appropriate mathematical model. Various types of mathematical models are used for such analysis depending on the geometry of the boundaries and interfaces between different material properties as well as in the form of the foundation medium supporting the structure.

Hence this phenomenon of soil structure interaction has been recognised by research workers and many designers all over the world, to be an

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important factor in the behaviour of structures during seismic excitation. Several methods have been proposed for incorporating the effect of the interaction into the analysis and design of structural systems. However, the m·jority of these methods are based on highly emperical derivatives which have severe short comings or inconsistancies that limit their ability to properly model the phenomenon.

Many of the investigators dealing with the influence of flexibility of foundation on the seismic response of a structure have assumed the foundation to be represented by a homogeneous, isotropic elastic half space. The structure is assumed to be situated on the surface of the foundation medium and the interaction forces at the soil structure interface are produced by both the horizontal translation and rocking of the elastic foundation medium.

Parmelee, et al. (1967) studied this interaction problem on a similar structural system, however, the input motion was assumed to be harmonic; thus yielding only steady state response.

The purpose of the investigation presented in this study deals with the dynamic coupling or interaction between a single storey elastic structure and the flexible elastic foundation medium, when the system is subjected to an earthquake motion. The foundation medium is represented by an isotropic, homogeneous elastic half space. The earthquake effect resulting in ground acceleration is simulated by a transient time dependent function which exhibits the characteristics of the system resembling those of strong motion seismic disturbances.

2. REVIEW OF LITERATURE

Numerous analytical studies have been conducted to investigate the dynamic response of the building, however the majority of these investigators have assumed the structure to be attached to a rigid foundation medium (Berg 1960, Clough 1961, Lee 1966, Penzien 1960, Tung and Newmark 1955).

In reality, most of the buildings are situated on flexible foundations and the results have shown that the response of the structure is influenced by its supporting foundation medium [Kyoshi, et al. (1949), Suheyiro (1932) and Tanabashi, et al. (1953)] Within the past few years several investigators have directed their efforts towards the problem of determining the manner and extent of this influence on the response of structures.

Several methods have been proposed to represent the mass, damping and stiffness characteristics of the foundation media by discrete parameter models. The difficulty arisen with this approach is that there appears to be no rational approach for evaluating the approximate parametric constants.

Reissner (1936) studied the vertical translation of a rigid circular mass by assuming a uniform pressure distribution between the mass and the medium. Sung (1953) studied the problem but considered three different pressure distributions. Toriumi (1955) in addition to vertical translation included the horizontal translation and rotation of the plate about its diameter Bycroft (1956) considered these three modes in addition to torsional mode. He assumed stress distribution corresponding to static loading conditions and obtained approximations to the displacements of rigid plate by taking a weighted average of the resulting displacements taken at the centre and periphery of the plate. Paramelee et al. (1968) presented a method for the seismic response of a single storey elastic structure situated on elastic foundation medium which are represented by an elastic half space. The dynamtc properties of the half space are defined by steady state motion. The solution to the equation of the motion is obtained by harmonic analysis, by subjecting the system to a psuedo strong motion earthquake by a method similar to harmonic analysis, with the foundation properties redefined for each harmonic component. By inspecting the frequency dependent nature of these properties, it is found that good approximations to the seismic response of the interaction system may be obtained by taking the average values for the foundation properties in the range of the harmonic frequencies of the earthquake excitation. The results indicate that the response of the structure may be increased or decreased, when compared to the response of the same structure on rigid foundation. The review of the above research investigations reveals the effect of response with respect to the treatment of the foundation being considered as rigid or flexible. This leads to no intermodel coupling or coupling of the system.

Hence, from the practical point of view of an engineer, designing for seismic soil-structural systems, the most important basic assumptions used in the development of the elasticity solutions are (i) the structure is situated on the surface rather than having a partial embedment into the foundation medium, and (ii) no intermodel coupling has been assumed in the derivation of the solution for the various model responses (which are not so npractice).

A situation is encountered with increase in frequency in recent years, about the seismic design of a massive structures embedded at a considerable depth in a soil deposit; This is often the case for the design of nuclear power plants and pumping plants. A typical example is shown in Fig.5 and 6 where a massive structure, with a base pressure of about 9000 psf, a height of 120 feet; and a natural period of vibration of about 0.25 sec. is embedded at a depth of about 75 feet below the ground surface.

This example leads to an important aspect of the seismic design of a structure wherein the evaluation and importance of the dynamic interaction between the structure and surrounding soil play an important role. This has been generally considered by representing the effects of the soil on the structural response by a series of springs and dashpots (Fig.5) by modelling the soil-structure interaction system by the discrete mass approach model (Fig. 3) or finite element model (Fig. 4). These are the usual methods which are generally used to model the system and each method has its advantages and limitations and sometimes leads to different evaluation of the seismic response of the structure.

Parmelee and Kudder (1973) studied the dynamic response characteristics of tall buildings having their bases embedeed on a flexible foundation media using discrete parameter model for the soil structure system. The dynamic parameters of the foundation system in the form of their dynamic stiffness and associated radiation damping constants are determined using the dynamically loaded half space developed by Karasuddhi, Lee and Keer (1968). These interaction coefficients were developed from elasticity solutions generated by Bycroft (1956) and Gladwell (1967), wherein no intermodal coupling was assumed. These coefficients are used in the evaluation of the seismic response of a building embedded into an elastic foundation medium.

Hence the object of the present study is to consider the recently developed solution for the dynamic response of embedded structures and a solution which considers the intermodal coupling. The results are used in a convenient form with respect to the evaluation of the response of building foundation interaction systems. The response of the system is expressed in the form of displacements resulting due to stiffness coefficients and associated radiation damping coefficients. These coefficients characterise the dynamical properties of the elastic, semi-infinite foundation medium.

The response of the building-foundation system is further studied by introducing the embedment effect into the dyanamic interaction coefficients.

3. FORMULATION OF THE EQUATIONS OF MOTION

The response of the structure is studied by an equivalent dynamic model having fewer degrees of freedom. Full scale structures of actual structures subjected to transient loading indicate that the general method of replacing a complex structure by a suitable dynamic model yields satisfactory results for engineering purposes. Thus this study will utilise a mathematical model whose properties can be defined so that it will represent the dynamic response of any given building in its generalised, or normal co-ordinates. This simple model is shown in Figs. 1 and 2 with three degrees of freedom, (horizontal translation of the basemass m_o, horizontal translation of the topmass m, and rocking or rotation about an axis which bisects the cross sectional area of the base and lies on the horizontal boundary plans of the half space, i.e. an axis passing through point 'b').

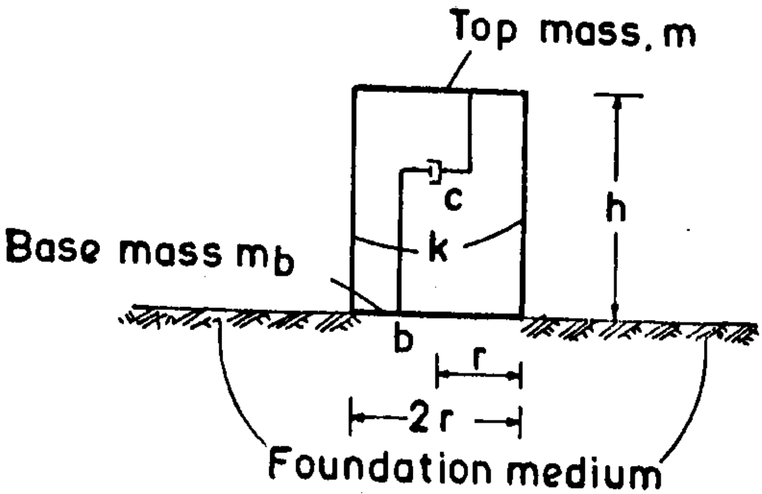

Fig. 1 Dynamic Model

To have the dynamic model compatible with the conditions of Bycroft solutions, some assumptions are necessary with respect to the foundation medium, the building model and the seismic waves.

4. ASSUMPTIONS

The basic assumptions on which this analysis is based is as follows :

(i) The ground or foundation medium, is a semi infinite, isotropic linearly elastic body.

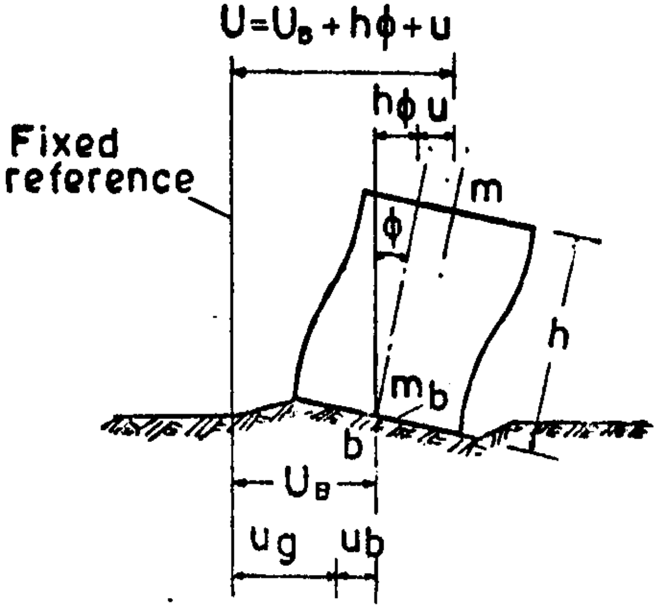

Fig. 2 Deflected Position

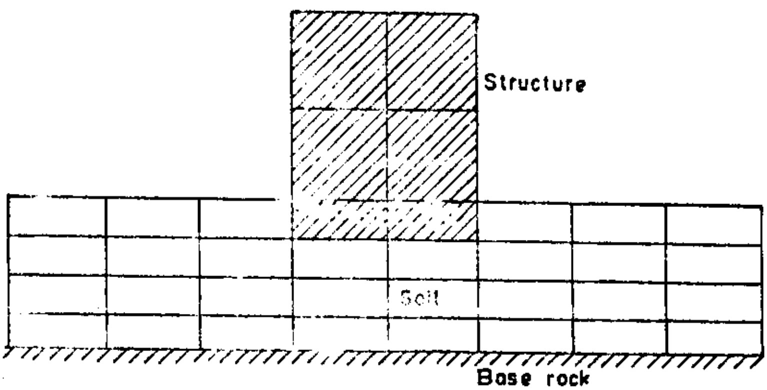

Fig. 3 Discrete Structure Soil Model

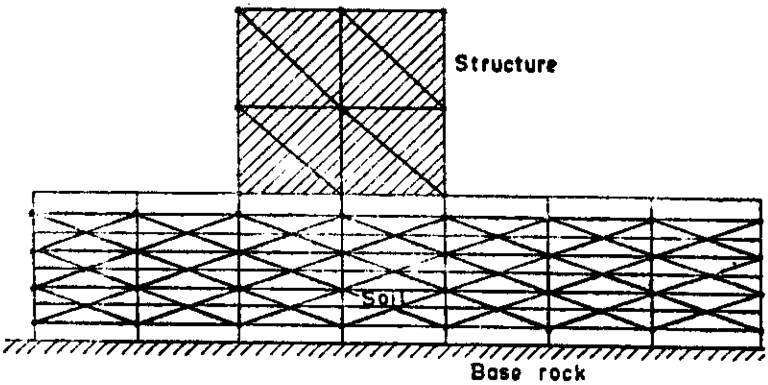

Fig. 4 Finite Element Model

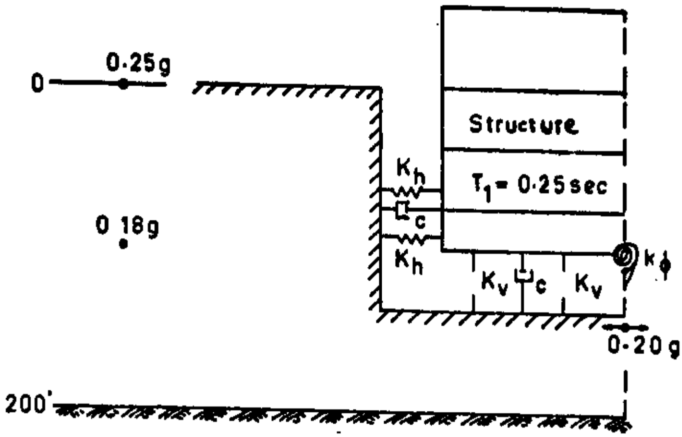

Fig. 5 Soil Structure Interaction Model

(ii) The soil properties do not change during the vibration.

