

# **A COMBINATION METHOD FOR RESPONSE OF IRREGULAR BUILDINGS UNDER SIMULTANEOUS ACTION OF TWO HORIZONTAL COMPONENTS OF EARTHQUAKE MOTION**

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## **ABSTRACT**

This paper proposes the use of the square root of the sum of squares (SRSS) spatial combination rule in combination with a newly defined critical response spectrum, SACRIT, to arrive at improved estimates of the maximum values of the various response quantities of an irregular building with no principal axes. The SACRIT represents the maximum amplitudes on the orbit plots of a bi-directional oscillator with different natural periods excited simultaneously by two horizontal components of ground motion, similar to the pendulum in a structural response recorder (SRR). Application of this response spectrum along any two arbitrarily selected orthogonal axes of a single story irregular building has been shown to provide the maximum response amplitudes in very good agreement with the exact time history solution for several sets of real accelerograms with widely differing amplitude and frequency characteristics. The application of the proposed method is illustrated for a very simple building just for the purpose of brevity to minimize the influence of the mode superposition method used to estimate the partial contribution to the response due to each component of motion. On theoretical grounds, the proposed method is expected to be equally applicable to more complex multi-degree-of-freedom buildings also, which will be illustrated in a future paper.

**KEYWORDS:** Critical Response Spectrum, Irregular Building, Maximum Response, Principal Components, Response Spectrum Analysis

## **INTRODUCTION**

The seismic response of real buildings, in general, depends on the simultaneous action of the three translational and three rotational components of ground excitation [1, 2]. However, most of the studies have analyzed the effect of the simultaneous action of two horizontal components only, which are expected to have the most significant influence on the structural response. The maximum response of a building under the simultaneous action of two horizontal components of excitation occurs only for a specific orientation of the input ground motion with respect to the principal axes of the building, which need not necessarily be the orientation of the recording instrument. For use in response spectrum analysis under multi-component excitation, several different types of response spectra have been defined by different investigators, most common amongst which are the response spectra of the two principal components of the horizontal ground motion defined by Penzien and Watabe [3].

To combine the partial contributions to a response quantity of a building under earthquake excitations in two horizontal directions, several different rules have been proposed by different investigators which can broadly be grouped into three broad categories as: percentage rule, square root of the sum of square (SRSS) rule, and the complete quadratic combination (CQC-3) rule. Wang et al. [4] have presented an excellent review on the existing combination rules used to account for orthogonal seismic effects.

In the percentage rule, the maximum value of a response quantity is taken equal to the maximum value due to one component of excitation plus a specified percentage of the maximum value of the same response quantity due to excitation in the other orthogonal direction. The maximum response due to each component of motion is estimated by a suitable response spectrum superposition method like SRSS or CQC method. Newmark [5] was perhaps the first to propose the (Max+40%) rule in an Adhoc manner, which was rationalized to (Max+30%) rule by Rosenbluth and Contreras [6] on the basis of minimization of errors in the response envelope. The Indian code IS:1893 Part-1 [7] and ATC-3 [8] provisions have recommended the use of the 30% rule.

In the SRSS method, the maximum value of a response quantity is taken as the square root of the sum of squares of the maximum values of the response quantity due to each of the two orthogonal horizontal components of motion. This rule was proposed by Chu et al. [9] and has been adopted in the USNRC [10] guide. The Indian code IS:1893 Part-1 [7] also recommends the use of the SRSS method as an alternate procedure.

Both the percentage and the SRSS rules do not consider the correlation between the two horizontal components of excitation, due to which the values of the maximum response obtained are generally not very accurate. To improve upon these methods, based on a study by Smeby and Der Kiureghian [11], Menun and Der Kiureghian [12] proposed the CQC3 rule for multi-component seismic analysis. The maximum value of a response quantity in the CQC3 rule is obtained by applying the response spectra of the major and intermediate principal components of input excitation at a critical angle to the structural principal axes. This rule is based on the assumption of identical shapes of the two principal response spectra of horizontal motion. Lopez et al. [13] have carried out detailed investigations to propose the optimum values of the ratio between minor and the major principal horizontal spectra.

Several investigators [14-17] have analyzed the performance of the various combination rules by taking the results of the CQC3 rule as a benchmark. However, the accuracy of the CQC3 rule compared to the exact time history results has not been analyzed adequately in any of the past studies. Gao et al. [18] have studied a curved bridge under simultaneous excitation by the recorded horizontal components (EW and NS) of the El Centro earthquake of May 18, 1940 applied at different angles between  $0^\circ$  and  $180^\circ$  with respect to the selected structural axes. They have concluded that though the CQC3 rule is better than the percentage and the SRSS rules, more attention is needed to verify the CQC3 method by practical time history analysis. Also, to find the maximum response amplitudes for irregular structures, it is necessary to carry out the response analysis for a large number of the choices of the structural axes.

To simplify the problem of response analysis for irregular buildings with no fixed axes, Wilson et al. [19] have used the SRSS rule with an identical response spectrum for both horizontal directions of input excitation. However, they provide no guidelines on what this response spectrum should be. For an identical response spectrum in two horizontal directions, the CQC3 rule reduces to the SRSS rule, but percentage rules still depend on the direction of application of the input excitation. To get very accurate estimates of the maximum values of the response quantities of irregular buildings, we propose to use the SRSS rule with a newly defined critical response spectrum, SACRIT. This has been shown to provide much more accurate results for a simple single-story irregular building under excitation by the response spectra of seven different sets of recorded ground acceleration time histories with widely differing characteristics. Use of the more commonly used geometric mean and the major principal response spectra are found to underestimate the exact time history results in the majority of cases. Thus, the proposed method can be considered to provide a better alternative to the existing methods. On physical grounds, the proposed method is expected to provide equally accurate results for more complex multistory irregular buildings also.

## **DEFINITION OF THE PROBLEM**

The input excitation for simultaneous action of two horizontal components of motion is commonly defined in terms of the response spectra of the major and the minor horizontal components of the motion. The maximum response of a building is obtained by applying the principal response spectra at a critical angle of rotation with respect to the two principal axes of the building. Further, both the response spectra are assumed to have the same shape with the response spectrum of minor principal components taken as a constant fraction of the spectrum of the major principal component. However, in spite of similar shape,

the maximum values of the response quantities of an irregular building with no natural principle axes depend strongly on the choice of the two axes of the building, which cannot be done in a unique way.

As the orientation of input excitation leading to the maximum value of the response for an irregular building cannot be decided a priori, it becomes necessary to compute the response for all possible orientations of the building axes. Also, the maximum value of different response quantities will, in general, occur for different orientations of the building axes. To obviate this inconvenient situation for irregular buildings with no fixed principal axes, Wilson et al. [19] proposed the use of an identical spectrum for both the horizontal directions of input excitation. With identical response spectra, the problem becomes independent of the choice of the structural axes, and the CQC3 rule is simplified to the SRSS rule. However, no guidelines exist on the type of response spectrum to be used in such cases. It has been found that even the use of the major principal response spectrum in both the horizontal directions underestimates the maximum value of the response amplitudes significantly.

To get realistic and accurate estimates of the maximum response amplitudes of irregular buildings under the simultaneous action of two horizontal components of motion, this paper proposes to use a newly defined critical response spectrum, SACRIT, which is defined as the response spectrum of a two-dimensional oscillator subjected simultaneously to both the horizontal components of ground motion. The use of the SACRIT along with the SRSS combination rule has been shown to provide much more accurate results compared to any other type of response spectrum.

### EXAMPLE BUILDING AND THE INPUT EXCITATIONS

A simple single-bay single-story irregular building with no natural principle axes as shown in Figure 1 is considered to illustrate the applicability of the method proposed in the present study. The building comprises four steel columns of type ISLB 200 and a 150 mm thick RCC slab. The building is assumed to have a rigid diaphragm with lumped mass at floor level only. The diaphragm is considered to have an eccentricity of 0.11 m along the y-axis only, and it is thus characterized by a rotational d.o.f.,  $\theta$  and the displacements along x- and y-axes of the building. The damping ratio is assumed to be 5% of critical damping. The frequency for three modes of vibration in example building are obtained as 1.432 Hz, 4.790 Hz, and 4.989 Hz, respectively.

