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EXTRAPOLATION OF AMPLITUDE FREQUENCY PLOT

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INTRODUCTION

As per IS : 5249–1969[1] the natural frequency of soil-block system and hence the values of dynamic properties of soils can only be obtained if well defined peak in frequency amplitude plot is obtained by experimental data. But resonance peak may not be reached, in some cases such as in case of small block on stiff soils. In such cases results need be analysed by fitting frequency-amplitude plot by extrapolation and peak so obtained *will be workable peak and not the true peak*. A method is proposed here to fit the frequency-amplitude plot by extrapolation.

An index to check as to which data would give a reliable peak has also been developed. Field records from a few sites have been analysed to check the validity of the proposed method.

THEORY

From elementary theory of mechanical vibrations, the relation between amplitude (A_Z) and frequency (ω) in forced vertical vibration test is given by

$$A_Z=\frac{F_0}{\sqrt{(K-m\omega^2)^2+(c\omega)^2}}$$

or

$$A_Z=\frac{m_0 e\omega^2}{\sqrt{(K-m\omega^2)^2+(c\omega)^2}} \tag{1}$$

where : A_Z = dynamic amplitude,
$F_0 = m_0 e\omega^2$, dynamic force,
e = eccentricity of rotating mass,
ω = operating frequency of machine,
K = spring constant of soil,
m = mass of block and mountings,
c = damping of the system.

If 'f' is the operating frequency in cycles per second, then

$$\omega=2\pi f$$

Equation (1) can be put in the form

$$A_Z=\frac{f^2}{\sqrt{\frac{m^2}{(m_0 e)^2}f^4+\frac{(c^2-2km)^2}{(m_0 e)^2(2\pi)^2}f^2+\frac{K^2}{(m_0 e)^2(2\pi)^4}}}$$

or

$$A_Z=\frac{f^2}{\sqrt{A_1 f^4+A_2 f^2+A_3}} \tag{2}$$

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where A_1, A_2 and A_3 are constants for a particular block-soil system such that

$$A_1=\left(\frac{m}{m_0e}\right)^2,\ A_2=\left(\frac{c^2-2km}{2\pi\ m_0e}\right)^2,\ \text{and } A_3=\left(\frac{K}{(2\pi)^2\ m_0e}\right)^2$$

Equation (2) can now be put in the form as below

$$A_1 f^4+A_2 f^2+A_3=\frac{f^4}{(A_Z)^2} \tag{3}$$

For obtaining the relation between amplitude and frequency, A_1, A_2 and A_3 are determined by the method given below from the observed field data. Thus from equation (3) the values of amplitude (A_Z) at different frequencies may be obtained to extrapolate the curve beyond the frequency range of field tests.

METHOD OF EXTRAPOLATION

Equation (3) is non-linear type of equation of fourth degree. Fot determining constants A_1, A_2 and A_3 a simple method is proposed here in which it is assumed that the experimental points on the rising curve of frequency amplitude lie very close to the actual curve.

If there are 'n' experimental points on the rising curve of frequency-amplitude plot, the three constants, can be obtained by choosing the three points in n_{c_3} ways. In the proposed method the constants A_1, A_2 and A_3 are determined from all possible combinations of three points and the final values of constants are obtained by taking the average of values determined by different combinations. For one set of 'f' and 'A_Z', equ. 3 becomes

$$\begin{aligned} A_1\times f_1^4+A_2\times f_1^2+A_3&=\frac{f_1^4}{(A_{Z_1})^2}\\ A_1\times f_2^4+A_2\times f_2^3+A_3&=\frac{f_2^4}{(A_{Z_2})^2}\\ A_1\times f_3^4+A_2\times f_3^2+A_3&=\frac{f_3^4}{(A_{Z_3})^2} \end{aligned} \tag{4}$$

These equations can be put in matrix form as below:

$$\begin{bmatrix} f_1^4 & f_1^2 & 1\\ f_2^4 & f_2^2 & 1\\ f_3^4 & f_3^2 & 1 \end{bmatrix}\times\begin{Bmatrix} A_1\\ A_2\\ A_3 \end{Bmatrix}=\begin{Bmatrix} \dfrac{f_1^4}{(A_{Z_1})^2}\\ \dfrac{f_2^4}{(A_{Z_2})^2}\\ \dfrac{f_3^4}{(A_{Z_3})^2} \end{Bmatrix} \tag{5}$$

Solution of equation (5) gives values of A_1, A_2, A_3 for all possible combination, and an average is taken.