(iii) There is no slippage between the base and foundation.

(iv) The base of the building model rests on the surface of the ground.

(v) The base of the building model is flexible/rigid and circular in plan.

(vi) The seismic motion is not violent enough to produce plastic strains or separation between the base of the model and the foundation medium.

(vii) The seismic waves come vertically upward from minus infinity to the boundary plane of half space, and

(viii) The seismic waves induce a free-field motion of the foundation medium which is purely horizontal.

5. EQUATIONS OF MOTION

The structure-foundation system as shown in Fig. 1 is an idealised model representing a single storey structure, which is circular in plan and has three degree of freedom i.e. horizontal translation of the top mass, 'm', horizontal translation of the rigid base mass, m_b, and ratation of the system about an axis on the horizontal boundary plane through the pint 'b'.

The deformed shape of the building is as shown in Fig. 1 where U_B and U are respectively the absolute horizontal displacements of the centre of the basemass and top mass with respect to a fixed reference. These quantities can be expressed in terms of the free field earthquake displacement, u_g and the relative displacement of the system as

$$U_B = u_g + u_b \tag{1}$$

$$U = u_g + u_b + h + \phi + u \tag{2}$$

in which u_b=the interaction translation of the base mass, ϕ, the interaction rotation of the base mass, u the flexural displacement of the top mass relative to the base mass and 'h', the storey height. Taking into consideration the effect of cross coupling, the equations of motion of the system as shown in Fig. 1 (b) would be

$$m(\ddot{u} + \ddot{u}_b + \ddot{u}_\phi) + C\dot{u} + Ku = -m\ddot{u}_g \tag{3}$$

$$m(\ddot{u} + \alpha\ddot{u}_b + \ddot{u}) + d_T\dot{u}_b + d_{hT}\dot{u}_\phi + K_T u_b + K_{RT} u_\phi = -m\alpha\ddot{u}_g \tag{4}$$

$$mh^2(u + u_b + \eta\ddot{u}_\phi) + d_{RT}\dot{u}_b + d_R\dot{u}_\phi + K_{RT}u_b + K_R u_\phi = -mh^2\ddot{u}_g \tag{5}$$

in which

$$\alpha = \frac{(m + m_b)}{-m} \tag{6}$$

and

$$\eta = [1 + \alpha\,(b/2h)^2] \tag{7}$$

and K_T and d_T are the translational dynamic stiffness and the associated translational radiation damping coefficients representing the foundation media, while K_R and d_R are the coefficients of rotational mode, K_{RT} and d_{RT} are the intermodal coupling coefficients. The other expression in Eqn. (1—5) represents a single storey interaction system as shown in Fig. 1

which has a base width of 2b and rests on a surface of a homogeneous elastic half space.

Dividing the Eqn. (3), Eqn. (4) and Eqn. (5) by m, m_b and mh^2 the Eqn (3), (4) and (5) reduce to

$$\ddot{u} + \ddot{u}_b + \ddot{u}_\phi + 2\omega_\gamma \lambda \dot{u} + \omega^2_\gamma u = -\ddot{u}_g \tag{8}$$

$$\ddot{u} + \alpha \ddot{u}_b + \ddot{u}_\phi + \frac{dT}{m}\dot{u}_b + \frac{d_{RT}}{m}\dot{u}_\phi + \frac{K_T}{m}u_b + \frac{K_{RT}}{m}u_\phi = -\alpha \ddot{u}_g \tag{9}$$

$$\ddot{u} + \ddot{u}_b + \eta \ddot{u}_\phi + \frac{d_{RT}}{mh^2}\dot{u}_b + \frac{D_R}{mh^2}\dot{u}_\phi + \frac{K_{RT}}{mh^2}u_b + \frac{K_R}{mh^2}u = -\ddot{u}_g \tag{10}$$

The interaction coefficients parameters are given by

$A_1 = d_T\ /m$ (11)

$A_2 = d_{RT}\ /m$ (12)

$A_3 = K_T/m$ (13)

$A_4 = K_{RT}/m$ (14)

$A_5 = d_R/mh^2$ (15)

$A_6 = K_R\ /mh^2$ (16)

$A_7 = A_2/h^2 = d_{RT}\ /\ mh^2$ (17)

$A_8 = A_4/h^2 = K_{RT}\ /\ mh^2$ (18)

Introducing these termimology and substituting in place of ω_r^2 and λ being equal to K/M and C/2 $\sqrt{}$ km, the Eqns. (8), (9), and (10) get simplified to

$$\ddot{u} + \ddot{u}_b + \ddot{u}_\phi + 2\omega_r \lambda \dot{u} + \omega_r^2 = -\ddot{u}_g \tag{19}$$

$$\ddot{u} + \alpha \ddot{u}_b + \ddot{u}_\phi + A_1 \dot{u}_b + A_2 \dot{u}_\phi + A_3 u_b + A_4 u_\phi = -\alpha \ddot{u}_g \tag{20}$$

$$\ddot{u} + \ddot{u}_b + \eta \ddot{u}_\phi + A_7 \dot{u}_b + A_5 \dot{u}_b + A_8 u_b + A_6 u_\phi = -\ddot{u}_g \tag{21}$$

where

$\alpha = [(m+m_b)/m]$ (22)

$\eta = [1+(b/2h)^2]$ (23)

$\ddot{u}_g$ = the ground acceleration

It should be observed that A_i obtained for steady state harmonic motion, are frequency dependent. Consequently Eqns. (19) to (21) can only deal with harmonic excitation: i.e. A_i are functions of the frequency of the input function $\ddot{u}_g$ which must be harmonic.

5.1 Earthquake Acceleration Function

To study the seismic response of the interaction system an approximate acceleration function is simulated from the actual earthquake acceleration function. This function is represented by series of harmonic components by Bogdanoff, Goldberg and Bernard (1960). Goldberg, Bogdanoff and Sharpe (1964) suggested a random acceleration function of the form :

$$\ddot{u}_g(t) = 0 \qquad t<0 \qquad (24)$$

$$\ddot{u}_g(t) = \sum_{J=1}^{J} \xi_J \; t \; e^{\zeta jt} \cos(\omega_J t + \Psi_J t) \qquad t \geqslant 0 \qquad (25)$$

in which ξ_J, ζ_J are real positive numbers with $\omega_1 < \omega_2 < \omega_J$ and $\Psi_1, \Psi_2, \ldots \ldots \Psi_J$ are J independent real random variables uniformly distributed over the interval 0 to 2π.

To reduce the amount of computational time Bogdanoff et al. (1960) suggested ten terms which would be quite sufficient assuming ξ_J and Ψ_J are constants.

Hence the acceleration funciton used in the present study is given by

$$\ddot{u}_g(t) = 0 \qquad t<0 \qquad (26)$$

$$\ddot{u}_g(t) = 0.50\, t_e - 0.333t \sum_{J=1}^{10} \cos(\omega_J t + \Psi_J) \qquad t \geqslant 0 \qquad (27)$$

Expanding each term in Eqn. (27) in to a Fourier series, the earthquake acceleration function takes the form

$$\ddot{u}_g(t) = \sum_{J=1}^{10} \sum_{n=1}^{N} b_{nJ} \sin \Omega_n t \qquad t \geq 0 \qquad (28)$$

in which

$$\Omega_n = n\pi/T$$

and b_{nJ} = the Fourier coetficient of n^{th} term of the Fourier series corresponding to the J^{th} term of the acceleration function.

T = time interval over which the function is to be represented

N = Number of terms required to represent the function

$$\ddot{u}_g(t) = \sum_{n=1}^{N} \Sigma \ddot{u}_{gn} C_n \sin \Omega_n t, \quad t \geq 0 \tag{30}$$

in which

$$\ddot{u}_{gn}(t) = C_n \sin \Omega_n t \tag{31}$$

and

$$C_n = \sum_{J=1}^{10} b_{nJ} \tag{32}$$

$(n=1, 2, \ldots\ldots N)$

For the investigations, the time interval 'T' is selected to be 20 sec. and 200 terms to represent the function with sufficient accuracy.

5 2 Solutions to the Equations of Motion

The equations of motion (19 – 21) can be solved 'N' times once for each harmonic of $\ddot{u}_{gn}(t)$ of the input function $\ddot{u}_g(t)$. Since the system is linearly elastic the total response will be the superposition of 'N' components.

$$u_b(t) = \sum_{n=1}^{N} u_{bn}(t) \tag{33}$$

$$u_\phi(t) = \sum_{n=1}^{N} u_{\phi n}(t) \tag{34}$$

$$u(t) = \sum_{n=1}^{N} u_n(t) \tag{35}$$

For the n^{th} component, the Eqns. (19, 20, 21) get modified when substituting $\ddot{u}_{gn}(t)$ into the Eqn. (19), (20) and (21), and are given by

$$\ddot{u}_{bn} + \ddot{u}_{\phi n} + \ddot{u}_n + 2\omega_r \lambda \dot{u}_n + \omega^2_r u_n = -\ddot{u}_{gn} \tag{36}$$

$$\ddot{u}_{bn} + \ddot{u}_{\phi n} + \ddot{u}_n - A_1 \dot{u}_{bn} + A_2 \dot{u}_{bn} + A_3 u_{bn} + A_4 u_{\phi n} = -\alpha \ddot{u}_{gn} \tag{37}$$

$$\ddot{u}_{bn} + \eta \ddot{u}_n + \ddot{u}_n + A_7 \dot{u}_{bn} + A_5 \ddot{u}\phi_n + A_8 u_{bn} + A_6 u_{\phi n} = -\ddot{u}_{gn} \tag{38}$$

Introducing the operator $D = d/dt$

$$\ddot{D} = D^2, \dot{D} = D, D = D \tag{39}$$

These form the operators of the equation.

The equations are further reduced to

$$xD^2 + yD^2 + zD^2 + 2\omega_r \lambda Dz + \omega^2_r z = -\ddot{u}_g \tag{40}$$

$$\alpha x D^2 + yD^2 + zD^2 + A_1 Dx + A_2 Dy + A_3 x + A_4 y = -\alpha \ddot{u}_g \qquad (41)$$

$$xD^2 + \eta y D^2 + D^2 z + A_7 Dx + A_5 Dy + A_8 x + A_6 y = -\ddot{u}_g \qquad (42)$$

Rearranging the terms,

$$D^2 x + D^2 y + (D^2 + 2\omega_r \lambda D + \omega^2_r)\, z = -\ddot{u}_g \qquad (43)$$

$$(\alpha D^2 + A_1 D + A_3)\, x + (D^2 + A_2 D + A_4)\, y + D^2 z = -\alpha \ddot{u}_g \qquad (44)$$

$$(D^2 + A_7 D + A_8) x + (\eta D^2 + A_5 D + A_6) y + D^2 z = -\ddot{u}_g \qquad (45)$$

Let

$$\begin{aligned} J &= (D^2 + 2\,\omega_r \lambda\, D + \omega^2_r) \\ K &= (\alpha D^2 + A_1 D + A_3) \\ L &= (D^2 + A_2 D + A_4) \\ M &= (D^2 + A_7 D + A_8) \\ N &= (\eta D^2 + A_5 D + A_6) \end{aligned} \qquad (46)$$

$$P = -\ddot{u}_{gn},\ x = u_{bn},\ y = u_{\phi_n},\ z = u_n$$

Then Eqns. (43) to (45) reduce to the form

$$D^2 x + D^2 y + Jz = -P \qquad (47)$$

$$Kx + Ly + D^2 z = -\alpha P \qquad (48)$$

$$Mx + Ny + D^2 z = -P \qquad (49)$$

The Eqns (47) to (49) represent three simultaneous equations with x, y, and z as unknowns representing the response parameters u_{bn}, u_{ϕ_n}, and u_n. After solving these equations, we get

$$x = \frac{[(D^2 - \alpha J) - (D^4 - JL) y] P}{(D^4 - KJ)} \qquad (50)$$

$$y = [A/B]\, P$$

and

$$z = \frac{P}{D^2}\left[\frac{(\alpha B - LA)}{NB} - \frac{K[B(\alpha - 1) - A(L - N)]}{B(K - M)}\right] \qquad (51)$$

where

$$\begin{aligned} A &= \alpha D^4 - D^4 + KJ - KD^2 + MD^2 - M\alpha J \\ B &= D^4(L - N) + K(NJ - D^4) + M(D^4 - JL) \end{aligned} \qquad (52)$$