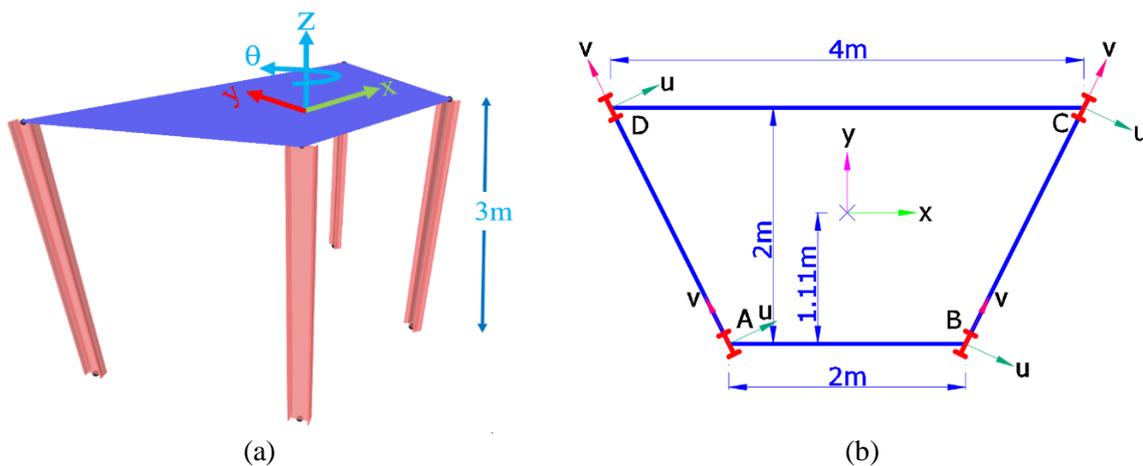


Fig. 1 Example irregular building considered for multicomponent response analysis with (a) 3D isometric view, and (b) Plan view

To compare the results of the proposed combination rule with the exact time history solutions for the maximum values of the various response quantities of the example irregular building under the simultaneous action of two horizontal components of ground motion, we have used three different types of response spectra of seven sets of real accelerograms representing wide variations in the ground motion characteristics. Details of these accelerograms are given in Table 1, which lists the name, date, magnitude, focal depth, and epicentral distance of the contributing earthquake as well as the name of the recording site, components of motion, and recorded peak ground acceleration, PGA. The plots of all the seven sets of selected accelerograms are shown in Figure 2, in which the title above each set indicates the details of the contributing earthquake and the name of the recording site.

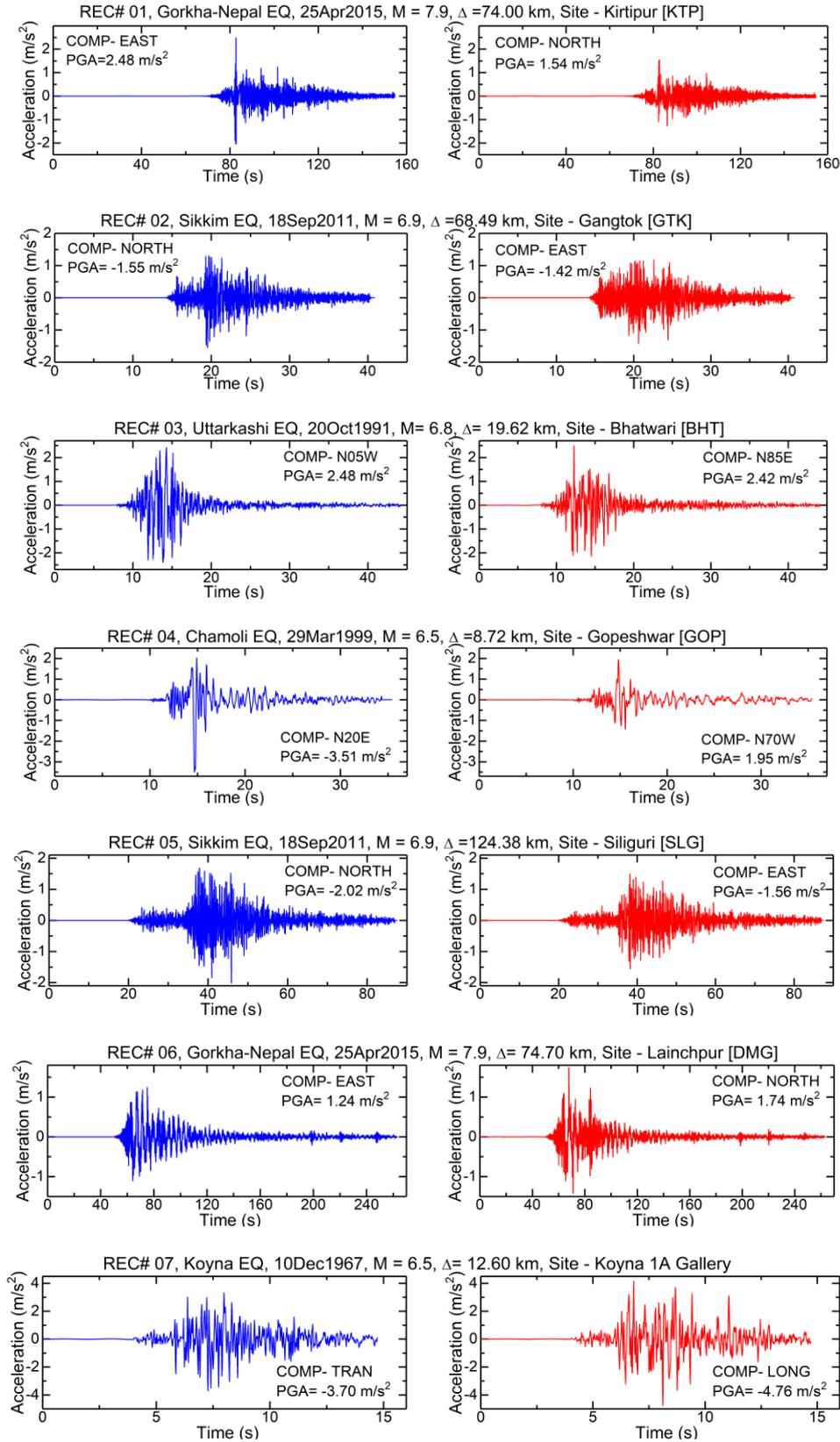


Fig. 2 Plots of the seven sets of the recorded time histories of two horizontal components of ground accelerations used to compute the example numerical results

**Table 1: Details of the accelerograms selected for computation of the exact time history response of the example building**

REC#	EQ. Name	Date	Moment magnitude (M <sub>w</sub> )	Focal depth (km)	Epicenter distance (km)	Recording station	Comp.	PGA (m/s <sup>2</sup> )
1	Gorkha Nepal	25/04/2015	7.9	13.4	74.00	Kirtipur (KTP)	EAST NORT	2.484 1.539
2	Sikkim	18/09/2011	6.9	29.60	68.49	Gangtok (GTK)	NORT EAST	1.545 1.421
3	Uttarkashi	20/10/1991	6.8	19.0	19.62	Bhatwari (BHAT)	N05W N85E	2.483 2.417
4	Chamoli	29/03/1999	6.5	21.0	8.72	Gopeshwar (GOP)	N20E N70W	3.511 1.945
5	Sikkim	18/09/2011	6.9	29.60	124.38	Silliguri (SLG)	NORT EAST	2.016 1.561
6	Gorkha Nepal	25/04/2015	7.9	13.40	74.70	Lainchpur (DMG)	EAST NORT	1.243 1.738
7	Koyna	10/12/1967	6.5	12.0	12.60	Koyna 1A Gallery	TRAN LONG	3.702 4.759

To compute the maximum response amplitudes of the example building by the SRSS combination rule using identical response spectrum for both horizontal components of excitation, we have used the conventional geometrical mean response spectrum of the response spectra of two recorded horizontal components of motion [SAGM], response spectrum of the major principal component of motion [SAXP], and a newly defined critical response spectrum [SACRIT]. Methods to obtain these three types of response spectra for the selected seven sets of accelerograms are described briefly in the following.

