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INVESTIGATIONS FOR EVALUATION OF DYNAMIC SHEAR MODULUS OF SOILS

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INTRODUCTION

The determination of the dynamic shear modulus '*G*' of soils, accounting for the various parameters affecting its value, has assumed a special significance in recent years in view of the advancements in the techniques for analysing dynamic soil-structure interaction problems. Needless to say, the predicted response of the system will only be as dependable as the precision with which the value of '*G*' has been evaluated. The process of evaluation of the dynamic shear modulus is however quite complicated because of its dependence on a large number of different parameters. Laboratory tests such as Resonance column tests, Dynamic Triaxial tests or oscillatory shear tests and field tests such as Block vibration tests, wave propagation tests, Cyclic plate load tests and Dynamic load bearing tests on footings have been suggested for the determination of the value of dynamic shear modulus by various investigators Drnevich, Hall & Richart (1967), Whitman (1970), Praksh et al. (1973), Arya et al. (1975). It is generally observed that the values obtained by different methods differ widely, Cunny and Fry (1973), and the order of difference may range upto several hundred percent. This is essentially because of the fact that the dynamic shear modulus is affected by factors such as effective confining pressure, strain amplitude, void ratio, degree of saturation, soil structure and time effects and these parameters differ in the different methods used for evaluation of '*G*'. Factors such as void ratio, time effects, soil structure and degree of saturation may be assumed to be fairly constant for a given location. And the largest effect, on '*G*' values will therefore be exerted by the confining pressure and strain amplitude. Since these conditions differ from test to test, the values of '*G*' are accordingly affected. In selecting the value of '*G*' from laboratory or in-situ tests, due consideration should be given to the conditions of confinement and strain associated with the test conducted and those occurring in the prototype whose response is being predicted. An approach taking care of these factors is therefore necessary.

The basic aim of this investigation was to determine a rational value of '*G*' for use in analysis under seismic loading conditions. However, it will also be desirable to obtain the variation of '*G*' in the entire range of shear strains of interest in practice. A method of determination of strain dependent value of '*G*' from field tests is rather necessary in case of cohesionless soils below water table where undisturbed sampling is rather difficult if not impossible. Moreover, in-situ testing has the further advantage that the unknown disturbances associated with sampling from the site and during transportation to the laboratory do not come into the picture at all.

SOIL DESCRIPTION AND TESTING PROGRAMME

A typical bore log of the site where the field investigations were carried out for evaluation of '*G*' is shown in figure 1. The alluvial deposits at the site are about 400 m deep. The following in-situ and laboratory tests were conducted.

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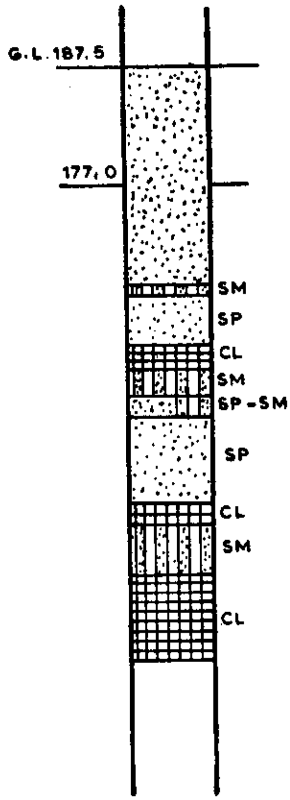

Fig. 1. Bore Log

INSITU TESTS

1. Block Vibration Test (As per IS 5249-1969)
   (A) Forced Vibration Tests
   (B) Free Vibration Tests
2. Wave Propagation Tests
3. Dynamic Load Bearing Test on Footing

LABORATORY TESTS

1. Oscillatory Shear Tests

INSITU TESTS

1. Block Vibration Tests: These tests were conducted on (i) a rectangular block 1.5 m$\times$0.75 m$\times$0.70 m high and (ii) a circular block, 1.0 m dia$\times$0.70 m high, of plain concrete (M-150) cast at suitable locations at the site.

The block was excited to resonance with the help of a mechanical oscillator mounted on the top surface of the block and driven by a speed controlled D.C. Motor.

The vibrations were sensed using acceleration pick ups, the signal of which was amplified through a universal amplifier and recorded directly on an ink writing oscillograph. The blocks were excited both with vertical and horizontal sinusoidal forces of varying magnitudes.

Free vibration tests on the blocks were conducted by exciting the block by hitting it with a hammer and recording the vibrations with acceleration pick ups suitably mounted in relation to the direction of vibrations. Only horizontal vibration of the block were generated in free vibration tests.

2. Wave Propagation Tests: The wave propagation tests were conducted for determination of in-situ dynamic moduli of soil by measuring the velocity of propagation of seismic waves. The waves were generaged by impact of a 5 kg hammer on a steel plate resting on the ground surface. A velocity pick up was kept fixed very near to the point of impact and an acceleration pick up was kept at a known distance along a predetermined line. The hammer was dropped through a height of 2m. near the first pick up. The output of the two pickups were amplified through universal amplifiers and fed to a dual channel recorder. Figure 2 shows a typical record. The test was repeated in a grid pattern for different distances between the pickups. Four different locations were explored to get a fairly representative average value for the area.

