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MODELING CONSIDERATIONS OF FRICTION DAMPER ASSEMBLY FOR SEISMIC ANALYSIS

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ABSTRACT

Earlier literature used elasto-plastic model for numerical modeling of friction dampers. However, the friction phenomenon is velocity dependent and elasto-plastic model is displacement dependent. Hence, their equivalence is not obvious and not established based on a systematic study. The present study evaluates three modeling approaches for friction dampers - (a) rigidly connected damper (b) friction component and spring in series and (c) elasto-plastic model. The response is calculated and compared for harmonic and earthquake excitations. Frequency response and response spectra are used to see effect of friction damper on response of SDOF. It is observed that the response of the systems have a higher sensitivity to the stiffness of the friction damper only upto a specific value. It is also shown that the optimum value of the slip load of the friction damper cannot be dependent on only system properties or only ground motion characteristics as considered in earlier literature.

KEYWORDS: Coulomb Friction; Energy Dissipation; Friction Damper; Vibration Control; Friction Modeling

INTRODUCTION

Structural response during an earthquake depends upon amount of energy it receives from the event and rate at which the energy is dissipated. A moment resisting frame (MRF) usually dissipates energy during a strong earthquake by the development of plastic hinges at select locations. However, the rate of dissipation with this hysteretic mechanism is low. Also the member once yielded cannot dissipate any more energy subsequently. As a result, over the past few years, supplemental damping devices have become increasingly popular for controlling seismic response of frames. Friction dampers as supplemental devices are known to reduce the seismic response of structures. Such dampers are often composed of two components

- friction interface: where energy dissipation takes place
- staging: which integrates friction interface with the structure. Bracings are often used as staging.

For numerical modeling of the friction damper in a structural system, modeling of both components is necessary.

Various attempts have been made to model the energy dissipation through friction and its effect on the response of single-degree-of-freedom (SDOF) system. Hartog (1930) was among the first to provide analytical solution of a SDOF system resting on a sliding support subjected to forced vibration. Crandall et al. (1974) and Westermo and Udwadia (1983) studied the response of sliding rigid block subjected to harmonic excitation. Westermo and Udwadia (1983) also considered the response of an oscillator on a sliding support. For the rigid block case, the initiation of slippage was found to depend on forcing frequency of periodic excitation. Moreover, in the case of oscillator, the harmonic response was characterized by several subharmonic resonant frequencies. Mostaghel et al. (1983) studied the response of SDOF systems to harmonic and earthquake excitation. The system in these cases was considered to be sliding on the support. Vafai et al. (2001) and Yang et al. (1990) studied the response of multi-degree-of-freedom (MDOF) system on a sliding floor subjected to harmonic and earthquake excitation, respectively. They considered a spring connected between base of the structure and support and the spring was modeled to have infinite and zero stiffness in the stick phase and the sliding phase, respectively. Numerically modeling involved using a rigid-plastic link between the sliding surfaces. Although the studies provide an insight into the behavior of friction component, the rigid-plastic link modeling approach is inappropriate for the modeling of friction damper assembly due to finite stiffness of the staging.

The studies by Pall and Marsh (1982) and Filiatrault and Cherry (1987), are among the first attempts to consider the staging stiffness for modeling friction damper. Both studies report numerical and experimental investigations into the performance of friction dampers under seismic excitation. Experimental results suggest significant amount of energy dissipation due to friction dampers. Pall and Marsh (1982) assumed the hysteretic behavior of the damper which is accurate only if slip occurs at every loading cycle. For the case of earthquakes this is rarely true. The model also overestimated the energy dissipation due to assumption of straightening of buckled brace instantaneously upon load reversal. In an effort to eliminate these inaccuracies, Filiatrault and Cherry (1987) proposed an alternate way to model the friction damper using additional link and pad elements at the cost of additional degrees of freedom. Both the studies in principle have considered the modeling of friction damper assembly using elasto-plastic behavior where the initial stiffness corresponds to the stiffness of staging.

Study by Lu et al. (2006) considered the dynamic analysis of MDOF structure fitted with friction dampers. The friction dampers are modelled using elastoplastic model. The authors proposed a new method for estimation of damper force and compared it with that of obtained from elastoplastic model. Borhan et al. (2021) studied the rotational friction dampers in building frames using numerical and experimental investigations. The numerical investigation considered the bilinear material model for modelling the friction damper. Yadav and Vyas (2021) studied SDOF systems with friction component, where the friction component is modelled using Bengisu and Akay (1994) friction model. Qiu et at. (2022) studied the effect of shape memory alloy friction damper on a multi storey concentrically braced frame. The authors modelled the friction damper as an elastoplastic material. Nabid et at. (2019) provided a simplified method for the design of friction damper slip load. The authors provided the methodology based on the energy dissipation parameter defined by Nabid et al. (2017). A detailed state-of-the-art review of the friction damper and its applications is provided by Jaisee et al. (2021).

All the above-mentioned studies (excluding Yadav and Vyas (2021)) consider behavior of friction component (not the damper) as rigid-plastic. Whereas, the study by Yadav and Vyas (2021) refers to only the friction component and not friction damper. As per the Coulomb friction model (see Figure1), the friction force is given by $f_s \mathrm{sgn}(\dot{u})$, where f_s is sliding force of the friction component, $\dot{u}$ is relative velocity between the sliding surfaces and sgn() represents signum function. It is not obvious to consider this behavior as rigid-plastic as the rigid-plastic behavior defines relation and the phase difference between force, deformation and velocity. Whereas, elastoplastic model defines the relation between force and deformation alone.