**1. Geometric Mean Response Spectrum**

The geometric mean response spectrum, SAGM, of two recorded orthogonal horizontal components of the ground motion is used commonly in specifying the input excitation in practical engineering applications, which is defined as

$$SAGM = \sqrt{SA_x \cdot SA_y} \tag{1}$$

where,  $SA_x$  and  $SA_y$  are the response spectra of two horizontal components of ground acceleration.

**2 Response Spectrum in Major Principal Direction**

The principal components of ground acceleration can be obtained by resolving the recorded components along two horizontal directions to have no correlation between them [3]. The correlation  $\sigma_{ij}$  between the  $i^{th}$  and  $j^{th}$  component of ground motion is defined as

$$\sigma_{ij} = \sigma_{ji} = \frac{1}{T} \int_0^T a_i(t) a_j(t) dt \tag{2}$$

where,  $T$  is the total duration of ground motion,  $i$  and  $j$  represents the x- or y-component of ground motion. This gives the symmetric correlation coefficient matrix for the recorded horizontal components of ground acceleration as

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \tag{3}$$

By definition, the cross-correlation terms in this matrix should be close to zero along the principal directions  $x_p$  and  $y_p$ . The principal directions can be obtained by diagonalizing the above correlation matrix by solving the eigenvalue problem to get the eigenvalues  $\lambda_1$ ,  $\lambda_2$  and the corresponding eigenvectors  $\phi_1$  and  $\phi_2$ . The eigenvector corresponding to the larger eigenvalue represents the major

principal direction  $x_p$  and that corresponding to the smaller eigenvalue represents the minor principal direction  $y_p$ . If the major principal direction makes an angle  $\theta$  with the x-component of the recorded accelerogram, the accelerogram in the major principal direction is given by

$$a_{xp}(t) = a_x(t) \cos(\theta) + a_y(t) \sin(\theta) \quad (4)$$

The response spectrum of  $a_{xp}(t)$  gives the major principal response spectrum, SAXP.

### 3 Critical Response Spectrum

The critical response spectrum, SACRIT, is defined as the maximum absolute response of a bi-directional SDOF oscillator under the simultaneous action of both x- and y-components of a recorded accelerogram. If  $R_x(t)$  and  $R_y(t)$  are the response time histories of the oscillator to the x- and y-components of ground acceleration, the response to the simultaneous action of both the components is given by orbit plot representing the trace of all the points  $(R_x(t), R_y(t))$  in the (x, y) plane as shown in Figure 3. The maximum distance on this trace from the origin for each natural period ( $T$ ) and a selected damping ratio ( $\xi$ ) gives the SACRIT spectral amplitude.

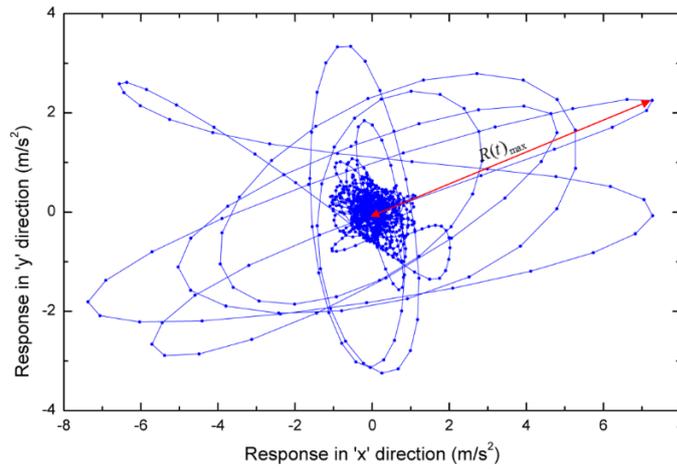


Fig. 3 Orbit plot for the trace of the response of a bi-directional oscillator excited simultaneously by two horizontal components of motion

The three types of response spectra computed as above for all the seven selected sets of accelerograms are shown in Figure 4. The SACRIT is seen to envelop the other two types of response spectra. However, SAXP is exceeded even by the SAGM at some of the natural periods in some cases.

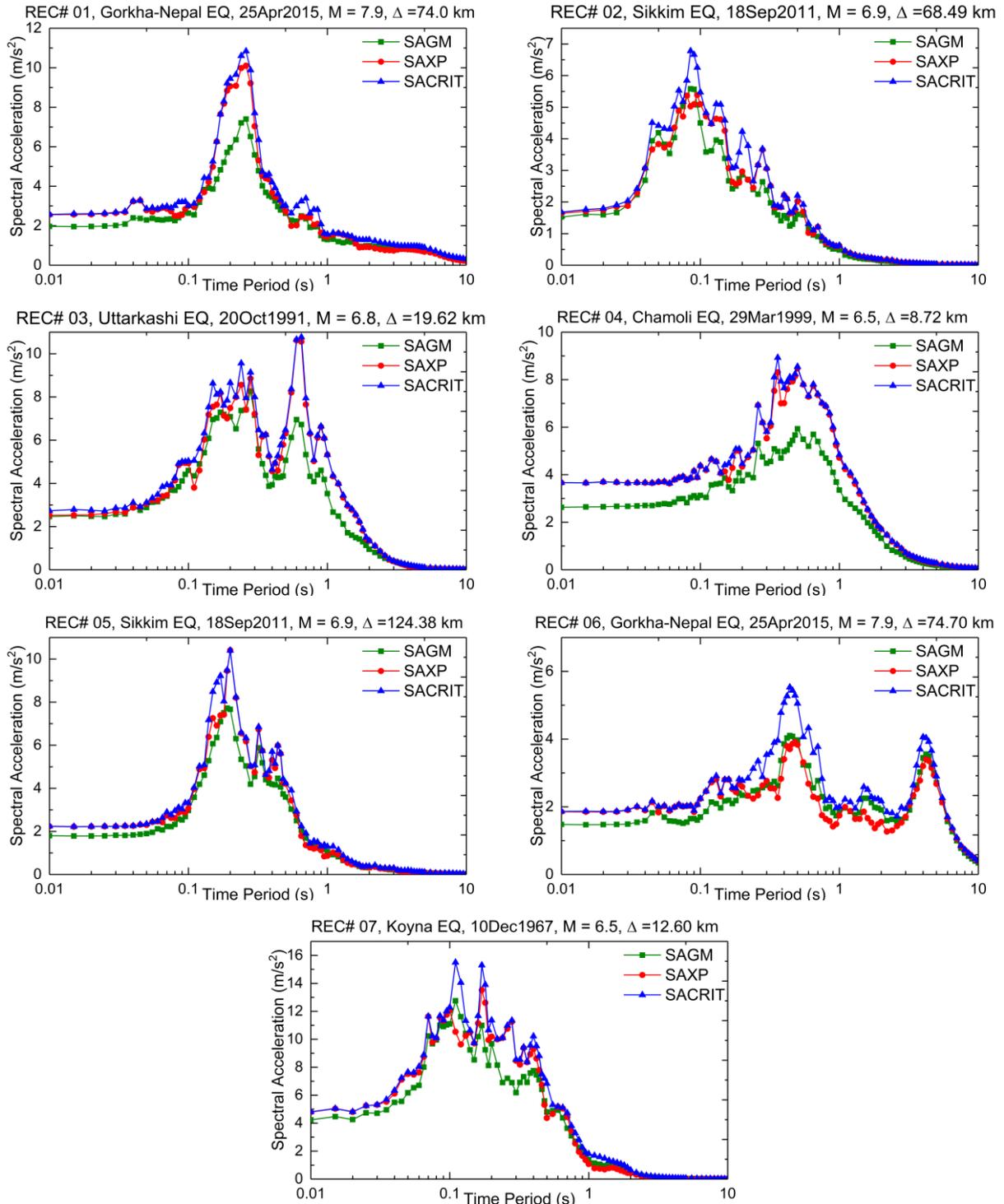


Fig. 4 Comparison of the different types of response spectra for the selected seven sets of accelerograms

**NUMERICAL RESULTS AND DISCUSSION**

The response of the example irregular building has been estimated in terms of the floor displacements along the selected x and y-axis of the building and the base shear along and the bending moment about local u and v axes of each element of the building under simultaneous action of two horizontal components of the selected seven sets of accelerograms. Maximum values of the response quantities are computed by the response spectrum method for the three different types of response spectra defined in the

previous section and compared with the corresponding exact solution obtained by detailed time history response analysis.