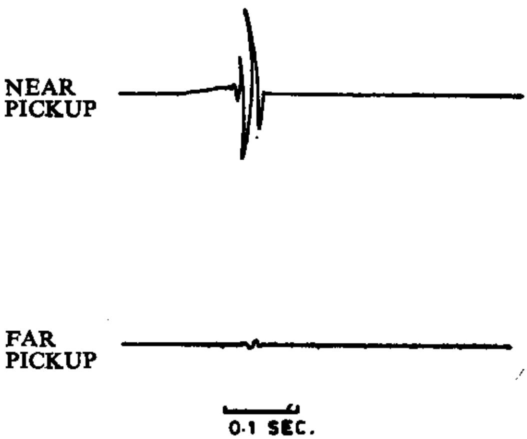

Fig. 2. Typical Record from Wave Propagation Tests

3. Dynamic Load Bearing Test on Footing: The test was conducted with the object of in-situ determination of residual soil settlements which the soil might undergo when the dynamic loads are applied in addition to the sustained static loads. The dynamic soil moduli are determined in the test from the resulting load settlement plots.

Test was conducted on a 1.0 m×1.0 m concrete slab 20 cm thick having two way reinforcement at top and bottom faces. Figure 3 shows the test set up. A kentledge supported on six masonry columns was erected centrally above the slab and loaded to 20 tonnes. The static loads were applied to the slab with the help of a 50 tonnes remote controlled hydraulic jack acting against the central girder of the kentledge and the reaction was transmitted to the slab through a loading frame held in position over the slab with foundation bolts.

Fig. 3. Test Setup for Dynamic Load Bearing Test

The dynamic vertical load was applied with the help of a motor oscillator assembly mounted on a base plate attached to the slab with the help of foundation bolts. It was ensured that the centre of slab, loading frame and line of action of the dynamic force were along the same vertical line. The settlements were observed with the help of four dial gauges, one at each corner of the slab. In order to ensure that the reference marks for measuring settlements do not get disturbed as a result of vibration being imparted to the slab, the dial gauges were fixed to a well designed mechanical arrangement anchored to the ground at a sufficient distance away from the slab.

After taking the initial readings of the dial gauges, a static load of 4 tonnes was applied and maintained at a constant value. When the settlement became constant, the readings of the dial gauges were recorded. Spindles of the dial gauges were then raised to avoid any damage to them during vibrations. The oscillator was run at a constant speed of 5 c/s for 30 seconds. After stopping the oscillator the dial gauges were released and their readings recorded. The process was repeated for dynamic loads applied at 10, 15, 20, 25 and 30 c/s each for 30 seconds and for different angles of setting of eccentric masses. The values of static loads were increased in succession to 6.5, 9.5, 12.25 and 14 tonnes. After each static load, dynamic load was applied as described earlier. Settlement observations were made in each case.

LABORATORY TESTS

4. Oscillatory Shear Tests: The oscillatory shear tests were conducted using the oscillatory shear test device developed at SRTEE, Prakash, Nandakumaran and Joshi (1973), on sand samples at reproduced field densities. Figure 4 shows the oscillatory