Substituting the corresponding values of A,B,K,L,M, and N, in the Eqns (50) to (52), the equations get modified to

$$x(B) = [D^4(1 - \alpha) + \alpha NJ - JL + LD^2 - ND^2] P \qquad (53)$$

$$y(B) = [\alpha D^4 - D^4 + KJ - KD^2 + MD^2 - M\alpha J] P \qquad (54)$$

and

$$z(B) = \left[(\alpha B - LA) - K(D^4 + \alpha NJ - \alpha D^4 - JL + LD^2 - ND^2)\right] \frac{P}{D^2} \qquad (55)$$

By referring to Eqns. (53) to (55), it can be observed that x, y and z are functions of 'B'. To obtain the values of the response parameters u_{bn}, $u_{\phi n}$ and u_n, both the left hand side and right hand side of the Eqns. (53)— (55) are to be expanded. Therefore by expanding the left hand side of the expression of the eqn. (53) we get into the form

$$x(B) = xD^4(L-N)+K(NJ-D^4)+M(D^4-JL) \quad (56)$$

Substituting the values of J,K,L,M,N it gets into the form

$$\begin{aligned}&[D^6(-\alpha+1-\eta+\alpha\eta)+D^5(-A_1-A_5-\alpha A_5+\eta A_1+2\alpha\eta\,\omega_r\lambda-2\omega_r\lambda)\\&+D^4(A_4-A_6+\alpha A_6+A_1A_5+\eta A_3+2\alpha\omega_r\lambda A_5+2\omega_r\lambda\eta A_1+a\eta\omega^2_r-2A_7\omega_r\lambda\\&-\omega^2_r-A_2A_7\ \ A_3-2\omega_rA_2\lambda\ \ A_4)+D^3(A_1A_6+A_3A_5+2\omega_r\lambda\alpha A_6+2\omega_r\lambda A_1A_5\\&+2\omega_r\lambda\eta A_3+\omega^2_r\alpha A_5+\omega^2_r\eta A_1-2A_6\omega_r-A_7\omega^2_r-A_8A_2-2A_7A_2\omega_r\lambda-\omega^2_r\\&-A_3-A_4A_7-2A_4\omega_r\lambda)+D^2(A_6A_3+2\omega_r\lambda A_1A_6+2\omega_r\lambda A_3A_5+\omega^2_r\ \alpha A_6+\omega^2_rA_1A_5\\&+\omega^2_r\eta A_3-A_3\,\omega^2_r-2A_8\,\omega_r\lambda A_2-A_7A_3\,\omega^2_r-A_4A_8-2A_4A_7\,\omega_r\lambda\\&-A_4\,\omega^2_r)+D\,(2\omega_r\lambda\ A_6A_3+\omega^2_r\ A_1A_6+\omega^2_r\ A_3A_5-A_8A_2\,\omega^2_r\\&-2A_4A_8\,\omega_r\lambda-A_4A_7\,\omega^2_r)+(A_6A_3\,\omega^2_r-A_4A_8\,\omega^2_r)]\end{aligned} \quad (57)$$

This can be written in a simplified way as

$$[\beta_1D^6+\beta_2D^5+\beta_3D^4+\beta_4D^3+\beta_5D^2+\beta_6D+\beta_7]\,u_{bn}=L(u_{bn}) \quad (58)$$

This remains same even for $u_{\phi n}$ and u_n.

Similarly

$$y(B)=L(y)=L(u_{\phi n}) \quad (59)$$

and

$$z(B)=L(z)=L(u_n) \quad (60)$$

$L(u_{bn})$, $L(u_{\phi_n})$ and $L(u_n)$ should be equated to the corresponding expanded form of the R H S of the expressions associated with x(B) and z(B).

Hence for the n^{th} component u_{bn}, u_{ϕ_n} and u_n, substituting $\ddot{u}_{gn}(t)$ in Eqns. (53) — (55) leeds to

$$\begin{aligned}L\,(u_{bn}) = {}&[D^4(1-\alpha+\alpha\eta-\eta)+D^3(\alpha A_5+2\omega_r\lambda\,\alpha\eta-2\omega_r\lambda-A_5)\\&+D^2(\alpha A_6+2\omega_r\lambda\alpha A_5+\alpha\eta\omega^2_r-\omega^2_r-2\omega_r\lambda-A_6)+D(2\omega_r\lambda\alpha A_6\\&+\alpha A_5\omega^2_r-A_2\omega^2_r-2\omega_r\lambda\,A_4)+(\alpha A_6\,\omega^2_r-\omega^2_rA_4)]\,\ddot{u}_{gn}\end{aligned} \quad (61)$$

$$\begin{aligned}L\,(u_{\phi_n}) = {}&[(A_7(1-\alpha)D^3)+D^2(2\omega_r\lambda A_1+A_2-2\omega_r\lambda A_7-\alpha A_8)\\&+D(2\omega_r\lambda A_3+\omega^2_rA_1-\alpha A_7\omega^2_r-\alpha A_8\,2\omega_r\lambda)+(\omega^2_rA_3-\alpha A_8\,\omega^2_r)]\,\ddot{u}_{gn}\end{aligned} \quad (62)$$

$$\begin{aligned}L(u_n) = {}&[D^3(\alpha A_7-A_7-A_1+A_1\eta)+D^2(\alpha A_8-A_2A_7-A_8-A_3\\&+A_3\eta+A_1A_5)+D(-A_7A_4-A_2A_8+A_3A_5+A_1A_6)+(-A_4A_8\\&+A_3A_6)]\,\ddot{u}_{gn}\end{aligned} \quad (63)$$

in which the operator

$$L=\beta_1D^6+\beta_2D^5+\beta_3D^4+\beta_4D^3+\beta_5D^2+\beta_6D+\beta_7 \quad (64)$$

where

$$\beta_1 = [(1-\alpha) + \eta\ (\alpha-1)] \tag{65}$$

$$\beta_2 = [(A_1(\eta-1) - A_5\ (1+\alpha) + 2\alpha\eta\omega_r - 2\omega_r\lambda)] \tag{66}$$

$$\begin{aligned}\beta_3 = [(A_4 - A_6 + \alpha A_6 + A_1A_5 + \eta A_3 + 2\alpha\omega_r\lambda A_5 \\ + 2\omega_r\lambda\ \eta A_1 + \alpha\eta\omega^2_r - 2\ A_7\ \omega_r\lambda\ - \omega^2_r \\ - A_2A_7 - A_3\ -2\omega_r A_2\lambda - A_4)]\end{aligned} \tag{67}$$

$$\begin{aligned}\beta_4 = [(A_1A_6 + A_3A_5 + 2\omega_r\lambda\alpha A_6 + 2\omega_r\lambda A_1A_5 \\ + 2\omega_r\lambda\ \eta A_3 + \omega^2_r\ \alpha A_5\ - 2A_8\omega_r\lambda\ + \omega^2_r\eta A_1 \\ - A_7\omega^2_r - A_8A_2 - 2A_7A\omega_{2r}\lambda - \omega^2_r - A_2 - A_4A_7 - 2A_4\omega_r\lambda)]\end{aligned} \tag{68}$$

$$\begin{aligned}\beta_5 = [(A_6A_3 + 2\omega_r\lambda\ A_1A_6 + 2\omega_r\lambda\ A_3A_5 + \omega^2_r\ \alpha A_6 + \omega^2_r\ A_1A_5 \\ + \omega^2_r\ \eta A_3 - A_8\ \omega^2_r\ - 2A_8\omega_r\lambda A_2 - A_7A_2\omega^2_r - A_4A_8 \\ - 2A_4A_7\ \omega_r\lambda - A_4\ \omega^2_r))]\end{aligned} \tag{69}$$

$$\begin{aligned}\beta_6 = [(2\omega_r\lambda\ A_6A_3 + \omega^2_r\ A_1A_6 + \omega^2_r\ A_3A_5 - A_8A_2\omega^2_r \\ -2A_4A_8\omega_r\lambda\ - A_4A_7\omega^2_r)]\end{aligned} \tag{70}$$

$$\beta_7 = [(A_6A_3\omega^2_r - A_4A_8\ \omega^2_r)] \tag{71}$$

The particular integral of the n^{th} component may be obtained in the form

$$u^p_{bn}\ (t) = E_{bn}\ \cos\Omega\ n^t + F_{bn}\ \sin\ \Omega_n{}^t \tag{72}$$

$$u^p_n\ (t) = E\phi_n\ \cos\ \Omega n^t + F\phi_n\ \sin\ \Omega_n{}^t \tag{73}$$

$$u^p_n\ (t) = E_n\ \cos\ \Omega n^t + F_n\ \sin\ \Omega_n{}^t \tag{74}$$

where

$$\begin{aligned}E_{bn} &= \frac{a_2x_2 - a_4x_1}{a_2a_3 - a_1a_4} \\ F_{bn} &= \frac{a_3x_1 - a_1x_2}{a_2a_3 - a_1a_4} \\ E_{\phi n} &= \frac{a_2y_2 - {}_4ya}{a_2a_3 - a_1a_4} \\ F_{\phi n} &= \frac{a_3y_1 - a_1y_2}{a_2a_3 - a_1a_4} \\ E_n &= \frac{a_2z_2 - a_4z_1}{a_2a_3 - a_1a_4} \\ F_n &= \frac{a_3z_1 - a_1\ z_2}{a_2a_3 - a_1\ a_4}\end{aligned} \tag{75}$$

and

$$a_1 = -\ \Omega^6{}_n\ (XL6) + \Omega^4{}_n\ (XL4) - \Omega^2{}_n\ (XL2) + XL$$

$$a_2 = \Omega^5{}_n\ (XL5) - \Omega^3{}_n\ (XL3) + \Omega_n\ \ (XL1)$$

$$a_3 = -\ [\Omega^5{}_n\ (XL5) + \Omega^3{}_n\ (XL3) - \Omega_n\ (XL1)]$$

$$a_4 = -\ [\Omega^6{}_n\ (XL6) + \Omega^4{}_n\ (XL4) - \Omega^2{}_n\ (XL2)]$$

$$x_1 = C_n[-\Omega^3{}_n\ (XR3) + (XR1)\ \Omega_n]$$

$$x_2 = C_n[\Omega^4{}_n\ (XR4) - \Omega^2{}_n\ (XR2) + XR]$$

$$y_1 = C_n[\Omega^3{}_n\ (yR3) + (YR1)\ \Omega_n]$$

$$y_2 = C_n[-\Omega^2{}_n\ (YR2) + (YR)\ \Omega_n]$$

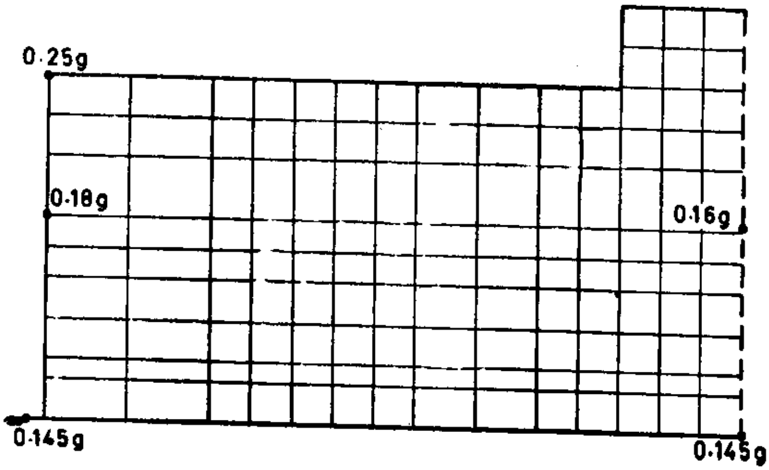

Fig. 6 Finite Element Model of Soil Structure System

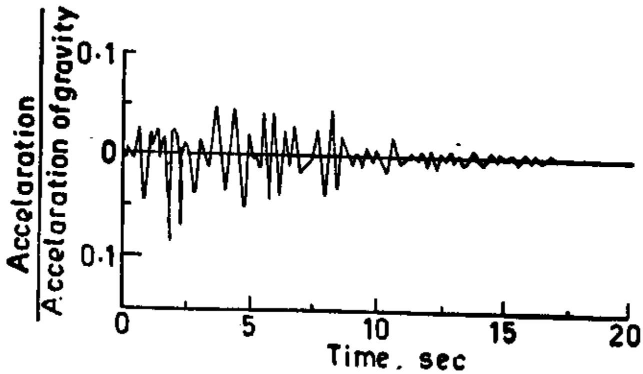

Fig. 7 Earthquake Acceleration Function as Defined by Eq (6-49) and (6-52)

$$z_1 = C_n[-\Omega^2_n\,(ZR3) + \Omega_n\,(ZR1)]$$
$$z_2 = C_n[-\Omega^2_n\,(ZR2) + (ZR)\,\Omega_n] \qquad (76)$$

Substituting the values of E_{bn}, F_{bn}, $E\phi_n$, $F\phi_n$, E_n and F_n in Eqns. (72), (73) and (74), the values of the response parameters $u^p{}_{bn}$, $u^p\phi_n$ and $u^p{}_n$ are evaluated. To have the complete solution with respect to the total response of the system, complimentary solutions, has to be evaluated.

