Effect of the seismic oblique waves on dynamic response of an embedded foundation

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ABSTRACT

This study analyzes the influence of soil-structure interaction on the seismic response of a three-dimensional (3-D) rigid foundation on the surface of viscoelastic soil limited by bedrock. Vibrations result only from P, SV, SH, and R harmonic seismic waves. The key step is the characterization of the soil-foundation interaction with the impedance matrix on one hand and the input motion matrix on the other hand. The mathematical approach is based on the method of integral equations in the frequency domain using the formalism of Green's functions (Kausel and Peck, 1982) for layered soil. This approach has been applied to analyze the effect of soil-structure interaction on the seismic response of the foundation as a function of the kind of incident wave, the angles of wave incidence, the wave frequency and the embedding of foundation.

KEYWORDS : Waves propagation, BEM-TLM, Soil-Structure Interaction.

INTRODUCTION

The analysis of the behavior of foundations under dynamic loads has grown considerably over the past four decades. Stringent security requirements imposed on design of certain types of structures have played a particularly important role in the development of analytical methods. The key step in studying the dynamic response of foundation is the determination of the relationship between forces. This relationship which results in displacement is expressed using impedance functions (dynamic stiffness) or the compliance functions (dynamic flexibility). Consideration of the soil-structure interaction in the analysis of the dynamic behaviour of foundation allows to take realistically into account the influence of soil on its vibration.

A myriad of methods has been proposed to solve the soil-structure interaction problem. To simplify the problem linear-analysis techniques have been developed. One of the most commonly used approaches is the substructuring method that allows the problem to be analyzed in two parts (Kausel et al [12], Aubry et al [1] and Pecker [21]). In this approach the dynamic response of superstructure elements and substructure are examined separately. The analysis of foundation systems can be reduced to the study of the dynamic stiffness at the soil-foundation interface (known as impedance function) and driving forces from incident waves. The kinematic interaction of the foundation with incident waves is implemented in the form of a driving-force vector

Determining the foundation response thereby becomes a wave propagation problem. Due to the mixed-boundary conditions of the problem (displacement compatibility with stress distribution underneath the foundation and zero tension outside) solutions are complex. The determination of impedance functions and forces of movement related to the incident waves is a complex process. Several studies have been conducted on the dynamic response of foundation using the finite-element and boundary-element methods. Wong and Luco [30] have shown the importance of the effect of non-verticality of SV, SH harmonics on the response of a foundation.

Apsel and Luco [3] used an integral-equation approach based on Green's functions for multilayered soils determined to calculate the impedance functions of foundation. Using this approach, Wong and Luco [29] studied the dynamic interaction between rigid foundations resting on a half-space. Boumekik [5] studied the problem of 3-D foundations embedded in soil limited by a rigid substratum. The finite-element method was applied by Kausel et al [12], Kausel and Roesset [11] and Lin et al [13] to determine the behavior of rigid foundations placed on or embedded in soil layer limited by a rigid substratum. A formulation of the boundary-element method in the frequency domain has been developed to address wave-propagation problems of soil-structure interaction and structure-soil-structure which limits the discretization at the interface soil-foundation. In this approach, the field of displacement is formulated as integrals equation in terms of Green's functions Beskos [4], Aubry et al [1], Qian and Beskos [22], Karabalis & Mohammadi [9] and Mohammadi [20]. Celebi et al [6] used the boundary-element method with integral formulation (BIEM) to compute the dynamic impedance of foundations. In this context, the analytical solutions of 3-D wave equations in cylindrical coordinates in layered medium with satisfying the necessary boundary conditions are employed by Liou [14] and Liou & Chung [15]. Sbartai and Boumekik [23] used the BEM-TLM method to calculate the one hand, the dynamic impedance of rectangular foundations placed or embedded in the soil layered limited by a substratum and also the propagation of vibrations in the vicinity of a vibrating foundation. In this study, the method has been applied to analyze the effect of some parameters on the dynamic response of the foundations (depth of the substratum, embedding, masses and shape of the foundation, soil heterogeneity and frequency). However, Sbartai and Boumekik [24, 25] have studied the dynamic response of two square foundations placed or embedded in soil layered limited by a substratum. Spryakos and Xu [26] have developed a hybrid BEM-FEM and have conducted several studies for parametric analysis of soil-structure interaction.

Recently, McKay [18] used the reciprocity theorem based on the BIEM to analyze the influence of soil-structure interaction on the seismic response of foundations. However, Suarez et al [27] applied the BIEM to determine the seismic response of an L-shaped foundation. In addition, experimental work has been carried out by researchers in Japan to determine the effect of soil-structure interaction on the response of real structure Fujimori et al [7], Akino et al [2], Mizuhata et al [19], Watakabe et al [28] and Imamura et al [8].

In our study, the solution is derived from the BEM in the frequency domain with constant quadrilateral elements and the thin-layer method is used to analyze the influence of soil-structure interaction on the response of seismic foundation. The results are presented as coefficients of movement in matrix [S*] and in terms of displacement as a function of dimensionless frequency, angle of incidence (vertically and horizontally) and embedding of foundation. This paper represents a continuation of that paper was previously published in Sbartai and Boumekik [23] where the impedance functions have been well studied in details. Therefore, they will not be discussed in this article.

FOUNDATION RESPONSE TO WAVES OF SEISMIC ORIGIN

1. Physical Model and Basic Equations

The geometry of the calculation model is shown Fig 1. Consider 3-D, rigid, massless, surface foundation of arbitrary shape S in full contact with a homogeneous, isotropic, and linearly-elastic soil that is limited by bedrock. The soil is characterized by its density ρ, shear modulus G, damping coefficient β and Poisson's ratio ν. The foundation is subject to harmonic oblique-incident waves that are time-dependent: P, SV, SH and R.

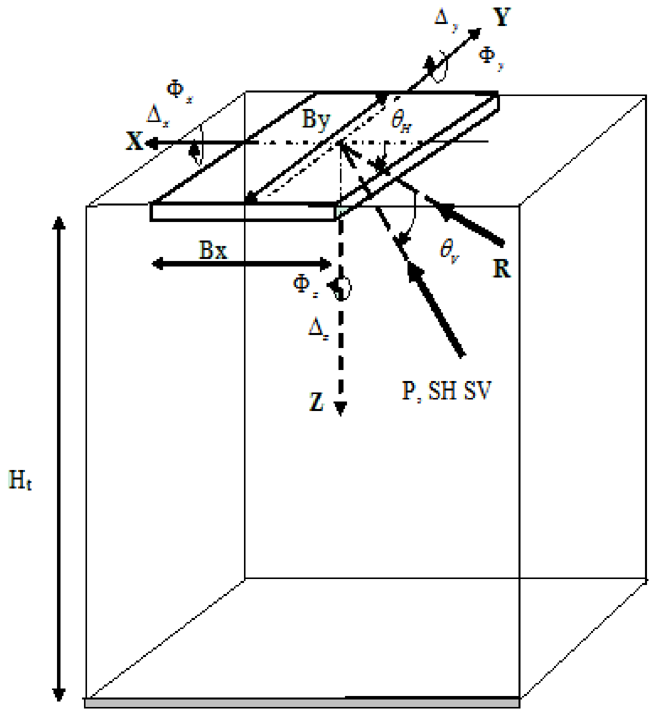

Fig .1 Geometry of a foundation subjected to harmonic seismic waves.