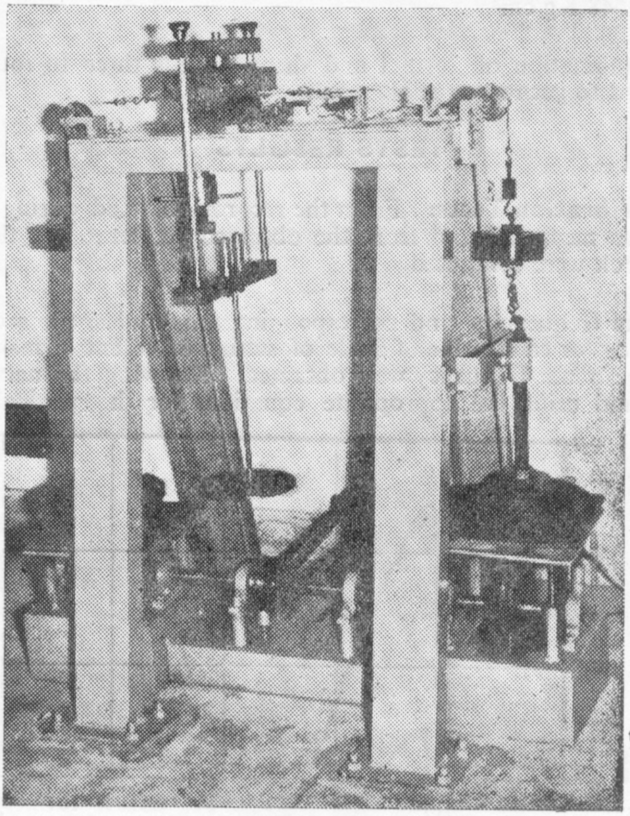

Fig. 4. Oscillatory Shear Test Apparatus

shear test apparatus which is capable to simulating simple shear deformation conditions in the soil sample which is also the case when elements of soil mass are deformed by the upward propagation of shear waves due to an earthquake. The oscillatory shear box houses a sample 60 mm × 60 mm × 20 mm high enclosed in a rubber membrane. The box consists of a top plate resting on the soil sample, a movable base plate resting on a pair of ball trains, two fixed vertical sides, and two tilting side plates capable of rotating in either direction from their mean vertical position and hinged to the fixed vertical sides. Normal stress can be applied on the sample by weights placed on the hanger of a yoke resting on the top plate. Oscillatory shear stresses are applied on the sample by applying on either sides of the base plate equal forces by means of weights hung on flexible cords and then by lifting these weights alternately by a pair of cams with 180° phase difference and fixed on the same shaft driven at a desired speed by a geared motor through a chain-sprocket system. This gives rise to an oscillatory shear stress with rectangular wave shape. The applied shear loads and the shear deformations are measured by employing strain gauge mounted load and displacement transducers connected to universal amplifiers and ink-writing oscillographs.

For conducting the oscillatory shear tests, field density was reproduced in the shear box by vibrating a known weight of dry sand to a known volume by mounting the box on a shaking table and by employing suitable surcharge weights. This was accomplished by a few trial runs. The tests were conducted on samples, from two bore holes, taken from depths roughly from 5 m to 22 m respectively. The normal stress used in the tests were 0.987, 1.403 and 1.802 Kg/cm² and the shear stresses were 0.0906, 0.231, 0.36, 0.509 and 0.648 Kg/cm².

For each combination of normal and shear stresses three to four repetetions of tests were conducted to get reproducible results.

TESTS RESULTS

I. Block Vibration Tests: From the records obtained during forced vibration tests, frequency was measured and from the observed accelerations, the amplitudes of vibration of the block were computed.

Knowing the frequencies and corresponding amplitudes, plots of amplitude Vs frequency were made for each value of angle of setting of eccentric masses. From such plots the value of resonant frequency were obtained. Knowing the natural frequency of block-soil-system and using the appropriate equations for the vertical and horizontal

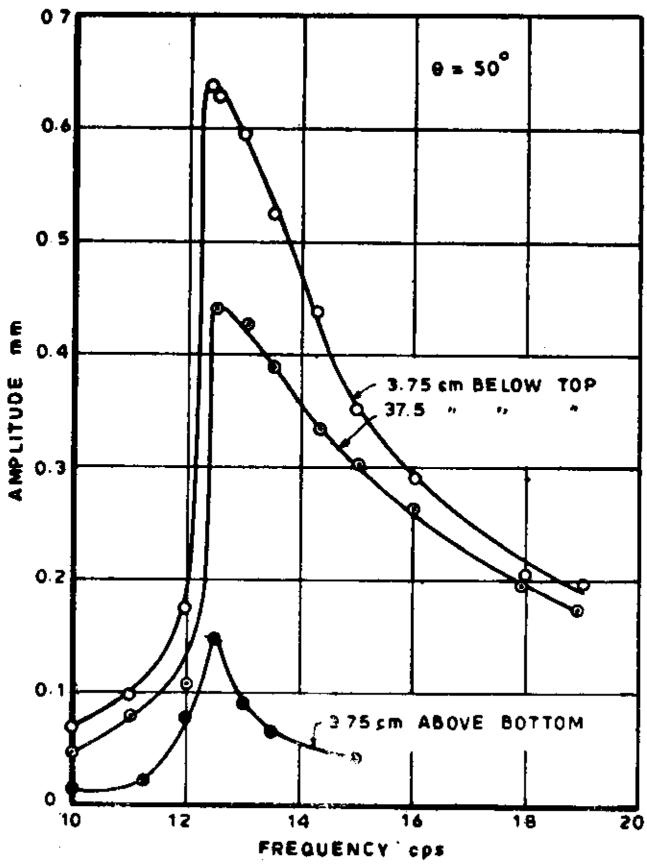

Fig. 5. Amplitude VS Frequency for Circular Block (Horizontal Vibration Test)

vibrations given by Barkan (1962) the values of coefficient of elastic uniform compression 'Cu', the coefficient of elastic uniform shear 'C_T' and coefficient of elastic non-uniform compression 'C_ϕ' can be computed. In case of horizontal vibrations the mode of vibrations was first ascertained, by studying the variation of amplitudes of vibration along the height of the block. The values of 'Cu' from the vertical vibration tests are shown in table 1. The values of 'C_T' from horizontal vibration tests and the corresponding values of Cu, assuming $Cu=2\,C_T$, are shown in table 2.

From free vibration tests, the natural frequency of free vibrations was measured and the rest of the computations were made as in the case of forced vibration tests. Knowing the values 'C_u' the corresponding values of 'G' were computed using equation 1.