VALIDITY OF METHOD AND DISCUSSION

Nineteen sets of experimental data of different sites from various sources (Table 1) are taken for checking the validity of the method. In these sites full frequency amplitude plots were obtained experimentally. Extrapolation of the curves are made by proposed method taking five experimental points on the initial portion of the curve. In Figures 1, 2, 3 and 4 are plotted both experimental and extrapolated graphs for serial nos. 1, 4, 5 and 9 respectively of Table 1. All plots, except the one in Fig. 4 are for vertical vibrations. The plot of Fig. 4 is for H-vibrations.

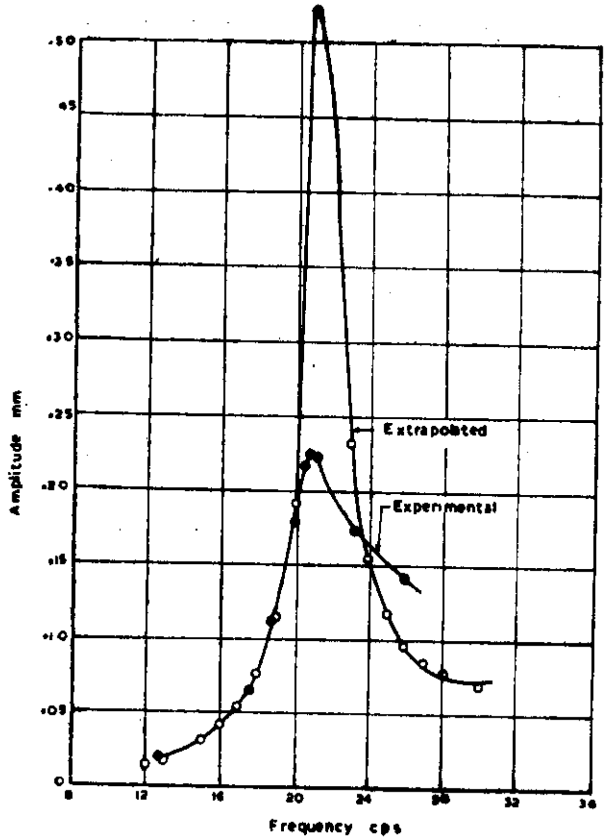

Fig. 1—Amplitude versus frequency plots—Sirhind (Prakash and Gupta 1971 a)

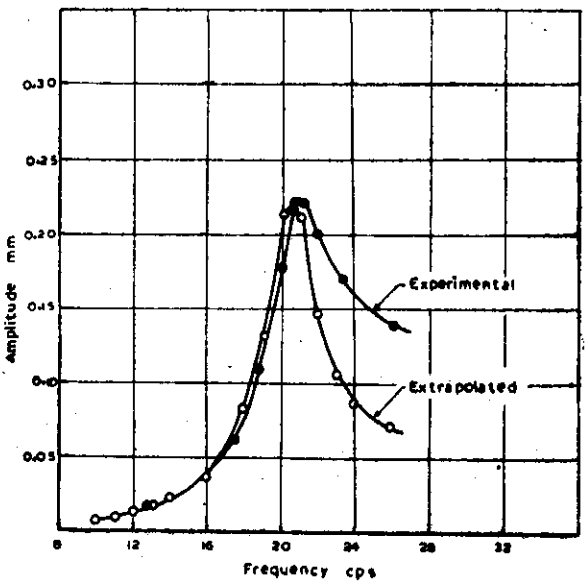

Fig. 2—Amplitude versus frequency plots for Nakodar (Prakash and Gupta 1971 b)

