Paper No. 110, Bull. ISET, Vol. 8, No. 3, September 1971, pp. 109–123

ASEISMIC DESIGN OF REINFORCED MULTISTORIED STRUCTURE WITH THE HELP OF STORY RELATIVE RIGIDITIES

BY

MIHAIL IFRIM* AND DAN LUNGU**

INTRODUCTION

The elastic behaviour of multistoried structures acted on by lateral forces, including seismic forces, may be characterized by means of the concept of "level relative rigidity". The relative rigidity between the consecutive levels is of fundamental significance for the analysis and design of earthquake resistant sturctures.

The importance of the story rigidity is noticeable in many of its applications such as the aseismic design of structures, the determination of the natural periods of vibration, the distribution of the basic shearing force along the height of the structure, the distribution of the story seismic force between the resistant members of the respective story, the computation of the horizontal displacements of the structrure. The whole theory of free and forced oscillations of structures may be stated in terms of the above quantities. In general, it may be said that with the help of the matrix of relative rigidities, the dynamic response of structures acted on by lateral forces caused by seismic motion, wind or explosions, is fully determined.

In the following we shall refer only to the elastic behaviour of multistoried frames acted on by the lateral forces due to earthquake motion.

The story relative rigidities may be determined by some of the methods of the static analysis of structures known in the literature. Among these and besides the exact methods (the stress method or the displacement method), many widely used methods may be also applied, such as the methods of the transmission of rotations distribution of moments or relaxation. However, these methods are extremely difficult to apply and of small practical value for the design of complex structures, when no digital computer is available.

For this reason, in current design practice, approximate and direct methods are preferably used for the calculation of structures acted on at every story by horizontal forces. Some of these methods, of easy application may be mentioned here : the cantiliver and the portal method, as well as the methods due to Witmer, Spurr, Wilbur, Maney-Goldberg, Morris, Grinter etc. These methods are largely used for the calulation of wind loads on high buildings and have been worked out for that purpose. With the exception of the first two, the above methods take into account the elastic characteristics of the structural members both for finding the distribution of horizontal shearing forces in columns, and for determining the stresses in columns and beams.

By using the concept of "story relative rigidity" an expeditious method can be devised for the treatment of lateral loads (horizontal loads due to earthquakes, wind, explosions); the method is efficient, practical, rapid and advanced and is particularly suited to aseismic design of structures.

*Civ. Eng. Professor at the Institute of Civil Engineering, Bd, Lacul Tei, 124, Bucarest, Sector 2, Romania.

**Structural Eng., Assistant Lecturer at the Institute of Civ. Eng., Bd. Lacul Tei, 124, Bucarest, Sector 2, Romania.

The principle of the method appeared in the Japanese literature as far back as 1921. The method has been steadily improved and perfected and may be considered as a principal tool for the treatment of horizontal loads on hyperstatic structures.

EFFECT OF THE FLEXIBILITY OF COLUMN CONNECTIONS AT THE FRAME JOINTS

The relative rigidity between two consecutive stories $i+1$, i, noted K is the force which applied at the story $i+1$ causes a unit relative displacement with respect to the story i (fig. 1) i. e.

$$K=\frac{V}{\Delta} \tag{1}$$

The story relative rigidity is usually obtained as the sum of the relative rigidities of the columns of the story considered,

$$K=\sum_{\text{STORY}} K_C \tag{2}$$

where K_C is defined, for a column considered independent, as the force required to effect a unit displacement of the upper end of the column with respect to its lower end.

For infinitely rigid connections (beams, slabs, foundations) at both ends of the column, we have

$$K_{C,\infty}=\frac{12EI_C}{H^3} \tag{3}$$

which represents the shearing force corresponding to a unit relative displacement of the column ends. In the above relation E is the modulus of elasticity of the material, I_C is the moment of inertia of the cross section of the bar and H is the height of the column.

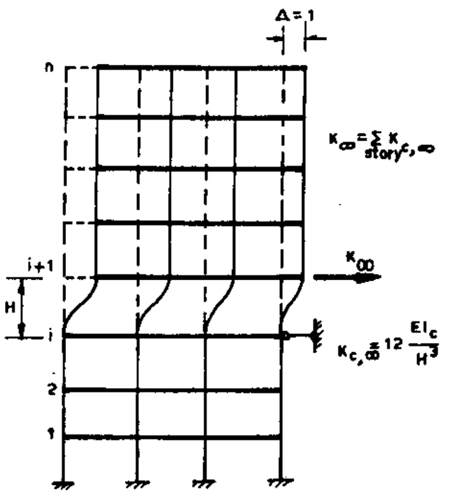


Fig. 1. Relative Rigidity Fixed Joints.

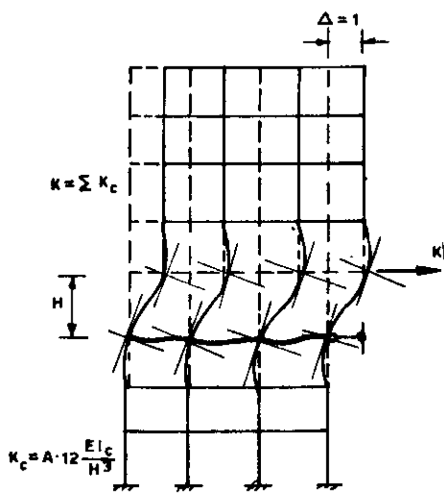


Fig. 2. Relative Rigidity Flexible Joints.