The movement of a not-specified-point " ζ " can be obtained from solving the wave equation:

$$(C_P^2 - C_S^2)\, u_{j,ij} + C_S^2\, u_{i,jj} - \omega^2 u_i = 0\,, \qquad (1)$$

Where Cs, and Cp are the velocities of shear and compression waves and ω the angular frequency of excitation.

u_i is the component of the harmonic displacement-vector in the x-direction;

$u_{j,ij}$ is the partial derivative of the displacement field with respect to x and y;

$u_{i,jj}$ is the second partial derivative of the displacement field with respect to y.

The solution of equation (1) may be expressed by the following integral equation:

$$u_j(x,\omega) = \int_S G_{ij}(x,\xi,\omega)\ t_i(\xi,\omega)\ ds(\xi), \tag{2}$$

With G_{ij} denoting the Green's functions at point i due to unit-harmonic load (vertical and horizontal) of the ground at point j and t_i being a load (traction) distributed over an area of soil.

The medium is continuous so this relationship is very difficult to assess. However, if the soil mass is discretized appropriately, this relationship can be made algebraic and displacement can be calculated. The key step of this study is to determine the impedance matrix linking the harmonic forces applied to the resulting harmonic displacement. Even with a continuous medium the determination of the impedance matrix is still very difficult, if not impossible, due to the propagation problem and its mixed-boundary conditions. However, if the medium is discretized vertically and horizontally then it is possible to making the problem algebraic by considering that the variation of interface displacement is a linear function.

1.1. Discretization of the model

The principle of horizontal and vertical discretization of soil mass is shown Fig 2. The principle of vertical discretization based on the division of every soil layer into a number of sub-layers of height h_j with similar physical characteristics. Each sublayer is assumed to be horizontal, viscoelastic, and isotropic, and characterized by constant Lame λ_j, a shear modulus μ_i and a density ρ_j. The bedrock at depth H_t is considered infinitely rigid and is not discretized. The reflection wave is assumed to be total and the displacements null.

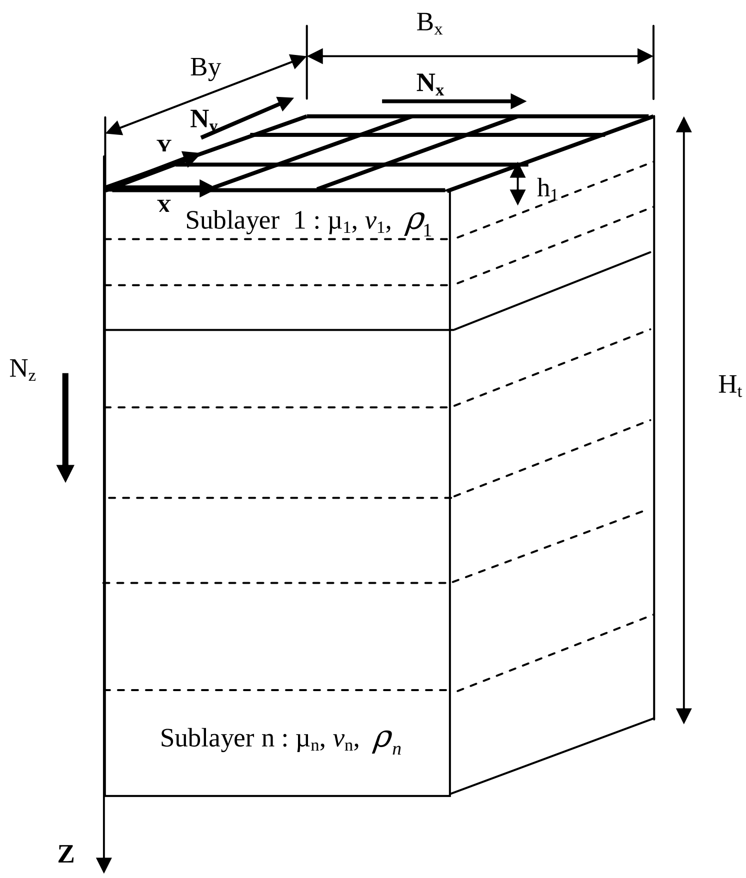

Fig .2 The model of calculation

H_t: height of soil mass;

h_1: height of the sublayer 1;

N_x: number of elements in the x-direction for a horizontal plane;

N_y: number of elements in the y-direction for a horizontal plane;

N_z: number of soil layers.

N is the total number of elements in soil-foundation interface and B_x and B_y are the dimensions of the foundation.

Within a given sublayer, the displacement is assumed to be a linear function of interface displacement above and below. This is true when the height of the sublayer is small in relation to the wavelength considered (in the order of $\lambda/10$). This method is comparable to the FEM in the sense that the movements within each sublayer are completely defined from the displacements in the middle of the interfaces. The interaction between the elements is done only through the nodes. The degrees of freedom of the soil mass are reduced to the degrees of freedom of the nodes. The stiffness matrix of soil mass is obtained in a similar manner to how it is determined in the FEM.

This technique has been developed by Lysmer and Waas [17] and is known as the thin-layer method (TLM) and is used mainly for horizontal soil layers. This method has the advantage to making the problem algebraic and thus obtains the Green's functions by applying the boundary elements method in the soil-foundation interface. For this reason a horizontal discretization of the interface soil-foundation is established.