TABLE—I

VALUES OF DYNAMIC SOIL CONSTANTS FROM VERTICAL VIBRATION TESTS

Block Size	Angle of Setting of Eccentric Masses '0'	Observed Natural Frequency of Vertical Vibrations cps	Value of Cu Kg/cm^3 (For Contact Area of Block)	Value of Dynamic Shear Modulus G Kg/cm^2
1	2	3	4	5
1.5m×0.75m ×0.7m High	50	21.2	3.16	102.0
	100	20.9	3.08	98.5
	150	19.1	2.57	82.5
	180	17.9	2.26	73.0
1.0m Dia ×0.7m High	50	22.0	3.26	87.5
	75	19.0	2.44	66.0
	100	19.0	2.44	66.0
	125	19.0	2.44	66.0
	150	19.0	2.44	66.0
	180	18.4	2.31	61.0

$$G=\frac{(1-\nu)\,A.Cu}{2K} \qquad \ldots(1)$$

where G=shear modulus

ν=Poisson's ratio, taken as 0.3

A=Area of contact

K=Constant depending upon geometry of the contact area (Barkan 1962)

The computed values of 'G' are also shown in Tables 1 and 2.

TABLE—2

VALUES OF DYNAMIC SOIL CONSTANTS FROM HORIZONTAL VIBRATION TESTS

Block Size	Angle of Setting of Eccentric Masses 'θ'	Observed Natural Frequency cps	Value of C_T Kg/cm³ (For Contact Area of Block)	Value of 'C_u' Kg/cm³ (For Contact Area of Block)	Value of Dynamic Shear Modulus G Kg/cm²
1	2	3	4	5	6
1.5m×0.75m ×0.7m High	50	15.8	2.19	4.38	141.0
	100	15.7	2.13	4.26	137.0
	150	14.3	1.79	3.58	115.0
	180	14.0	1.72	3.44	110.0
	Free Vibration Test	20.5	3.60	7.20	226.0
1.0m Dia ×0.70m High	25	12.6	1.98	3.96	106.0
	50	12.4	1.90	3.80	102.5
	75	12.3	1.89	3.78	102.0
	100	12.3	1.89	3.78	102.0
	125	12.0	1.74	3.48	94.0
	150	11.5	1.65	3.30	87.5
	180	11.0	1.51	3.02	87.5
	Free Vibration Test	16.0	3.125	6.25	160.0

2. Wave Propagation Tests: From the records obtained during wave propagation tests, the time taken by the waves to travel a known distance can be obtained. Usually from the records of ground vibrations, the time of travel of the compression waves can be more easily distinguished. In this case also, the travel time of the direct compression wave was measured from the records and the results are shown in figure 6. The values of the average time for each location were obtained to make this plot. The slope of the line gives the value of compression wave velocity as 456 m/sec. The value of 'G' can be determined using the relation given below:

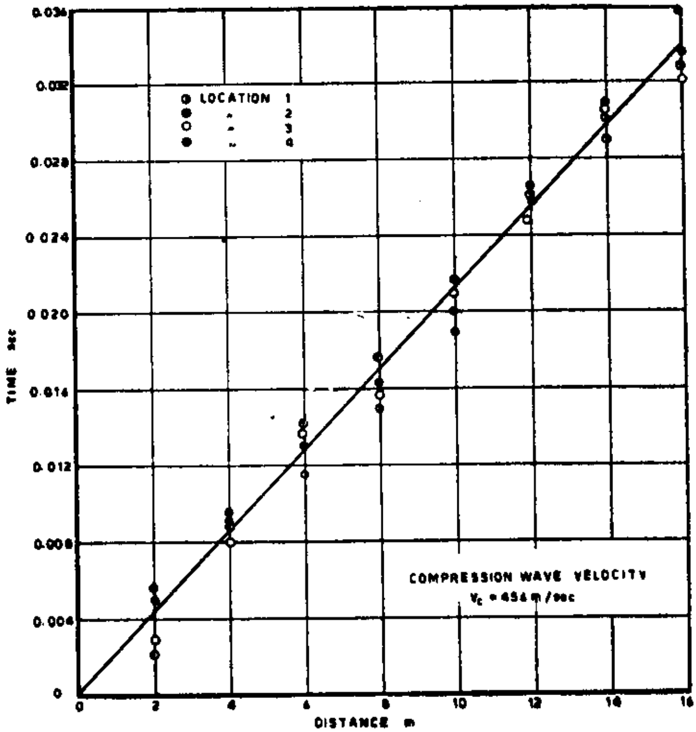

Fig. 6. Time VS Distance

$$G = Vc^2 \cdot \rho \cdot \frac{(1-2\nu)}{(1-\nu)} \qquad \ldots(2)$$

where V_C=velocity of compression waves,

ρ=mass density of the medium.

The value of 'G' works out to be 885 kg/cm^2 for an assigned value of ν=0.3.