The rigidity $K_{C,\infty}$ is an ideal magnitude, since in fact, owing to the flexibility of the column connections (beams, foundations) at the joint, the latter rotates and the relative rigidity of the column and therefore the story relative rigidity is smaller (fig. 2).

The magnitude of the relative rigidity being affected by the flexibility of the bars at the joints, it will be expressed by a relation of the form :

$$K_C = A\frac{12EI_C}{H^3} \tag{4}$$

where A is a subunitary correction factor which depends on the ratio of the rigidities of the bars.

The method for determination of A is important since formally we may write :

$$V \simeq \sqrt{A} \tag{5}$$

V being the total shearing force at the base of the structure, caused by the earthquake action.

By means of the usual methods and after making certain simplifying assumptions, an approximate evaluation of the correction factor A may be effected; the value thus obtained covers the design requirements of most types of structures. Without insisting on the details and peculiarities pertaining to each method, table 1 gives the formulas for the computation of the factor A as proposed by different authors for the design of structures acted on by lateral forces (Blume[3], Ifrim[5], Muto[6] and Wilbur[7]).

A direct expression for the computation of the correction factor corresponding to a given story can also be obtained. However, the expression is not discussed here since, as owing to its general character, it contains no elements common to the other formulas. Nevertheless it has been included in table 1 because it may be useful in design computation.

The concept of story relative rigidity has been investigated in Rumania nearly 15 years ago by Titaru and Cismigiu where a method for interpreting the linear seismic response by means of the relative rigidities has been presented 8.

In order to compare the formulas given in table 1, the correction factor A has been expressed as a function of the distribution factors of the column at the joint only, according to the procedure proposed by Ifrim. Thus, the following notations have been introduced :

$d_T = \dfrac{K_C}{\sum\limits_{TOP} K}$ —distribution factor of the column at the top joint

$d_B = \dfrac{K_C}{\sum\limits_{BOTTOM} K}$ —distribution factor of the column at the bottom joint

$\sum\limits_{TOP} K$ —sum of the rigidities of all the bars converging in the top point

$\sum\limits_{BOTTOM} K$ —sum of the rigidities of all the bars converging in the bottom point.

In the case of the Wilbur and Muto formulas, an additional assumption is required namely, that the cross sections of the columns belonging to the stories adjacent to the one considered should be constant.

A graphical representation of the analytical expressions of the correlation factor A is suggestive and at the same time constitutes a direct criterion for comparison. Fig. 3 shows the graphs of the functions A of table 1 ($0 \leqslant A \leqslant 1$) for every value of the independent parameters d_T and d_B ($0 \leqslant d \leqslant 1$). To every pair of values d_T and d_B there corresponds a point on the surface A.

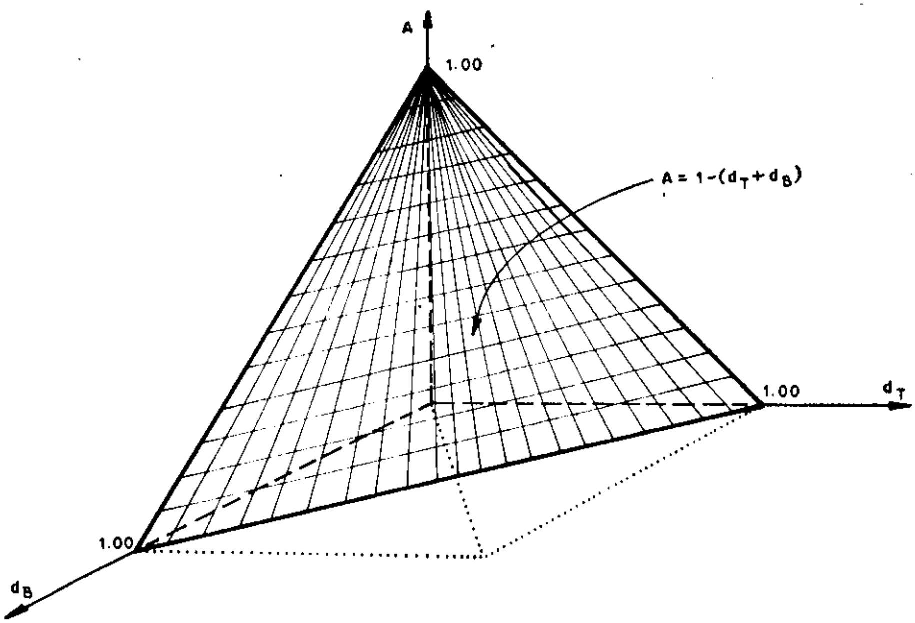


Fig. 3a. Surface A (d_T ; d_B) After Blume, Newmark and Corning's Equation.