The horizontal discretization permits to subdivide any horizontal interface of soil-foundation by elements of square-sections S_k. These elements where the constant-moving average is replaced by the movement of the center, assumes that stress distribution is uniform and is shown in Fig 2. Seeking simplicity of the integration calculation and economy of computing time the square elements are approximated disc elements. If the units loads (along the direction x, y, z) are applied to disc j, the Green's functions at the center of disc i can be determined. By successively applying these loads on all discs, the flexibility matrix of soil complex at a frequency of a given ω can be formed. The discretized model to calculate the impedance functions of the foundation is also presented in Figure (2). In the discrete model, Equation (2) is expressed in algebraic form as follows:

$$u_j = \sum_{i=1}^{N} \int_S G_{ij}\ t_i\ ds \qquad (3)$$

2. Determination of Green's Functions by The TLM

The Green's function for a layered stratum is obtained by an inversion of the thin-layer stiffness matrix using a spectral-decomposition procedure of Kausel and Peek [10]. The advantage of the thin-layer-stiffness matrix technique over the classical transfer-matrix technique for finite layers and the finite-layer-stiffness matrix technique Kausel and Roesset [11] is that the transcendental functions in the layered-stiffness matrix are linearized.

In this work, the body (B) represents a layered stratum resting on a substratum base with n horizontal layer interfaces defined by $z = z_1, z_2,..., z_n$ and with layer j defined by $z_n < z < z_{n+1}$, as shown in Figure (3). The medium of each n layer of h_n thickness is assumed to be homogeneous, isotropic, and linearly elastic. For this body, the Green's function frequency domain is obtained with help of the TLM.

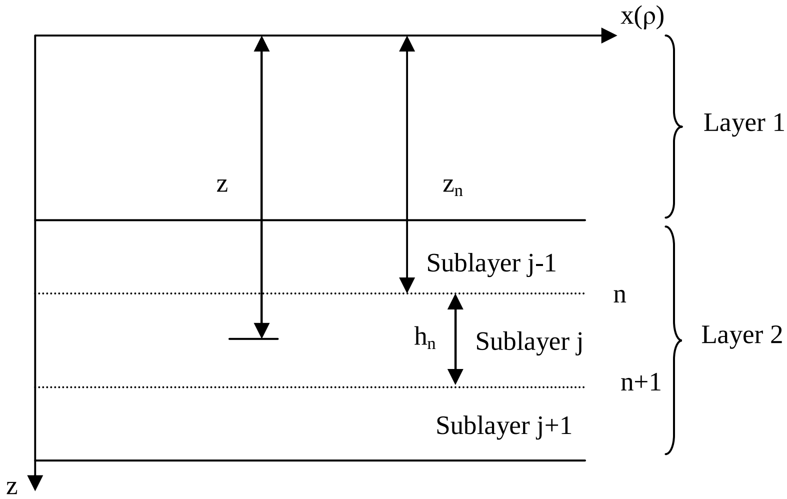

Fig. 3 Geometry of soil layers (B).

According to the thin-layers theory of Lisper and Waas [17], displacements in each sub-layer vary linearly from one plane to another and still continue in the relevant direction (x, y, z). Thus the displacements in each sublayer are obtained by linear interpolation of nodal displacements at the interface of the sublayer (n) as follows:

$$U^{(n)}(z) = (1-\eta)U^{n} + \eta\ U^{n+1} \tag{3a}$$

$$V^{(n)}(z) = (1-\eta)\ V^{n} + \eta\ V^{n+1} \tag{4b}$$

$$W^{(n)}(z) = (1-\eta)\ W^{n} + \eta\ W^{n+1} \tag{4c}$$

where $\eta = \frac{(z - Zn)}{h_n}$ et (0 ≤ η ≤ 1), and, $U^{(n)}$, $V^{(n)}$ et $W^{(n)}$ are the displacements along the x-axis, the y-axis and the z-axis as functions of z in the layer j, and with U^{n}, V^{n} et W^{n} which are their nodal values at the interface layer z = Z_n.

$$G_{ij}^{mn} = \sum_{l=1}^{2N} \frac{a_{\alpha\beta} \cdot \phi_i^{ml} \cdot \phi_j^{nl}}{k^2 - k_l^2} \tag{5}$$

with:

$a_{\alpha\beta} = 1$ si $\alpha = \beta$ and $a_{\alpha\beta} = \frac{k}{k_l}$ si $\alpha \neq \beta$ i, j = x, y, z.

k, k_l: wave number ;

m: represents the interface where the load is applied;

n: represents the interface where Green’s functions are calculated.

The obtained Green's functions are complex and constitute the starting point for the determination of the flexibility matrix of an arbitrary soil volume. However, considering the geometry of the foundation, a system of Cartesian coordinates was adopted. The obtained U, V, and W Green's functions are in fact the terms of the flexibility matrix of the soil. The determination of this flexibility matrix gives the impedance function of one or several foundations and the amplitudes of vibrations in the neighborhood of a foundation Sbartai and Boumekik [23, 24].

Viscoelastic soil behavior can be easily introduced in the present formulation by simply replacing the elastic constants λ and G with their complex values:

$$\lambda^{*}(Z) = \lambda(1+2i\beta) \tag{6a}$$

$$\mu^{*}(Z) = \mu\ (1+2i\beta)\ , \tag{6b}$$

β is the hysteretic damping coefficient.

CALCULATION MODEL

The total displacement of the soil matrix is obtained by successive application of unit loads on the constituents of the discretized solid ground. The displacements in the soil are then expressed by

$$\{u\} = [G]\{t\} \tag{7}$$

The vectors $\{u\}$ and $\{t\}$ are the nodal values of the amplitudes of displacements and tractions respectively at the interface soil-foundation. [G] is the flexibility matrix of the soil.

When the foundation is in place, it requires different components of soil displacement consistent with rigid body motions. Compatibility of displacements at the contact area *S* between the soil and the rigid foundation leads to the matrix equation

$$\{u\} = [R]\{\Delta\} \tag{8}$$

[R] is the transformation matrix

$$[R] = \begin{bmatrix} 1 & 0 & 0 & 0 & z & y \\ 0 & 0 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix} \tag{9}$$

$\{\Delta\} = \{\Delta_x, \Delta_y, \Delta_z, \Phi_x, \Phi_y, \Phi_z\}^t$ is the displacement vector; $\Delta_i (i = x, y, z)$ represents translations and $\Phi_i (i = x, y, z)$ rotations (Fig 1).

If we denote $\{P\}$ the vector of load applied to the foundation, the equilibrium between the vector of loads applied and the forces (tractions) distributed over the elements discretizing the volume of the foundation is expressed by the following equation:

$$\{P\} = [R]^t \{t\} \tag{10}$$

Combining equations (7), (8) and (10) we obtain the following equation:

$$\{P\} = \left([R]^t [G]^{-1} [R]\right)\{\Delta\} = [K(\omega)]\{\Delta\} \tag{11}$$

with ω is the circular frequency of vibration and $[K(\omega)]$ the impedance or dynamic-stiffness matrix of the rigid foundation.

