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ESTIMATION OF NORMAL MODES VIA SYNCHROSQUEEZED TRANSFORM

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ABSTRACT

The identification of mode shapes is an important but challenging task of vibration based system identification procedures. We propose a simple and robust procedure for extracting mode shape information from the vibration signatures of structures with particular reference to the earthquake excitation cases. A wavelet based synchrosqueezed transform technique is used to decompose the measured vibration response into individual modal response components. The modal frequencies and mode shapes are obtained from the analysis of these isolated modal components. The proposed scheme is validated by considering the response of UCLA Factor Building for a suite of three different ground motions.

KEYWORDS: Blind Source Separation, Modal Identification, System Identification, Synchrosqueezed Transform, Wavelet Transform

INTRODUCTION

Civil engineering systems comprise of massive structures, which have to withstand a variety of dynamic loads like wind, earthquakes, traffic, etc. Continuous operation of these systems, including several life line facilities, requires rigorous maintenance regime and also a mechanism to ascertain any changes in structural properties during regular operating conditions and also in extreme events such as strong earthquakes, high speed winds, etc. Several methods based on the analysis of vibration signatures of structures for assessing its health have been proposed in recent times to detect and identify damage in a structural system. The aim is to provide an early indication of physical damage for facilitating preventive maintenance (Doebling et al., 1996).

System identification is the process of estimating structural system parameters (physical, ormodal) through analysis of input and/or output data (Shiradhonkar and Shrikhande, 2011; Shrikhande, 2014). Due to difficulty in recording the input data in several instances such as wind, or traffic induced vibrations, some output only system identification techniques have been developed, namely, random decrement (Ibrahim, 1977), natural excitation (James III et al., 1993), eigensystem realization algorithm (Juang and Pappa, 1985), subspace identification (van Overschee and de Moor, 2012) frequency domain decomposition (Brincker et al., 2001), and a few wavelet-based techniques (Staszewski, 1997; Lardies and Gouttebroze, 2002; Yang and Nagarajaiah, 2015). Hilbert-Huang transform (HHT) has also been used for iterative empirical mode decomposition to extract intrinsic mode functions from the measured system response (Huang et al., 1998). A wavelet based Hilbert-Huang transform (HHT) technique has also been used in modal identification (Yang et al., 2003a, b).

While most of the processes provide a robust estimation of the natural frequency and damping, the estimation of mode shapes has remained a challenging proposition. The problem of modal identification is structurally similar to the problem of blind source separation (BSS), wherein a set of independent sources are sought to be separated from an uncertain mixture of these sources as shown in Figure 1. The general BSS problem statement can be defined as:

$$\boldsymbol{x}(t) = \boldsymbol{A}\,\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

where, $\boldsymbol{x}(t)$ is the vector of observed output, $\boldsymbol{s}(t)$ is the vector of independent source signals, $\boldsymbol{A}$ is the mixing matrix, and $\boldsymbol{n}(t)$ represents the measurement noise.

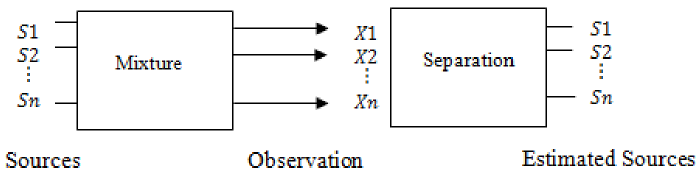


Fig. 1 Basic blind source separation framework

The BSS technique has shown its application in the various field, like artificial intelligence, image processing, biomedical engineering, telecommunication and structural dynamics. Application of BSS in structural dynamics identification has been studied earlier based on independent component analysis (ICA) (Hyvarinen and Oja, 2000; Comon, 1994), second order blind identification (SOBI) (Belouchrani et al., 1997) and a complexity pursuit (CP) as a generalization of the SOBI procedure (Antoni et al., 2017). Modal identification by using BSS has been considered for a variety of structural systems with varying degrees of success, particularly with reference to the estimation of mode shapes (Zang et al., 2004; Kerschen et al., 2007; Poncelet et al., 2007; Zhou and Chelidze, 2007; Hazra and Narasimhan, 2009; Hazra et al., 2010, 2012; Sadhu et al., 2012; Sadhu and Hazra, 2013; Sadhu et al., 2017).

We present a new BSS approach to modal identification based on synchrosqueezed transform (SST) in a non-statistical framework for empirical mode decomposition. The empirical mode decomposition scheme, originally proposed by Huang et al. (1998), is designed to extract slowly varying nearly harmonic components meeting the criteria for existence of instantaneous frequency based on Hilbert-Huang transform (HHT). The suggested process, however, is an iterative numerical sifting procedure with no connection to the mechanics of the problem. The decomposition, therefore, is not unique and individual components may not be representative of the system properties. The synchrosqueezed trans-form based empirical mode decomposition proposed by Daubechies et al. (2011) allows for unambiguous extraction of slowly varying harmonic components of different instantaneous frequencies which are representatives of the underlying physical system. This wavelet based tool allows for extraction of independent sources from the observed data (Daubechies et al., 2011; Brevdo et al., 2011; Thakur et al., 2013) and we use it for extraction of modal responses from the recorded vibration signature. We propose an elegant and robust procedure to extract the mode shapes from the separated modal response components. The modal parameters of UCLA Factor Building (Skolnik et al., 2006) are identified for validation of the proposed scheme using the noisy response data for a suite of earthquake time histories.