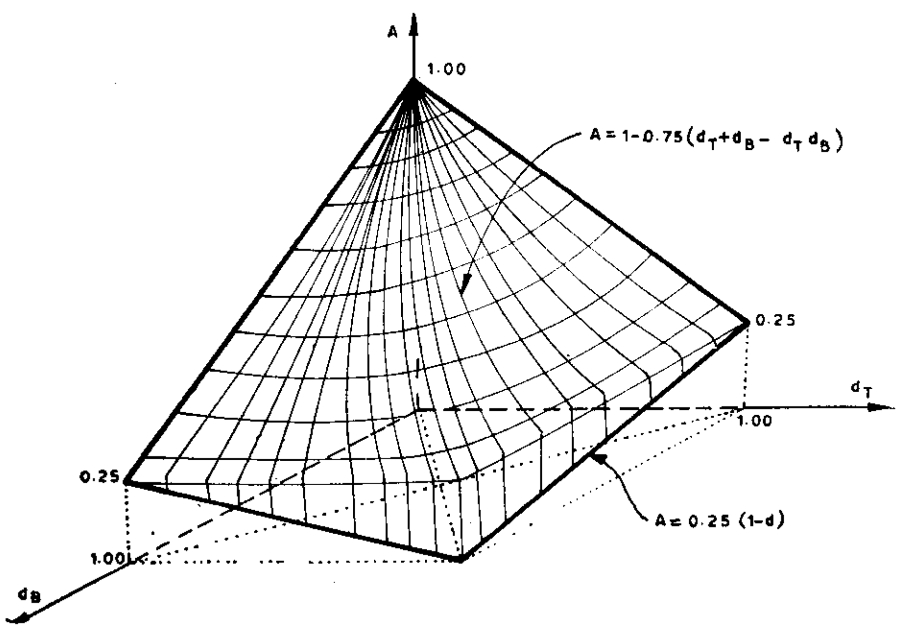


Fig. 3b. Surface A (d_T ; d_B) After Ifrim's Equation.

The geometric aspect of the surface A (fig. 3), the compliance to the boundary conditions and the exact numerical computations effected lead to the conclusion that the surface A, continuous and uniform, is of the hyperbolic paraboloid type. The authors consider that this surface characterizes the real variation of the correction factor for the story relative rigidity.

Referring to fig. 4 the following important remarks should be noted :

—the value $A=1$ for $d_T=d_B=1$ corresponds to a column with built in ends;

—the value $A=0, 25$ for $d_T=1$ and $d_B=0$, or for $d_B=1$ and $d_T=0$, corresponds to a column with one built-in end and one hinged end;

—the value $A=0$ for $d_T=d_B=1$, corresponds to a bar with both ends hinged, i.o. of the "mechanism" type (virtual case);

—the surface A rests in four curves on the planes $d=0$ and $d=1$;

—the curves on the planes $d=0$ express the variation of A in the case, where the column has one end built-in and the other is a frame joint;

—the curves of planes $d=1$ express the variation of A in the case where one end of the column is hinged and the other end is a frame joint;

—the current zone of the surface expresses the variation of the correction factor A of a current column of a multistoried structure both ends of which are frame joints.

The built-in joints or the hinges are assimilated to sliding joints, i.e. they allow the relative lateral displacement of the column ends.

The analytical expression of the surface A, fig. 4, is fairly complicated and difficult to establish for practical purposes.

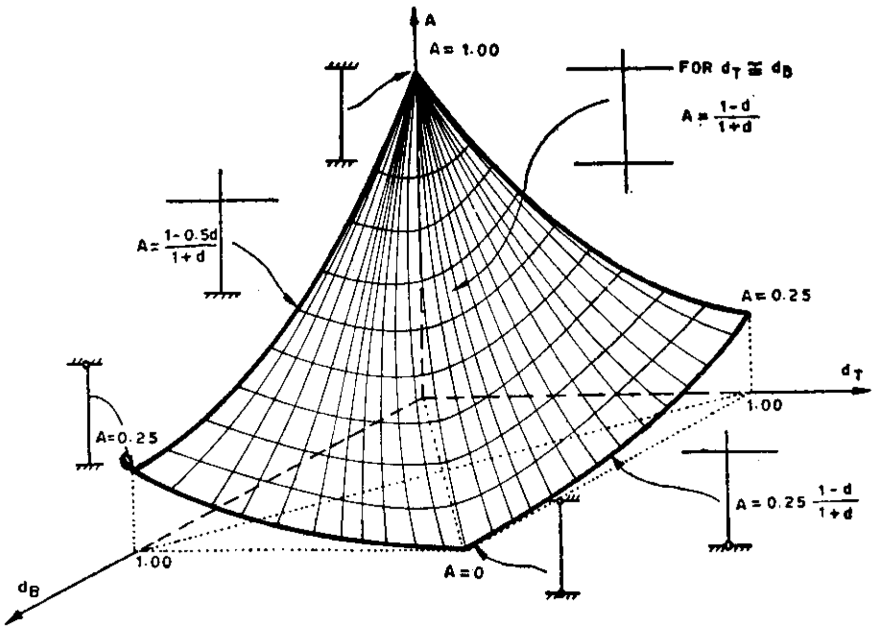
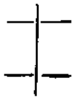


Fig. 4. Surface A (d_T, d_B) proposed by Lungu.