Considering an incident plane, SH, P, SV and R harmonic waves are characterized by the vertical and horizontal angles of incidence θ_V and θ_H respectively, as shown in Fig 1. The motion of the half-space due to these seismic waves can be expressed by the following equation:

$$\{u^f\} = \{U^f\} e^{[-i\omega\,(x.\cos\theta_H + y.\sin\theta_H)/c]}. \tag{12}$$

$\{U^f\} = \{U_x^f, U_y^f, U_z^f\}^t$, is known as the vector of amplitudes of the soil, that depends on the z coordinate if we want to study the embedded foundations case. However, in the case of surface foundations (z = 0), it is known as the vector of amplitudes of the free field. c is the apparent velocity of the incident waves having the form $c = \dfrac{c_1}{\cos\theta_V}$ *or* $c = \dfrac{c_2}{\cos\theta_V}$ for P or S waves, respectively, and being equal to the R-wave. The explicit expressions of the vector $\{U^f\}$ of waves SH, P, SV and R may be found in Wong and Luco [30].

The presence of a rigid foundation on the surface of the half-space results in diffraction of the above waves so that the total displacement field $\{u\}$ is expressed by the following equation:

$$\{u\} = \{u^f\} + \{u^s\} \tag{13}$$

where $\{u^s\}$ represents the scattered wave field that satisfies the equation of motion (7). Also, the total displacement field in the contact region between the foundation and the half space must be equal to the rigid body motion of the foundation.

Substituting equation (8) into equation (13), written in terms of the scattered field leads to the force-displacement relation:

$$\{u^s\} = [R]\{\Delta\} - \{u^f\} \tag{14}$$

Substituting equation (7) into equation (14), written in terms of the traction forces:

$$[G]\{t\} = [R]\{\Delta\} - \{u^f\} \tag{15}$$

hence

$$\{t\} = [G]^{-1} [R]\{\Delta\} - [G]^{-1}\{u^f\} \tag{16}$$

Multiplying both sides of the equation (16) by the transpose of the transformation matrix and combining with equations (10) and (11) yields the external forces:

$$[R]^t \{t\} = [R]^t [G]^{-1} [R]\{\Delta\} - [R]^t [G]^{-1}\{u^f\} \tag{17}$$

The equilibrium between external forces and seismic forces can be as follows:

$$\{P\} = [K]\{\Delta\} - [K^*]\{U^f\}, \tag{18}$$

with $[K^*]$ is the driving force matrix given by the following formula:

$$[K^*] = [R]^t [G^{-1}] e^{[-i\omega (x.\cos\theta_H + y.\sin\theta_H)/c]} \; . \tag{19}$$

Equation (14) can be replaced by the alternative form:

$$\{\Delta\} = [C]\{P\} + [S^*]\{U^f\} \tag{20}$$

where

$$[C] = [K]^{-1}$$

$[C] = [K]^{-1}$ is the dynamic compliance matrix and $[S*]$ is the input motion matrix given by the following formula:

$$[S^*] = [C][K^*] \tag{21}$$

When the rigid foundation is acted upon by seismic waves only, the external forces are zero (P = 0), and the seismic response of the foundation is obtained from equation (18) or (20) by the following expression:

$$\{\Delta\} = [S^*]\{U^f\} \tag{22}$$

with

$$[S^*] = \begin{bmatrix} S_{xx} & 0 & 0 \\ 0 & S_{yy} & S_{yz} \\ 0 & S_{zy} & S_{zz} \\ 0 & R_{xy} & S_{xz} \\ R_{yx} & 0 & 0 \\ S_{zx} & 0 & 0 \end{bmatrix} \tag{23}$$

When the mass of the foundation is not zero, one simply has to replace $[K]$ by $[K] - \omega^2 [M]$ in the above equations, where $[M]$ is mass matrix of the foundation.

VALIDATION OF THE METHOD

The accuracy of the method BEM-TLM used to study the 3D-response of foundations subject to plane-harmonic waves with variable angles of incidence and vibration frequency a_o in this section is validated through comparisons with results obtained by Luco and Wong [16] and Qian and Beskos [22] for a semi-infinite ground. A parametric study was conducted to define the parameters of the calculation model. The influence of the discretization of the soil-foundation interface was studied. The thickness of a sublayer h must be small enough that the discrete model can transmit waves in an appropriate manner and without numerical distortion. This size depends on the frequencies involved and the velocity of wave propagation. The frequency of loading and velocity of wave propagation affect the precision of the numerical solution. Kausel and Peek [10] showed that the thickness of sub-layer must be smaller than a quarter of the wavelength λ.

Consequently, the maximum dimensionless frequency must not exceed the number of sub-layer N divided by four.

Consider a rigid, massless, square foundation of side B_x = 2a on the surface of the half-space with a Poisson's ratio of v = 1/3 subjected to plane P, SV, or SH harmonic waves θ_H = 90 ° and θ_V= 45 °. Figure (4) shows the variation of the real and the imaginary part of coefficient Sxx movement based on the dimensionless frequency $a_0 = \frac{\omega}{C_s}\frac{B}{2}$. The results obtained by the proposed method are in agreement with those obtained by the method used by Qian and Beskos [22]. Considering the same foundation subjected to a Rayleigh wave where the angle of incidence is horizontal (θ_H = 0) and the corresponding velocity taken is equal to c_R = 0.9325c for a Poisson's ratio v= 1/3.

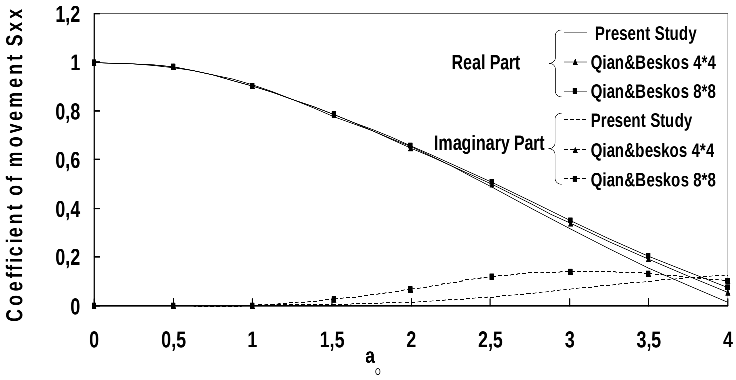

Fig. 4 The coefficient of movement Sxx a square foundation *(θ_H=0°, θ_V=45° and c_s/c=0.70711)*.