In the response spectrum analysis, the partial contribution to the maximum response due to each horizontal component of motion is obtained by the CQC method of the modal response spectrum superposition and the maximum response due to both components of excitation is obtained by the SRSS rule. Due to the use of identical response spectra in both the horizontal directions of motion, the response estimation becomes independent of the direction of application of the input excitation. The example results are therefore computed only by applying the input response spectra along the selected two axes of the irregular building. However, the recorded acceleration time histories of two horizontal components being different, the response amplitudes will vary with directions along which the input accelerograms are applied with respect to the selected structural axes of an irregular building. To find the maximum values of the various response quantities, the time history response analysis has been thus carried out by applying the input accelerograms at different angles between 0° and 180° at intervals of 10°.

Figure 5 shows typical examples of the comparison between the maximum values of the shear force and bending moment responses of column ‘C’ of the example irregular building obtained by response spectrum method and the time history analysis for all seven selected sets of the accelerograms. It is seen that the maximum response amplitudes based on the proposed critical response spectrum SACRIT are the closest and in very good matching with the maximum values of the exact time history solutions in all the cases. On the other hand, the response amplitudes based on the geometrical mean spectrum SAGM are seen to be grossly underestimated in all the cases. Though, the results based on the principal response spectrum SAXP show reasonably good agreement in a few cases, the maximum values of the exact time history response amplitudes are underestimated in the following 10 cases out of 14 cases.

Rec #	Response quantities underestimated	No. of cases
1	Shear force-u and B.M. about v	1
2	Shear force-v and B.M. about u	1
3	Shear force-v and B.M. about u	1
4	Shear force-v and B.M. about u	1
5	Shear force-u & v and B.M. about u & v	2
6	Shear force-u & v and B.M. about u & v	2
7	Shear force-u & v and B.M. about u & v	2
Total no. of cases		10

REC# 01, Gorkha-Nepal EQ, 25Apr2015, M = 7.9, Δ =74.00 km, Site - Kirtipur [KTP]

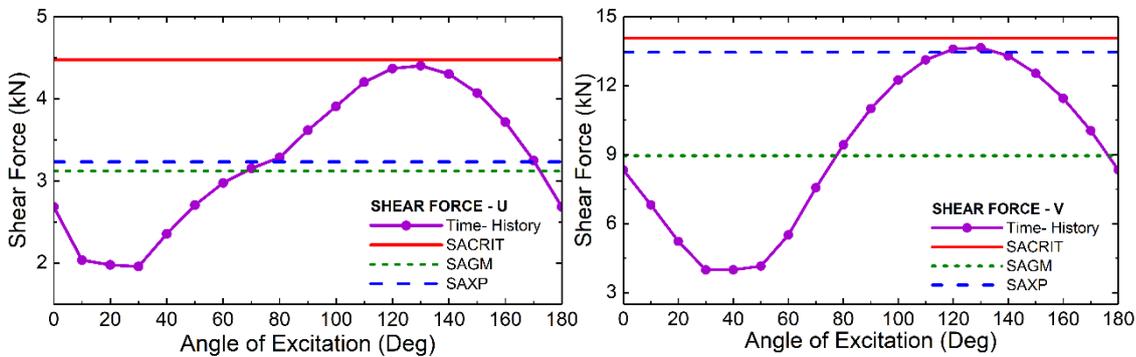


Fig. 5 Typical comparisons of the shear force along the local u- and v-axes of column ‘C’ of the simple example building

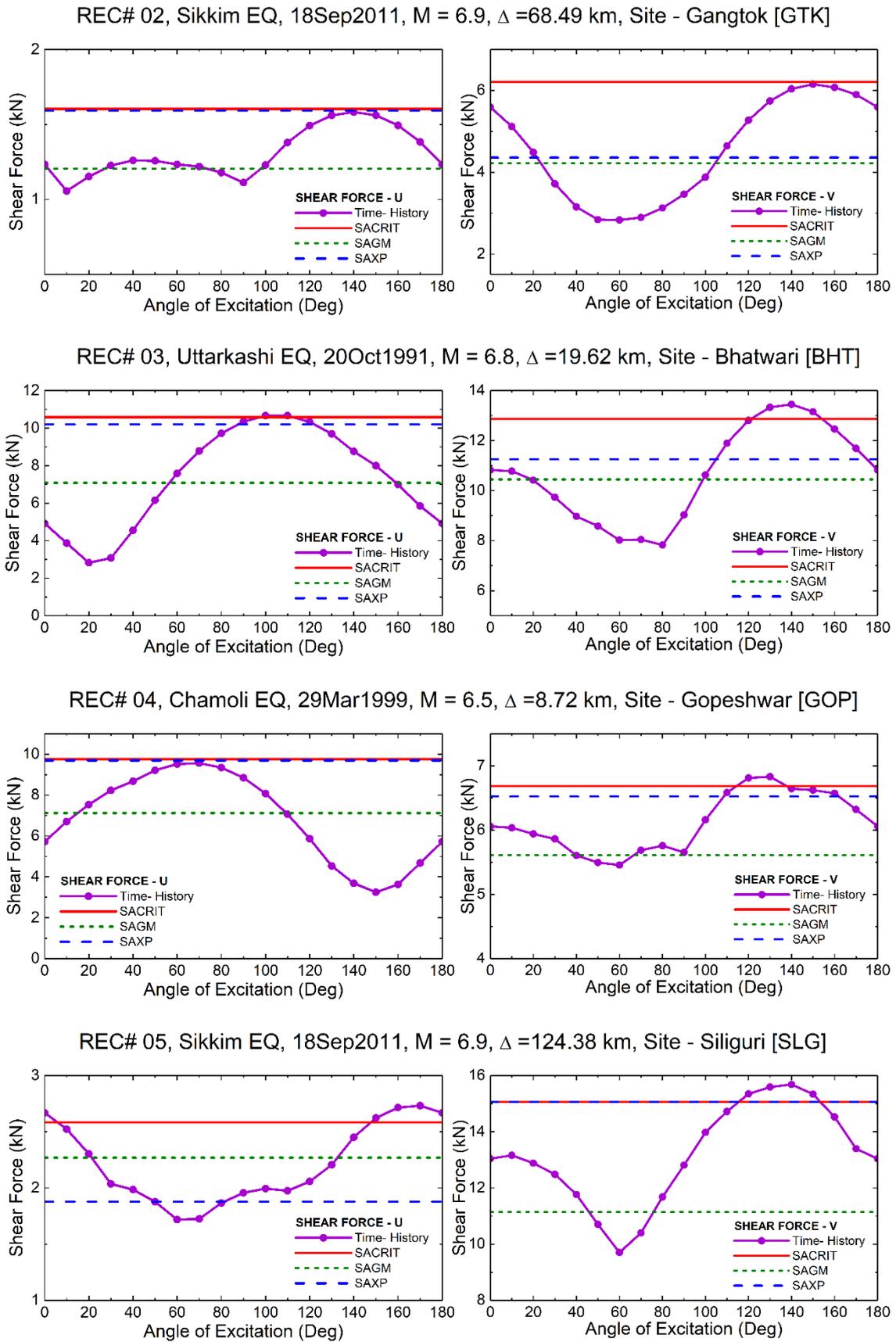
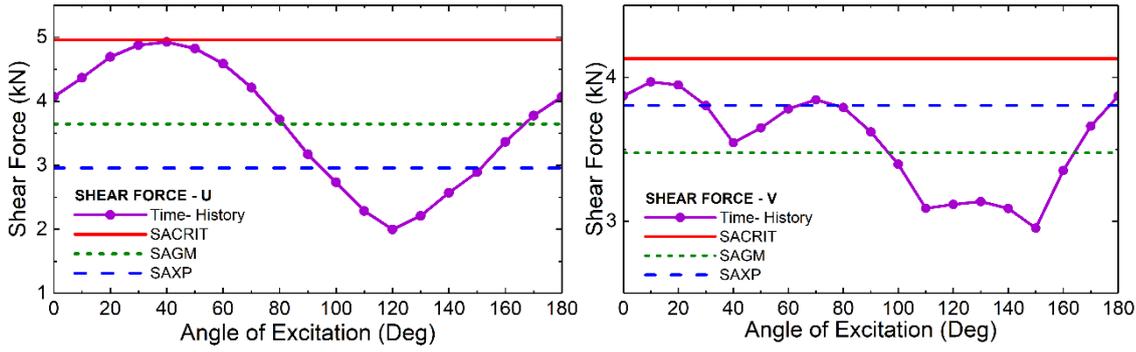