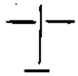
However, for current design calculations the analytical expressions given in fig. 4 may be applied; they describe with sufficient accuracy the form suggested for the surface A (d_T d_B), namely :



for $d_T \simeq d_B = d$

or $d = \frac{d_T + d_B}{2}$

$$A = \frac{1-d}{1+d} \qquad (6)$$



$$A = \frac{1-0{\cdot}5d}{1+d} \qquad (7)$$

$$A = 0{\cdot}25\frac{1-d}{1+d} \qquad (8)$$

Finally it should be pointed out that in the case of frame structures with closely approaching rigidities of the bars (columns, beams), the results obtained by applying the expressions given in table 1, are convergent.

MODULUS OF ELASTICITY OF CONCRETE

The correct evaluation of the rigidity of a structure and therefore of the basic shearing force, is governed by the way in which the modulus of elasticity has been determined.

In reinforced concrete design widely different expressions are used in which the modulus of elasticity and the compressive strength are related in different ways. Some of the more important are given in table 2 and fig. 5.

The following remarks should be taken into account when applying the formulas :

—the initial slope of the stress-strain curve defines the tangent modulus (initial or instantaneous). The formulas of Ros, Graf, the values given by the german standards DIN 1045, the french REGLES B. A. 68 and C. E. B., refer to that modulus, denoted by E_O (fig. 6);

—the slope of the chord drawn through the origin and the point corresponding to a compressive stress of about 0, ϕ, 4 f_C' defines the secant or elasto-plastic modulus. The american formulas refer usually to that modulus, which is denoted by E (Fig. 6);

—the modulus of elasticity decreases with increasing compressive stresses applied according to an approximately linear relation represented in Fig. 7; it should be mentioned that the modulus of elasticity varies nearly linearly with the specific gravity of the concrete;

—the effects of creep such as the two-or threefold amplification of the elastic strains, or the redistribution of stresses between concrete and reinforcements, are sometimes interpreted and elucidated by the concept of reduced elastic modulus (E_r). In the linear calculation of the story relative rigidities the concept of reduced elastic modulus is physically not justified;

—through ageing the compressive strength of concrete increases by about 10–20% with respect to the standard strength after 28 days; this results in a higher (by about 10%) value of the elastic modulus in the case where the linear relation between the modulus and the square root of the compressive strength is complied with.

—Under dynamic loads the static stress-strain relation becomes modified. Fig. 8 shows the variation of the compressive strength of concrete as a function of the loading rate. The increase of the compressive strength varies between 10 and 30% (the higher values corresponding to rigid elements and the lower values to flexible ones). Owing to the dynamic action of earthquake motion we may consider for the elastic modulus of concrete an increase of about 10–20%.