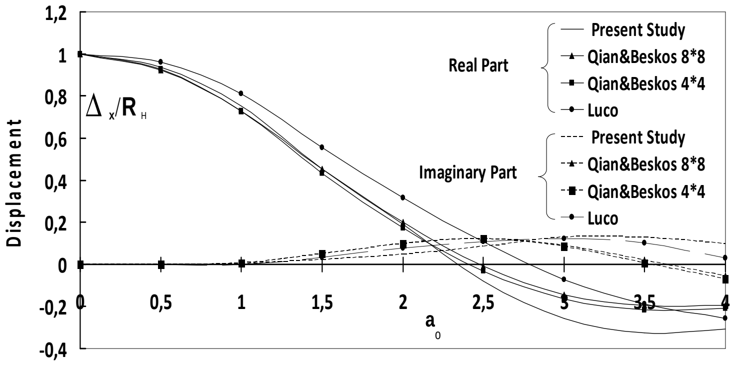

Fig. 5 The response a square foundation under the Rayleigh wave *c_R/c=0.9325.*

Figure (5) shows the real and the imaginary part of dimensionless displacement $\Delta x/H_R$ function of frequency a_o. The results of this study, using the BEM-TLM, method were compared with those of Qian and Beskos [22] (BEM) and those of Luco and Wong (BEM) [16].

The results obtained are in agreement with those of Beskos and Qian [22] and those of Luco and Wong [16]. However, a difference is present for dimensionless frequencies a_o higher than 2.5. This difference can be explained in two ways:

1) Qian and Beskos isoparametric elements were used to determine the soil-foundation interface. These types of elements are more accurate than constant elements used by the method developed in this article.
2) Qian and Beskos and Luco and Wong use the Green's functions of a semi-infinite soil whereas our method used the Green's functions of soil bounded by bedrock.

PARAMETRIC STUDY AND DISCUSSION

1. Surface Foundation

In this section, a parametric analysis was performed by studying a square foundation of the side (B_X =2a) subjected to plane-harmonic waves with variable angles of incidence and vibration frequency a_o, as shown in Fig 1 is presented. The results are presented in terms of coefficients of motion as functions of the dimensionless frequency $a_0 = \frac{\omega}{C_s}\frac{B}{2}$. The soil is characterized by the height H_t = 10m of the bedrock to simulate a semi-infinite, its Poisson's ratio is ν = 1/3, the coefficient of the hysteretic damping β = 0.05, the shear modulus μ = 1, and its density $\rho = 1$. The terms of -coefficient of motion are presented in the following figures for the shear S-wave , with an horizontal angle of incidence θ_H = 90° and vertical angle of incidence θ_V = 0°, 30°, 45°, 60° and 90°

1.1. Coefficients of movements in translation

For an angle of incidence θv = 90 °, the coefficients of translational movement Sxx, Syy, S_{zz}, are equal to unity for all frequencies. Typically this value is adopted in the study of a structure subjected to seismic loading. This value induces oversized foundations. Figures 6 and 7 show that for other angles these coefficients vary with the dimensionless frequency. The amplitude of the response also depends on the vertical angle of incidence.

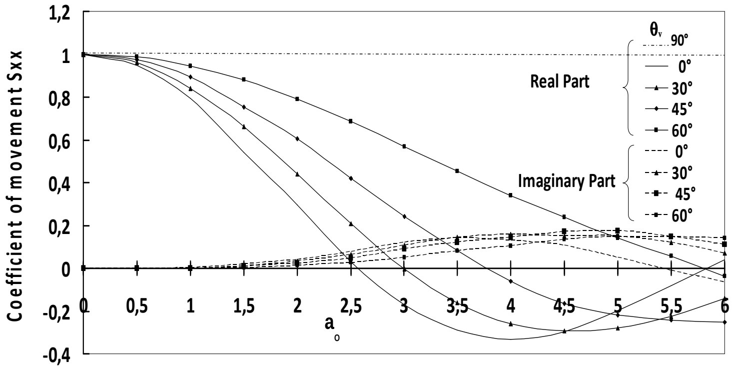

Fig. 6 The coefficient of movement Sxx *(θ_H = 90 °).*

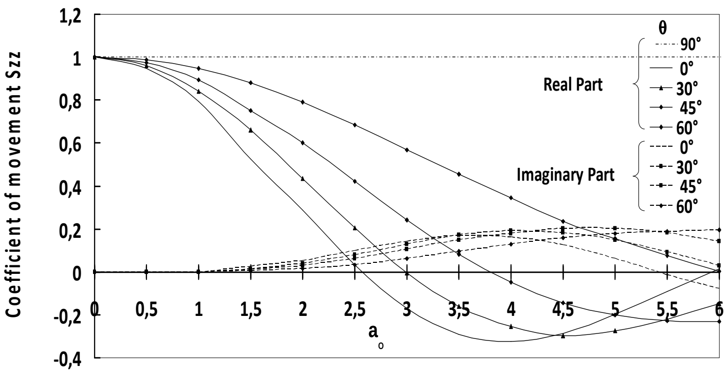

Fig. 7 The coefficient of movement Szz (*θ_H = 90 °*).

The results show that the real parts of S_{xx} and S_{zz} have a higher magnitude than the value of the imaginary parts. With low frequencies, the response is in phase with the free-field motion. These coefficients filter low frequencies and therefore behave as low-pass filters.

1.2. Coefficients of movement in rotation and torsion

Figures 8 and 9 present the relative coefficients of the rotational movement R_{xy} and R_{yx} to the x-axis and y-axis respectively. For an angle of incidence θv = 90 °, the coefficients of rotational movement (R_{xy}, and R_{yx}) are zero and maximum for θv = 0°. Figure (10) shows the relative coefficient of torsion S_{zx} to the axis of z as a function of dimensionless frequency and the vertical angle of incidence θv. The results obtained show that the value of the imaginary part of S_{zx} is dominant, which shows a large damping. Thus, the answer is out of step with the free-field motion in the center of the foundation.

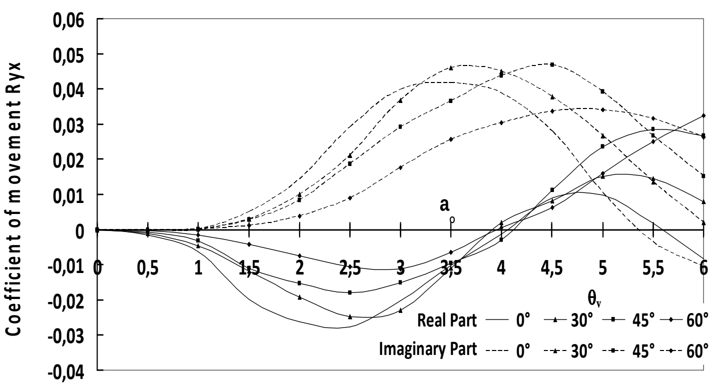

Fig. 8 The coefficient of movement R_{yx} (*θ_H = 90 °*).