Moreover, when such a friction component is connected in series with the staging, the overall behavior need to be investigated for validating modeling assumptions. Since friction component is an integral part of the friction damper assembly, the phenomenological modeling of friction component along with staging is considered in this paper. The phenomenological modeling is limited to Coulomb friction model although several models have been proposed in the literature, such as, Dahl model, LuGre model, Bengisu-Akay model etc (Bengisu and Akay 1994; Dahl 1968; Johanastrom and Canudas-de-Wit 2008; Leine et al. 1998). Modeling of Coulomb friction phenomenon poses numerical difficulties due to discontinuous nature of signum function. To alleviate this difficulty, Mostaghel and Davis (1997) proposed four continuous functions as alternatives to signum function. We choose $(2/\pi) \cdot \arctan(\alpha\dot{u})$ as an approximation to the signum function in this study, where α is arbitrarily high real number (with dimensions [L-1 T]). Higher the value of α, better will be the approximation to signum function.

To address these conceptual issues in the mechanics of functioning of a friction damper, we investigate modelling of friction damper assembly using three different modelling approaches - a) damper with infinite staging stiffness b) damper as a Maxwell model with finite staging stiffness and c) damper as an elastoplastic model. An ensemble of SDOF systems with damper is subjected to harmonic excitations and ground motions to understand the behaviour of each damper model to realistic inputs. An attempt is also made to identify the optimal values of damper parameters for seismic response reduction. The study does not consider the effect of environmental factors such as ambient moisture content, temperature variation and ageing while modeling the friction phenomenon. The study does not consider the effect of environmental factors such as ambient moisture content, temperature variation and ageing while modeling the friction phenomenon.

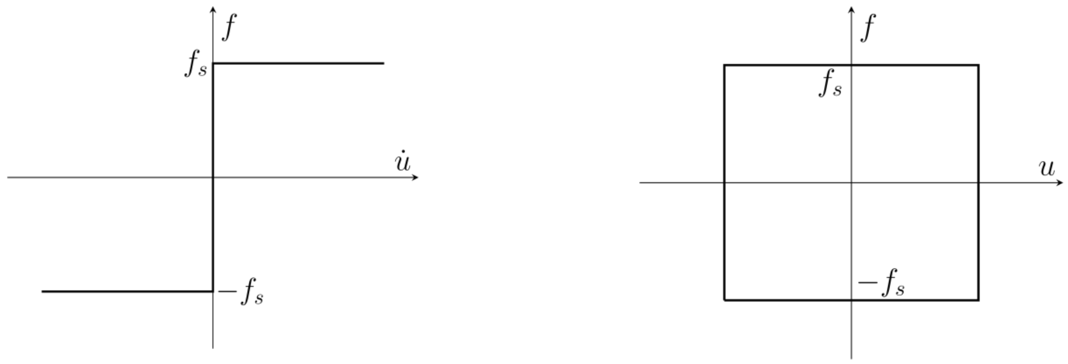
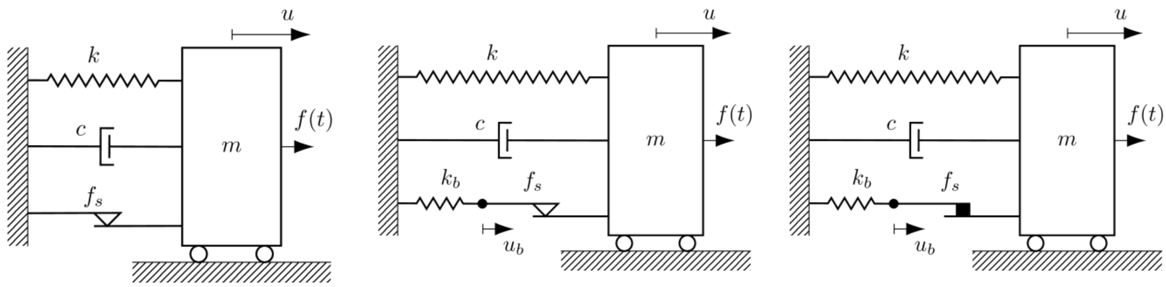


Fig. 1 Modeling of friction behavior



(a) (b) (c)

Fig. 2 A SDOF system with (a) friction component only (b) friction damper assembly and (c) friction damper assembly as elasto-plastic model

MODELING OF FRICTION DAMPER ASSEMBLY

A typical single-degree-of-freedom (SDOF) system with an additional friction damper assembly arranged in different configurations is shown in Figure *2*. In the case (a) (hereafter referred to as "friction only" modeling), the friction damper assembly is considered as composed of only friction component. This signifies that the stiffness of the damper assembly is very large (theoretically infinite). The governing differential equation in this case is given by Equation (1) where, f_f is force due to friction component and other symbols carry their usual meaning.

$$m\ddot{u} + c\dot{u} + ku + f_f = f(t) \tag{1}$$

The friction force f_f can be modeled by using Mostaghel and Davis (1997) model as:

$$f_f = f_s \frac{2}{\pi} \arctan(\alpha \dot{u}) \tag{2}$$

where, f_s denotes the slip force at the sliding interfaces. Dividing Equation (1) by m and using Equation (2), the governing differential equation can be rewritten as

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u + \tau g \frac{2}{\pi} \arctan(\alpha \dot{u}) = \frac{1}{m} f(t) \tag{3}$$

where, $\tau = \frac{f_s}{mg}$ is the friction sliding force per unit weight of the system and is the acceleration due to gravity.