Fig. 5 Typical comparisons of the shear force along the local u- and v-axes of column ‘C’ of the simple example building (continued from previous page)

REC# 06, Gorkha-Nepal EQ, 25Apr2015, M = 7.9, Δ =74.70 km, Site - Lainchpur [DMG]



REC# 07, Koyna EQ, 10Dec1967, M = 6.5, Δ =12.60 km, Site - Koyna 1A Gallery

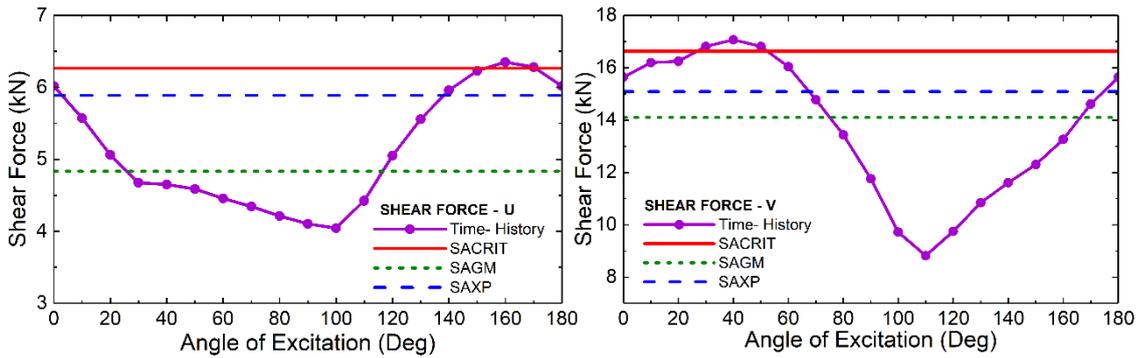


Fig. 5 Typical comparisons of the shear force along the local u- and v-axes of column ‘C’ of the simple example building (continued from previous page)

Table 2: Displacements in all the four columns along selected x- and y-axes of the example irregular building under seven different cases of input excitations

REC#	EQ. Name	Input Excitation	Displacement (mm) along structural							
			x-axis in column				y-axis in column			
			A	B	C	D	A	B	C	D
1	Nepal	Time History	20.22	20.22	39.08	39.08	11.47	16.91	25.65	20.08
		SACRIT	20.14	20.14	38.89	38.89	13.95	13.95	22.36	22.36
		SAGM	13.97	13.97	27.14	27.14	9.30	9.30	15.29	15.29
		SAXP	14.92	14.92	27.99	27.99	11.84	11.84	17.47	17.47
2	Sikkim	Time History	7.02	7.02	13.84	13.84	5.83	5.67	7.23	7.64
		SACRIT	7.31	7.31	13.90	13.90	5.64	5.64	8.47	8.47
		SAGM	5.46	5.46	10.47	10.47	3.99	3.99	6.19	6.19
		SAXP	7.09	7.09	13.86	13.86	4.67	4.67	7.74	7.74
3	Uttarkashi	Time History	44.53	44.53	91.51	91.51	25.21	26.72	50.51	48.35
		SACRIT	46.16	46.16	92.43	92.43	25.01	25.01	47.59	47.59
		SAGM	30.94	30.94	61.78	61.78	17.24	17.24	32.13	32.13
		SAXP	44.45	44.45	89.08	89.08	23.78	23.78	45.69	45.69
4	Chamoli	Time History	42.02	42.02	84.44	84.44	23.94	20.51	40.76	45.37
		SACRIT	42.37	42.37	85.19	85.19	21.96	21.96	43.22	43.22
		SAGM	30.95	30.95	62.19	62.19	16.15	16.15	31.62	31.62
		SAXP	42.13	42.13	84.71	84.71	21.82	21.82	42.97	42.97
5	Sikkim	Time History	11.13	11.13	21.97	21.97	11.60	13.31	17.18	14.17
		SACRIT	12.37	12.37	22.17	22.17	12.17	12.17	16.09	16.09
		SAGM	10.59	10.59	19.56	19.56	9.38	9.38	13.06	13.06

		SAXP	9.66	9.66	15.82	15.82	11.52	11.52	14.07	14.07
6	Nepal	Time History	21.67	21.67	43.08	43.08	11.27	11.15	21.70	21.82
		SACRIT	21.58	21.58	43.35	43.35	11.30	11.30	22.06	22.06
		SAGM	15.86	15.86	31.83	31.83	8.38	8.38	16.25	16.25
		SAXP	12.90	12.90	25.82	25.82	7.05	7.05	13.33	13.33
7	Koyna	Time History	26.29	26.29	53.88	53.88	17.38	20.80	33.98	30.78
		SACRIT	27.83	27.83	54.54	54.54	18.15	18.15	30.29	30.29
		SAGM	21.56	21.56	42.04	42.04	14.62	14.62	23.78	23.78
		SAXP	26.17	26.17	51.28	51.28	16.76	16.76	28.30	28.30

The results in Figure 5 indicate that the use of the SACRIT response spectrum in combination with the SRSS rule for the combination of component responses is able to provide a highly improved and practically very convenient method for estimation of the maximum values of the various response quantities of irregular buildings without fixed principal axes. This corroboration is further confirmed by the results on all the response quantities for all four columns of the example building as listed in Table 2 to 4. Table 2 gives the displacement amplitudes in the directions of the selected x- and y-axes of the structure, whereas Tables 3 and 4 give the shear force along and bending moment about the local u-and v-axis of each column of the example building. The maximum response amplitudes are based on three different types of response spectra viz. SACRIT, SAGM, and SAXP are compared with the exact time history solutions in each of the tables.