3. Dynamic Load Bearing Test: From the observed data, the load Vs settlement curves for the static load and also for total load (static+dynamic) Vs total settlement (static+increased settlement due to dynamic load) have been plotted, since the incremental settlement because of any load application has been observed. Since the additional dynamic settlements were recorded at different eccentricities as well as at different frequencies of load application, it is possible to plot the total load Vs settlement curves at different frequencies. Typical load settlement curves for the static case and for dynamic load at 5 c/s are shown in figure 7. In order to determine the value of dynamic shear modulus from this test, the depth of soil mass which shares the settlement is taken as the depth at which the vertical stress intensity becomes 0.01 kg/cm^2. This depth has been determined using the well known Boussinesq equation. Shear modulus 'G' is then determined from equation

$$G = \frac{PD}{2A.\Delta h.(1+\nu)} \qquad \ldots(3)$$

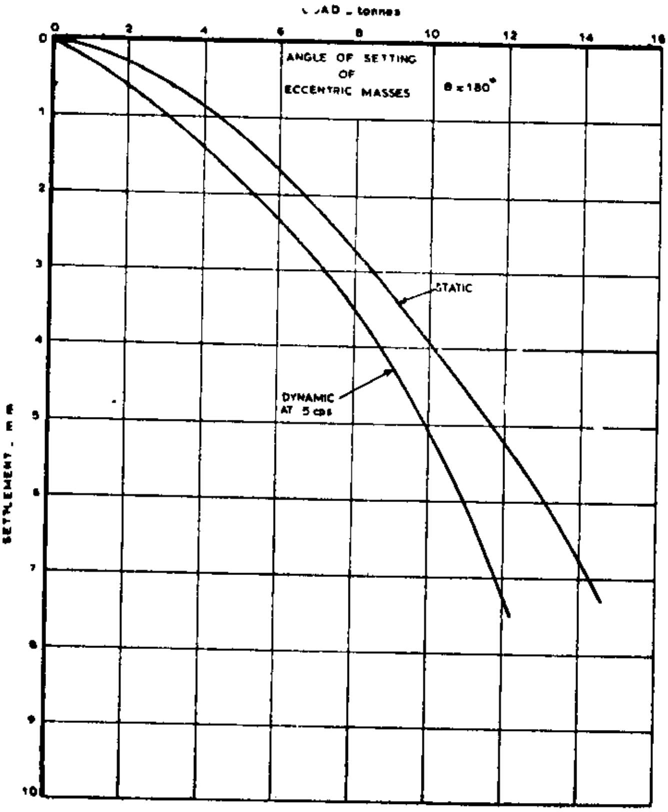

Fig. 7. Load VS Settlement

where P=Load applied on the Plate

A=Area of contact of the Plate

D=Depth at which stress intensity becomes 0.01 kg/cm²

and Δh=Settlement

The values of G were observed to vary from 87.5 to 187 kg/cm².

4. **Oscillatory Shear Tests:** From the records of the deformation gauges, the shear deformation corresponsing to any cycle or at any stage of loading can be read. It was observed that for the dry soil tested, the shear deformations did not change very much with number of cycles for 10 to 12 cycles. The effective number of equivalent uniform cycles in the design earthquake for the site was also between 10 to 12. The shear modulus value will therefore be independent of the number of cycles of loading likely to occur in case of the probable chosen earthquake for the site. The value of 'G' can be determined using the equation below:

$$G=\frac{T}{\gamma}=\frac{T}{\Delta s/h} \qquad \ldots(4)$$

where T=oscillatory shear stress

γ=Shear strain

Δs=Shear displacement

and h=Height of sample

The values dynamic shear modulus Vs strain for different normal pressures used in the tests are shown in figure 8. Figure 9 shows the variation of shear modulus with normal stress at different strain levels.