Hence the total solution for the n^{th} component is then given by

$$u_{bn}\,(t) = u^p{}_{bn}\,(t) + u^c{}_{bn}\,(t) \qquad (77)$$

$$u_n\,(t) = u^p{}_{\phi_n}\,(t) + u^c{}_{\phi_n}\,(t) \qquad (78)$$

$$u_n\,(t) = u^p{}_n\,(t) + u^c{}_n\,(t) \qquad (79)$$

6. RESULTS AND DISCUSSION

The numerical computation for the solution of the problem has been carried out in IBM-360 and DEC-1090 Computer of Indian Institute of Science, Bangalore. In obtaining the complimentary solution for each harmonic, the value of A_i, β_i in the operator are those associated with the frequency Ω_n, and $\ddot{u}_{gn}$. The validity of the complimentary solution so obtained is verified for a representative test problem. From the ressults it was found that although each of the seven complimentary solution are of the same order of magnitude as their particular solutions, the sum of the seven complimentary solutions are approximately eight of magnitude smaller than the total particular solution i.e. the complimentary solution vanish for practical purposes. Hence to evaluate the response of the system the particular solution is used. The interaction system is subjected to an acceleration function defined by equation 6.50 with N=200. The exact solution is obtained by superimposing two hundred particular integrants and their associated complimentary solutions.

To compare the results the example problem studied by Parmelee, et al, (1967) is taken into the analysis, wherein the dynamic interaction coefficients are introduced resulting due to intermodal coupling. Further the response study has been made for different values of Poisson's ratio (μ) equal to 0 0, 0.10, 0.25, 0.35, 0 50 and shear wave velocity (V_s) equal to 300 fps, 400 fps, 600 fps, 800 fps, 900 fps, 1000 fps, 1200fps and 2000 fps. The response parameters u_b, u_ϕ and u are obtained without embedment of and with embedment of building / foundation respectively. The study has been made for a single storey building as assumed by Parmelee, et. al. (1967).

The results obtained are discussed in the following sections.

6 1 Comparison of the results with the example problem of Parmelee, et. al (1967).

Parmelee, et al (1967) studied the interaction problem of single-storey structure - foundation system subjected to seismic excitation. In the analysis the interaction dynamic coefficients resulting due to foundation media has not been taken into account. The corresponding response curves under the above data has been represented in Fig. 8. The results are independent of dynamic, coupling between the structure and its flexible foundation medium. To show the importance of the intermodal coupling, the dynamic interaction coefficients, are introduced in the problem studied

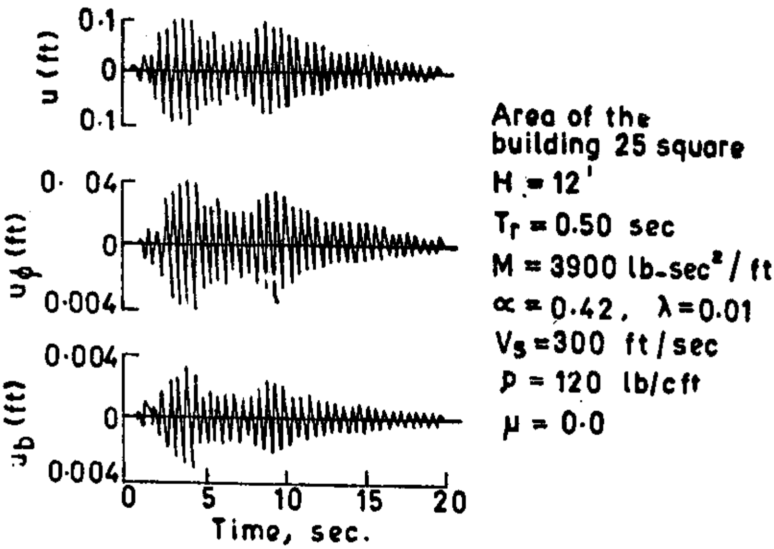

Fig. 8 Response Curves for Example Problem (After Parmele let. al)

by Parmelee et al (1967) and the results are presented in Fig. 9. By comparing the results it can be seen that the reponse parameters u_b, $u\phi$ and u representing the horizontal displacement of the base mass, interaction rotation of the base mass and flexural displacement of the top mass vary appreciably with respect to the response curves of response problem, studied by Parmelee, et. al (1967) respectively, (Fig. 8). From the results it can be seen that the intermodal coupling due to foundation media will give rise to an important aspect of the economy in the design of structures.

The parametric study has been made with reference to single-storey structures for using about the importance of dynamic interaction coefficient resulting due to soil structure interaction. The parameters considered, and their values are discussed in section 6. Their importance on the response of the system (u_b, $u\phi$, u) are briefly discussed and concluded in section 7.1, 7.2, 7.3.

7. DISCUSSIONS AND CONCLUSIONS

A method is presented for investigating the seismic response of soil-structure system in which the building is embedded into the elastic foundation medium. The imporatance of the foundation media as the integral part of the stuctural system has been considered, since the flexible

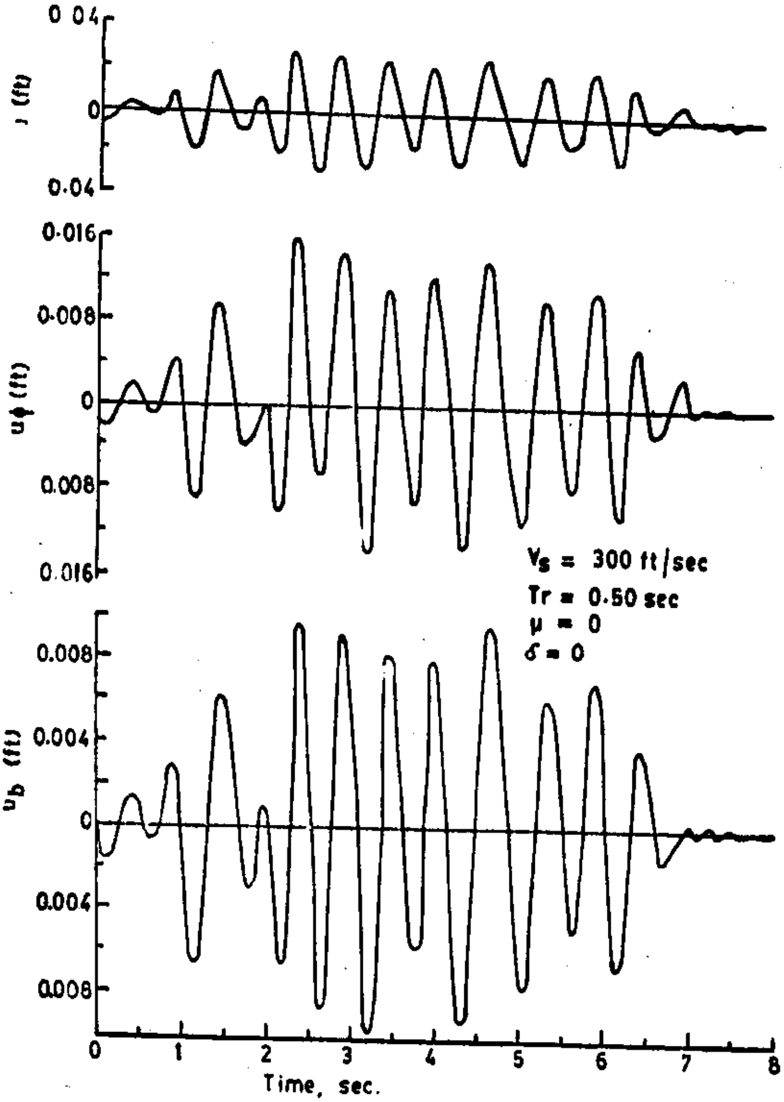

Fig. 9 Response Curves

foundation can attenuate or amplify the flexural response of a structure depending upon the characteristics of the building and the foundation media as well as the embedment of the building. The procedure proposed in the proceding sections incorporating the effect of soil-structure interaction provides sufficient flexibility and accuracy in evaluating the structural response parameters for practical applications. Hence depending on the characteristics of the structure and he ground motion under considaration the effect of soil structure interaction may increase or decrease or have no effect on the magnitudes of the maximum forces induced in the structure itself. From the results discussed in the preceding sections, it can be seen that the soil-structure interaction will reduce the design values of the base shear and moment from the levels applicable to a rigid base condition. Becuase of the influence of the foundation Rocking, however the horizontal displacement relative to the base of elastically supported structure may be larger than those of the fixed base structure.

For the problem studied on the dynamics of soil structure interaction, the importance of dynamic coupling due to dynamic interaction coefficients are considered and its imporatance can be seen from the analysis of the results. The response parameters u_b, u_ϕ and u are studied under the effect of Poission's ratio (μ) shear wave velocity (V_s) and embedment ratio (δ). The results are discussed in detail in sections 6 1. The major discussion and conclusions inferred from the results are as follows.

7.1 Response parameter u_b (horizontal interaction displacement of the base mass)

From the results it can be seen that the response parameters u_b, u_ϕ and u decrease with the increasing shear wave velocities. Decrease in value of the response parameter u_b reveals that the horizontal interaction displacement of the base mass decreases as the shear wave velocity increases and its effect is not much predominant. The maximum response value can be seen for a shear wave velocity of 300 ft/sec., $\mu=0.00$ and $\delta=0.00$. The response parameter u_b further decreases with increase in embedment ratio. The influence of the shear wave velocity and Poisson's ratio with increase in embedment ratio will not have any influence on the values of the response parameter (Figs. 10-29).

7.2 Response parameter u_ϕ (interaction rotation of the base mass)

Herein also, the response parameter u_ϕ represents a similar behaviour as being observed in the case of response parameter u_b. The maximum value of the rotational displacement is observed for shear wave velocity of 300 fps, $\mu = 0\ 0$ and $\delta = 0.00$. The value is much lower than that of the horizontal displacement of the basemass u_b. At higher velocities the value reduces to a very low value of the order of 10^{-4} to 10^{-5} feet. Even with increasing, Poisson's ratio the value of the response parameter u_ϕ decreases.

From the magnitude of the values it can be seen that the effect of the inter modal coupling will not have any influence on the response of the system due to interaction rotation of the base mass. Comparing the values with that of the structure-foundation system resting on a rigid base, it can be seen that the base moments and shears are reduced considerably, hence the advantage of flexible structure-foundation system. (Ref. Figs. 10-29).