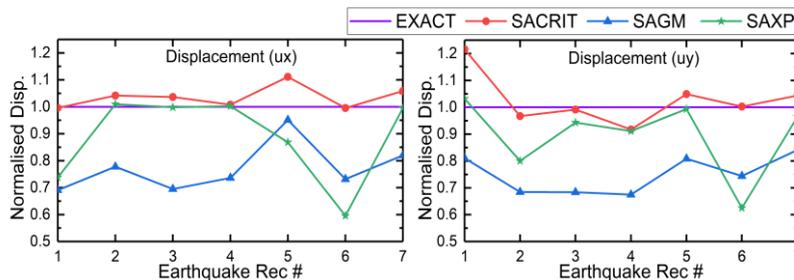
**Table 3: Shear forces along the local u- and v-axes of all the four columns of the example irregular building under seven different cases of input excitations**

REC#	EQ. Name	Input Excitation	Shear Force (kN) along							
			local u-axis in column				local v-axis in column			
			A	B	C	D	A	B	C	D
1	Nepal	Time History	2.32	2.25	4.40	4.47	14.15	13.65	13.65	14.15
		SACRIT	2.29	2.29	4.48	4.48	14.07	14.07	14.07	14.07
		SAGM	1.59	1.59	3.12	3.12	8.95	8.95	8.95	8.95
		SAXP	1.69	1.69	3.23	3.23	13.46	13.46	13.46	13.46
2	Sikkim	Time History	0.79	0.80	1.58	1.57	6.10	6.15	6.15	6.10
		SACRIT	0.83	0.83	1.60	1.60	6.21	6.21	6.21	6.21
		SAGM	0.62	0.62	1.21	1.21	4.22	4.22	4.22	4.22
		SAXP	0.81	0.81	1.59	1.59	4.36	4.36	4.36	4.36
3	Uttarkashi	Time History	5.09	5.22	10.67	10.48	12.17	13.45	13.45	12.17
		SACRIT	5.29	5.29	10.59	10.59	12.87	12.87	12.87	12.87
		SAGM	3.54	3.54	7.08	7.08	10.45	10.45	10.45	10.45
		SAXP	5.09	5.09	10.20	10.20	11.25	11.25	11.25	11.25
4	Chamoli	Time History	4.91	4.72	9.57	9.76	6.66	6.83	6.83	6.66
		SACRIT	4.86	4.86	9.76	9.76	6.69	6.69	6.69	6.69
		SAGM	3.55	3.55	7.12	7.12	5.61	5.61	5.61	5.61
		SAXP	4.83	4.83	9.70	9.70	6.53	6.53	6.53	6.53
5	Sikkim	Time History	1.31	1.39	2.73	2.43	13.50	15.69	15.69	13.50
		SACRIT	1.41	1.41	2.59	2.59	15.07	15.07	15.07	15.07
		SAGM	1.21	1.21	2.27	2.27	11.14	11.14	11.14	11.14
		SAXP	1.10	1.10	1.88	1.88	15.06	15.06	15.06	15.06
6	Nepal	Time History	2.47	2.49	4.93	4.92	3.98	3.97	3.97	3.98
		SACRIT	2.47	2.47	4.96	4.96	4.13	4.13	4.13	4.13
		SAGM	1.82	1.82	3.65	3.65	3.48	3.48	3.48	3.48
		SAXP	1.48	1.48	2.96	2.96	3.81	3.81	3.81	3.81
7	Koyna	Time History	3.14	2.92	6.35	6.29	15.08	17.07	17.07	15.08
		SACRIT	3.18	3.18	6.27	6.27	16.63	16.63	16.63	16.63
		SAGM	2.47	2.47	4.84	4.84	14.10	14.10	14.10	14.10
		SAXP	2.99	2.99	5.89	5.89	15.09	15.09	15.09	15.09

**Table 4: Bending moment about the local u- and v-axes of all the four columns of the example irregular building under seven different cases of input excitations**

REC#	EQ. Name	Input Excitation	Bending Moment (kNm) about							
			local u-axis in column				local v-axis in column			
			A	B	C	D	A	B	C	D
1	Nepal	Time History	21.22	20.48	20.48	21.22	3.48	3.38	6.60	6.71
		SACRIT	21.10	21.10	21.10	21.10	3.44	3.44	6.71	6.71
		SAGM	13.43	13.43	13.43	13.43	2.39	2.39	4.68	4.68
		SAXP	20.18	20.18	20.18	20.18	2.54	2.54	4.85	4.85
2	Sikkim	Time History	9.15	9.22	9.22	9.15	1.19	1.20	2.37	2.36
		SACRIT	9.31	9.31	9.31	9.31	1.25	1.25	2.41	2.41
		SAGM	6.34	6.34	6.34	6.34	0.93	0.93	1.81	1.81
		SAXP	6.54	6.54	6.54	6.54	1.21	1.21	2.39	2.39
3	Uttarkashi	Time History	18.25	20.17	20.17	18.25	7.63	7.83	16.00	15.72
		SACRIT	19.30	19.30	19.30	19.30	7.93	7.93	15.88	15.88
		SAGM	15.67	15.67	15.67	15.67	5.32	5.32	10.62	10.62
		SAXP	16.87	16.87	16.87	16.87	7.64	7.64	15.31	15.31
4	Chamoli	Time History	9.98	10.25	10.25	9.98	7.36	7.07	14.35	14.64
		SACRIT	10.03	10.03	10.03	10.03	7.28	7.28	14.63	14.63
		SAGM	8.41	8.41	8.41	8.41	5.32	5.32	10.68	10.68
		SAXP	9.79	9.79	9.79	9.79	7.24	7.24	14.55	14.55
5	Sikkim	Time History	20.25	23.53	23.53	20.25	1.97	2.08	4.10	3.65
		SACRIT	22.60	22.60	22.60	22.60	2.11	2.11	3.88	3.88
		SAGM	16.71	16.71	16.71	16.71	1.81	1.81	3.40	3.40
		SAXP	22.59	22.59	22.59	22.59	1.65	1.65	2.82	2.82
6	Nepal	Time History	5.97	5.95	5.95	5.97	3.71	3.73	7.39	7.39
		SACRIT	6.19	6.19	6.19	6.19	3.71	3.71	7.45	7.45
		SAGM	5.21	5.21	5.21	5.21	2.73	2.73	5.47	5.47
		SAXP	5.71	5.71	5.71	5.71	2.22	2.22	4.44	4.44
7	Koyna	Time History	22.62	25.60	25.60	22.62	4.71	4.38	9.53	9.43
		SACRIT	24.95	24.95	24.95	24.95	4.77	4.77	9.40	9.40
		SAGM	21.15	21.15	21.15	21.15	3.70	3.70	7.25	7.25
		SAXP	22.63	22.63	22.63	22.63	4.48	4.48	8.84	8.84

To get an idea at a glance about the extent to which the maximum response amplitudes based on the three different types of response spectra differ from the exact time history values, Figures 6-9 present the ratios of the displacement, shear force, and bending moment responses for all the four columns obtained using three different types of response spectrum to the corresponding exact time history values for all the seven sets of ground motion. The deviation of this ratio from 1.0 indicates the error in the results based on the response spectrum method, with values higher than 1.0 indicating conservatism and those lower than 1.0 indicating unsafe design. It is seen that the proposed use of SACRIT gives the most accurate estimates of multi-component response, which is mostly within 5% of the exact time history estimates.



**Fig. 6** The response amplitudes obtained from three different types of response spectra normalized by the exact time history response amplitudes plotted versus the seven different input excitations for Column-A

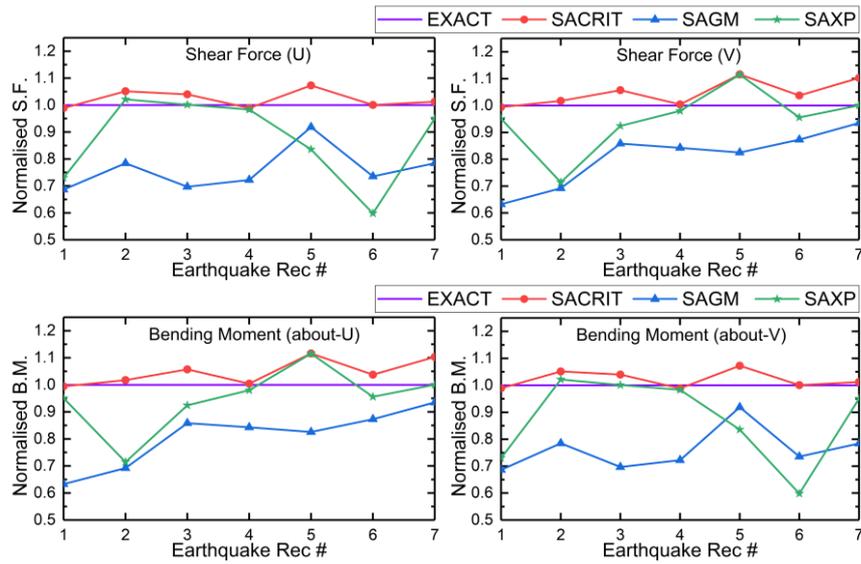


Fig. 6 The response amplitudes obtained from three different types of response spectra normalized by the exact time history response amplitudes plotted versus the seven different input excitations for Column-A (continued from previous page)