BLIND SOURCE SEPARATION AND MODAL IDENTIFICATION

The governing differential equation of structural dynamics is given as:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{f} \tag{2}$$

where, $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and the stiffness matrices, $\boldsymbol{u}$ denotes the displacement response, $\boldsymbol{f}$ is the excitation, and a dot indicates time derivative. The system response can be given by a linear combination of the normal modes of the system:

$$\boldsymbol{u}(t) = \boldsymbol{\Phi}\, \boldsymbol{q}(t) = \textstyle\sum_{i=1}^{n} \phi_i\, q_i(t) \tag{3}$$

where, $\boldsymbol{q}$ denotes modal coordinates, and $\boldsymbol{\Phi}$ is the mode shape matrix. Kerschen et al. (2007) proposed that modal coordinates vectors $\boldsymbol{q}(t)$ can be viewed as independent sources, if the ratios of system natural frequencies are not integers. Thus the modal expansion of system response Eq. (3) can be seen as being analogous to the BSS statement Eq. (1) with modal coordinates $\boldsymbol{q}$ being proportional to virtual sources s regardless of the number and type of the physical excitation force and, the mixing matrix $\boldsymbol{A}$ is similar to the mode shape matrix $\boldsymbol{\Phi}$. The modal parameters (natural frequencies and modal damping) are identified from the post-processing of the identified source time histories, and the mode shapes are provided by the columns of the mixing matrix.

For a base excited structure, such as during earthquake excitation, the equations of motion are given in terms of the relative displacement of the mass with respect to the base:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = -\boldsymbol{M}\boldsymbol{R}\ddot{\boldsymbol{u}}_g \tag{4}$$

where, $\ddot{\boldsymbol{u}}_g$ represents the ground acceleration, $\boldsymbol{R}$ is matrix of rigid body influence coefficients, and $\boldsymbol{u}$ denotes the displacement of inertia points relative to the base. The solution by mode superposition proceeds with the modal expansion as in Eq. (3). However, the relative motion can not be measured directly and the inertial accelerometers used for building instrumentation pickup total acceleration of the point (Celebi, 2002). Therefore the modal expansion theorem is simplified for base excited structures and is given as (Kaloni and Shrikhande, 2016, 2017a):

$$\ddot{\boldsymbol{u}}^t = -\boldsymbol{M}^{-1}(\boldsymbol{K}\boldsymbol{u} + \boldsymbol{C}\dot{\boldsymbol{u}}) \approx -\boldsymbol{\Phi}(\boldsymbol{\Lambda}\boldsymbol{q}) \qquad (5)$$

where, $\ddot{\boldsymbol{u}}^t$ is the total acceleration of inertia points, and $\boldsymbol{\Lambda}$ is the diagonal matrix of system eigenvalues. The weighted modal coordinates are the sources to be identified while the mode shapes can be inferred from the mixing matrix. The weighting of modal coordinates by the respective eigenvalues enhances the energy of higher mode components which helps in the identification of higher modes as well.

1 Synchrosqueezed Transform and Source Separation

Synchrosqueezed transform (SST) is a wavelet based time-frequency analysis tool for identifying different harmonic components in a time series (Daubechies and Maes, 1996). Being non-iterative in nature, it offers significant advantages over the Empirical Mode Decomposition (EMD) algorithm (Huang et al., 1998). Synchrosqueezed transform of a signal involves three steps: (i) computation of wavelet transform, (ii) the synchrosqueezing operation to map the wavelet coefficients from the time-scale plane to time-frequency plane, and (iii) reconstruction of the time-domain signal from its synchrosqueezed transform. It is possible to extract individual harmonics by using ideal band-pass filter with unit gain over the desired frequency band after the synchrosqueezing operation and before the time domain reconstruction. The continuous wavelet coefficients (CWT) $W_y(a,b)$ are estimated as (Daubechies, 1992):

$$W_y(a,b) = \frac{1}{\sqrt{a}}\int y(t)\psi^*\left(\frac{t-b}{a}\right)dt \qquad (6)$$

where $\psi^*(t)$ is the complex conjugate of the mother wavelet $\psi(t)$. The parameters aa and bb control the scale and time-shift of the transform process. The wavelet transform provides information regarding the instantaneous frequencies (Daubechies et al., 2011). The instantaneous frequency $\omega(a,b)$ of the signal y for any point in time-scale plane (a,b) is given by:

$$\omega(a,b) = \begin{cases} \frac{-i}{W_y(a,b)}\frac{\partial[W_y(a,b)]}{\partial b}; & |W_y(a,b)| \neq 0 \\ \infty & |W_y(a,b)| = 0 \end{cases} \qquad (7)$$