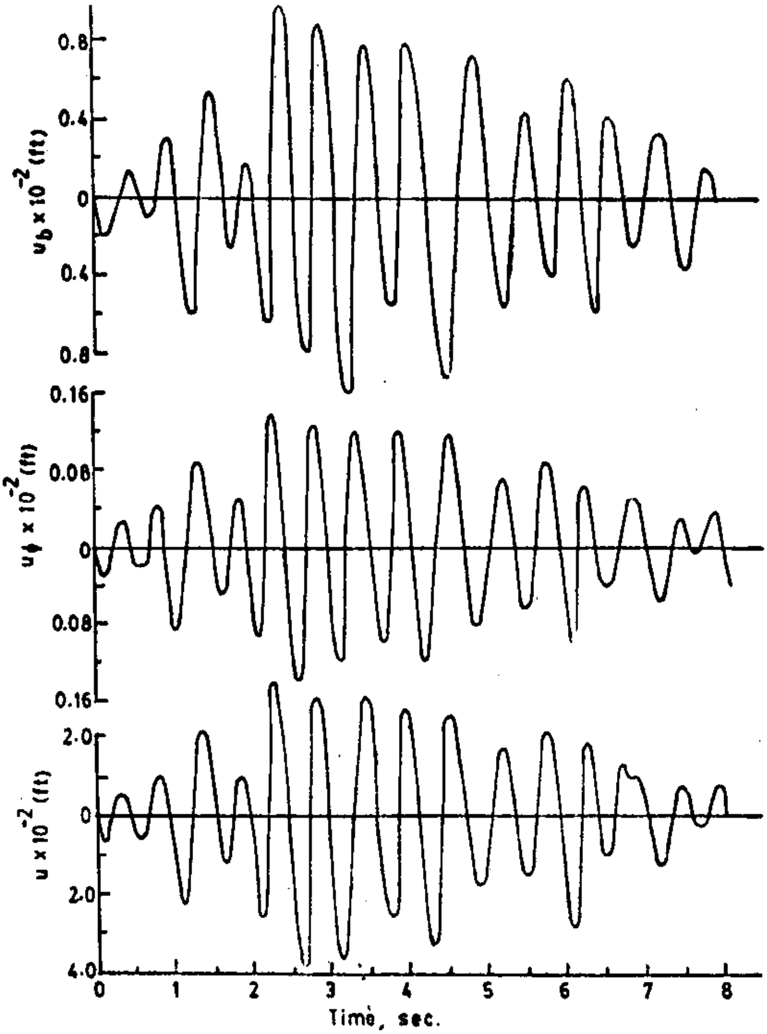

Fig. 10 Response Curves (μ=0, V_s=300 ft/sec, δ=0.2)

7.3 Response parameter u (flexural displacement of the top mass).

From the results it is seen that the value of the response parameter u

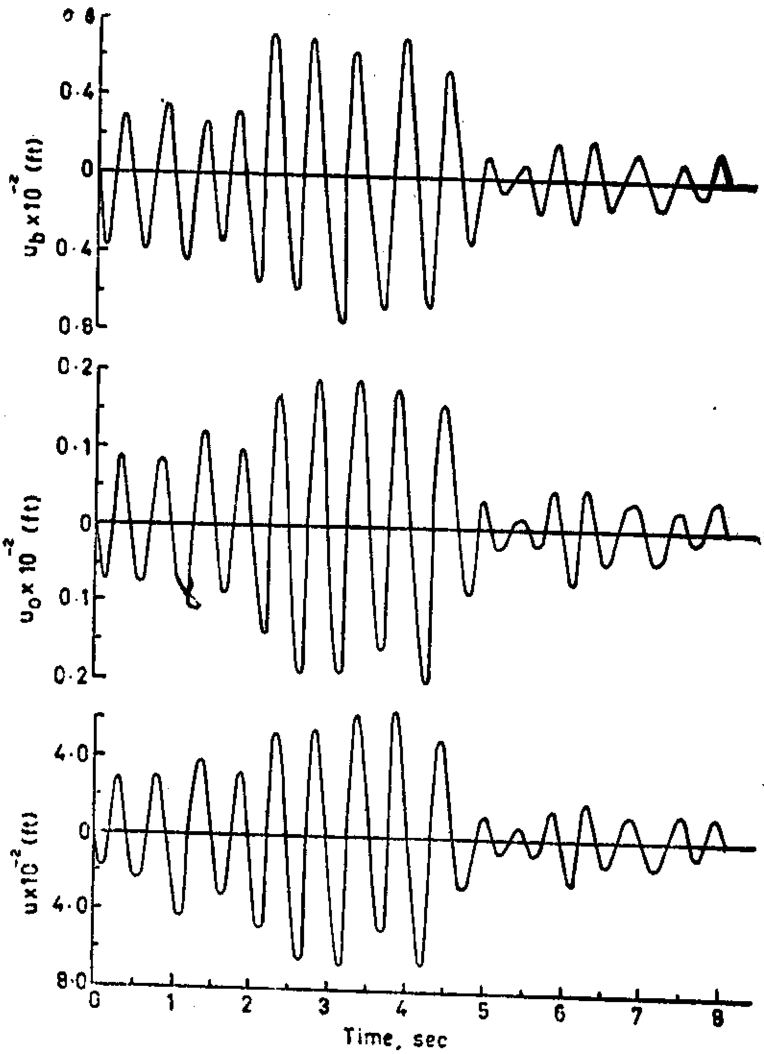

Fig. 11 Response Curves (μ=0.25, V_s=300ft/sec, δ=0.0)

increases for shear wave velocity of 300 fps, and 600 fps. For velocities higher than these velocities, the response behaviour almost remains constant. Hence from the results it can be concluded that the effect of shear wave velocity will not have any appreciable effect on the flexural displacement of the structure. The same conclusion is valid even in case of increasing Poisson's ratio and embedment ratio. (Ref. Figs. 10-29)

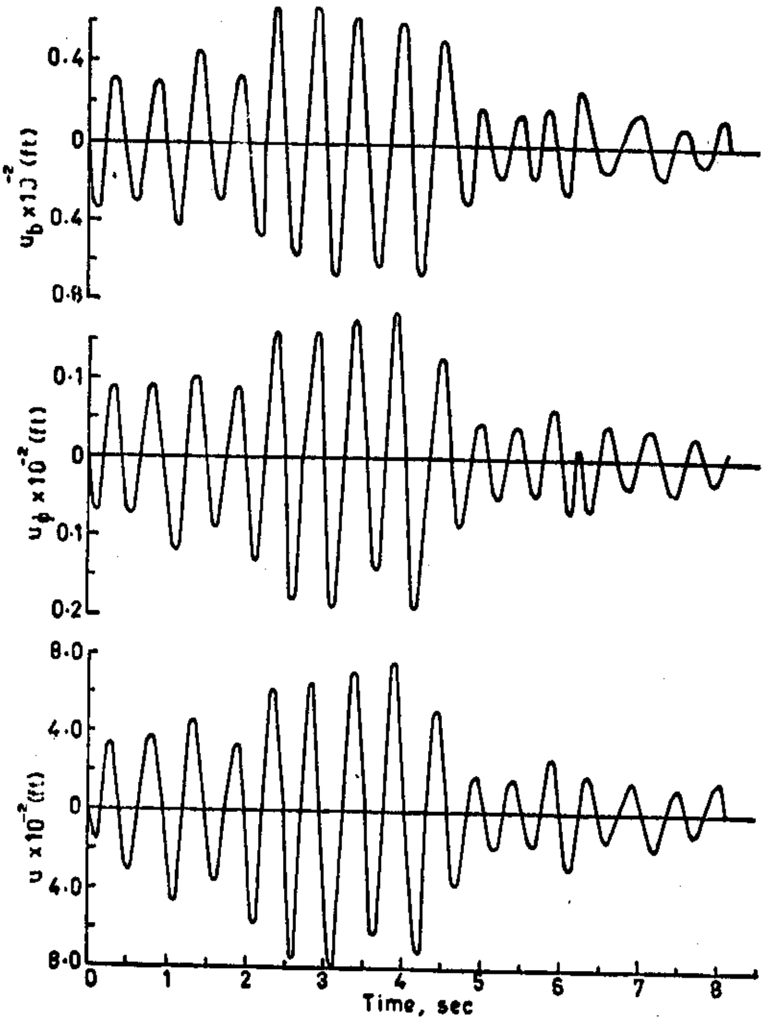

Fig. 12 Response Curves (μ=0.25, V_s=300ft/sec, δ=0.2)

Hence by comparing the response of the structural systems under a steady state excitation, with dynamic coupling due to dynamic interaction coefficients and in comparison with no intermodal coupling, it was found that the inclusion of the intermodal coupling terms results in variation of the response by considerable percentage reduction in the values of the response parameters. Therefore it can be concluded that the intermodal coupling terms K_{RT} and d_{RT} significantly alter the response of the interaction system of a building type structure resting on an elastic foundation medium. It can be concluded further, that for practical purposes the dynamic stiffness

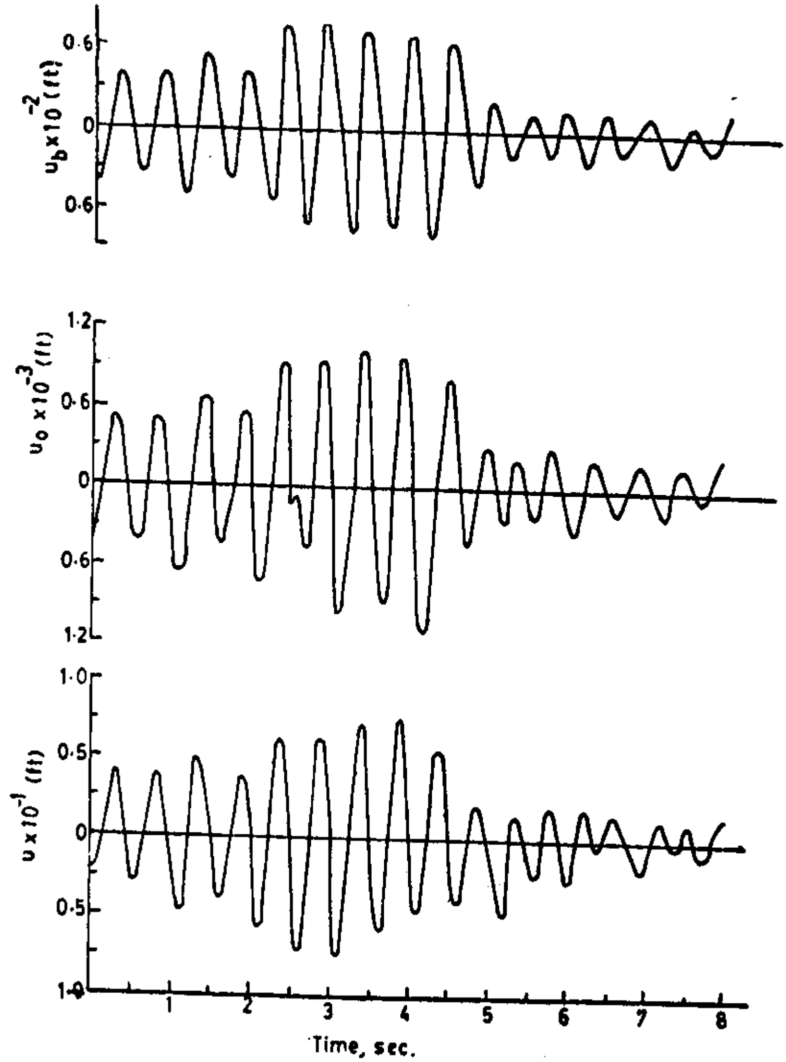

Fig. 13 Response Curves (μ=0.0, V_s=600 ft/sec, δ=0.0)

and radiation damping characteristics of an elastic foundation media can be assumed to be constant and are determined by the physical properties of the system. b, δ, v_s and μ. In the discussion of the various curves shown in Fig. 9 to 29, the comparison of the flexural response between flexible and rigid foundation media is done point by point along the response curves, it was noted that the foundation media with a higher shear wave velocity, say, 6000 fps closely approximate a rigid foundation. The maximum flexural response with reference to Fig. 30. indicates that foundation

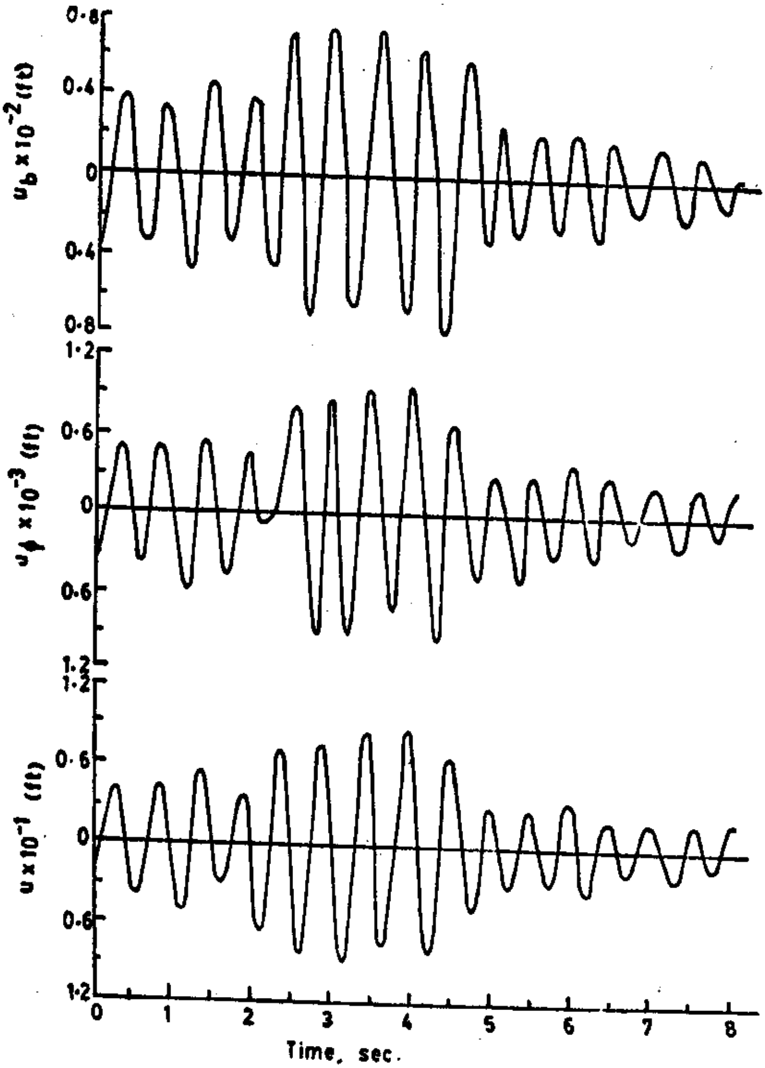

Fig. 14 Response Curve (μ=0.0, V_s=600 ft/sec, δ=0.02)

media with a shear wave velocity of 1000 fps may be approximated by a rigid foundation for practical purposes.