However, in case of friction damper assembly, there will always be a spring with finite stiffness connected in series with friction component as shown in case (b) (hereafter referred to as "damper assembly" modeling). In this case, the governing differential equation of motion for the system is given by Equation (4).

$$m\ddot{u} + c\dot{u} + ku + k_b(u - u_p) = f(t)$$
$$f_f = k_b(u - u_p) = f_s\left(\frac{2}{\pi}\right)\arctan(\alpha\dot{u}_p) \tag{4}$$

where k_b is stiffness of damper assembly representing the staging stiffness, $u_p(u - u_b)$ is relative displacement of sliding surfaces of friction component, u_b is elastic component of the total deformation in damper assembly and f_f is the force in friction component. Dividing Equation (4) by m and introducing new parameters $\frac{k_b}{m} = {\omega_b}^2$ and $\frac{\omega_b}{\omega} = \gamma$; $\left(\text{or}, \frac{k_b}{k} = \gamma^2\right)$:

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u + \gamma^2\omega^2(u - u_p) = \frac{1}{m}f(t)$$
$$\frac{f_f}{mg} = \tau\frac{2}{\pi}\arctan(\alpha\dot{u}_p) \tag{5}$$

Hence, in the state space, the governing differential equation can be written as Equation (6) where $u_1 = u$, $u_2 = \dot{u}$ and $u_3 = u_p$.

$$\begin{pmatrix}\dot{u}_1\\ \dot{u}_2\\ \dot{u}_3\end{pmatrix} = \begin{pmatrix} u_2 \\ \frac{1}{m}f(t) - 2\xi\omega u_2 - \omega^2 u_1 - \gamma^2\omega^2(u_1 - u_3) \\ \frac{1}{\alpha}\tan\left(\frac{\gamma^2\omega^2(u_1-u_3)}{\tau mg}\frac{\pi}{2}\right)\end{pmatrix} \tag{6}$$

If the friction damper assembly is modeled as an elasto-plastic system as shown in case (c) (hereafter referred to as "elasto-plastic" modeling), the governing differential equation of motion and its state space representation is given by Equations (7, 8).

$$m\ddot{u} + c\dot{u} + ku + k_b(u - u_p) = f(t)$$
$$f_f \vee k_b(u - u_p)\begin{cases}\leq f_s & \text{if in elastic phase}\\ f_s & \text{if in plastic phase}\end{cases} \tag{7}$$

$$\begin{pmatrix}\dot{u}_1\\ \dot{u}_2\\ \dot{u}_3\end{pmatrix} = \begin{pmatrix} u_2 \\ \frac{1}{m}f(t) - 2\xi\omega u_2 - \omega^2 u_1 - \gamma^2\omega^2(u_1 - u_3) \\ \begin{cases}0 & \text{elastic phase}\\ u_2 & \text{plastic phase}\end{cases}\end{pmatrix} \tag{8}$$

where, $u_p(u - u_b)$ is plastic deformation with u_b being elastic deformation. From the schematic representations shown in Figure *2*, it can be seen that response of the friction damper assembly system (b) should approach system (a) if k_b is very high compared to k i.e. if γ is high. However, it is expected that the response will show significant deviation if k_b is of the same order of magnitude as that of k. The value τ indicates how easily the slider is expected to slip. Lower value of τ signifies lower threshold for sliding while for higher τ the sliding may fail to initiate.

VALIDATION OF MODELS

The Equations (1, 6 and 8) are solved using Runge-Kutta fourth order scheme (implemented in Julia programming language) for varying values of γ and τ with two types of input load - harmonic force and ground motion base excitation. Results provided by Mostaghel and Davis (1997) are used for validation of implementation of SDOF system models. The numerical values used for validation are : $\omega = 1.0$ rad/s, $\xi = 0.05$, $\tau = 0.4$ and $f(t) = 1.0\sin(0.4t)$ N. Response obtained for three cases viz. friction only model, damper assembly model and elasto-plastic model are shown in Figure *3* and Figure *4*.

Response of damper assembly and elasto-plastic case is obtained for $\gamma^2 = 100$ and $\alpha = 100$. As a result, damper assembly model becomes nearly equivalent to friction only model (since damper stiffness is 100 times that of elastic system stiffness). From the responses obtained, it is clear that the response of all the cases is almost same. However, it should be noted that the velocity response shows no stick phase (zero velocity phase) in either damper assembly case or elasto-plastic case in contrast to friction only model. This is expected behavior since the friction component is connected in series with spring. Hysteresis behavior for all the cases is almost same with the exception of sharp edges in the case of elasto-plastic model. Friction force vs. velocity plot can be seen as reflecting Coulomb friction model which can be brought arbitrarily close to Coulomb friction model by increasing parameter α in case of "friction only" model and "damper assembly" model.

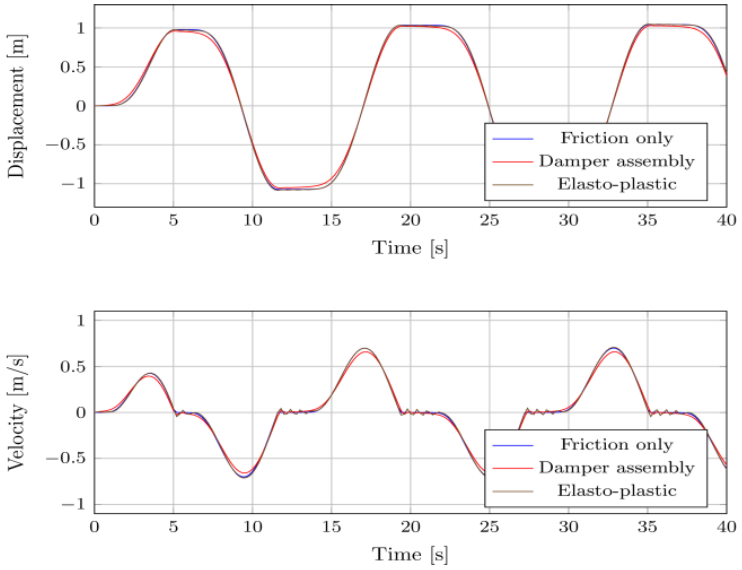


Fig. 3 Displacement and velocity response time histories for validation cases

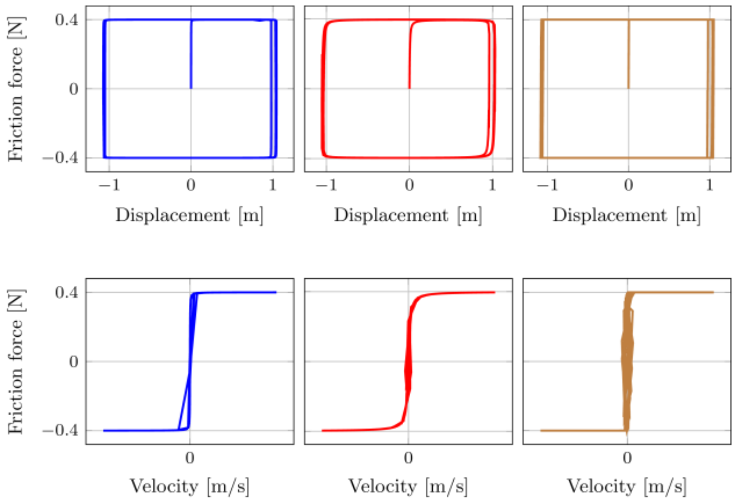


Fig. 4 Hysteresis loops and friction force Vs velocity plots for friction only (left), damper assembly (middle) and elastoplastic (right modelling respectively