After conversion of signals in time-scale plane, synchrosqueezing operation is applied, where wavelet coefficients are mapped from time-scale plane to the time-frequency plane, i.e., *(b,a)→(b,ω(a,b)).* Let us assume that the wavelet coefficient $W_y(a,b)$ is evaluated at discrete values of the scale parameter, a_k, with $(\Delta a)_k = a_{k-}a_k - 1$ and its synchrosqueezed transform $T_y(\omega, b)$ is determined at centre ω_l of the frequency interval $[\omega_l - \Delta\omega/2,\ \omega_l + \Delta\omega/2]$, with $[\omega_l - \omega_{l-1} = \Delta\omega]$, by adding different contributions (Daubechies et al., 2011):

$$T_y(\omega_l, b) = \frac{1}{\Delta\omega}\sum_{a_k:|\omega(a_k,b)-\omega_l|\leq\frac{\Delta\omega}{2}} W_y(a_k,b)a_k^{-3/2}(\Delta a)_k \qquad (8)$$

This procedure facilitates the refinement of the signal representation in time-frequency plane around frequency, ω_l. The final step involves reconstruction of original signal from its synchrosqueezed transform given by:

$$y(t) = \Re\left[\frac{1}{C_\psi}\sum_l T_y(\omega_l, t)\right] = \sum_l \Re(y_l(t)) \qquad (9)$$

where, $\Re(\cdot)$denotes the real part of a complex quantity, and the normalizing constant C_ψ is related to the mother wavelet $\psi(t)$ as:

$$C_\psi = \frac{1}{2}\int_0^\infty \frac{\hat{\psi}^*(\xi)}{\xi}d\xi \qquad (10)$$

with $\hat{\psi}(\xi)$ being the Fourier transform of $\psi(t)$ and the superscript $(\cdot)^*$ indicating complex conjugate. Alternatively, it is possible to extract components in different non-overlapping frequency bands by using ideal band-pass filter with unit gain in the pass-band to extract the relevant information from the time-frequency plane which is then transformed to time-domain by using Eq. (9). These time-domain constituents can be added to recover the original time-domain signal, e.g., $\Re\big(y_l(t)\big)$denotes the constituent signal corresponding to the band centered at frequency ω_l.

2. Identification of Mode Shapes

The harmonic constituents of a vibration signature can be separated by the process of synchrosqueezing transform, band-pass filtering and reconstruction [Eqs. (6–9)]. Thus the recorded vibration signature at any degree of freedom in a structural system can be given by:

$$u_k(t) = \sum_{i=1}^{m} s_{ki}(t) \tag{11}$$

where, $u_k(t)$ represents the response of *k*th degree of freedom, and $s_{ki}(t)$ denotes the *i*th SST identified harmonic component of the *k*th signal. The estimated independent source components are used for the estimation of modal parameters of *n* degree of freedom structural system. Considering all dynamic degrees of freedom, we have:

$$\begin{array}{ccc} u_n(t) & = & s_{n1}(t) + s_{n2}(t) + \cdots + s_{nm}(t) \\ \vdots & = & \vdots \\ u_2(t) & = & s_{n1}(t) + s_{n2}(t) + \cdots + s_{nm}(t) \\ u_1(t) & = & s_{n1}(t) + s_{n2}(t) + \cdots + s_{nm}(t) \end{array} \tag{12}$$

Considering the modal expansion of the dynamic response with *m* number of dominant modes, we have:

$$\begin{bmatrix} u_n(t) \\ \vdots \\ u_2(t) \\ u_1(t) \end{bmatrix} = \begin{bmatrix} \phi_{n1} & \phi_{n2} & \cdots & \phi_{nm} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2m} \\ \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_m(t) \end{bmatrix} \tag{13}$$

Comparing Eqs. (12 and 13), the one-to-one relationship between the separated harmonic components and the mode shape vectors and modal coordinates can be established. Kaloni and Shrikhande [2017b] have proposed estimating the mode shape coefficients as the average of instantaneous ratio of SST identified sources with respect to the corresponding components of the signal recorded at the *n*th degree of freedom. It is possible that for some modes, the amplitude of response at the *n*th degree of freedom might be very small and then using this small amplitude as a normalization factor may lead to numerical instability in the computation of ratio. We now define the mode shape coefficients as the time average of instantaneous ratio (normalization) of SST identified sources with respect to the component with maximum energy for that harmonic:

$$\phi_{ki} = \underset{t}{\text{mean}} \left(\frac{s_{ki}(t)}{s_{pi}(t)}\right) \tag{14}$$

where $s_{pi}(t)$ is the source component having maximum energy for *i*th harmonic, E_i out of all degree of freedom *k*=1, 2, …, n. Where E_i is defined as:

$$E_i \left(= \int_0^t \big(s_{pi}(t)\big)^2 \, dt\right) \tag{15}$$

For the case of seismic excitation through base motion, the estimated mode shape ordinates have a bias due to contribution of ground component in addition to structural vibration. This bias (b) given by Eq. (16) need to remove from the estimated coefficient to get structural mode shapes.

$$b = \underset{t}{\text{mean}} \left({\ddot{u}_g(t)} \big/ {s_{pi}(t)} \right) \tag{16}$$

The extracted mode shape coefficients can be further normalized either to unit length, or mass orthonormal as required. The other modal parameters (natural frequency and modal damping) are estimated from post-processing of maximum energy source components as modal coordinates. The modal frequencies are identified by the peak picking method from the Fourier transform of the identified modal responses. The modal damping can be estimated by curve fitting, or logarithmic decrement applied to the tail portion of the modal response since the response is dominated by the free vibration owing to negligible energy of earthquake excitation in the tail portion.