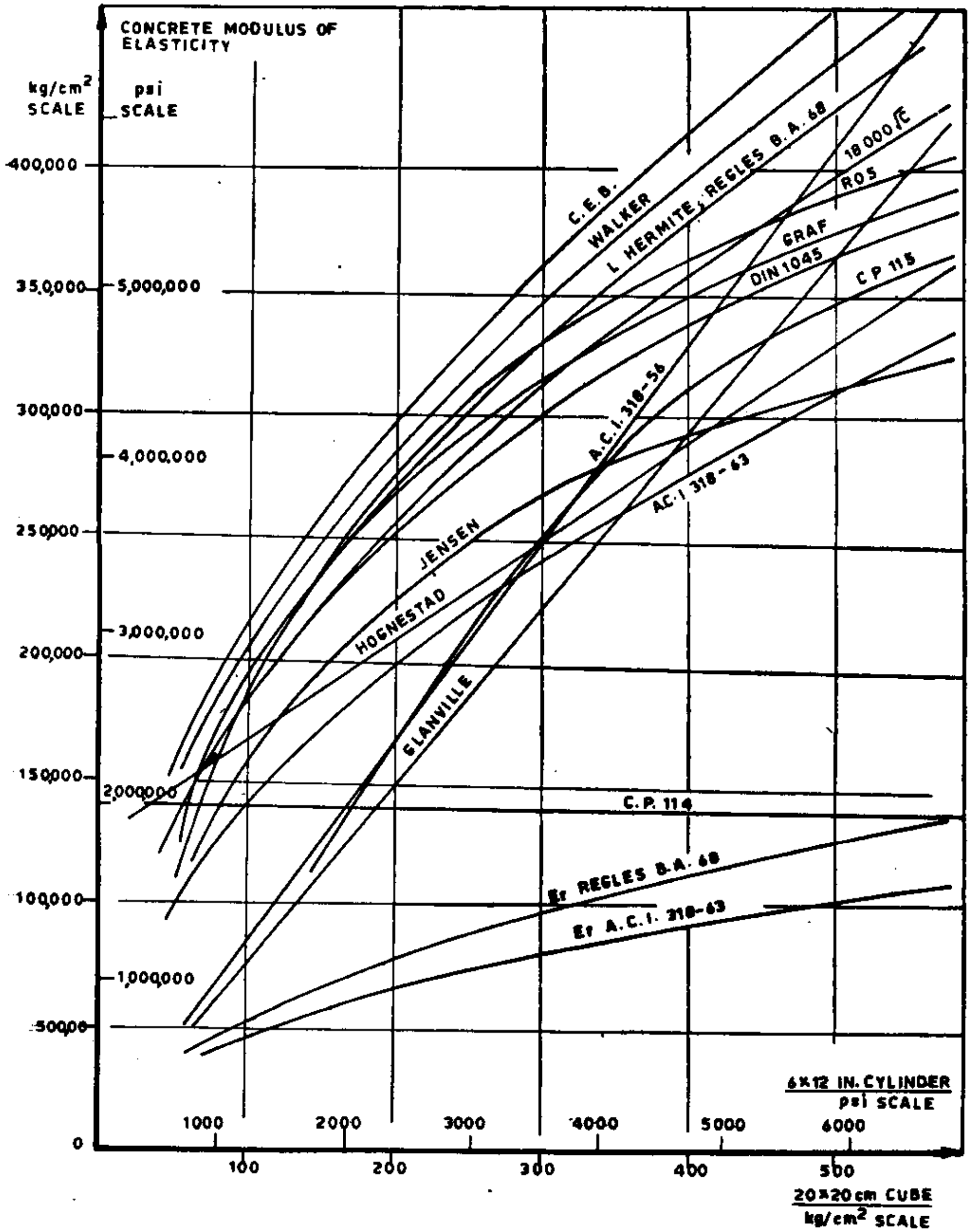


Fig. 5. Modulus of Elasticity for Various Concrete Strength.

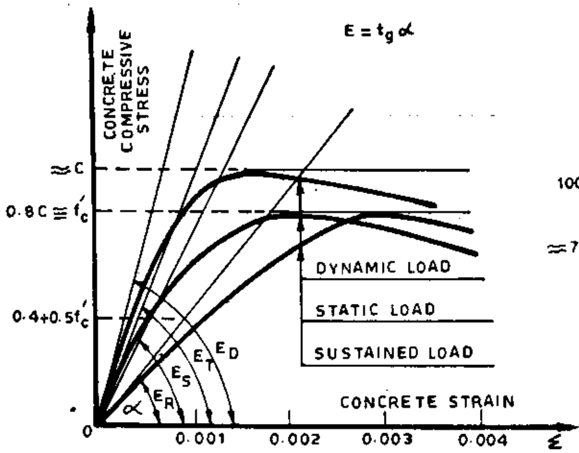


Fig. 6. Tangent, Secant, Dynamic and Reduced Modulus of Elasticity.

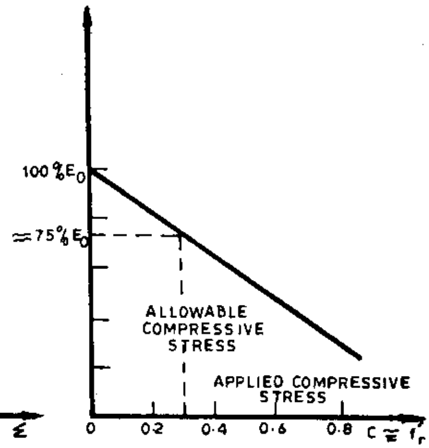


Fig. 7. Variation of Modulus with Applied Stress.

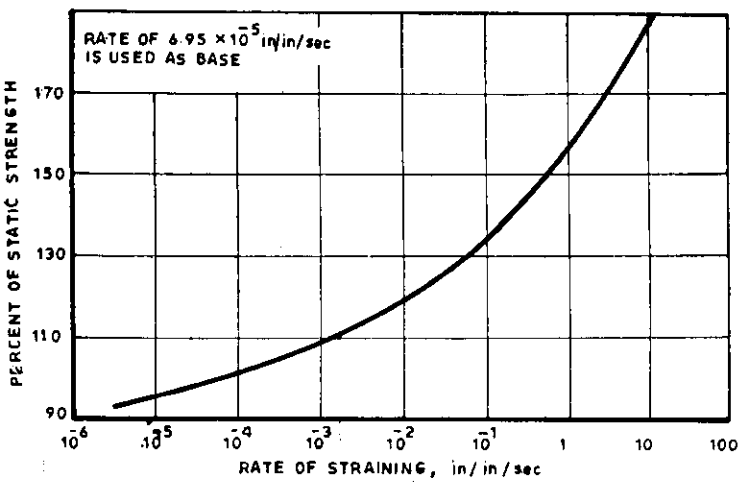


Fig. 8. Effect of Rate of Straining on the Compressive Strength of Concrete.

Taking into account the foregoing it is advisable that in the calculation of story relative rigidities one should use for the elastic modulus of concrete the value corresponding to the existing working stresses (i.e. $E_{secant} = 14{,}000\sqrt{C}$ or $57{,}000\sqrt{f_c'}$ and not $E_{tangent}$). The dynamic character of the stresses applied and the strength increase of the concrete through ageing, lead to a higher value (by about 30%) of the elastic modulus, so that one may take

$$E \simeq 1{\cdot}3 \times 14{,}000\sqrt{C} \simeq 18000\sqrt{C} = 75000\sqrt[5]{f'_c} \qquad (9)$$

C in Kg/cm²	E in Kg/cm²
f'_c in P.S.I.	E in P.S.I.

The value $18{,}000\sqrt{C}$ for the elastic modulus is frequently used in european literature but it refers always to the tangent or initial compressive modulus.

EFFECTIVE LENGTH OF BARS

Usually, for calculation purposes, the length of the bar of a structure is considered to be that of the interaxis. The japanese assumption which considers a rigid zone at the intersection of the axes of the members of reinforced concrete structures, is justified by the analysis of damaged structures and is a much closer approach to reality (Fig. 9).

