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ANALYTICAL PROSPECTS IN NONLINEAR STRUCTURAL DYNAMICS

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ABSTRACT

Currently, the dynamic response of nonlinear structures is generally determined by numerical integration of the incremental coupled second order differential equations of motion involving their tangent stiffness matrices. However, the dominant paradigm being computational, the initial value problems (IVPs) being solved are rarely stated in an analytical form. Three new formulations of analytical IVPs for nonlinear structures are presented in this Paper. Firstly, it is argued that only the IVPs based upon coupled third order differential equations of motion are proper for nonlinear MDOF dynamical systems. Secondly, it is shown that, for homogeneous dynamical systems, coupled second order differential equations are applicable. Finally, for general nonlinear dynamical systems, it is proposed to formulate the IVPs using decoupled nonlinear second order differential equations. Theoretical significance of these three analytical prospects in nonlinear structural dynamics is discussed.

KEYWORDS: Nonlinear Structural Dynamics, Third-Order Equations of Motion, Homogeneous Systems, Decoupled MDOF Systems

INTRODUCTION

As in rest of classical mechanics, Newton's second law of motion is the basis of structural dynamics. Starting from their basic formulation in the year 1877 by Rayleigh [1], the matrix form of equations of motion for linear MDOF structures was proposed by Collar, Duncan and Frazer in the third decade of twentieth century [2]. The following second-order coupled linear differential equations of motion for vibrating MDOF linear elastic structures with nodal masses and forces are popular till now in civil, mechanical and aeronautical engineering disciplines:

$$M\ddot{u} + C\dot{u} + Ku = F(t) \tag{1}$$

Here, the symbols M, C and K represent the constant mass, damping and stiffness matrices of the structure while $u(t)$ represent its instantaneous nodal displacement response to the applied forcing function $F(t)$. As usual, superposed dot implies time derivative. For complete formulation of initial value problem (IVP), the initial nodal displacements $u(0)$ and velocities $\dot{u}(0)$ are required to be specified. Under the assumption of classical damping, the damping matrix of these structures is generally formulated in terms of their mass and stiffness matrices. Linear mode theory valid for these structures is well-developed [3, 4, 5].

Nonlinear dynamical systems theory having applications across the disciplines deals primarily with SDOF systems. Duffing oscillators are known to exhibit sub-harmonic resonances, extreme sensitivity to initial conditions, exciting forces and system parameters, chaotic motions, etc. Self-excited vibrations are predicted for van der Pol oscillators. Analytical solutions being rarely possible, the nonlinear dynamic response is generally predicted by numerical integration of the equations of motion [6].

The theory of MDOF nonlinear dynamical systems is not so well developed. According to Lyapunov, for an n-DOF conservative system without internal resonances, there exist at least n periodic solutions about the equilibrium state. These stable periodic motions constitute the normal modes for dynamical systems. Analogous to normal modes in linear dynamical systems, these invariant periodic motions are called nonlinear normal modes for nonlinear systems, even though these modes lack orthogonality and violate the principle of superposition [7, 8]. However, it has been recognised [9] that these nonlinear modes vary with instantaneous total amplitude of vibration. Also, in view of the pure uncoupling of linear modes, the concept of 'mode interactions' is considered by some researchers to be an oxymoron [10].

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