From the detailed analysis of the single storey buildings, considering the intermodal coupling, it appears that the interaction of the structure-foundation system is significant only for shear wave velocities lower than 1000 fps. This is found to be true even in case of single-storey buildings where in the intermodal coupling was not taken into the analysis of the problem. (Fig. 30).

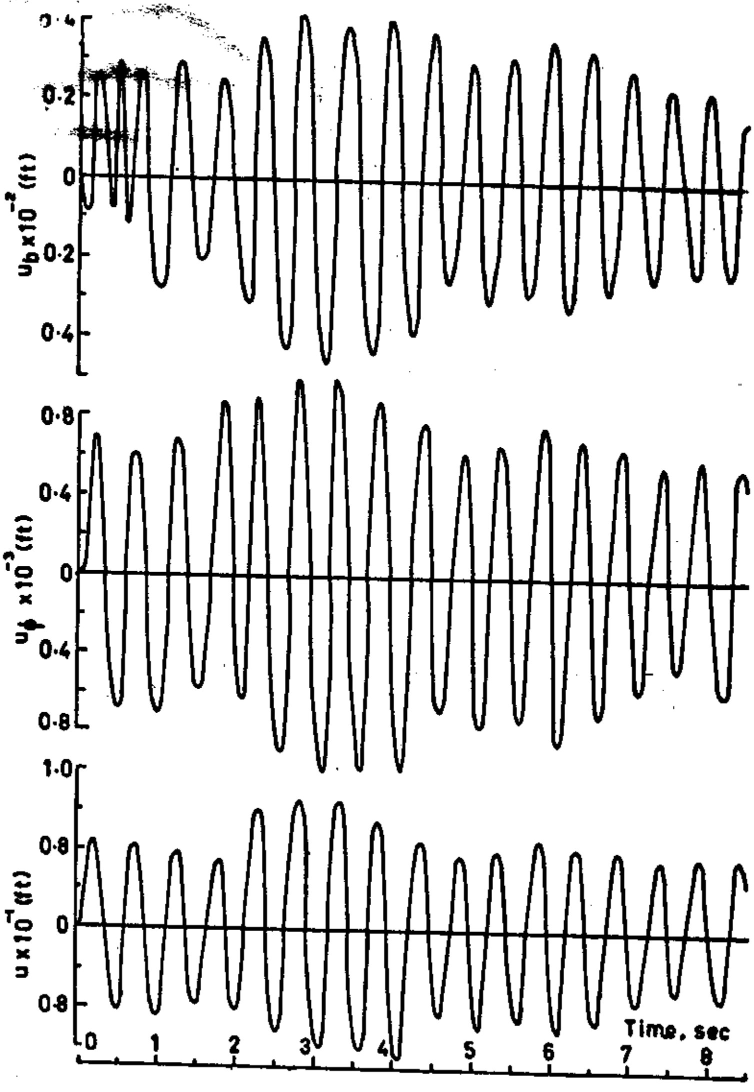

Fig. 15 Response Curves (μ=0.25, V_s=600ft/sec. δ=0.0)

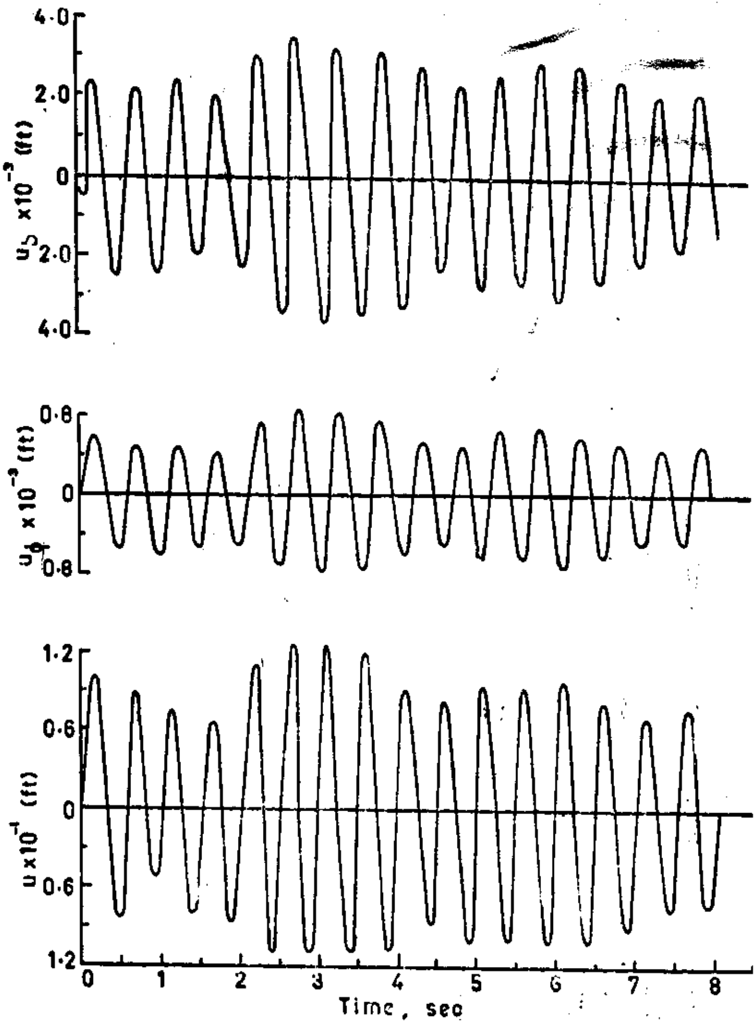

Fig. 16 Response Curves (μ=0.25, V_s=600 ft/sec, δ=0.2)

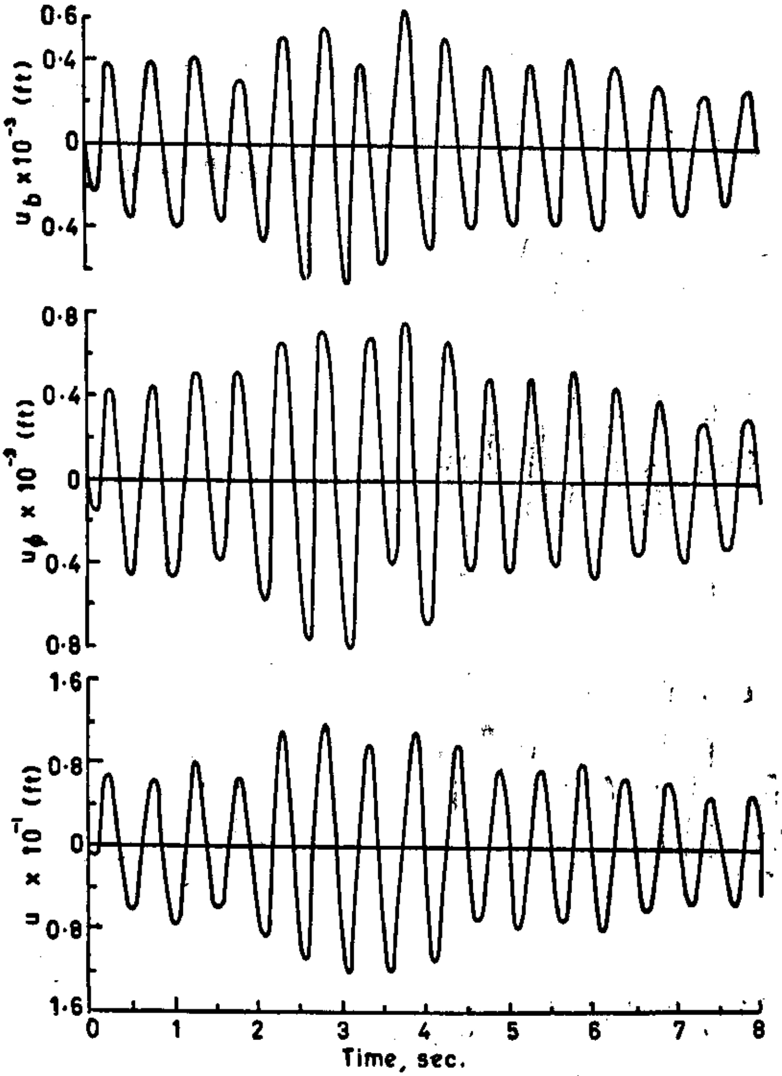

Fig. 17 Response Curves (μ=0.0, V_s=900ft/sec., δ=0.0)

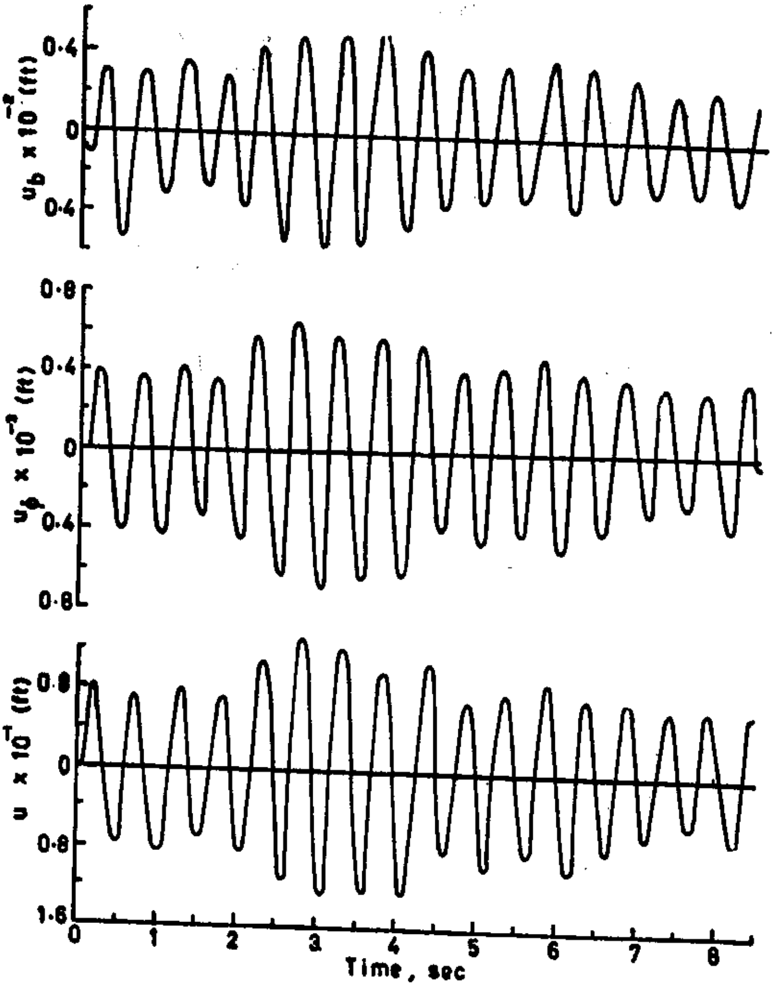

Fig. 18 Response Curves (μ=0.0, V_s=900 ft/sec, δ=0.2)