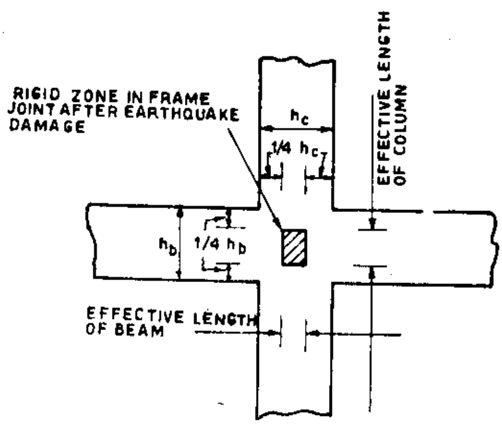


Fig. 9. Effective Length of Members.

Thus, the effective length of the columns is equal to the span length plus 1/4 of the height of the beam cross section along the direction of computation, at each end of the beam. The effective lengths of the beams which appear in the evaluation of the coefficient A, are computed in a similar way.

EFFECTIVE MOMENT OF INERTIA OF THE BAR SECTION

All the analysis methods of statically undetermined structures are based on the assumption that the bending rigidity of the bars is known beforehand. Under these circumstances, and considering that the flexural rigidity and implicitly the moment of inertia are approximately estimated, the exactness of the competitions effected has only a mathematical significance. The state of stress or strain of the structures resulting from the computation will be qualitatively and quantitatively affected.

When high strength concrete and reinforcement are used the resulting members are slender and the occurrence of cracks may affect appreciably the moments of inertia of the cross section.

In general, the compression stress in the columns prevents the formation of cracks so that the "I column" may be computed by considering a section free of cracks.

In case of strongly reinforced columns one may reckon with a 10–20% increase of the moment of inertia contributed by the reinforcement.

Fig. 10 shows the variation of the flexural rigidity as a function of the external bending moment applied, for different reinforcement ratios of the section. It will be noted that at the stress of 1/4 of the ultimate moment there occurs a marked drop of the rigidity EI. As the external moment increases the rigidity continues to decrease; for a given moment the rigidity diminishes as the reinforcement ratio becomes smaller.

When acted on by a seismic shock stresses approaching the ultimate bending moment may appear in the structure bars which may explain why the rigidity of a construction is reduced after an earthquake. It is obvious that on a subsequent earthquake the seismic forces acting on the structure will be substantially different.

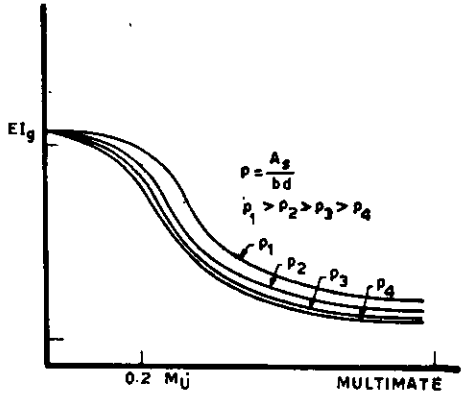


Fig. 10. Typical Variation of Flexural Rigidity with Applied Bending Moment.

The effect of cracks in the computation of the effective moment of inertia of the beam may be expressed by the relations given in reference 1 and 2. The relations refer to the ratio of the moment of inertia of the transformed cracked section I_{cr} and the section free of cracks I_g, at working stresses of the sections in the range of the elastic behaviour of the section.

For a singly reinforced rectangular section we have :[1]

$$\frac{I_{cr}}{I_g} = 8{\cdot}75\left[\frac{\bar{k}^3}{3} + np(1-\bar{k})^2\right] \tag{10}$$

$$\frac{I_{cr}}{I_g} = 12\,np\left(1-\frac{\bar{k}}{1}\right)\left(1-\bar{k}\right) \tag{11}$$

where
$$\bar{k} = \sqrt{(pn)^2 + 2pn} - pn \tag{12}$$

represents the ratio of the distance between the compressed outermost fibre and the neutral axis (in the elastic stage) and the effective height of the section;

—p is the reinforcement ratio;

—n is the ratio of the elastic moduli of steel and concrete.

In Fig. 11 only eq (10) is represented.

For "T" sections the moment of inertia depends on the active width of the slab. Table 3 gives for single—and double flanged "T" sections the effective flange widths of the slabs in accordance with different prescriptions for reinforced concrete.

Fig. 12 shows the approximate relation for a "T" beam between $\frac{I_{cr}}{I_g}$ and the reinforcement ratio at different values of ratio n. The curves refer to a continuous T beam and have been obtained from the mean of the span and support values. The similitude of the curves of Fig. 12 and of Fig. 11, which refer to rectangular sections, should be noticed.