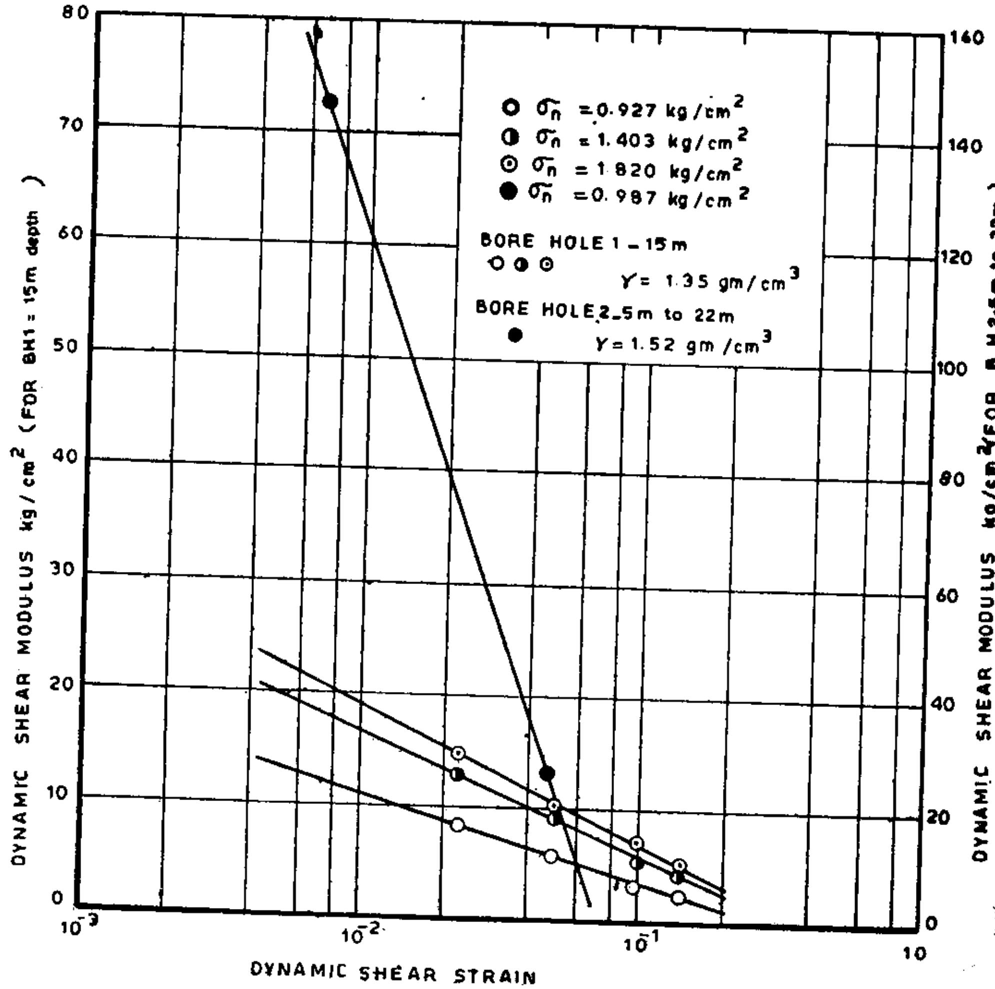

Fig. 8. Variation of Shear Modulus with Shear Strain Obtained from Oscillatory Shear Tests

DISCUSSION AND INTERPRETATION

The values of dynamic shear modulus obtained from the different field and laboratory tests differ considerably from one another. For example, the values as

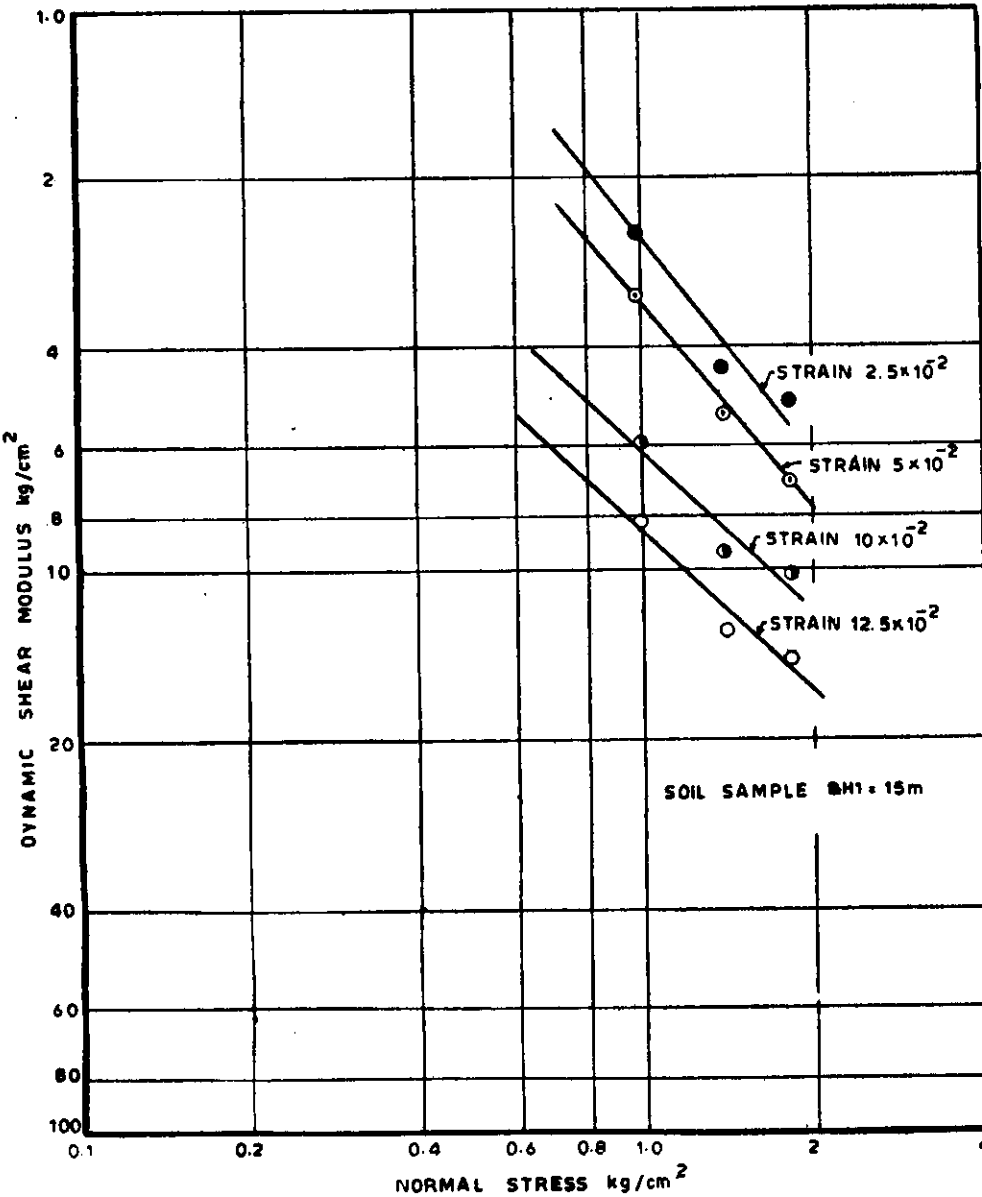

Fig. 9. Variations of Shear Modulus with Normal Stress

obtained from forced vertical vibration tests vary from 61 kg/cm² to 102.0 kg/cm² and the forced horizontal vibration tests give a value of 'G', of 160 kg/cm² to 220 kg/cm². The wave propagation tests give a value of 885 kg/cm². In case of oscillatory shear test they vary from 2.5 kg/cm² to 73 kg/cm² and in dynamic plate bearing test they are seen to vary from 87.5 kg/cm² to 187 kg/cm².