VALIDATION OF THE PROPOSED METHOD

We consider the dynamic response of UCLA Doris and Louis Factor building (UCLAFB) for validating the proposed method for modal identification. The UCLAFB is a seventeen storey special moment resisting

(SMF) steel frame structure including two ground floors. There are 12 SMF bays in both the EW and NS directions of the building. Due to a slight overhang on the east and west sides of the building, the floor are a for floors 10 through 16 increases by 13.5 percent approximate. The building is symmetric about the East-West axis and slightly asymmetric about the North-South axis. The modal properties of the UCLAFB have been estimated by using the recorded response data to earthquake excitation (Skolnik et al., 2006; Hazra and Narasimhan, 2009; Hazra et al., 2010, 2012; Sadhu and Hazra, 2013). The finite element model of the UCLAFB has been developed using SAP-2000 and has been validated against the reported modal properties of this building. First six translation modes correspond to the natural frequencies ranging from 0.39 Hz to 2.13 Hz. The dynamic response of the UCLAFB finite element model has been computed for a suite of ground motions from Loma Prieta, Hector Mine and Northridge earthquakes. The details of the ground motion suite used for analysis is given in Table 1. The total acceleration response at each floor is considered as the vibration signature recorded by accelerometers for further processing. The source components are estimated from observed data as discussed in the previous section with Morlet wavelet (Thakur et al., 2013) as the mother wavelet for wavelet transform. The response along the EW axis at top floor for the Northridge event, its synchrosqueezed transform and the extracted harmonics are shown in Figure 2. The four identified sources add up to 98% of the total energy of the measured total acceleration response. The modal frequencies are identified from the Fourier transform of these extracted components while the damping ratio can be estimated from the analysis of their tail portions.

Table 1: Ground Motion Details

S. No.	**Event Name, Date and Station**	M_w	***R*** **(km)**	***PGA*** **(g)**	f_c **(Hz)**	**Sampling rate**
1	Loma Prieta, Oct. 18, 1989, Hollister south & Pine	7.1	15	0.37 (H1) 0.18 (H2) 0.19 (V)	1.03	200
2	Northridge, Jan. 17, 1994 Simi Valley Katherine Rd	6.7	48	0.80 (H1) 0.54 (H2) 0.40 (V)	2.04	100
3	Hector Mine, Oct. 16, 1999, Hector	7.1	27	0.27 (H1) 0.33 (H2) 0.14 (V)	3.44	100
M_w: Moment magnitude, *R*: Epicentral distance, *PGA*: Peak ground acceleration, f_c: Characteristic frequency						

The mode shapes are estimated from the identified harmonic components by Eq. (14). These identified mode shapes are compared with those obtained from the calibrated finite element model of the UCLAFB. The modal assurance criterion (MAC) (Allemang, 2003) is used to compute the fidelity of the SST identified mode shapes with respect to the reference mode shapes obtained from the calibrated finite element model. The modal assurance criteria is defined as:

$$\text{MAC} = \frac{|\hat{\phi}^{\mathrm{T}}\phi|^2}{(\hat{\phi}^{\mathrm{T}}\hat{\phi})(\phi^{\mathrm{T}}\phi)} \tag{17}$$

where, $\hat{\phi}$ and ϕ are the estimated and reference mode shape vector respectively. The MAC value ranges from 1 to 0 with values close to unity representing a good correlation between the identified and reference mode shapes.

The mode shapes estimated by the proposed SST-based procedure and those from the analytical model are shown in Figure 3 for the Northridge event with the respective MAC values. These correspond to the worst identification with respect to the MAC values. The agreement between the identified mode shapes with the mode shapes of the analytical model are much better for other ground motions considered in this study. Their respective MAC values are listed in Table (2) and the identified modal frequencies are shown in Table (3). It is possible to identify up to three modes in each direction with high fidelity as indicated by the high MAC values. The vibration energy is smeared across a wider frequency band for higher frequencies and it is difficult to isolate a single harmonic component from the synchrosqueezed transform as seen in Figure 2. The estimated mode shapes for higher modes are therefore less strongly correlated with the mode shapes of analytical model. This is due to the lack of adequate energy in the ground motion in the requisite frequency band for exciting the higher modes of vibration.