It is worth mentioning that the values chosen above indicate a very flexible system with natural frequency being $\frac{1}{2\pi}$ Hz (excluding friction damper assembly). Building frames usually have natural frequency an order of magnitude higher that this. Moreover, the stiffness of spring in damper assembly is usually of the same order of magnitude as that of other components.

RESULTS AND DISCUSSION

Responses for harmonic loading and base excitation are obtained for all modelings. In case of harmonic loading, frequency responses are obtained to see the effect of damper stiffness and sliding force on response. The input force used is $f(t) = A\sin(\lambda t)$ with $A = 1.0$ N and $\lambda = 10.0$ rad/s. Frequency ratio $\beta = \frac{\lambda}{\omega}$ is varied from 0.05 to 10.0. The values of $\gamma^2 = 0.25, 0.50, 0.75, 1.0, 1.5, 2.0, 3.0, 5.0$ and $\tau = 2\%, 4\%, 6\%, 8\%, 10\%, 12\%, 14\%$ are used. Whereas, for the base excitation case, response spectra are obtained for the same values of γ^2 and τ. All the responses are normalized by the response of corresponding elastic system without friction damper assembly. Only a few representative responses are shown and discussed in detail.

1. Harmonic Excitation

The computed dynamic response for three models of damper assembly are compared for varying γ and τ in Figure *6*.

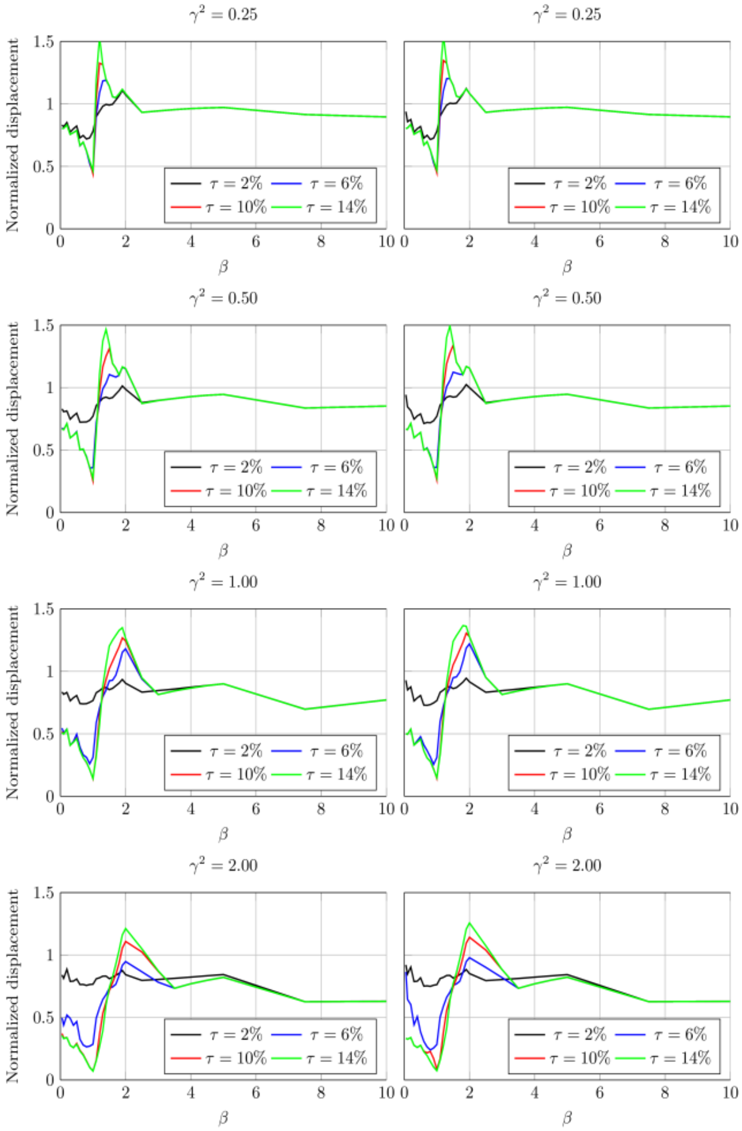


Fig. 5 Comparison of responses for “damper assembly model” (left column) and “elasto-plastic model” (right column)

It is clear that the response due to friction component alone deviates significantly from the other two cases where damper assembly has a finite stiffness. The response predicted by “friction only” case significantly underestimates the response. However, response predicted by other two models almost overlap. The “damper assembly” model, which models a Coulomb friction component in series with a linear elastic spring is a phenomenological model and can be made arbitrarily close to realistic behavior by increasing the value of parameter α. On the other hand, the “elasto-plastic” model is often used in numerical simulations for modeling friction damper assembly. In all the cases, the maximum response reduction is seen at $\beta = 1.0$ but response amplification is also seen in almost all the cases at some other specific β. Location of this new peak is dependent on the value of γ where peak shifts towards right with increasing γ, that is with relative increase in the staging stiffness in comparison with structural stiffness. This indicates that increase in staging stiffness leads to early slip in friction damper. One can also observe that with increasing τ, the response reduction and amplification both increases. On the other hand, with increasing γ, response keeps decreasing. As the responses obtained from “friction only” model aren’t realistic and severely underestimated, the model is not discussed here onward.