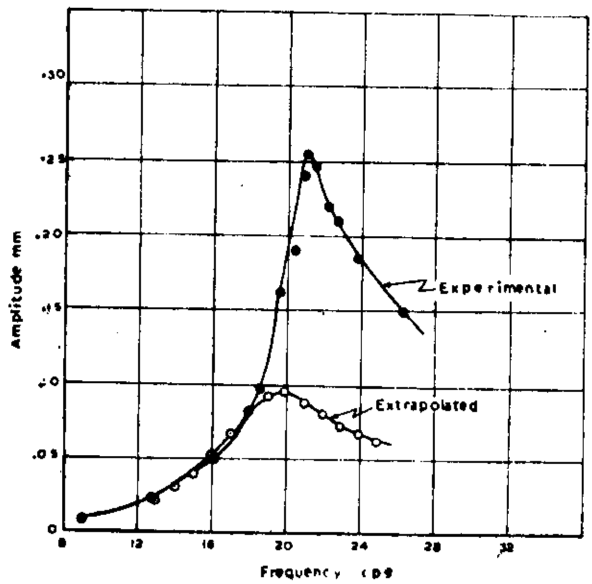

Fig. 3—Amplitude versus frequency plots from Nakodar (Prakash and Gupta 1971 b)

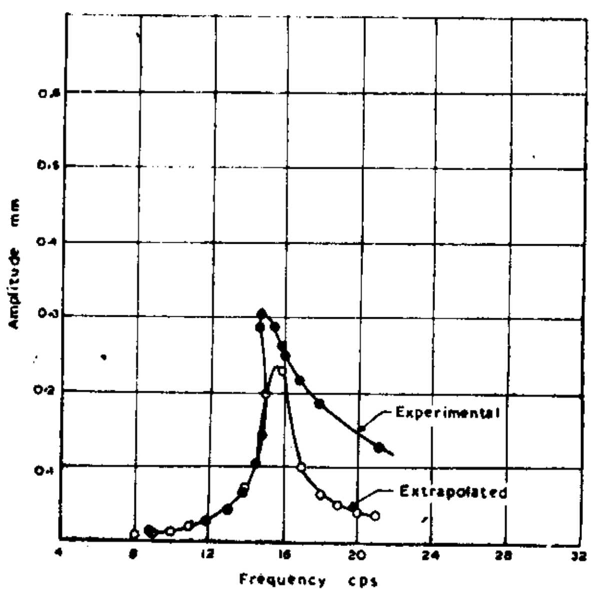

Fig. 4—Amplitude versus frequency plot for H-vibrations from Sirhind (Ref. Prakash and Gupta 1971 a)

TABLE I. AMPLITUDE FREQUENCY DATA USED FOR EXTRAPOLATION

S. No.		1	2	3	4	5	Remark
	Az10^{-2}mm	1.875	6.25	11.0	19.375	21.75	Vertical vibration Sirhind site
1	Frequency f cps	12.8	17.6	18.8	20.1	20.3	θ=105 (Ref. 1 Fig. 5)
	Az	2.25	5.0	9.5	18.875	23.875	Vertical vibration Sirhind site
2	f	12.7	16.0	18.5	20.3	20.8	θ=140 (Ref. 1 Fig. 6)
	Az	1.0	2.375	5.75	9.50	13.375	Vertical vibration Sirhind site
3	f	12.8	17.1	19.0	20.0	20.4	θ=70° (Ref. 1 Fig. 4)
	Az	1.625	3.75	6.25	11.0	17.625	Vertical vibration Nakodar site
4	f	12.8	16.0	17.5	18.8	20.0	θ=105° (Ref. 2 Fig. 5)
	Az	0.75	2.25	4.875	9.50	16.125	Vertical vibration Nakodar site
5	f	9.0	12.7	16.0	18.5	19.6	θ=140° (Ref. 2 Fig. 6)
	Az	2.30	2.95	5.30	8.0	9.0	Vertical vibration Chandigarh site
6	f	15.6	18.2	21.3	23.0	23.6	θ=70° (Ref. 3 Fig. 7)
	Az	1.40	1.90	2.60	6.0	9.30	Vertical vibration Chandigarh site
7	f	18.6	21.0	23.2	25.8	27.6	θ=35° (Ref. 3 Fig. 8)
	Az	0.75	2.5	4.5	6.5	10.5	Horizontal vibration Sirhind site
8	f	8.6	11.9	13.1	13.8	14.5	θ=35° Accn Pick up at top (Ref. 1 Fig. 7)

Table 1 (*Contd.*)