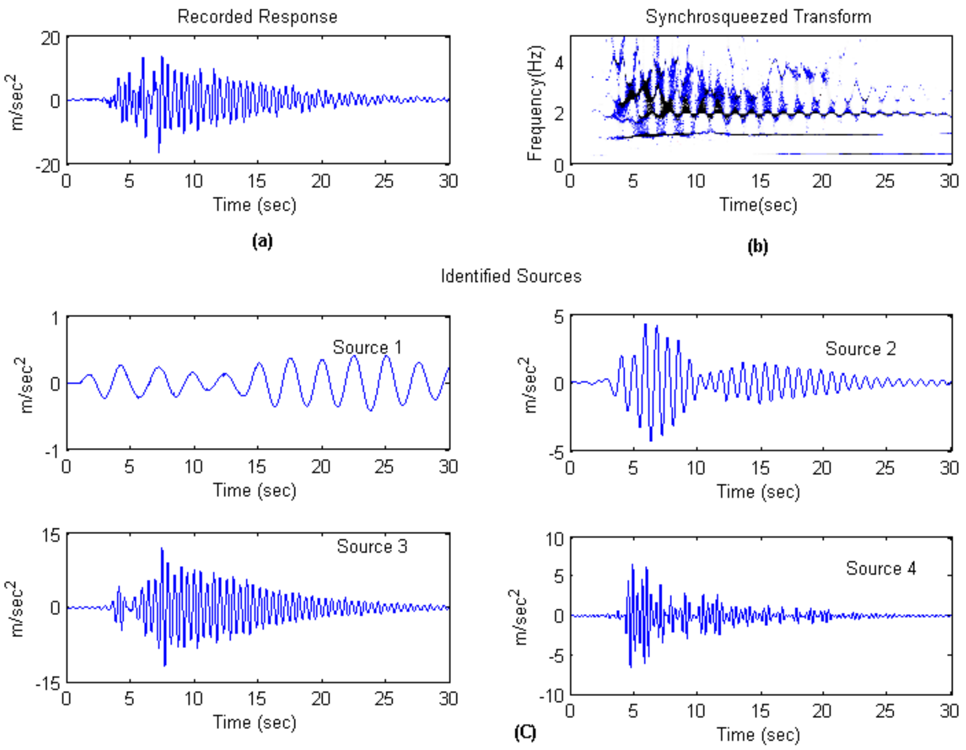


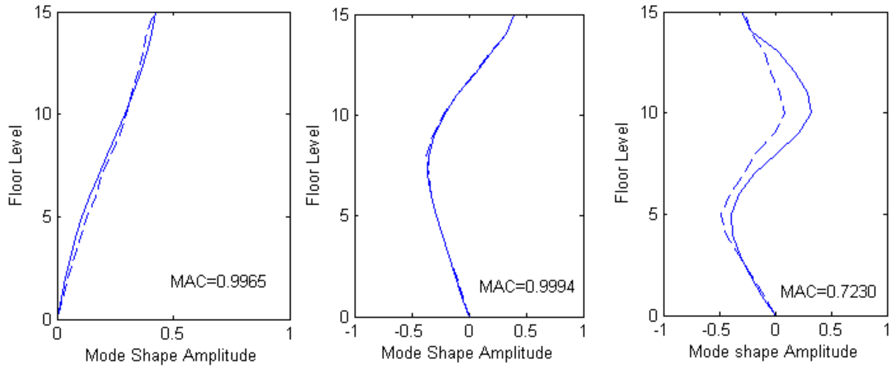
Fig. 2 Northridge event: (a) measured response at the top floor, (b) its synchrosqueezed transform, and (c) the extracted harmonic components

Table 2: Modal assurance criteria (MAC) values for different earthquake in NS and EW direction

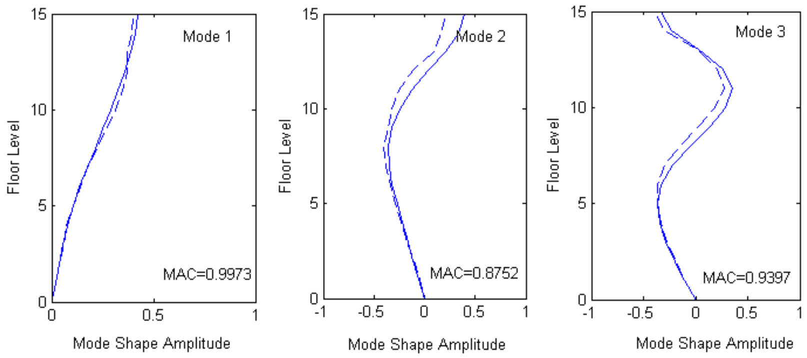
Ground Motion	Mode 1		Mode 2		Mode 3	
	NS	EW	NS	EW	NS	EW
Loma Prieta	0.9975	0.9990	0.7485	0.9856	0.8114	0.7765
Northridge	0.9965	0.9973	0.9994	0.8752	0.7230	0.9397
Hector Mine	0.9991	0.9990	0.7794	0.9228	0.9326	0.8954

Table 3: Natural frequencies (in Hz)