To see the effect of τ on response of the system, Figure *5* considers responses for a specific γ with varying τ for all the cases.

It reveals some very interesting behaviors. As noted earlier, maximum response reduction is seen at $\beta = 1.0$ and this is consistent and irrespective of value of τ. However, there is region in all the cases where response is higher than that of response of corresponding elastic system without damper. For very low value of γ or a high value of τ, the friction component of the assembly may not even slip. As a result, the system will behave as a linear elastic system with effective natural frequency $\Omega = \omega\sqrt{1+\gamma^2}$. Hence location of new peak response should be at $\beta \approx \sqrt{1+\gamma^2}$ for no slip cases. It may also be noted that τ ceases to have any effect on the response for long period structures, specifically for structures with $\beta > 1 + 2\gamma^2$. The observations so far suggests a lower τ and higher γ are preferred for design of friction damper assembly. This ensures that the slider will slip more during the vibration of the system.

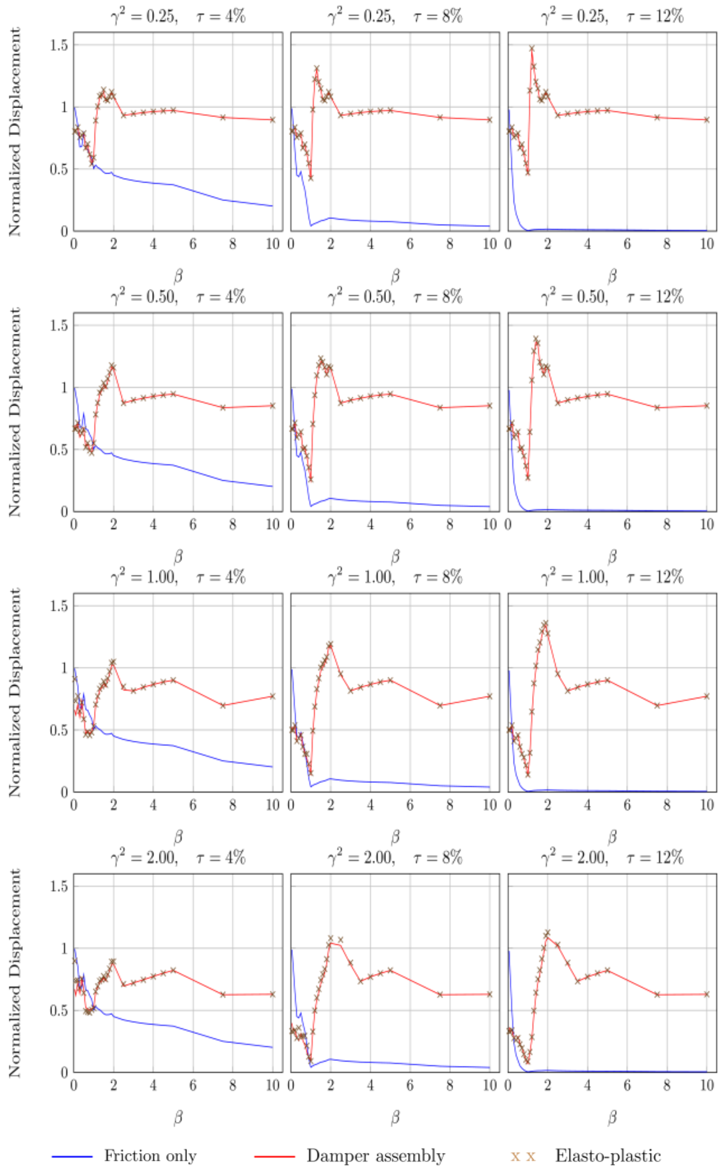


Fig. 6 Frequency response (displacement) of three damper assembly models for varying γ and τ

2. Base excitation with Ground Motion

Response spectrum is a valuable tool in characterizing seismic response. This section deals with response spectra of SDOF systems to see the effect of addition of friction damper assembly. The ground motions considered along with their necessary metadata are given in Table *1*. Corresponding time histories considered are plotted in Figure 7. All the data related to ground motions is obtained from PEER NGA-West2 database Ancheta et al. (2012).