S. No.		1	2	3	4	5	Remark
9	Az	2·0	5.75	11.5	15.5	19.75	Horizontal vibration Sirhind site
	f	9.90	12.4	13.4	13.8	14.0	θ=35° Pick up in middle (Ref. 1 Fig. 7)
10	Az	2.0	4.0	9.25	19.3	22.0	Horizontal vibration Sirhind site
	f	10.4	12.5	13.9	14.4	14.7	θ=35° pick up at bottom (Ref. 1 Fig. 7)
11	Az	3.0	6.25	13·25	27.0	39.75	Horizontal vibration Sirhind site
	f	8.8	10.4	11.8	12.5	13.0	θ=70° Pick up in middle (Ref. 1 Fig. 8)
12	Az	1.0	3.5	6.75	10.75	21.25	Horizontal vibration Sirhind site
	f	7.3	10.1	11.7	12.5	13.0	θ=70° Pick up at bottom (Ref. 1 Fig. 8)
13	Az	4.25	9.0	19.5	47.0	52.5	Horizontal vibration Sirhind site
	f	8.8	10.3	11.7	12.9	13.1	θ=70° Pick up at top (Ref. 1 Fig. 8)
14	Az	2.0	5.0	23.0	38.5	44.0	Horizontal vibration Sirhind site
	f	6.9	8.9	10.85	11.5	11.7	θ=140° Pick up at bottom (Ref. 1 Fig. 10)
15	Az	4.5	11.5	39.5	56.5	68.75	Horizontal vibration Sirhind site
	f	7.1	9.35	10.7	11.1	11.35	θ=140° Pick up at top (Ref. 1 Fig. 10)
16	Az	2.0	2.5	4.0	5.5	9.5	Horizontal vibrations Nakodar site
	f	10.1	11.0	12.4	13.0	13.9	θ=35° Pick up at bottom (Ref. 2 Fig. 7)
17	Az	2.25	6.25	11.25	15.5	20.0	Horizontal vibrations Nakodar site
	f	9.7	12.3	13.3	13.8	14.1	θ=35° Pick up in middle (Ref. 2 Fig. 7)

Table 1 (*Contd.*)

S. No.		1	2	3	4	5	Remark
18	Az	2.5	4.25	6.5	10.5	14.25	Horizontal vibration Nakodar site
	f	11.9	13.1	13.85	14.4	14.8	θ=35° Pick up at top (Ref. 2 Fig. 7)
19	Az	1.25	3.75	7.0	11.0	21.75	Horizontal vibration Nakodar site
	f	7.4	10.1	11.7	12.4	13.0	θ=70° Pick up at bottom (Ref. 2 Fig. 8)

Figure 5 shows plot of observed and computed natural frequency. A few points emerge distinctly from a perusal of these graphs:—