Ground Motion	Mode 1		Mode 2		Mode 3	
	NS	EW	NS	EW	NS	EW
Loma Prieta	0.400	0.38	1.250	1.134	2.118	1.915
Northridge	0.402	0.390	1.240	1.160	2.121	1.961
Hector Mine	0.397	0.400	1.236	1.169	2.120	1.964
Analytical	0.400	0.390	1.220	1.160	2.130	1.950



(a) NS Direction



(b) EW Direction

Fig. 3 Identified mode shapes for the Northridge event (solid: Analytical and dashed: SST-based identification)

1 Robustness to Noisy Data

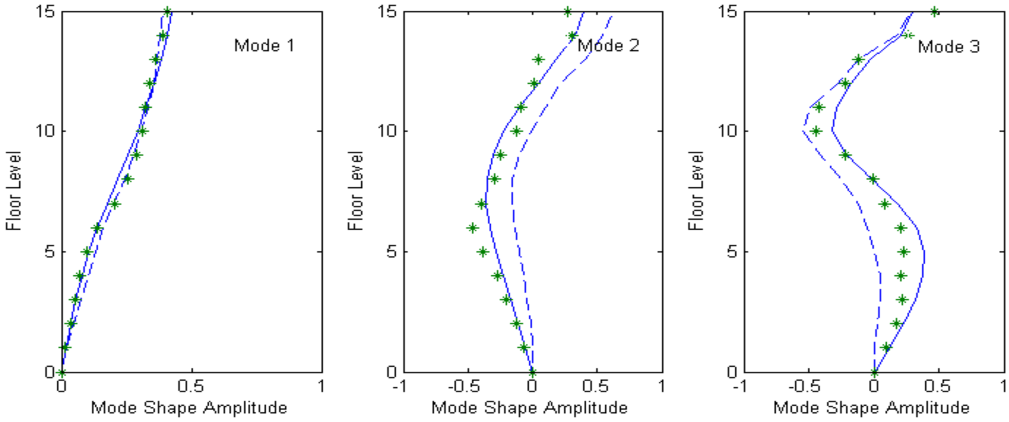
The robustness of SST-based modal identification procedure for noisy measurements is verified by adding a Gaussian white noise to analytically computed acceleration time histories at each level for all example cases considered in this study to have asignal to noise ratio (SNR) of 13 dB. The mode shapes are identified for the noisy measurements and the modal assurance criterion computed for noisy data are given in Table 4. There is a significant performance degradation for identification of mode shapes beyond the first mode. The noisy measurements are then processed through a wavelet-based denoising procedure before performing the synchrosqueezed transform and subsequent modal identification. A major improvement in the quality of the estimated mode shapes after noise removal can be seen in the Table 4. The mode shapes extracted from a noisy data for Northridge event are shown in Figure 4. The effect of noise is most significant in the Northridge case and the improvement after denoising is maximum in this case as well. The Northridge event is relative short duration event and a lower sampling rate produces fewer data points for discrete time processing of the signals. Fewer data points for processing generally lead to more errors in processing.

Table 4: MAC values for mode shapes identified from noisy data and after denoising

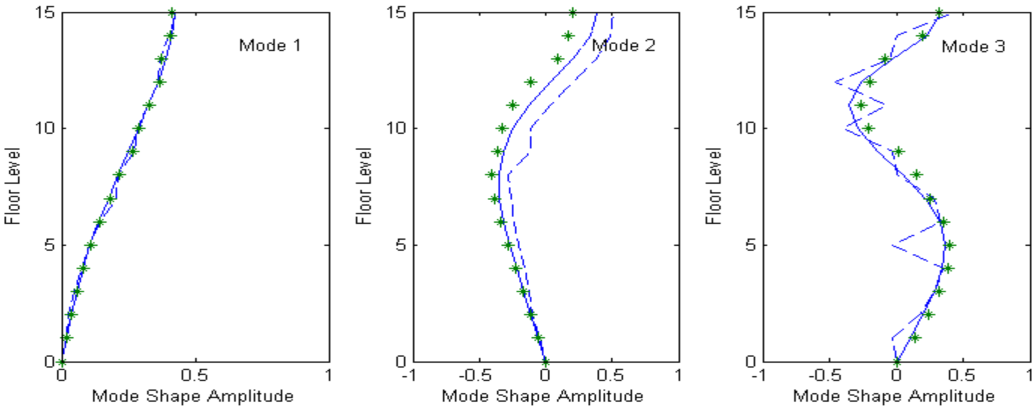
Ground Motion	MAC	Noisy Data		Denoised Data	
		NS	EW	NS	EW
Loma Prieta	Mode 1	0.9937	0.9997	0.9969	0.9999
	Mode 2	0.9873	0.8503	0.9633	0.9559
	Mode 3	0.9818	0.9783	0.9793	0.9794
Northridge	Mode 1	0.9935	0.9976	0.9949	0.9992
	Mode 2	0.5491	0.8010	0.9199	0.8752
	Mode 3	0.3539	0.6543	0.8531	0.9362
Hector Mine	Mode 1	0.9991	0.9994	0.9984	1.000
	Mode 2	0.9722	0.7071	0.9909	0.9089
	Mode 3	0.8373	0.5707	0.9447	0.9038