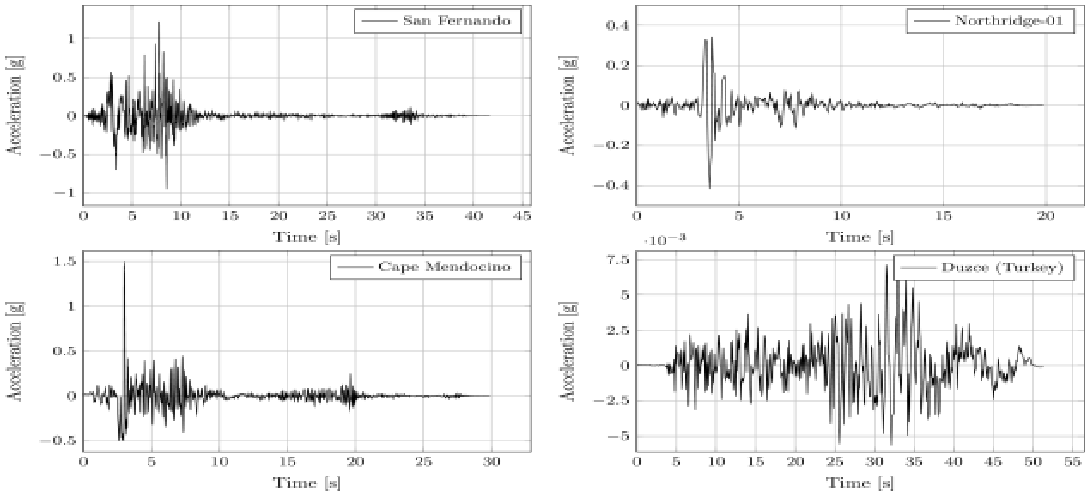


Fig. 7 Ground motion time histories

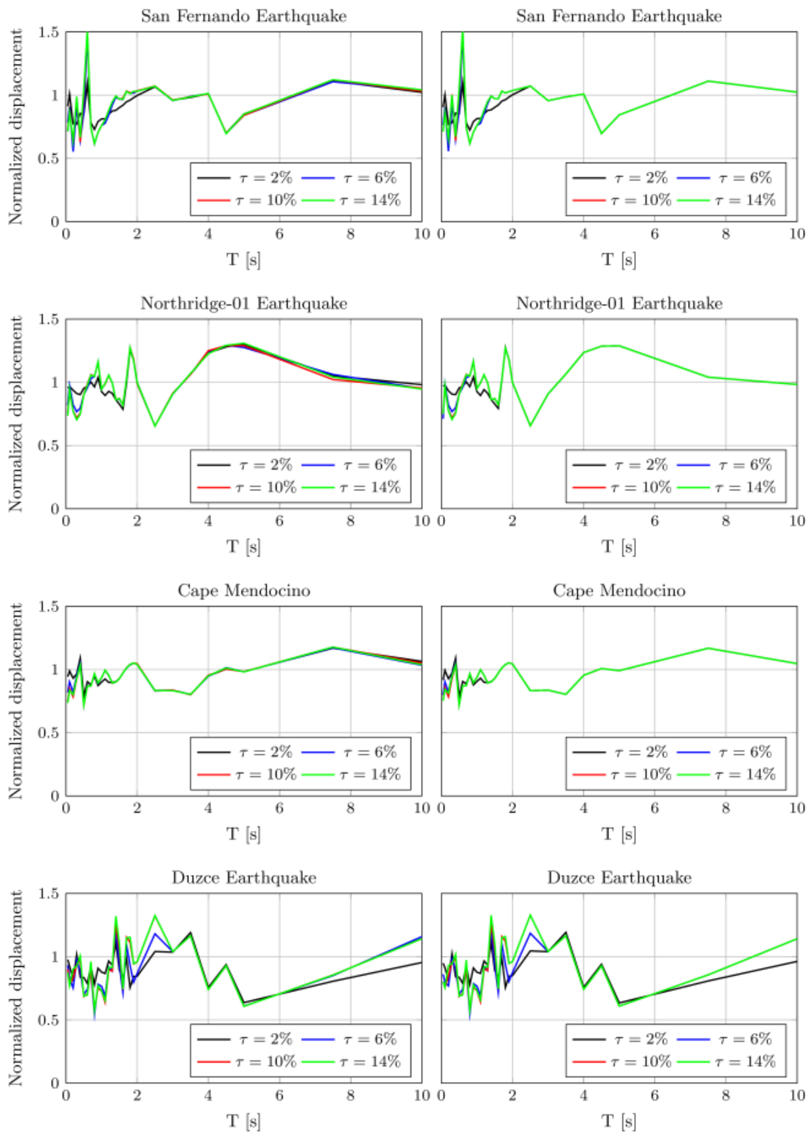


Fig. 8 Normalized displacement response spectra for $\gamma^2 = 0.25$ for "damper assembly model" (left column) and "elasto-plastic model" (right column)

For the purpose of facilitating the comparison of responses from different ground motions, the accelerograms are scaled to $0.36g$ PGA which corresponds to maximum considered earthquake level for the most severe seismic zone , zone V, in the Indian Standard Code IS1893 (2016). Just like the case of harmonic force excitation, all the response quantities are normalized with respect to the response of corresponding elastic system without friction damper assembly.

Table 1: Ground motions considered in the study

Sr. No	Earthquake name	Faultmechanism	Date	Station	Epicentral distance (km)	Component	Magnitude Mw	Shear wave velocity V_s30 (m/s)
1	San Fernando	Reverse	09Feb1971	Pacoima Dam (upper left abut)	11.86	PUL164	6.61	2016.13
2	Northridge-01	Reverse	17Jan1994	Pacoima Dam (downstr)	20.36	PAC175	6.69	2016.13
3	Cape Mendocino	Reverse	25Apr1992	Cape Mendocino	10.36	00	7.01	513.70
4	Duzke (Turkey)	Strike-slip	12Nov1999	Arcelik	149.38	00	7.14	523.00

Normalized response spectra for $\gamma^2 = 0.25,0.50,1.00,2.00$ for all time histories are given in Figure *8*, *9*, *10* and *11*, respectively. From the figures, it can be seen that the response due to both models are almost identical in all of the cases. However, the "damper assembly" model requires a much smaller time step for a stable solution. In all the solutions, the time step used for "damper assembly" model is $1/10^{th}$ as that of used for "elasto-plastic model". Moreover, increasing the value of α also affects the stability of the solution and dt needs to be reduced further for "stricter" α. Such cases show responses approaching to that of "elasto-plastic" model, upon reducing the time step.