It is observed that from the values of dynamic shear modulus as obtained, it is not possible to directly choose a value for the dynamic soil-structure interaction problem which could be the design of a machine foundation or analysis of a foundation-soil system subjected to seismic load.

It has been mentioned in the beginning that the dynamic shear modulus is considerably affected by confining pressure and strain amplitude. The authors made an attempt to interpret the observed data keeping in view these parameters and an approach for the same is outlined below:

(i) All the observed values should first be computed at a common confining pressure.

(ii) These values of G should then be plotted against the corresponding strains to study the variation with strains. Appropriate values of strains in various tests should be arrived on the basis of computations or judgement.

The confining pressures associated with the block vibration tests and the dynamic load bearing tests were found out by computing the depth 'Z' at which the stresses in the soil, because of superimposed loads, become negligible, say, as low as 0.01 kg/cm². The mean effective confining pressure corresponding to a depth of $Z/2$ was then computed. In the case of wave propagation tests, the average depth for which the measured wave velocity gives a representative value, varies from $L/8$ to $L/10$ where 'L' is the horizontal distance for which the value of wave velocities are representative. The effective confining pressure for the oscillatory shear tests is known from the applied normal pressure. After determining the confining pressure with each of the tests all the values of 'G' were computed for a confining pressure of 2 kg/cm² using the relation:

$$\frac{G_1}{G_2}=\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_2}\right)^m \qquad \ldots(5)$$

where G_1 and G_2 are the values of 'G' at effective confining pressures of $\bar{\sigma}_1$ and $\bar{\sigma}_2$ respectively. The value of 'm' observed in the laboratory was 0.9. According to Silver and Seed (1971) the typeial values of 'm' vary from 0.3 to 0.70. The values obtained in the laboratory are on the higher side as they pertain to much larger strains. On the basis of published data, a value of $m=0.5$ was adopted for computing the values of 'G' at different confining pressures, since the soil at site is cohesionless.

Fixing the order of representative strains likely to be associated with different field tests can only be done by comparison with known phenomenon or tests whose strains are known and using engineering judgement. This was done by comparison with figure 10 (Silver and Seed 1971) which shows the strains associated with different motion characteristics. It is well know that amplitudes at resonance in a block vibration test are much higher than for a properly designed machine foundation (strains 10^{-6} to 10^{-5}) and accordingly, the strains associated with these test were considered to be in the range 10^{-3} to 10^{-2}. Amplitudes in the free vibration tests are rather small compared to forced vibration tests and the oonditions correspond to much lower strain levels say of the order of 10^{-5} to 10^{-4}. In case of wave propagation tests, the strains can resonably be taken as 10^{-6}. The strains associated with oscillatory shear test are precisely known. In case of dynamic bearing test, the shear strains were assumed to be equal to normal strain. The shear strains are equal to $\frac{\sigma_1-\sigma_3}{2G}$ or $\frac{\sigma_1(1-K_0)}{2G}$ and the normal strains are $\frac{\sigma_1}{E}$ or $\frac{\sigma_1}{2G(1+\nu)}$. The ratio of shear strains to normal strains is therefore approximately equal to 1 for small values of ν. The values of 'G' from different tests after normalising them to the same confining pressure have been plotted against the associated shear strains. A curve has been drawn

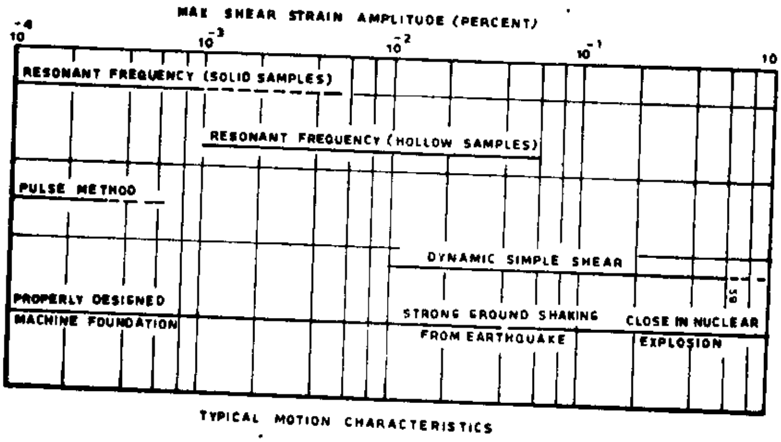

Fig. 10. Shear Strains Associated with Typical Motion Characteristics (Silver and Seed 1971)

giving due weightage to observed values as shown in figure 11. The curve when extrapolated gives a value of 'G' as 1300 kg/cm² at a strain of 10^{-6}, using this value of 'G', a plot has been made $\frac{G}{(G)_{\gamma=10^{-6}}}$ Vs strain amplitude in figure 12. The plot by Seed and Idris (1970b) has also been reproduced for comparison. It may be observed that the two curves agree rather closely. Using the plot given in figure 11, the value of 'G' corresponding to desired level can be picked up.