CONCLUSIONS

A blind modal identification scheme based on synchrosqueezed transform is developed within a non-statistical framework. Individual modal components are extracted from the measured acceleration response. Modal parameters are estimated from the further processing of these identified modal components. The validity of the proposed scheme is ascertained using simulation results of UCLA factor building from different earthquake records. The robustness of the proposed procedure for modal identification with noisy data is studied and it is shown that a simple denoising step enhances the fidelity of the modal identification. The proposed SST based blind modal identification can also provide an independent check and calibration for judging the suitability of chosen model structure in the model based system identification procedures, such as, eigensystem realization, subspace identification, etc.



(a) NS Direction



(b) EW Direction

Fig. 4 Identified mode shapes for the Northridge event (solid: Analytical, dashed: SST-based identification and Star: Denoised data)

REFERENCES

1. Allemang, R.J. (2003). “The Modal Assurance Criterion-Twenty Years of Use and Abuse”, Sound and Vibration, pp. 14-21.
2. Antoni, J., Castiglione, R. and Garibaldi, L. (2017). “Interpretation and Generalization of Complexity Pursuit for the Blind Separation of modal Contributions”, Mechanical Systems and Signal Processing, Vol. 85, pp. 773-788.
3. Belouchrani, A., Abed-Meraim, K., Cardoso, J-F. and Moulines, E. (1997). “A Blind Source Separation Technique Using Second-Order Statistics”, IEEE Transactions on Signal Processing, Vol. 45, pp. 434-444.
4. Brevdo, E., Fučkar, N.S., Thakur, G. and Wu, H-T. (2011). “The Synchrosqueezingal-Gorithm: Arobust Analysis Tool for Signals with Time-Varying Spectrum”, Technical Report arXiv:1105.0010, URL http://cds.cern.ch/record/1348072.

5. Brincker, R., Zhang, L. and Andersen, P. (2001). "Modal Identification of Output-Only Systems Using Frequency Domain Decomposition", Smart Materials and Structures, Vol. 10, pp. 441.
6. Celebi, M. (2002). "Seismic Instrumentation of Buildings (with Emphasis on Federal Buildings)", Technical Report 0-7460-68170, United States Geological Survey, Menlo Park, California, U.S.A.
7. Comon, P. (1994). "Independent Component Analysis, a New Concept?" Signal Processing, Vol. 36, pp. 287–314.
8. Daubechies, I. (1992). "Ten Lectures on Wavelets", Society for Industrial and Applied Mathematics, Philadelphia, U.S.A., Vol. 61.
9. Daubechies, I. and Maes, S. (1996). "A Nonlinear Squeezing of the Continuous Wavelet Transform Based on auditory Nerve Models", In Aldroubi A and Unser A (eds.) Wavelets in Medicine and Biology, CRC Press, pp. 527–546.
10. Daubechies, I., Lu, J. and Wu, H.-T. (2011). "Synchrosqueezed Wavelet Transforms: An Empirical Mode Decomposition-Like Tool", Applied and Computational Harmonic Analysis, Vol. 30, pp. 243–261.
11. Doebling, S.W., Farrar, C.R., Prime, M.B. and Shevitz, D.W. (1996). "Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in their Vibration Characteristics: A Literature Review", Technical Report LA-13070-MS, Los Alamos National Laboratory.
12. Hazra, B. and Narasimhan, S. (2009). "Wavelet-based blind identification of the UCLA Factor building using ambient and earthquake responses", Smart Materials and Structures, Vol. 19, pp. 025-005.
13. Hazra, B., Roffel, A.J., Narasimhan, S. and Pandey, M.D. (2010). "Modified Cross-Correlation Method for the Blind Identification of Structures", Journal of Engineering Mechanics, Vol. 136, pp. 889–897.
14. Hazra, B., Sadhu, A., Roffel, A.J., Paquet, P.E. and Narasimhan, S. (2012). Underdetermined Blind Identification of Structures by Using the Modified Cross-Correlation Method", Journal of Engineering Mechanics, Vol. 138, pp. 327–337.
15. Huang, N.E., Shen, Z., Long, S.R., Wu, M.C., Shih, H.H., Zheng, Q., Yen, N.-C., Tung, C.C. and Liu, H.H. (1998). "The Empirical Mode Decomposition and the Hilbert Spectrum for Nonlinear and Non-Stationary Time Series Analysis", Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, Vol. 454, pp. 903–995.
16. Hyvarinen, A. and Oja, E. (2000). "Independent Component Analysis: Algorithms and Applications", Neural Networks, Vol. 13, pp. 411–430.
17. Ibrahim, S.R. (1977). "Random Decrement Technique for Modal Identification of Structures", Journal of Spacecraft and Rockets, Vol. 14, pp. 696–700.
18. James III, G.H., Carne, T.G. and Lauffer, J.P. (1993). "The Natural Excitation Technique (NExT) for Modal Parameter Extraction from Operating Wind Turbines", Technical Report SAND–92-1666, Sandia National Laboratories, Al-buquerque, NM (United States).
19. Juang, J.-N. and Pappa, R.S. (1985). "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction", Journal of Guidance, Control and Dynamics, Vol. 8, pp. 620–627.
20. Kaloni, S. and Shrikhande, M. (2016). "Blind Source Separation Based System Identification of Earthquake Excited Structures", In Proceedings of 8th ISSS National Conference on MEMS, Smart Materials, Structures and Systems, September 28-30, Indian Institue of Technology Kanpur.
21. Kaloni, S. and Shrikhande, M. (2017a). "Damage Detection in a Structural System Via Blind Source Separation", In Proceedings of 16th World Conference on Earthquake Engineering, January 9-13, Santiago, Chile.
22. Kaloni, S. and Shrikhande, M. (2017b). "Output only System Identification Based on Synchrosqueezed Transform", Procedia Engineering, Vol. 199, pp. 1002–1007, doi: DOI:10.1016/j.proeng.2017.09.235.
23. Kerschen, G., Poncelet, F. and Golinval, J-C. (2007). "Physical Interpretation of Independent Component Analysis in Structural Dynamics", Mechanical Systems and Signal Processing, Vol. 21, pp. 1561–1575.
24. Lardies, J. and Gouttebroze, S. (2002). "Identification of Modal Parameters Using the Wavelet Transform", International Journal of Mechanical Sciences, Vol. 44, pp. 2263–2283.