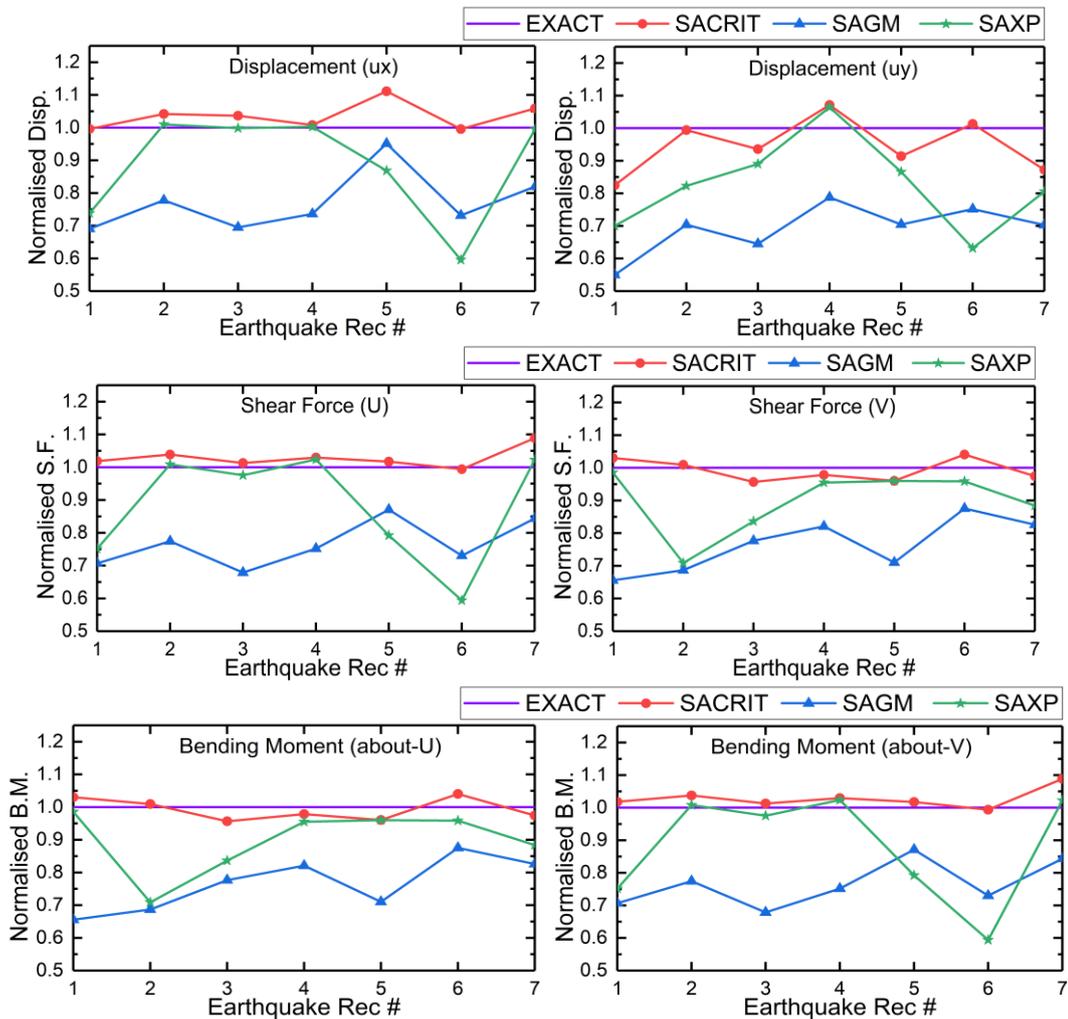


Fig. 7 The response amplitudes obtained from three different types of response spectra normalized by the exact time history response amplitudes plotted versus the seven different input excitations for Column-B

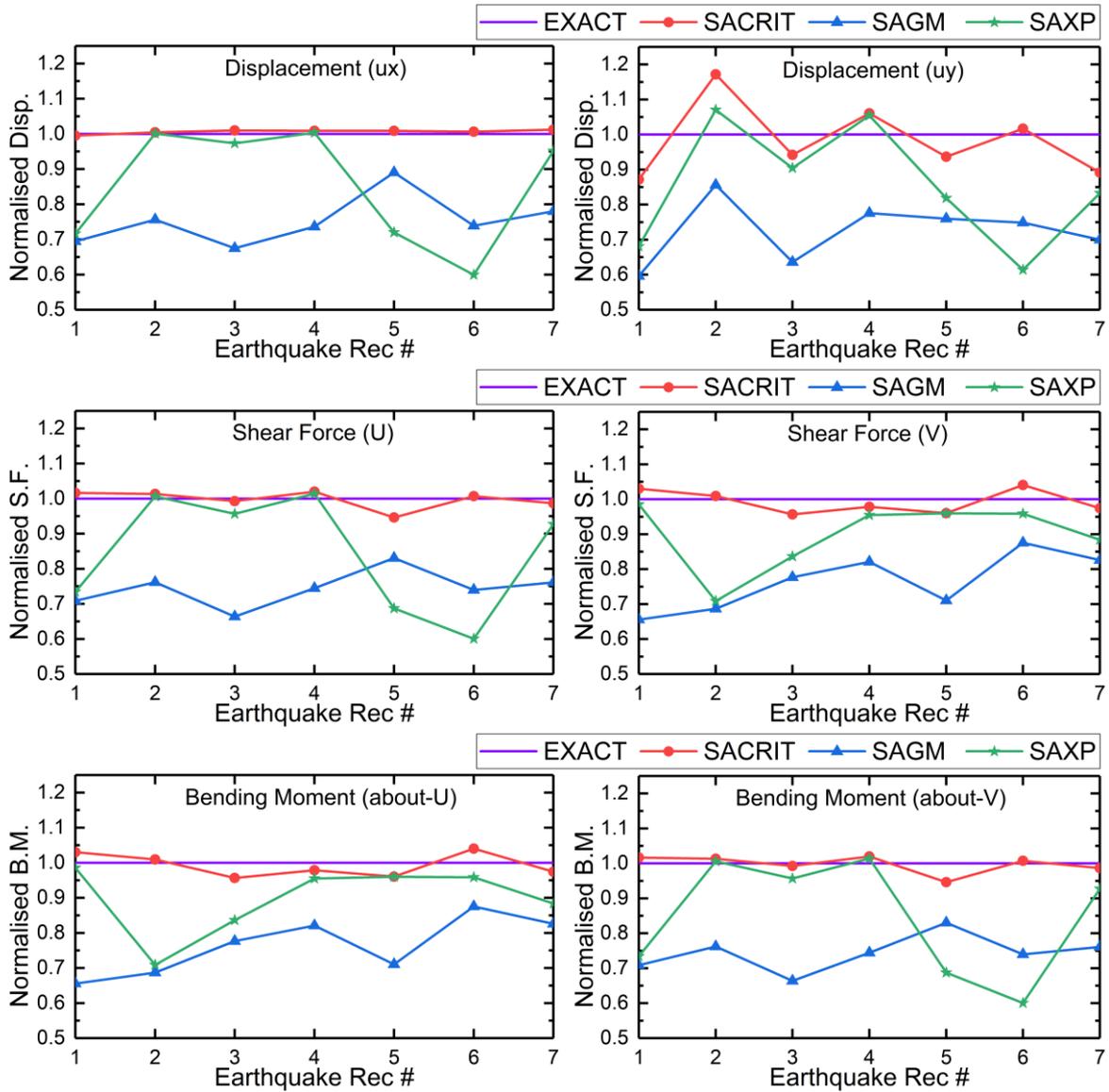


Fig. 8 The response amplitudes obtained from three different types of response spectra normalized by the exact time history response amplitudes plotted versus the seven different input excitations for Column-C