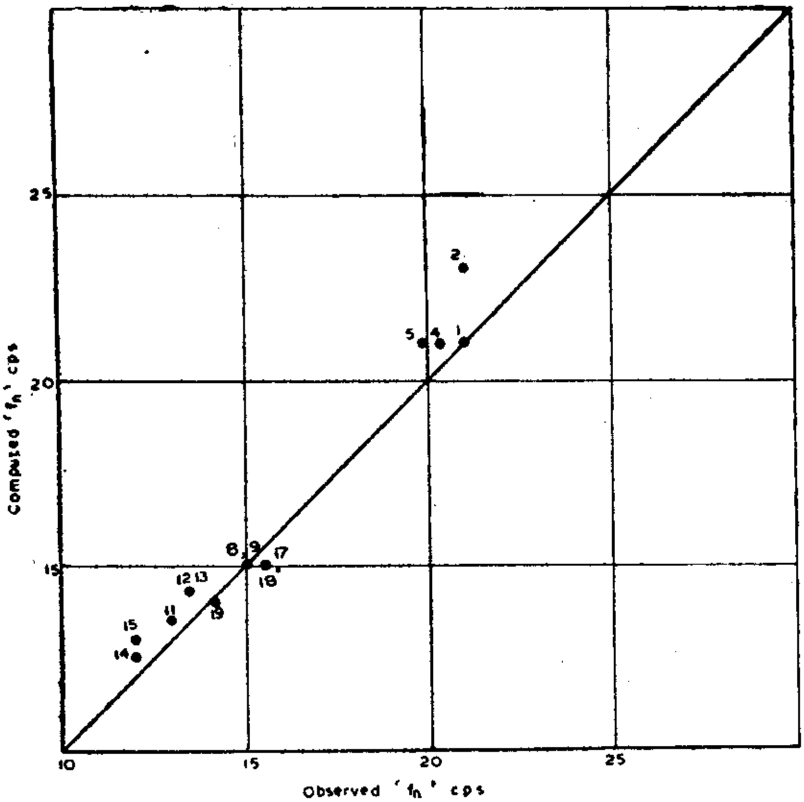

Fig. 5—Comparison of observed and computed natural frequencies

1. Prediction of natural frequency is within 7 to 8% of the observed value.

2. The shape of the amp-frequency is not truly reproduced and the departure from the measured plot is beyond the scope of possible experimental errors. It appears, in

equation (3), the value of A_Z are very sensitive to values of A-coefficients. However, the amplitudes measured in such a test are subject to certain errors and particularly so, close to the natural frequency. However, with the limited data at the disposal of the authors, it is not possible to draw any definite conclusions about amplitudes. Also, any worthwhile evaluation of the amplitudes is beyond the scope of present investigations.

In addition, the actual soil mass can at best be approximated by a non-linear spring, while equation (1) is based on linear spring theory.

For data of serial no. 3, 6, 7, 10 and 16, (Table-1) the curves could not be, extrapolated, since there was no resonance. This is physically incompatible. It was therefore, necessary to ascertain as to which data is likely to give incompatible results. It was first thought of if a possible index could be developed for the purpose. By simple examination of amp-frequency data no clearcut decision was possible. However, in case factor $\left(\frac{A_Z}{f^2}\right)$ increases with 'f', the prediction of f_n was reasonable. In other cases, if $\frac{A_Z}{f^2}$ either decreases with 'f' or oscillates with 'f', the solution obtained is incorrect. (See remarks in Table 2) Values of $\frac{A_Z}{f^2}$ with well defined frequency values has been computed for the rising portion of the graph, Table 2. On examination of this table, it shall be seen that this criterion is not satisfied only in serial no. 3, 6, 7, 10 and 16.

TABLE 2. COMPUTATION OF INDEX WHETHER FREQUENCY CURVE CAN BE CORRECTLY EXTRAPOLATED

S. No.	f	13	14	15	16	17	18	Remark
1	$A_Z \times 10^{-2}$	1.88	2.25	3.0	4.125	5.25	7.5	A_Z/f^2 is in increasing order, so ok.
	$A_Z/f^2 \times 10^{-3}$	0.111	0.115	.1332	0.156	0.181	0.2075	
2	$A_Z \times 10^{-2}$	2.625	3.25	4.0	5.0	6.25	8.00	A_Z/f^2 is in increasing order, so ok.
	$A_Z/f^2 \times 10^{-3}$	0.1554	0.166	0.1776	0.1953	0.2165	0.2215	
3	$A_Z \times 10^{-2}$	1.0	1.0	1.25	1.75	2.375		A_Z/f^2 is not in increasing order, so not correct.
	$A_Z/f^2 \times 10^{-3}$	0.0592	0.051	0.0556	0.0638			
4	$A_Z \times 10^{-2}$	1.625	2.1	2.75	3.75	5.25		A_Z/f^2 is in increasing order, so ok.
	$A_Z/f^2 \times 10^{-3}$	0.0962	0.1072	0.1223	.1412	0.1818		
	f	10	11	12	13	14		
5	$A_Z \times 10^{-2}$		1.25	1.75	2.30	3.0		A_Z/f^2 is in increasing order, so ok.
	$A_Z/f^2 \times 10^{-3}$	0.875	0.1032	0.1215	0.136	0.153		