25. Poncelet, F., Kerschen, G., Golinval, J-C. and Verhelst, D. (2007). "Output-Only Modal Analysis Using Blind Source Separation Techniques", Mechanical Systems and Signal Processing, Vol. 21, pp. 2335–2358.
26. Sadhu, A. and Hazra, B. (2013). "A novel Damage Detection Algorithm Using Time- Series Analysis-Based Blind Source Separation", Shock and Vibration, Vol. 20, pp. 423–438.
27. Sadhu, A., Hazra, B. and Narasimhan, S. (2012). "Blind Identification of Earthquake-Excited Structures", Smart Materials and Structures, Vol. 21, pp. 1–12.
28. Sadhu, A., Narasimhan, S. and Antoni, J. (2017). "A Review of Output-Only Structural Mode Identification Literature Employing Blind Source Separation Methods", Mechanical Systems and Signal Processing, Vol. 94, pp. 415–431.
29. Shiradhonkar, S.R. and Shrikhande, M. (2011). "Seismic Damage Detection in a Building Frame Via Finite Element Model Updating", Computers & Structures, Vol. 89, pp. 2425–2438.
30. Shrikhande, M. (2014). "Finite Element Method and Computational Structural Dynamics", PHI Learning Pvt. Ltd., New Delhi, India, ISBN 9788120349957.
31. Skolnik, D., Lei, Y., Yu, E. and Wallace, J.W. (2006). "Identification, Model Updating, and Response Prediction of an Instrumented 15-Story Steel-Frame Building", Earthquake Spectra, Vol. 22, pp. 781–802.
32. Staszewski, W.J. (1997). "Identification of Damping in MDOF Systems Using Time- Scale Decomposition", Journal of Sound and Vibration, Vol. 203, pp. 283–305.
33. Thakur, G., Brevdo. E., Fučkar, N.S. and Wu, H-T. (2013). "The Synchrosqueezing Algorithm for Time-Varying Spectral Analysis: Robustness Properties and New Paleoclimate Applications", Signal Processing, Vol. 93, pp. 1079–1094.
34. Van, Overschee P. and de Moor, B.L. (2012). "Subspace Identification for Linear Systems: Theory–Implementation–Applications", Springer Science & Business Media.
35. Yang, J.N., Lei,Y., Pan, S. and Huang, N.E. (2003a). "System Identification of Linear Structures Based on Hilbert–Huang Spectral Analysis", Part 1: Normal Modes, Earthquake Engineering & Structural Dynamics, Vol. 32, pp. 1443–1467.
36. Yang, J.N., Lei, Y., Pan, S. and Huang, N.E. (2003b). "System Identification of Linear Structures Based on Hilbert–Huang Spectral Analysis", Part 2: Complex Modes, Earthquake Engineering & Structural Dynamics, Vol. 32, pp. 1533–1554.
37. Yang, Y. and Nagarajaiah, S (2015). "Output-Only Modal Identification by Compressed Sensing: Non-Uniform Low-Rate Random Sampling", Mechanical Systems and Signal Processing, Vol. 56, pp. 15–34.
38. Zang, C., Friswell, M.I. and Imregun, M. (2004). "Structural Damage Detection Using Independent Component Analysis", Structural Health Monitoring, Vol. 3, pp. 69–83.
39. Zhou, W. and Chelidze, D. (2007). "Blind Source Separation Based Vibration Mode Identification", Mechanical Systems and Signal Processing, Vol. 21, pp. 3072–3087.