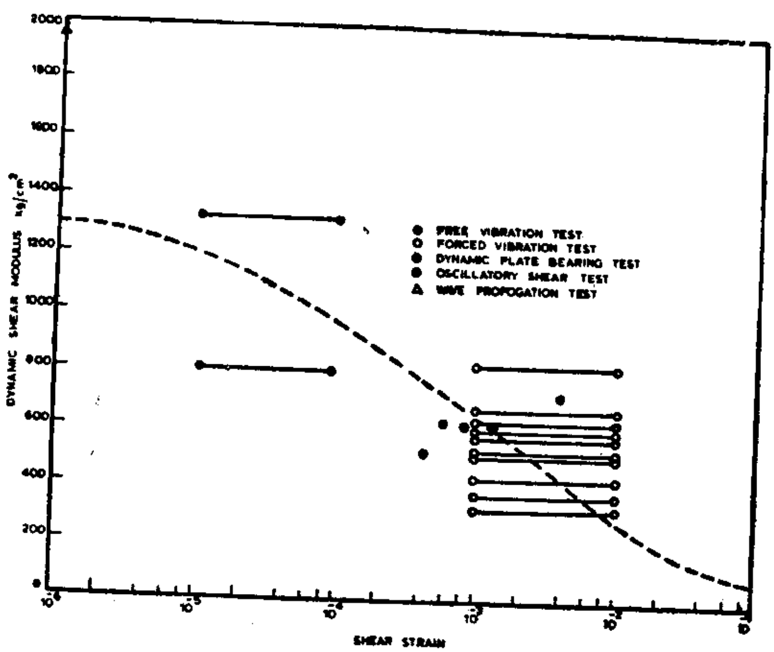

Fig. 11. Dynamic Shear Modulus VS Shear Strain

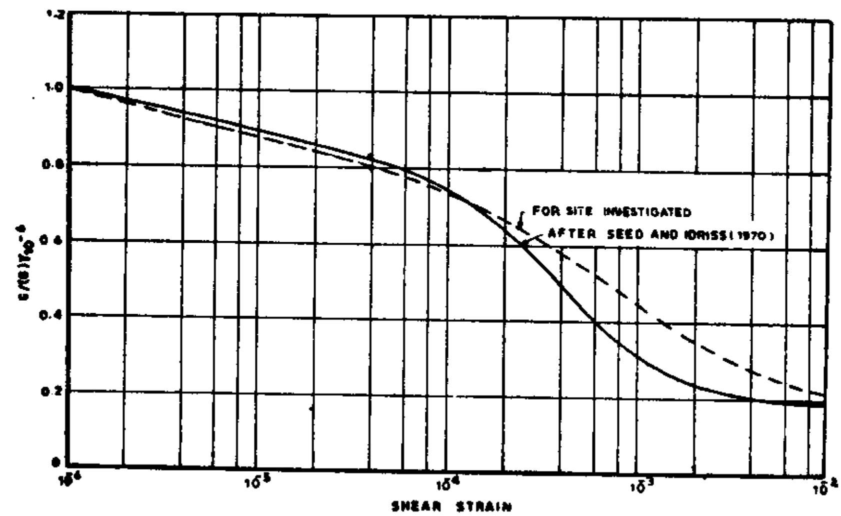

Fig. 12. $G/(G)_{\gamma=10^{-6}}$ VS Shear Strain

One aspect of the problem pertinent to the determination of strain dependent value of 'G' from the field tests is that although the normalized $\frac{G}{(G)_{\gamma=10^{6}}}$ Vs strain curve shows quite close agreement with laboratory test data of Seed and Idris (1970b) it may not be appropriate to trace out the variation of G Vs strain on the basis of just one type of field test. This point is obvious from figure 11 where the value of 'G' at a strain of 10^{-6}, as obtained by fitting an average curve giving due to weightage to observed data points, is much lower than the value from the wave propagation test. Consequently if G Vs strain for the site were established, just on the basis of the wave propagation test data and using the curve of seed and Idriss (1970b) the value of 'G' would be over estimated. It is therefore essential to conduct different types of tests so that the strains associated are in a fairly wide range.

CONCLUSIONS

1. A method has been proposed to rationally determine average values of shear modulus for a site for use in soil-structure interaction studies under dynamic loads.
2. It has been observed that taking the representative values of shear modulus on the basis of any single test may be considerably in error.
3. It has been demonstrated that a new test the dynamic load bearing test can be successfully carried out in the field to obtain the values of shear modulus of soils.
4. The need for field tests under conditions where sampling is difficult or impossible has been highlighted and use of field test to overcome these difficulties has been demonstrated.

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