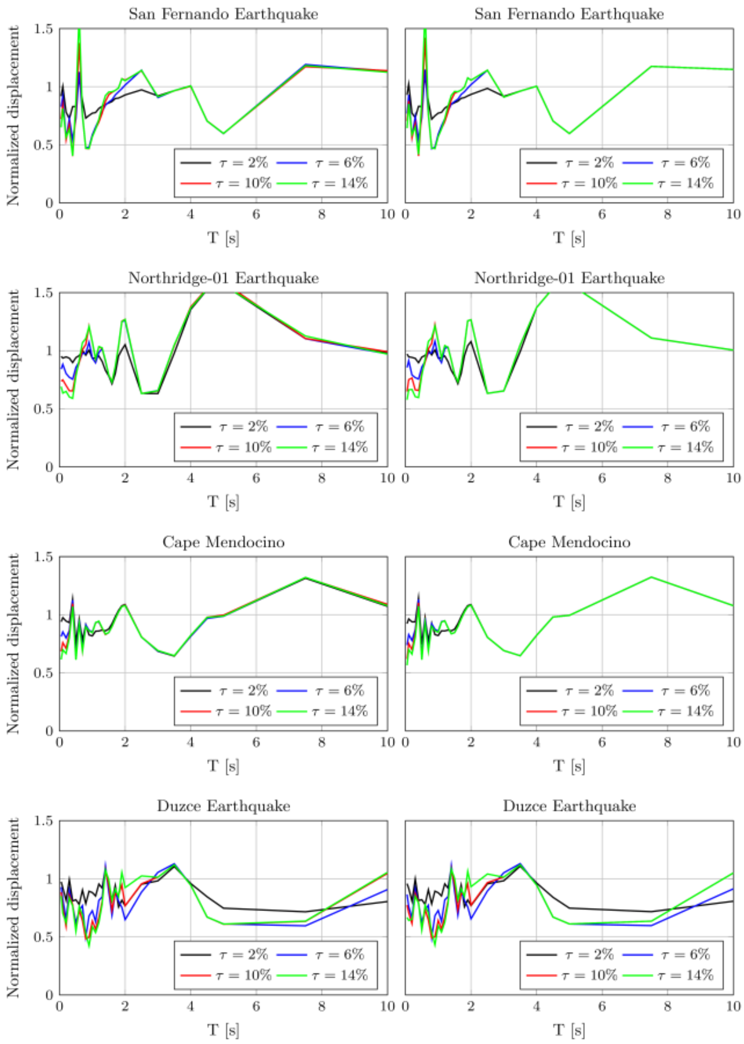


Fig. 9 Normalized displacement response spectra for $\gamma^2 = 0.50$ for "damper assembly model" (left column) and "elasto-plastic model" (right column)

Though the friction component model is dependent on velocity, the response of damper assembly model does not exhibit any dependence on velocity. The friction interface is in series with the staging/spring due to which both the components will have same force at any time. When the spring deformation is low corresponding to the force lower than the slip force, the friction component has to remain in the stick phase irrespective of the magnitude of the total velocity. In such cases, the compatibility condition ensures that the large component of the total velocity is shared by spring whereas only a very small fraction (ideally zero) of velocity is in between the sliding surfaces. Whereas, if the deformation is large corresponding to spring force larger than the slip force, the friction component is bound to slip to maintain the equilibrium. In other words, the friction component "activates" based on the deformation in the spring. As a result dependence of friction component on the velocity ceases to exist.

Unlike the case of harmonic loading, there isn't any specific pattern discernible in these plots. Increasing stiffness of the damper assembly leads to attracting more forces in it. This causes more number of sliding instances during the motion. As a result, effect of τ is seen improving with increasing γ^2 in all the plots. However, for a constant value of γ^2, τ seems to have no significant effect on the response with an exception of $\gamma^2 = 2.00$ case. Except for period 1.0 s, no region can be identified with consistent reduction or amplification of the responses. However, for each individual ground motion, there always exist a consistent zone for reduction and amplification of responses. Hence identifying the specific combination of γ and τ for reducing the response becomes a challenging task.

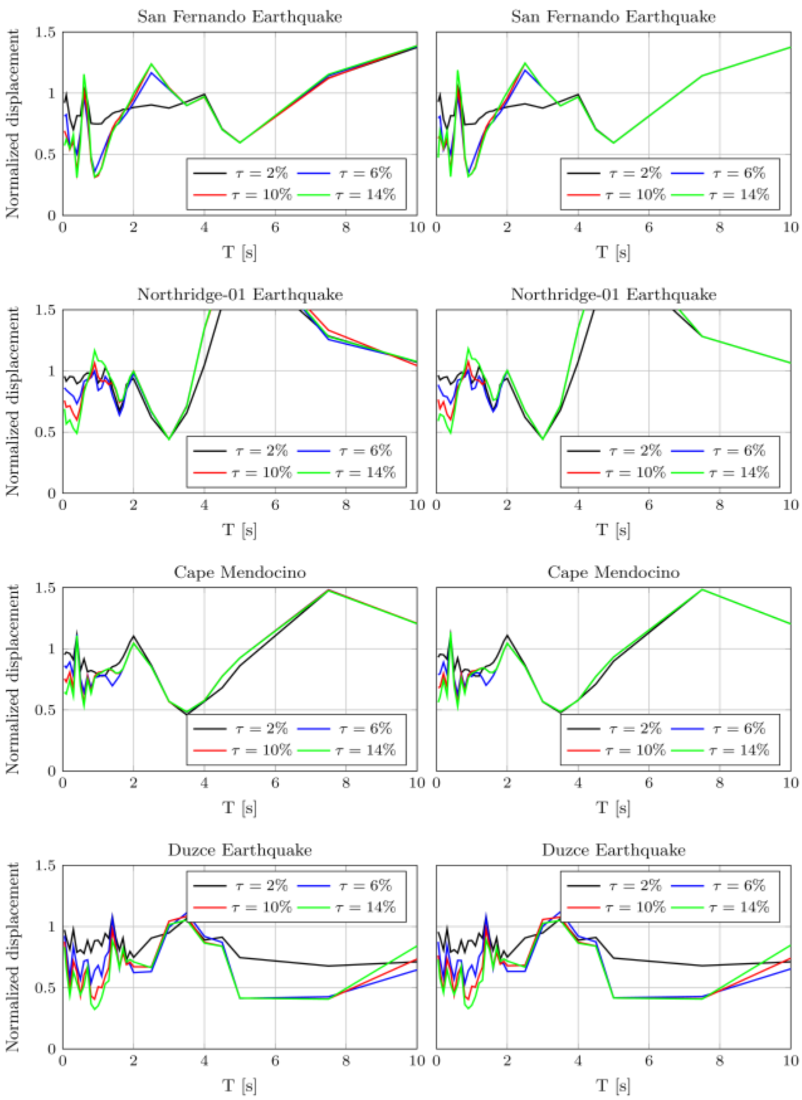


Fig. 10 Normalized displacement response spectra for $\gamma^2 = 1.00$ for "damper assembly model" (left column) and "elasto-plastic model" (right column)