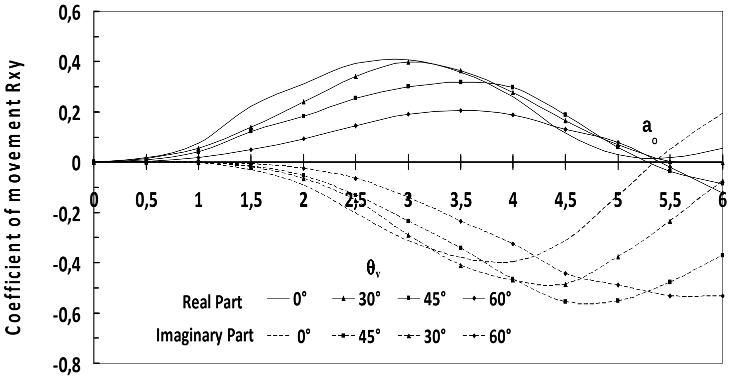

Fig. 9 The coefficient motion R_x *(θ_H = 90°).*

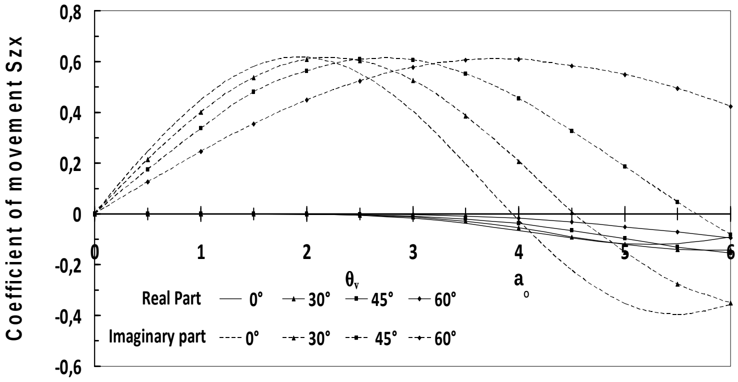

Fig. 10 The coefficient of movement S_{zx} (θ_H *= 90 °).*

Figures 8, 9, and 10 present these coefficients for other angles of incidence (0 °, 30°, 45°, 60° and 90°) and show that they vary depending on the dimensionless frequency. The amplitude of the response also depends on the vertical angle of incidence. The coefficient of movement of rotation and torsion filters high frequencies and therefore behaves as high-pass filters.

The matrix coefficients of movement shown in Figures 7 to 10 are valid only θ_H = 90° for S waves with a horizontal angle of incidence. These coefficients can be determined for other incidence angles and other types of waves.

2. Embedded Foundation

To present results that are easier to understand visually, the driving-force vectors are converted to input-motion vectors by multiplying by the inverse of the impedance matrix. Three different sets of results are given. The response of the massless foundation to incident body waves of type SH, P and SV are considered.

A square base dimension Bx = 2a embedded in a homogeneous viscoelastic soil to a depth (d) and subject to P, SV and SH-waves (Fig 11) is considered.

In this section, the influence of the embedding ratio (t = d/a = 0, 0.3, 0.6) on the seismic response of the foundation is studied. The results are presented in terms of displacements, rotations, and torsion; these terms are calculated with equation 18 as a function of dimensionless frequency a_0.

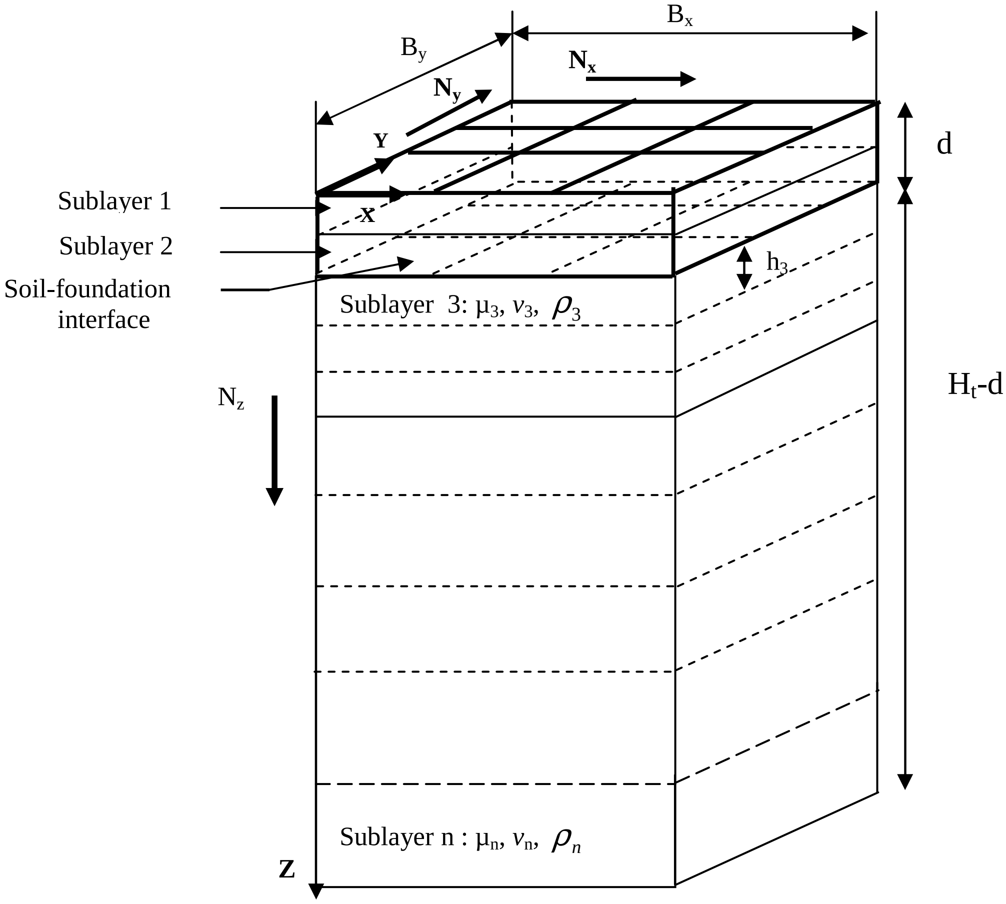

Fig. 11 Model of calculation of an embedded foundation subjected to harmonic seismic waves.

To simplify the presentation of this paper, only one direction of wave propagation is considered with the vertical incident angle θ_v = 45. Figures 12 to 19 represent the terms of displacement, rotation, and torsion caused by the incident P, SV and SH-waves. These curves are given for different values of relative embedding.