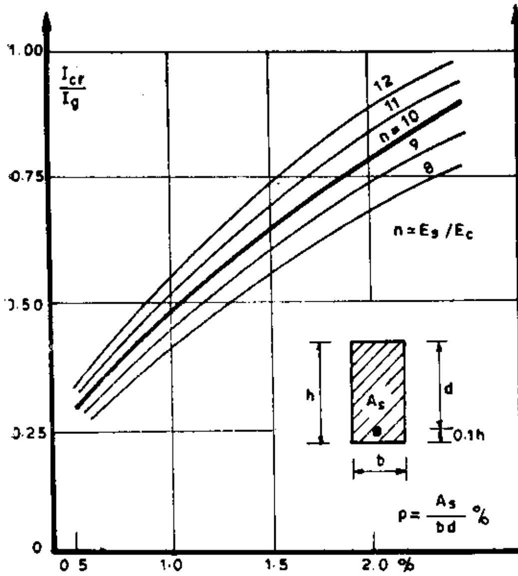


Fig. 11. Moments of Inertia of Transformed Cracked Section (Singly Reinforced Rectangular Beams)

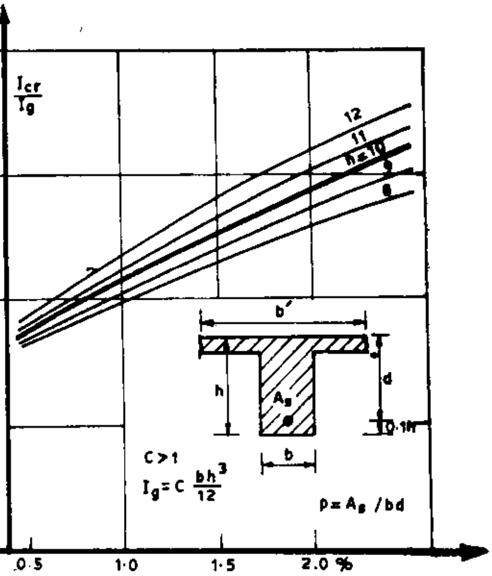


Fig. 12. Moment of Inertia of Transformed Cracked Section (Singly Reinforced T Beams)

In computing the correction factor A we may use in the case of beams the following relation :

$$I_{ef}=0{\cdot}75\ I_g \qquad (13)$$

this will simplify design calculations.

An upper limit has been preferred since, in general, the loads used in the calculation and on which the evaluation of the degree of cracking is based, do not occur in fact integrally. This applies also to the zone of the inflexion points of the beams, where $I_{ef}=I$. A more accurate expression of the effective moment of inertia of a cracked section is

$$I_{ef}=I_{cr}\left[\,1+\frac{1-\dfrac{I_{cr}}{I_g}}{\dfrac{I_{cr}}{I_g}}\left(\frac{M_{cr}}{M}\right)^3\right] \qquad (14)$$

where

$$M_{cr}\simeq\frac{1{\cdot}8\sqrt{C}\times I}{\dfrac{h_0}{2}}$$ is the cracking moment and

M is the bending moment of the section.

when $M\leqslant M_{cr}$ we have obviously $I_{ef}=I$.

The complex expression of the moment of inertia of the cracked section of beams complies to the recommendations of the ACI 435 Committee and is Intended for the computation of deflections.

It should be also added that in the case of high buildings, where usually a higher strength concrete is prescribed for columns than it is for beams and slabs, the flexural rigidity of beams at the frame joint must be corrected as follows :

$$K_{BEAM}=K_{BEAM}\frac{E_{BEAM}}{E_{COLUMN}}=K_{BEAM}\sqrt{\frac{C_{BEAM}}{C_{COLUMN}}}=K_{BEAM}\sqrt{\frac{f'_{c\ BEAM}}{f'_{c\ COLUMN}}} \qquad (15)$$

where

$$K_{BEAM}=\frac{I_{ef}}{l_{ef}} \quad \text{or} \quad 0{\cdot}75\,\frac{I_{ef}}{l_{ef}}$$

CONCLUSIONS

The elastic analysis of multistoried structures of reinforced concrete can be made using the concept of story relative rigidity.

The magnitude of the factor A which is used for evaluating the modified rigidity of the column owing to the flexibility of the bars at the joints, may be ascertained by the relations shown in Fig. 4.

The elastic modulus of the concrete may be estimated with the help of relation 4. The effective lengths of the bars are indicated in Fig. 9 and their moments of inertia shall be computed as explained in section 5.

It is considered that within the range of elastic behaviour of structures, the foregoing may constitute a sufficiently correct design criterion for horizontal and particularly for seismic loads.

REFERENCES

1. ACI Committee 317, "Reinforced Concrete Design Handbook-Working Stress Method" SP—3, 3rd Edition American Concrete Institute, Detroit, 1965.
2. ACI Committee 435, "Deflections of Reinforced Concrete Flexural Members", Journal of the American Concrete Institute, June 1966, No. 6, Proceedings, V. 63.
3. Blume, J. A., Newmark N. M,, Corning J. A,, Design of Multistory Reinforced Concrete Buildings for Earthquake Motions, published by Portland Cement Association, 1961.
4. Ferguson, P. M., Reinforced Concrete Fundamentals, 2-nd Edition, John Wiley & Sons, New York, 1965.
5. Ifrim, Mihail, Contributions to a unitary theory of the behaviour and design of multistoried structures for earthquake motions. Thesis for the doctor degree, Bucuresti, 1969.
6. Muto K., Seismic Analysis of Reinforced Concrete Buildings, Proceedings of the World Conference on Earthquake Engineering, Barkeley, California, 1956.
7. Norris, C. H. and Wilbur J. B., Elementary Structural Analysis, McGraw-Hill Co., Inc., New York, 1960.
8. Titaru, E. and Cismigiu Al., "The dynamic calculation of structures for earthquake and other lateral actions". Revista Construct iilor si materialelor de constructii nr. 12, 1959.