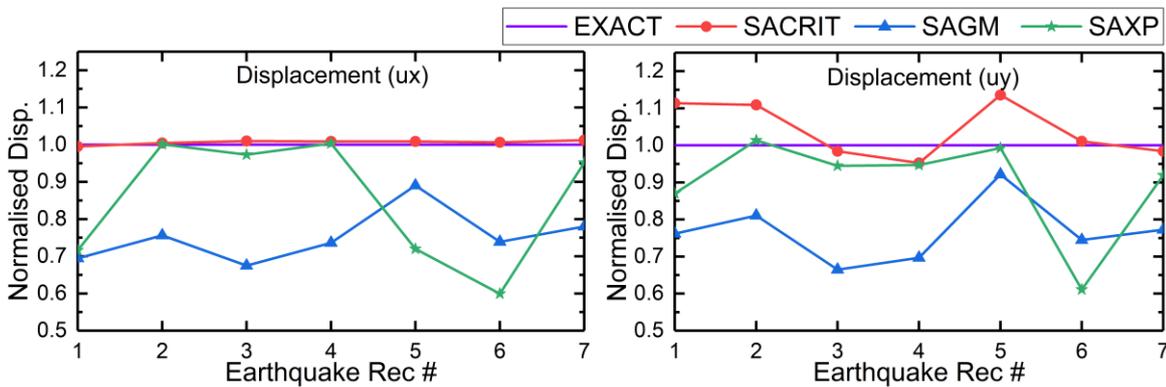


Fig. 9 The response amplitudes obtained from three different types of response spectra normalized by the exact time history response amplitudes plotted versus the seven different input excitations for Column-D

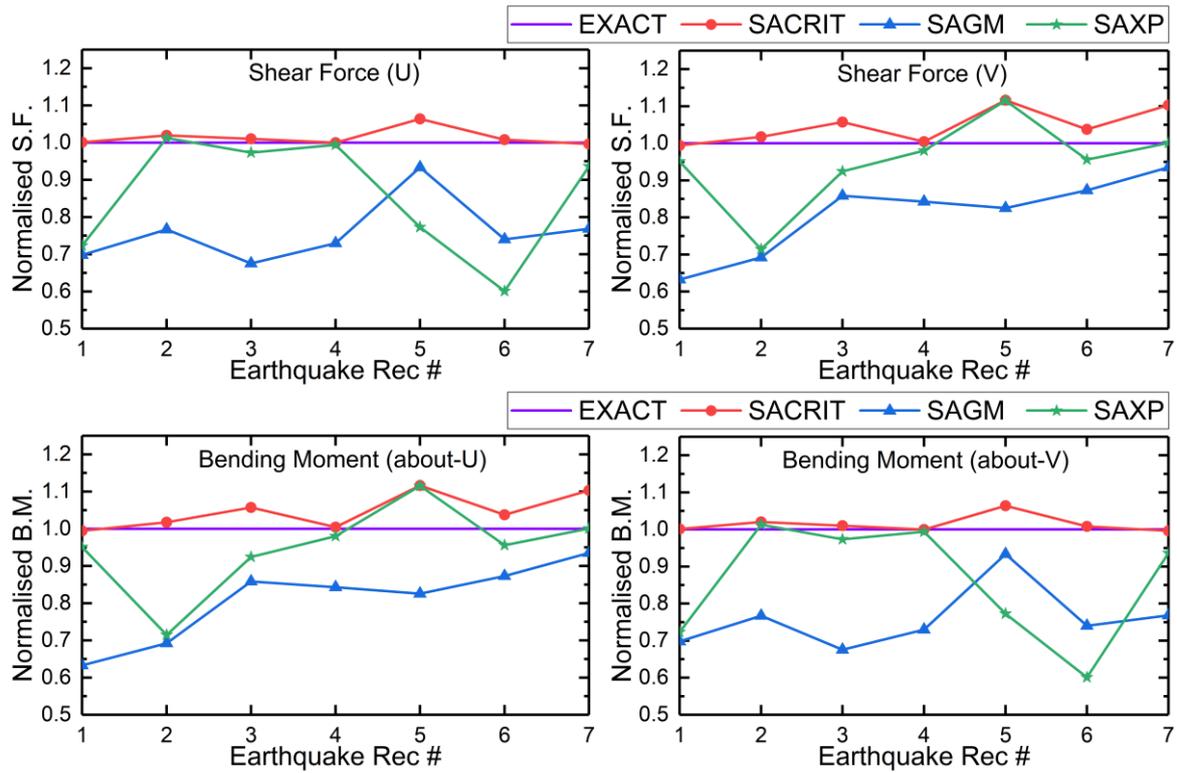


Fig. 9 Same as Fig. 6 for Column-D

Fig. 9 The response amplitudes obtained from three different types of response spectra normalized by the exact time history response amplitudes plotted versus the seven different input excitations for Column-D (continued from previous page)

The use of the geometric mean response spectrum, SAGM, underestimates all the response quantities to varying extents for all the seven cases of input excitations up to a maximum of 40%. The use of the major principal response spectrum, SAXP, also underestimates the response amplitudes significantly in the majority of cases. As the application of the major principal response spectrum along both the selected directions in combination with the SRSS rule can be considered equivalent to finding the maximum values of the response amplitudes by the CQC3 rule for all possible selections of the two principal axes of the example irregular building, the common practice of using the principal response spectra and critical direction of input excitation cannot be considered adequately safe for the multi-component response of irregular buildings. This is because the principal direction does not remain fixed during the complete duration of ground motion and hence the response spectra of principal components are not necessarily the true principal spectral amplitudes at all the natural periods, which may occur at different times. On the other hand, the critical spectral amplitudes represent the possible maximum response of an SDOF system under simultaneous action by two horizontal components of excitation without resolving the two components along any single direction. The use of SACRIT is thus able to provide the most accurate response amplitudes compared to the time history results. Also, this provides a much more convenient alternative to the prevalent use of the principal response spectra with the CQC3 combination rule.

## CONCLUSIONS

The major conclusions arrived at from the present study can be summarized as follows:

1. No specific method exists to estimate the maximum values of the various response quantities of an irregular building with no fixed principal axes under multi-component response spectra.
2. This paper has proposed the use of a newly defined critical response spectrum for both the horizontal components of motion and the SRSS combination rule to obtain very accurate estimates of the response amplitudes of irregular buildings without well defined structural axes.

3. The proposed method has been shown to provide the maximum values of various response quantities of a simple steel irregular building in excellent agreement with the exact time history estimates for seven different sets of the recorded accelerograms of the two horizontal components with widely differing characteristics. The estimates obtained are in general within 5% of the exact response amplitudes.
4. The corresponding response amplitudes based on the commonly used principal and the geometric mean response spectra are found to be significantly underestimated in the majority of cases and are thus unable result in adequately safe design.
5. The theoretical basis of the proposed combination rule for irregular buildings is quite generalized without any simplifying assumptions about the buildings to which can be applied. Illustrative numerical results in the present study are computed for a simple building only for the reasons of brevity, because the inaccuracies in mode superposition results for each component of motion may make it difficult to judge the accuracy of the proposed multi-component combination rule in a realistic way.
6. To arrive at the optimum and safe design of irregular buildings under the simultaneous action of two horizontal components of ground motion, it is thus recommended to replace the currently used CQC3 rule with the proposed SRSS rule with the critical response spectrum, SACRIT, introduced in the present paper. Also, this method is much simpler for implementation in practical design applications compared to the CQC3 rule.

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