2.1. Compression wave (P)

Figures 12 to 14 show the response of a massless foundation to an incident P-wave. The wave travels in the *x*-direction with its particle motion in the z and *x*-directions. Except for the vertical incidence θ_v = 90°, the incident P-wave causes a mode conversion and reflected SV-wave results. One angle of incidence is considered, and is 45° measured with respect to the *x*-axis. The wavelength of the incident P-wave is twice as long as that of the incident S-waves; therefore, the kinematic interaction is less prominent. In general, the P-wave induces displacement along the x and z-axes and rotation around the y-axis

Figures 12-14 show the variation of displacement and rotation as a function of dimensionless frequency and the influence of embedding coefficient t on the motion of foundation.

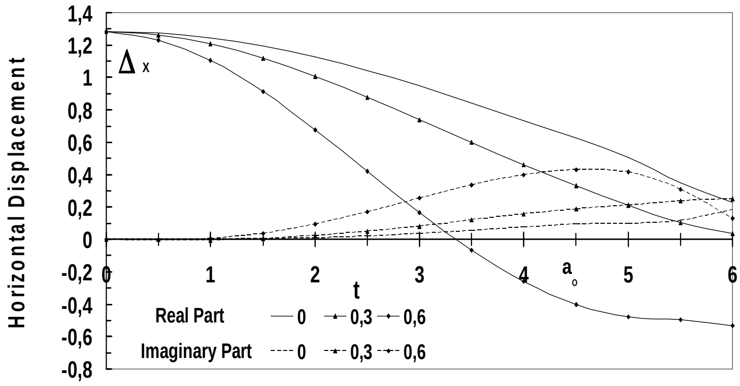

Fig. 12 Horizontal input motion Δ_x due to incident P-waves *(θ_v=45°, θ_H=0°).*

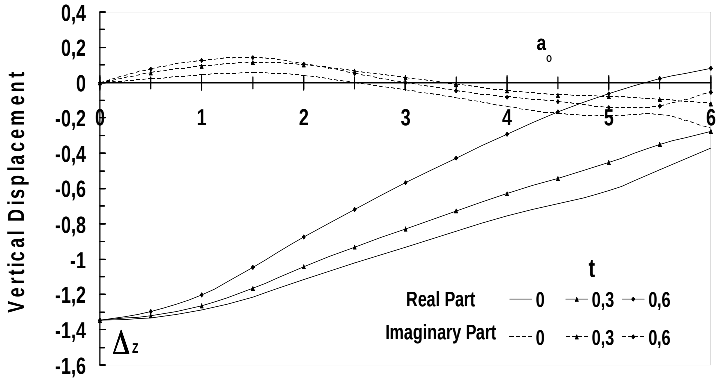

Fig. 13 Horizontal input motion Δ_z due to incident P-waves *(θ_v=45°, θ_H=0°)*

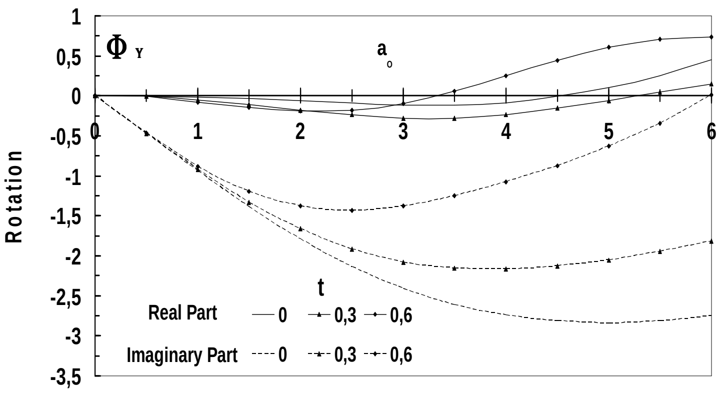

Fig. 14 Rocking input motion ϕ_y due to incident P-waves *(θ_v=45°, θ_H=0°).*

The displacements (Δ_x , Δ_z) and rotation (ϕ_y) are strongly attenuated due to the increase of embedded foundation. Another, the horizontal displacement Δ_x is more affected by the presence of embedding than the vertical displacement Δ_z and rotation ϕ_y. When increasing the embedding, it is noted that displacement and rotation are cancelled frequency decreases then the-signs change. The imaginary part of the two modes of translation (vertical and horizontal) is not affected by the increase of embedding. In contrast, the imaginary part of the rotation is strongly affected by the presence of embedding.

2.2. Shear Wave SV

Figures 15 to 17 show the response of a massless foundation subjected to an incident SV-wave. The wave travels in the *x*-direction with its particle motion in the *z* and x-directions. For a vertical incident angle θv = 45°, the free-field motion for the SV-wave in the direction of propagation is zero. Similar to the P-wave case, only the horizontal, vertical, and rocking components are excited.

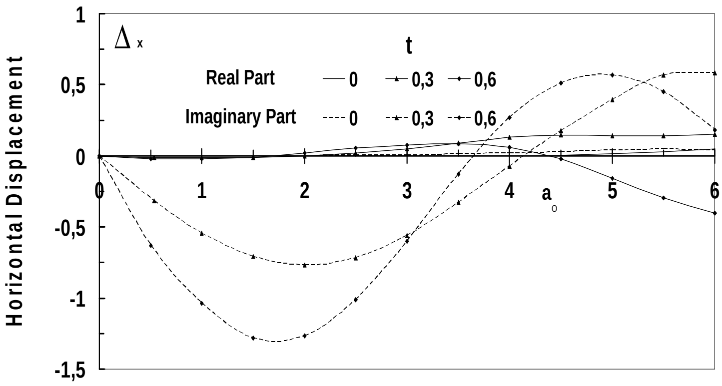

Fig. 15 Horizontal input motion Δ_x due to incident SV-waves *(θ_v=45°, θ_H=0°)*.

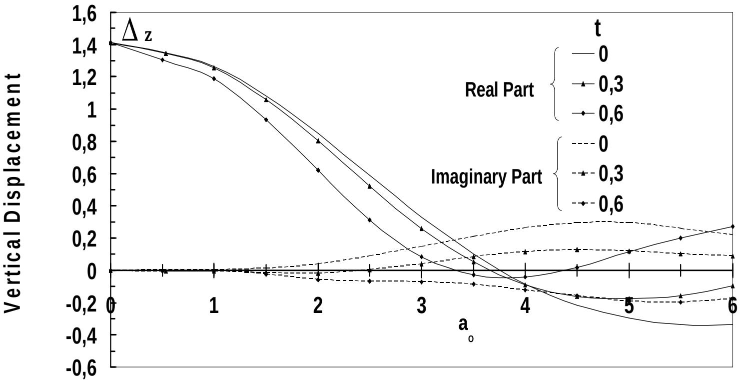

Fig. 16 Vertical input motion Δ_z due to incident SV-waves *(θ_v=45°, θ_H=0°)*.