TABLE I
THE A FACTOR FOR THE CALCULATION OF RELATIVE RIGIDITIES

$$K = A \cdot 12\frac{EI_c}{H^3}$$

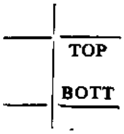
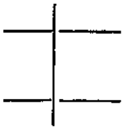
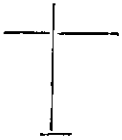
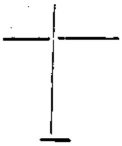
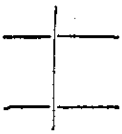
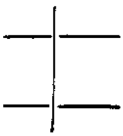
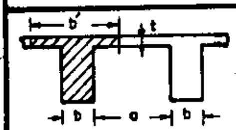
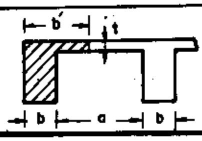
AUTHOR	CASE	A=f (Rotational Stiffnesses of Beams and Columns)	A=f (Distribution Factors)
WILBUR		$1-0.5\left[\frac{\sum_{\text{TOP}} K_C}{\sum_{\text{TOP}} K}+\frac{\sum_{\text{BOTTOM}} K_C}{\sum_{\text{BOTTOM}} K}\right]$	$4-(d_T+d_B)$
BLUME NEWMARK CORNING		$1-\left[\frac{K_C}{\sum_{\text{TOP}} K}+\frac{K_C}{\sum_{\text{BOTTOM}} K}\right]$	$1-(d_T+d_B)$
MUTO		$\dfrac{0.5+\frac{\Sigma K_b}{K_C}}{2+\frac{\Sigma K_b}{K_C}}$	$1-1.5d_T$ $d_B=0$
MUTO		$\dfrac{\frac{\sum_{\text{TOP \& BOTTOM}} K_b}{2K_C}}{2+\frac{\sum_{\text{TOP \& BOTTOM}} K_b}{2K_C}}$	$1-\frac{4d_T\,d_B}{d_T+d_B}$
MUTO		$\dfrac{0.5\,\frac{\Sigma K_b}{K_C}}{1+2\,\frac{\Sigma K_b}{K_C}}$	$\frac{1-2d}{4-6d}$ $d_B=1$
IFRIM		$1-0.75\left[\frac{K_C}{\Sigma K_{\text{TOP}}}+\frac{K_C}{\Sigma K_{\text{BOTT.}}}-\frac{K_C^2}{\Sigma K_{\text{TOP}}\,\Sigma K_{\text{BOTT.}}}\right]$	$1-0.75(d_T+d_B-d_Td_B)$
*GRINTER		$\dfrac{1}{1+0.5\left[\frac{\sum_{\text{STORY}} K_C}{\sum_{\text{STORY, BOTT.}} K_b}+\frac{\sum_{\text{STORY}} K_C}{\sum_{\text{STORY, TOP}} K_b}\right]}$	*THIS FACTOR IS THE SAME FOR ALL COLUMNS OF THE STORY,

TABLE 2
MODULUS OF ELASTICITY FOR CONCRETE

AUTHORS	EQUATION
A. C. I. 318-63	$33w^{\frac{3}{2}}\sqrt{f'_c}=57000\sqrt{f'_c}$
HOGNESTAD	$1{,}800{,}000+500\,f'_c$
A. C. I. 318-56	$1000\,f'_c$
GRAF	$\dfrac{1{,}000{,}000}{1{\cdot}7+\dfrac{360}{C}}$
ROŠ	$\dfrac{550{,}000\ C}{C+200}$
JENSEN	$\dfrac{6{,}000{,}000}{1+\dfrac{2000}{f'_c}}$
C. E. B.	$23{,}500\sqrt{f'_c}$; f'_c in Kg/cm²
REGLES B. A. 68	$21{,}000\sqrt{f'_c}$; f'_c in Kg/cm²
DIN 1045	Only numerical values
C. P. 114	$\frac{1}{15}E_{STEEL}$
GLANVILLE	$\dfrac{F_S}{E_c}=\dfrac{40000}{C}$ C. in P. S. I.
L'HERMITE	$19{,}000\sqrt{C}$
WALKER	$20{,}000\sqrt{C}$
DUTRON	$60{,}000\ C^{0{\cdot}28}$
C. P. 115	$2{\cdot}8\log_e\left(\dfrac{C}{1000}\right)\cdot 10^6$ C. IN P. S. I.

$C=20\times20$ cm. cube compressive strength; in Kg/cm²
$f'_c=6\times12$ in. cylinder compressive strength; in P. S. I.

TABLE 3
EFFECTIVE FLANGE WIDTH OF T BEAMS

	A.C.I. 318-63	DIN 1045	REGLES B A 68	C.P 114	P.C.A S.T. 70
	$\leq L/4$ $\leq b+2.8t$ $\leq a/2$	$\leq b'+2.6t$ $\leq a/2$	$\leq b+L/5$ $\leq a/2$	$\leq L/3$ $\leq b+2.6t$ $\leq a/2$	$\leq 6b$
	$\leq L/12$ $\leq b+6t$ $\leq a/2$	$\leq b+4{,}5t$	—	—	—

L = SPAN LENGTH OF THE BEAM