It is seen from the Figures 5, 6, 8, 9, 10, and 11 that the displacement response is sensitive to the γ^2 value when the $\gamma^2 < 1$. In this region, the response reduces significantly with increase in the γ^2 value. But for $\gamma^2 > 1$, the response is not very sensitive to it. For $\gamma^2 > 2$, the change in response is insignificant. Hence, beyond $\gamma^2 > 1$ the uncertainty in the stiffness has low effect on the response whereas for $\gamma^2 > 2$, the uncertainty in the stiffness plays almost no role. A similar behaviour is seen from normalized slip load τ, where for low values of τ the response is sensitive to it but for high values of τ the response does not show any dependence on it. This is because, beyond a certain value of τ, the slip load increases so much that the damper does not slip during the ground motion. Hence, beyond such a value of τ, the response quantities are identical. The threshold for this τ value where it ceases to have any effect on the response, varies with ground motion. It is found that this threshold value of τ is correlated with the pseudo spectral acceleration of the ground motion.

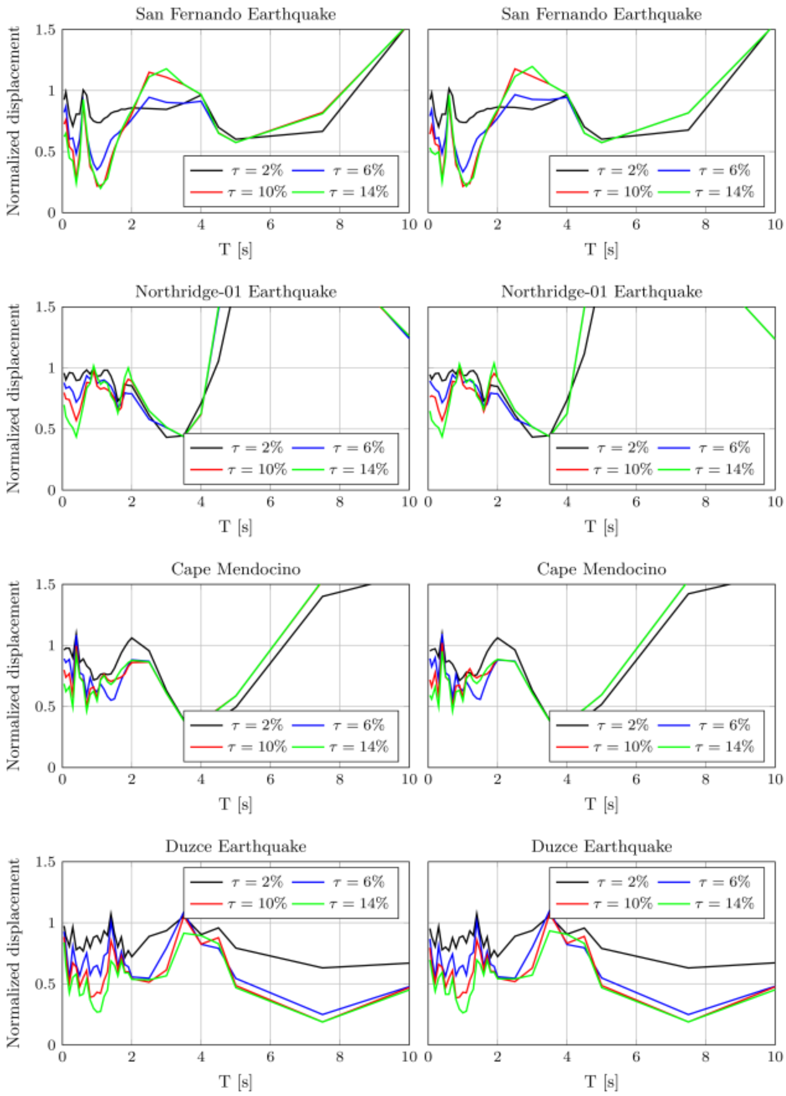


Fig. 11 Normalized displacement response spectra for $\gamma^2 = 2.00$ for "damper assembly model" (left column) and "elasto-plastic model" (right column)

CONCLUSIONS

Three modeling techniques for friction damper assembly are evaluated in the study. Among the three, the model referred to as "damper assembly" model is modeled using Coulomb friction model. This model is expected to be the closest phenomenological model although it suffers from stability issue for large time step.

- The study proves that the "friction only" model is insufficient for modeling friction damper assembly. It grossly underestimates the responses and hence cannot be used for decision making.
- Response obtained from "damper assembly" model and "elasto-plastic" model are almost identical. The "damper assembly" model requires a much smaller time step than that of "elasto-plastic" model. Moreover, with reducing time step, "damper assembly" model response approaches that of "elasto-plastic" model. This implies that "elasto-plastic" model is not only sufficient but also more

accurate and stable for a much larger time step. Hence for modeling friction damper assembly, one should only use “elasto-plastic” modeling as long as staging stiffness is linear elastic and friction interface is to be modeled with Coulomb friction.

- Even though the Coulomb friction is velocity dependent, the spring in series makes the entire assembly behavior independent of velocity.
- Stiffness of the damper assembly plays a very important role in deciding the overall response of the system. In general, higher γ is preferred although designing such an assembly may be very challenging.
- Sliding force per unit weight (τ) has no effect for higher values of β in case of harmonic loading.
- For harmonic loading, the response reduction and amplification zones are consistent and can be predicted based on system and loading characteristics.
- Though for each individual ground motion, zone of response reduction can be marked, there seems no apparent pattern in the location of such a zone. No region can be identified for consistent response reduction or amplification using the current methodology. This makes proposing methodology for designing friction damper a challenging task.

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