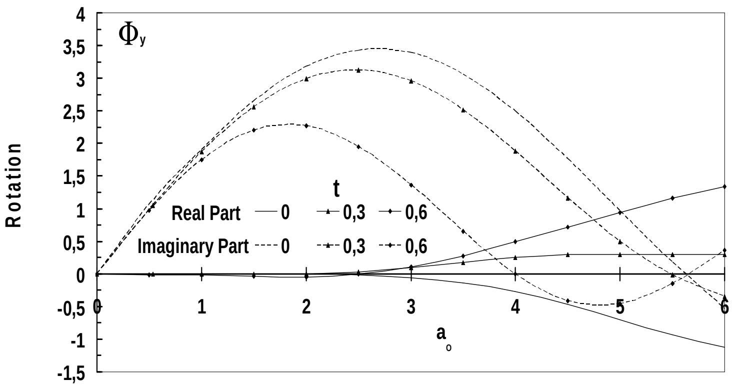

Fig. 17 Rocking input motion ϕ_y due to incident SV-waves *(θ_v=45°, θ_H=0°)*.

Figures 15 to 17 present the variation of displacement and rotation as a function of frequency, and show the influence of embedding on the motion of the foundation. Fig 15 shows that the displacement Δ_x is zero for a foundation on the surface (t = 0) and becomes non-zero for relative embedding (t = 0.3 and 0.6) with an imaginary part that is strongly affected, indicating that the foundation does not follow the free-field motion. Moreover the horizontal displacement Δ_x is more affected by the presence of embedding than the vertical displacement Δ_z and the rotation ϕ_y especially for low frequencies. The presence of the embedded foundation changes the sign of vertical displacement and rotation after frequency $a_o = 4$.

2.3. Shear Wave SH

The response of the square, massless foundation to a SH-wave is presented Figures 18 and 19, with a horizontal angle of incidence θ_H=90°. The incident wave travels in the *y*-direction; therefore, the particle motion of the wave is in the *x*-direction. The shear wave causes displacement and torsion. Figures 18 and 19 show the embedment influence on displacement and torsion.

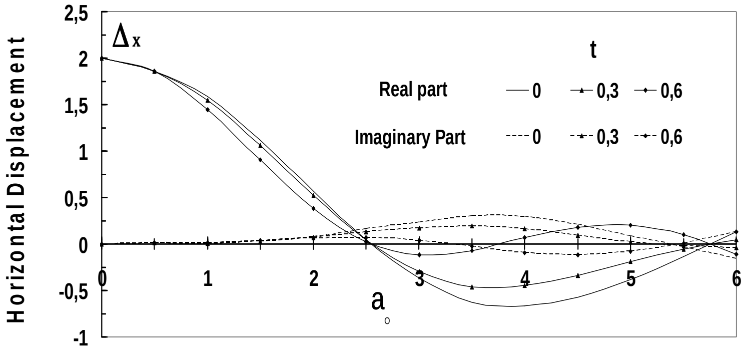

Fig. 18 Horizontal input motion Δ_x due to incident SH-waves *(θ_v=45°, θ_H=90°).*

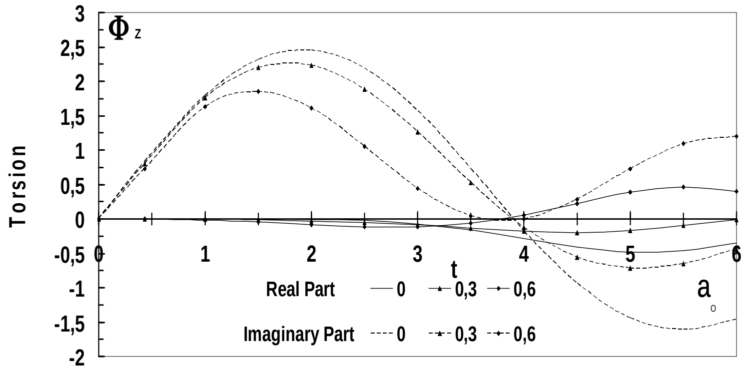

Fig. 19 Torsion-input motion ϕ_z due to incident SH-waves *(θ_v=45°, θ_H=90°).*

Figures 18 to 19 show that the displacement and the torsion are strongly affected by the embedment of the foundation. For dimensionless frequencies lower than 3, the horizontal Δ_x is not affected by increasing the relative embedding. This is not the case for torsion ϕ_z, which is strongly affected by an increase in the embedding. In contrast, for frequencies superior to 3, the presence of the foundation embedding causes a change of signs for Δ_x and ϕ_z.

Figures 12 to 19 show that the horizontal displacement caused by P-wave is more attenuated than the displacement caused by SH and SV-waves; while the rotation caused by SV-wave is more attenuated than the rotation caused by P-wave.

CONCLUSION:

The interaction of a seismic square-rigid foundation placed and embedded in a homogeneous viscoelastic soil and subjected to obliquely incident harmonic P, SV, SH and R-waves was implemented. A simplified BEM-TLM was developed and used to calculate the foundation-input motion under different travelling seismic waves. The solution was formulated by the boundary-element method in the frequency domain using the formalism of Green's functions. Constant quadrilateral elements were used to study the seismic response of a foundation. The efficiency of this technique was confirmed by comparison with previous studies. This remarkably simple technique was concluded to be both highly effective and economical to determine input motions for rigid foundations of arbitrary geometry. The originality of the method lies first in the insignificance of the number of elements used in the discretization of the model, and second, in the ability to simulate a limited height of bedrock.

This study shows the importance of the inclination of incident waves on the behavior of a foundation. The results indicate that:

- The response of a foundation subject to non-vertical incident waves is different from that of a foundation subject to vertical-incident waves.
- Non-vertical incident waves generate the torsion, translation, and rotation. Vertical incident waves cause the translation.
- A vertical angle of incidence equal to θ_V=0° leads to an oversizing of the foundations.
- Coefficients of translational movement filter low frequencies while coefficients of rotation filter at high frequencies.
- Embedment of foundation affects displacement, rotation, and torsion in more specific ways and acts as a favorable factor in the seismic response of foundations. The movement of the foundation is strongly attenuated for relatively deep embedded, especially at low frequencies.

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