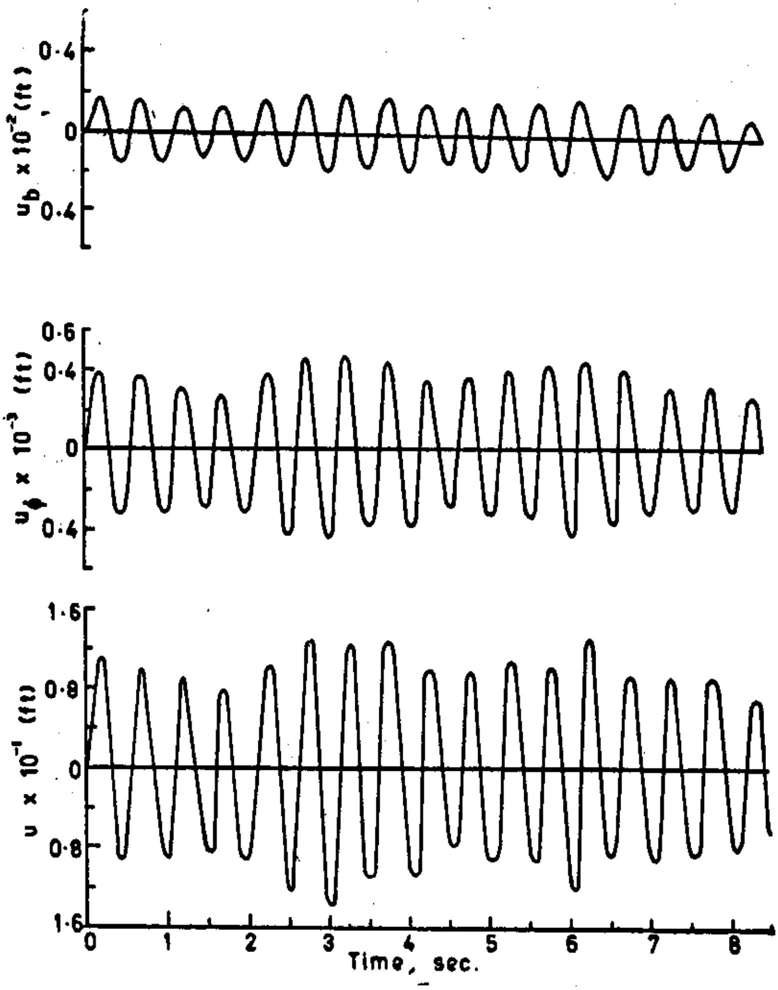

Fig. 19 Response Curves (μ=0.25, V_s=900ft/sec, δ=0.0)

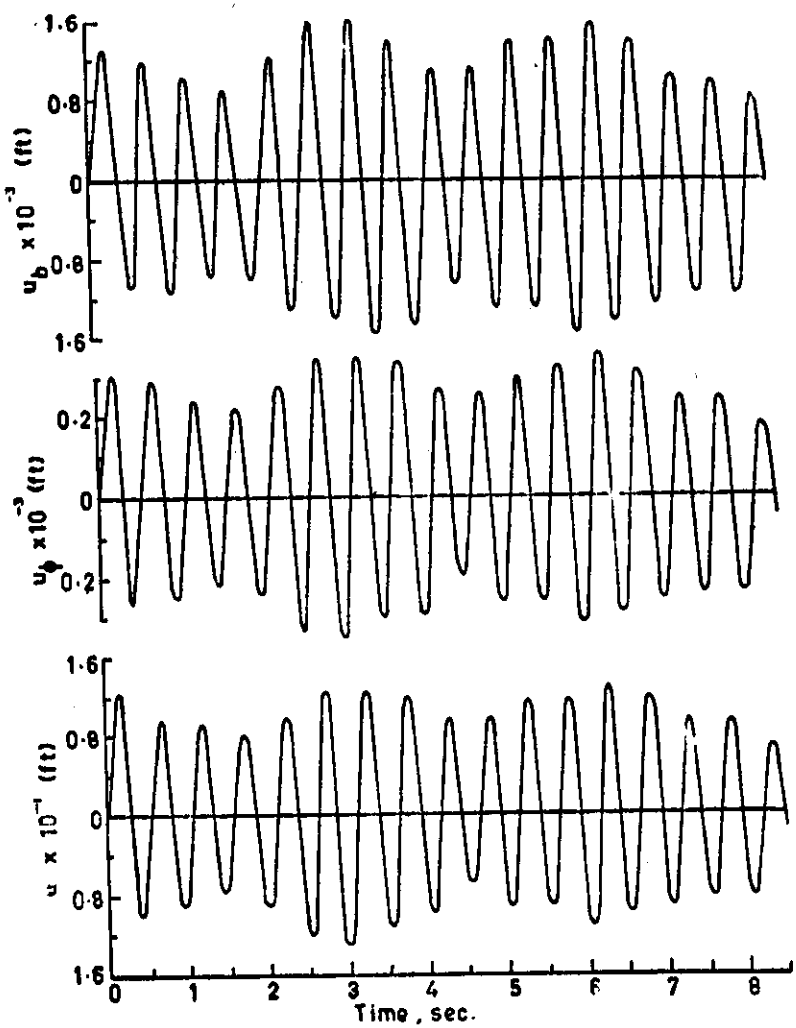

Fig. 20 Response Parameters (μ=0.25, V_s=900 ft/sec. δ=0.2)

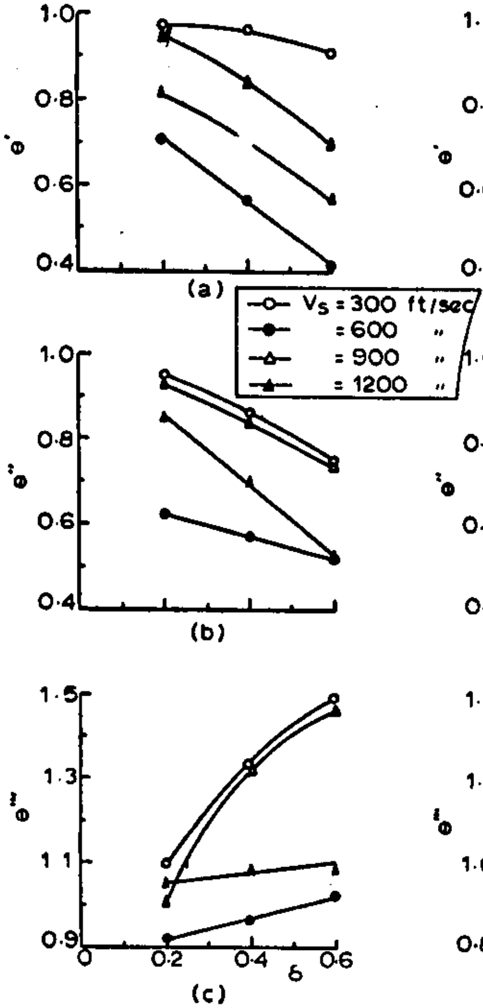

Fig. 21 Embedment Ratio (μ=0.0)

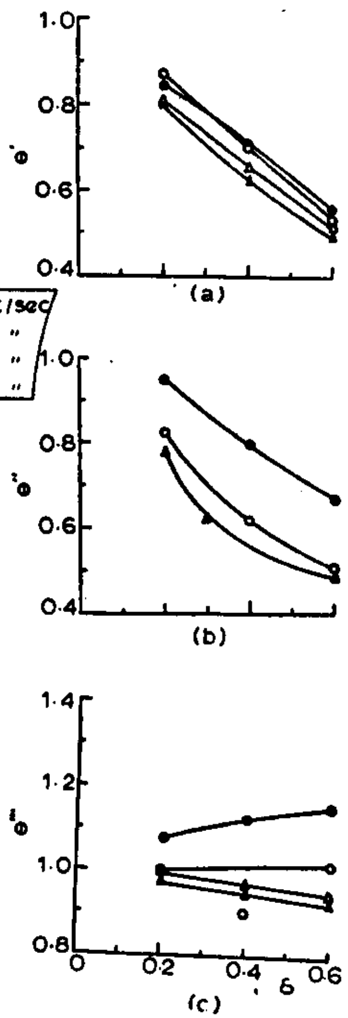

Fig. 22 Embedment Ratio (μ=0.25)

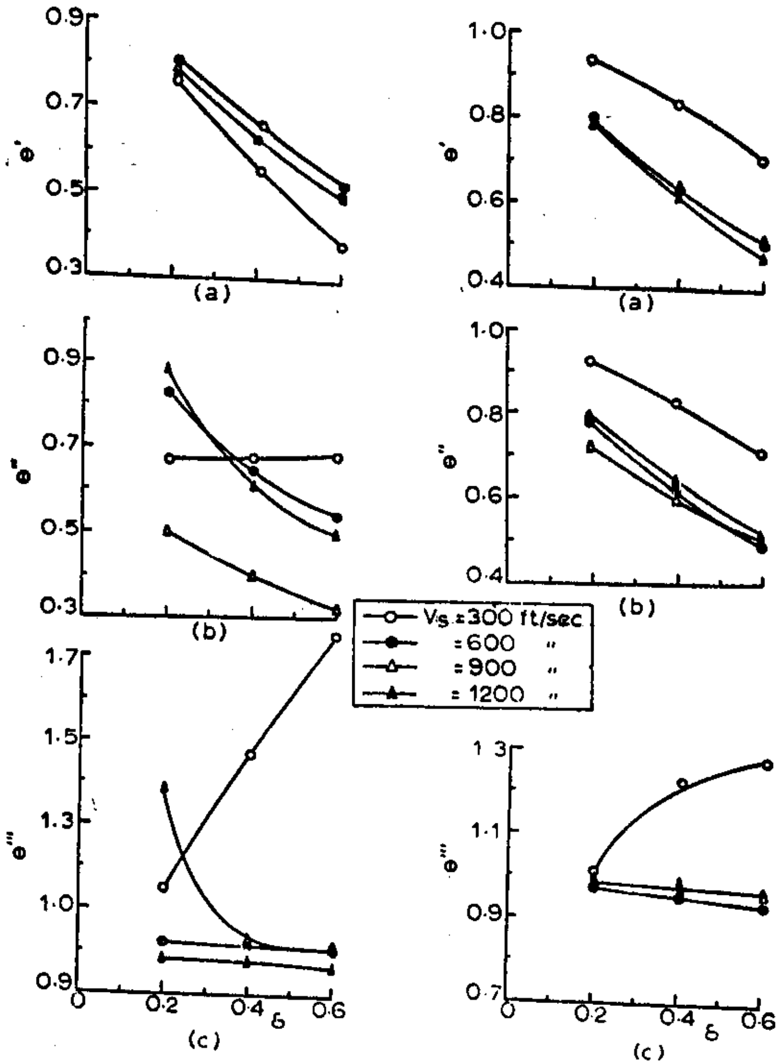

Fig. 23 Embedment Ratio (μ=0.35) *Fig. 24 Embedment Ratio (μ=0.50)*

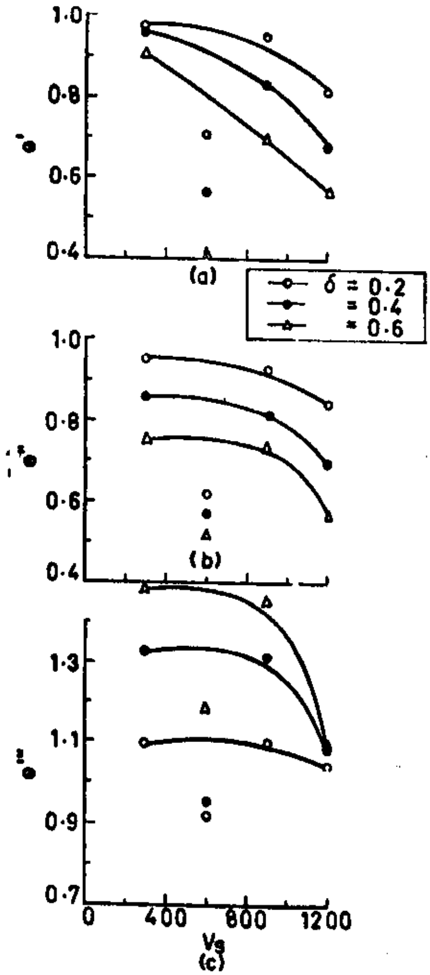

Fig. 25 Shear Wave Velocity (V_s) (μ=0 0)