	f	16	17	18	19	20	
6	$Az \times 10^{-2}$	2.340	2.58	2.90	3.36	4.10	$Az/f^2 \times 10^{-3}$ is not in increasing order, so not correct.
	$Az/f^2 \times 10^{-3}$	0.0915	0.895	0.0805	—	—	
	f	19	20	21	22		
7	$Az \times 10^{-2}$	1.45	1.60	1.80	2.20		$Az/f^2 \times 10^{-3}$ is not in increasing order, so not correct.
	$Az/f^2 \times 10^{-3}$	0.0402	0.040	0.048	0.0454		
	f	10	11	12	13	14	
8	$Az \times 10^{-2}$	1.25	2.0	2.75	4.5	7.5	$Az/f^2 \times 10^{-3}$ is in increasing order, so ok.
	$Az/f^2 \times 10^{-3}$	0.125	0.165	0.210	0.266	0.382	
9	$Az \times 10^{-2}$	2.0	3.5	5.2	8.5	15.5	$Az/f^2 \times 10^{-3}$ is in increasing order, so ok.
	$Az/f^2 \times 10^{-3}$	0.20	0.2895	0.361	0.503	0.792	
10	$Az \times 10^{-2}$	1.75	2.0	3.0			$Az \times 10^{-3}$ is not in increasing order, so not correct.
	$Az/f^2 \times 10^{-3}$	0.175	0.165	0.208			
	f	9	10	11	12		
11	$Az \times 10^{-2}$	2.75	5.5	9.0	17.5		$Az/f^2 \times 10^{-3}$ is in increasing order, so ok.
	$Az/f^2 \times 10^{-3}$	0.152	0.55	0.751	1.23		
	f	8	9	10	11	12	
12	$Az \times 10^{-3}$	1.25	2.0	3.0	5.0	8.0	$Az/f^2 \times 10^{-3}$ is in increasing order, so ok.
	$Az/f^2 \times 10^{-3}$	0.195	0.247	0.30	0.413	0.556	
	f	9	10	11	12	13	
13	$Az \times 10^{-2}$	4.5	7.5	12.0	22.5	—	$Az/f^2 \times 10^{-3}$ is in increasing order, so ok.
	$Az/f^2 \times 10^{-3}$	0.556	0.75	0.993	1.563		
	f	7	8	9	10	11	
14	$Az \times 10^{-2}$	2.0	3.2	5.5	10.0	20.3	$Az/f^3 \times 10^{-3}$ is in increasing order, so ok.
	$Az/f^2 \times 10^{-3}$	0.409	0.50	0.68	1.0	1.627	

15	$A_z \times 10^{-2}$	4.5	6.5	10	18	55		$A_z/f^2 \times 10^{-3}$ is in increasing order, so ok.
	$A_z/f^2 \times 10^{-3}$	0.918	1.015	1.234	1.80	4.55		
	f	10	11	12	13	14		
16	$A_z \times 10^{-2}$	2.0	2.25	3.25				$A_z/f^2 \times 10^{-3}$ is not in increasing order, so not correct.
	$A_z/f^2 \times 10^{-3}$	0.20	0.186	0.226				
17	$A_z \times 10^{-2}$	2.25	3.5	5.0	9.5	17.5		$A_z/f^2 \times 10^{-3}$ is in increasing order, so ok.
	$A_z/f^2 \times 10^{-3}$	0.225	0.289	0.347	0.563	0.892		
	f	9	10	11	12	13	14	
18	$A_z \times 10^{-2}$	1.0	1.5	2.0	2.5	4.5	7.0	$A_z/f^2 \times 10^{-3}$ is in increasing order, so ok.
	$A_z/f^2 \times 10^{-3}$	0.1235	0.150	0.165	0.1735	0.266	0.357	
	f	8	9	10	11	12	13	
19	$A_z \times 10^{-2}$	1.25	2.0	3.0	5.0	9.0	18	$A_z/f^2 \times 10^{-3}$ is in increasing order, so ok.
	$A_z/f^2 \times 10^{-3}$	0.195	0.247	0.30	0.455	0.625	—	

The procedure may therefore be summarized as follows:—

(1) Select at least '5' points on the rising portion of the amp-frequency plot.

(2) Check if factor $\frac{A_z}{f^2}$ is in increasing order with 'f' on rising position of the curve. If yes, proceed with steps 3 and 4, or else the data cannot be extrapolated with this method.

(3) Compute A_1, A_2, A_3 in equation (3), and take average values of A coefficients.

(4) Extrapolate amp-frequency plot beyond resonance from equation 3.

CONCLUSIONS

The following conclusions may be drawn based on this study:

(1) The extrapolation of natural frequency is reasonable from the rising portion of the curve.

(2) The method may be used for horizontal vibrations also.

(3) The validity of the method needs be checked for diverse soil conditions.

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