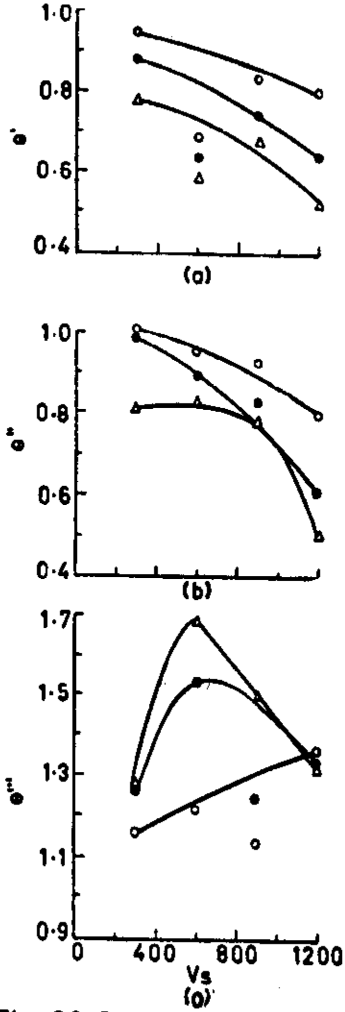

Fig. 26 Shear Wave Velocity (V_s) (μ=0.10)

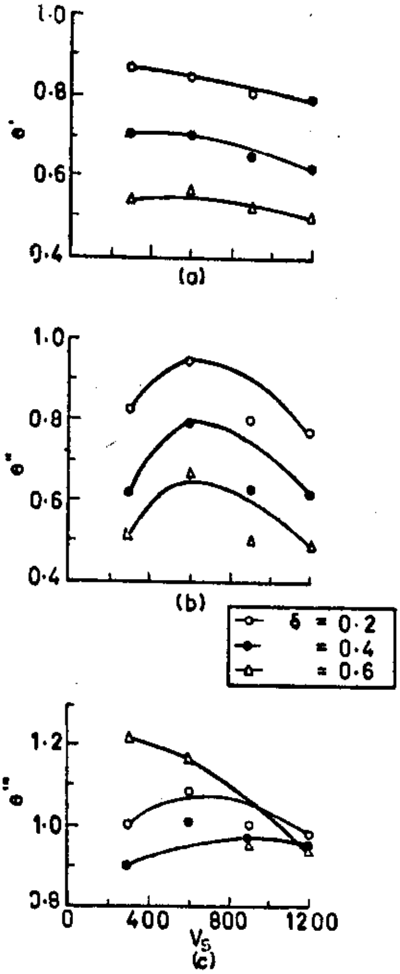

Fig. 27 Shear Wave Velocity (V_s) (μ=0.25)

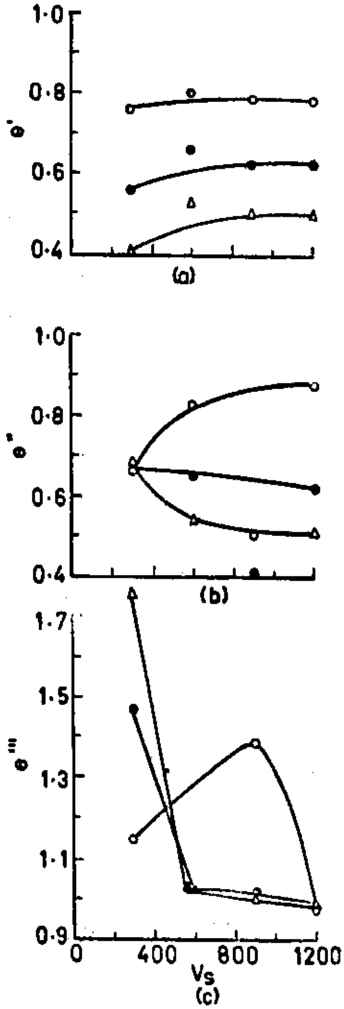

Fig. 28. Shear Wave Velocity (V_s) (μ=0 35)

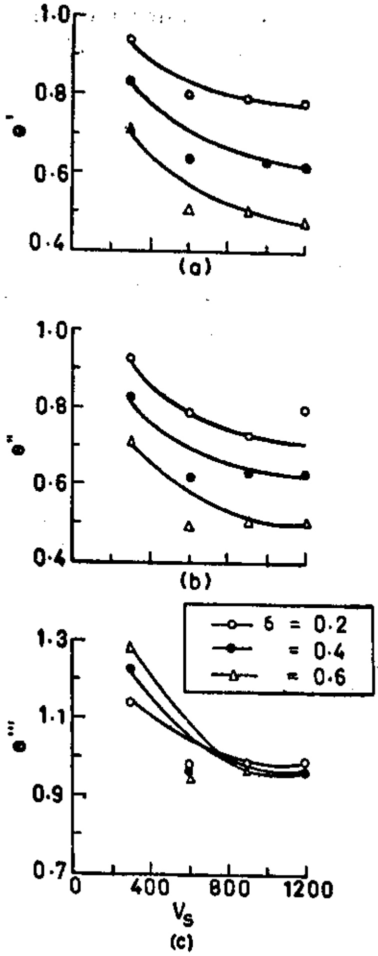

Fig. 29 Shear Wave Volocity (V_s) (μ=0.50)

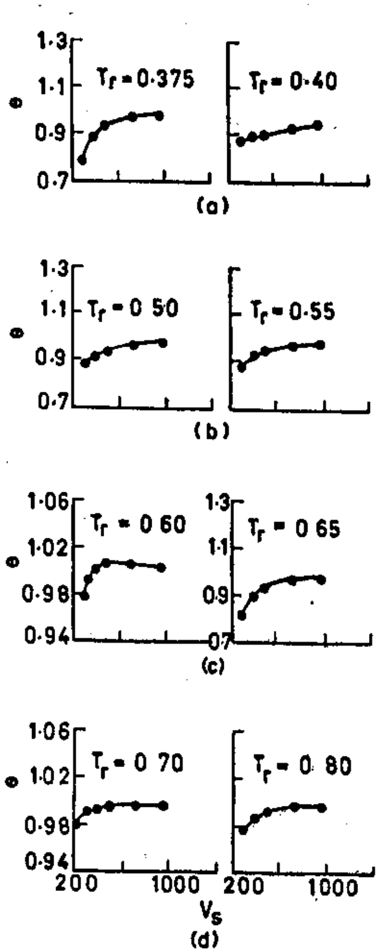

Fig. 30 Shear Wave Veiocity (V_s) (After Karasudhi et. al.)

The method of analysis proposed herein provides an approach for investigating the transient response of an elastic structure-foundation interaction system to seismic disturbances, Considering the dynamic interaction coefficients k_T, d_T, d_R, k_R, k_{RT}, and d_{RT} being the translational dynamic stiffness associated translational radiation damping coefficients of the foundation medium, coefficients of the rotational mode and the intermodal interaction coefficients respectively.

Further the effect of the embedment ratio ($\delta = H/b$) of the structure-foundation system has also been considered in the evaluation of the structural response parameters u_b, u_ϕ and u.

REFERENCES

1. Berg, G.V., and Dadeppo, D.A. (1960), "Dynamic Analysis of Elasto Plastic Structures", Journal of Engineering Mechanics Division, Proceedings, ASCE, Vol. 36, No. EM. 2, Proc. paper 2440, PP. 35-58,

2. Biggs, J.M. (1964), "Introduction to Structural Dynamics", Mc. Graw-Hill Book Company Inc., New York.

3. Bogdonoff, J.L., Goldberg, J.E., and Bernard. M.C. (1960), "Response of Simple Structure to a Random Earthquake Type Distrurbance", Bulletin, Seismological Society of America, Vol. 51, No. 2, PP. 293-310

4. Bycroft, G N. (1956), "Forced Vibration of a Rigid Circular Plate on a Semi-Infinite Elastic Space and on an Elastic Stratum", Philosophical Transactions, Royal Society of London, Series, A, No. 948, Vol. 248, PP. 327-368.

5 Clough, R.W. (1961), "Dynamic effects of Earthquakes, "Transactions, ASCE, Vol. 126, Part II, PP. 847-876.

6. Fleming, J.F., Screwvala, F.N., and Koudner, R.L (1965) "Foundation Super Structure Interaction Under Earthquake Motion", Proceedings, 3 WCEE, Newzealand, Vol. 1, PP. 122-130.

7. Gladwell, G.M.L. (1967), "Forced Tangential and Rotatory Vibration of a Rigid Circular Disc on Semi Infinite Solid", International Journal of Engineering Science", Vol. 6 PP. 591-602.

8. Goldberg, J.E., Bogdanoff, J.L, and Sharpe, D.R. (1964), "The Response of Simple Non-linear Systems to a Random disturbance of the Earthquake Type", Bulletin Seismological Society of America Vol. 54, No. 1, PP. 263-276.

9. Kano, Kyoshi, (1949), "Relation between the Earthquake Damage of Non-Woodden buildings and the Nature of the Ground", Bulletin, ERI, Vol. 27, PP. 97-100.
10. Karasudhi, P. Keer, L.M., and Lee, S.L. (1968), "Vibratory motion of a body on an Elastic Half Plane", Journal of Applied Mechanics, Trans, ASME., Vol. 35, Series E, No. 4, PP. 697-705.
11. Lee, S.L., Perelmann, D. S, and Fleming, J. F. (1966), "Earthquake Response of Inelastic Shear Buildings". Bulletin, Seismological Society of America, Vol. 56, No. 2, PP. 393-408.
12. Lycam, D. L., and Newmark, N.M. (1961), "Effect of Structure and Foundation Interaction", Journal of Engineering Mechanics Division, ASCE, Vol 95, No. EM 5, Proc. Paper 2960, PP. 1-31.
13. Merrit, R.G., and Housner, G.W. (1954), "Effect of Foundation compliance on Earthquake Stressed in Multistorey Buildings, Bulletin. Seismological Society of America, Vol. 55, No. r, PP. 551—
14. Parmelee, R. A. (1967), "Building-Foundation Interaction Effects" Journals of the Engineering Mechanics Division, ASCE, Vol. 93, No. EM. 2, Proc, Paper 5200, PP. 131-152.
15. Parmelee, R. A, Perelmann, D. S., Lee S.L, and Keer, L.M. (1968), Seismic Response of Structure-Foundation Systems", Journal of the Engineering Mechanics Division, ASCE, Vol. 94, No. EM. 6, Proc. paper. 6264,
16. Parmelee, R. A., and Kudder, R.J. (1973), "Seismic Soil-Structure Interaction of Embedded buildings", Proceedings, VWCEE, Section 5C, 1973, Rome.
17. Penzien, J. (1960), "Elasto-Plastic Response of Idealised Multistorey Structures Subjected to a Strong Motion Earthquakes", Proceedings, 2WCEE, Japan, PP 739-760
18. Roesset, J.M., Whitmann, R.V., and Destry, R. (1973), "Modal Analysis for Structures with Foundation Interaction", Journal of the Structural Division, Proceedings, ASCE, Vol. 99, No. ST 3, Proc. Paper 9603, PP. 399-416.
19. Sung, T.Y (1953), "Vibrations in Semi-Infinite Solids due to periodic surface Loading", Special Technical Publications, No. 156, ASTM, Symposium on Dynamic Testing of Soils, pp. 35-68.
20. Suyehiro, K. (1932), "Engineering Seismology", Notes on American Lectures Proceedings ASCE, Vol. 58, No. 4, Part 2, pp. 110.
21. Tanabashi, Ryo and Ishizaki, Hatsuo (1953), "Earthquake Damagcs and Elastic Properties of the ground", Bulletin No.4, Disaster Prevention Institute, Kyoro University, Kyoro, Japan.
22 Toriumi, I (1955), "Vibrations in Foundation of Mechanics on the Ground", Technology Reports of the Osaka University, No. 146, Vol.5, pp. 103-126
23. Tung, T.P, and Newmark, N.M. (1955), "Numerical Analysis of Earthquake Response of Tall Building", Bulletin, Seismological Society of America, Vol. 45, No. 4, pp. 269-278.