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THE SCALED NON-LINEAR DYNAMIC PROCEDURE: A PRACTICAL TECHNIQUE FOR OVERCOMING LIMITATIONS OF THE NON-LINEAR STATIC PROCEDURE

Mark A. Aschheim*, Tjen Tjhin** and Mehmet Inel***

*Civil Engineering Department, Santa Clara University, California, U.S.A.

**Department of Civil and Environmental Engineering, University of Illinois, U.S.A.

***Department of Civil Engineering, Pamukkale University, Denizli, Turkey

ABSTRACT

Non-linear Static Procedures (NSPs) have become widely accepted for use in seismic design and evaluation in recent years. While generally acceptable for peak displacement estimates, the accuracy of the NSPs is poor for quantities that are significantly affected by higher modes. In recent work performed for the ATC-55 project, a new approach was identified for determining those quantities that are significantly affected by higher modes. The Scaled Non-linear Dynamic Procedure (Scaled NDP) is easily used in conjunction with a NSP for performance-based seismic design and evaluation. The Scaled NDP appears to provide a valid basis for establishing design values at stated levels of confidence using the results of non-linear dynamic analysis. These results can be used to evaluate the performance of a given design as well as to determine the strengths required of members in order to ensure that ductile behavior develops. Suggestions are made for the use of the NSP and Scaled NDP in Performance-Based Earthquake Engineering.

KEYWORDS: Performance-Based Earthquake Engineering, Non-linear Analysis, Pushover Analysis

INTRODUCTION

Non-linear static procedures (NSPs) have become well known in the United States, Japan, and elsewhere with the implementation of procedures based on the Capacity Spectrum Method (CSM) in ATC-40 (ATC, 1996) and the Building Standard Law Enforcement Order (MOC, 2000), and the description of the Displacement Coefficient Method (DCM) in FEMA-273 (FEMA, 1997) and FEMA-356 (FEMA, 2000). In these implementations, a non-linear static (pushover) analysis is used to characterize the response of the structure typically by means of a "capacity curve", which is a plot of base shear versus roof displacement. The expected peak displacement, or "target displacement", is determined by means of an "equivalent" single-degree-of-freedom oscillator, whose properties are derived from the capacity curve. Values of various response quantities (e.g. story shears and plastic hinge rotations) are determined as the values computed in the non-linear static (pushover) analysis at the instant in the analysis at which the roof displacement is equal to the estimated (or target) displacement. Modifications being developed in the nearly completed ATC-55 project are expected to improve the accuracy of the target displacement estimates.

The load pattern used in pushover analyses generally is similar to or equal to a first mode pattern. Many researchers (Miranda, 1991; Collins et al., 1996; Cuesta and Aschheim, 2001; Chopra et al., 2003) have shown that such approaches can lead to good estimates of peak displacements. The success of these quasi-first mode pushover approaches can be attributed to the relatively small contribution of higher modes to displacements. This can be understood for elastic response by noting that the vector of peak displacements due to the i^{th} mode, $\mathbf{x}_i$, is given by

$$\mathbf{x}_i = \Gamma_i S_d(T_i)\boldsymbol{\varphi}_i \tag{1}$$

where $S_d(T_i)$ is the spectral displacement associated with the period T_i, Γ_i is the modal participation factor, and $\boldsymbol{\varphi}_i$ is the mode shape for the i^{th} mode. Higher mode contributions to displacements typically are minor because both Γ_i and S_d typically are much smaller for the 2^{nd} and higher modes, relative to their 1^{st} mode values.

Higher modes can contribute more significantly to other quantities, such as story shears and interstory drifts. This can be appreciated by noting for elastic response that the vector of lateral forces $\mathbf{F}_i$ associated with developing the peak displacements, $\mathbf{x}_i$, can be expressed as

$$\mathbf{F}_i = \Gamma_i S_a(T_i)\mathbf{M}\boldsymbol{\varphi}_i \tag{2}$$

where $\mathbf{M}$ is the mass matrix and $S_a(T_i)$ is the spectral pseudo-acceleration associated with the period T_i. This indicates the lateral forces will typically have more substantial contributions from the higher modes, because the shape of the response spectrum will often result in higher mode spectral pseudo-accelerations that are similar to or larger than those of the first mode.

Techniques to account for the contributions of higher modes have been proposed to improve the NSP (e.g. ATC, 1996; Chopra and Goel, 2002; Aydinoglu, 2003; Priestley and Amaris, 2003). These techniques introduce approximations to address the response of non-linear structures, such as considering response in each "mode" independently or in the manner in which response in multiple modes is considered in an incremental fashion. These approximations are absent in the Scaled Non-linear Dynamic Analysis procedure, which inherently represents capacity limits on the demands resulting during the dynamic response of the structural system.

ATC-55 MDOF STUDIES

The Scaled NDP developed from an observation made in studies of MDOF systems for the ATC-55 project, which focused on the development of improved inelastic analysis procedures. The MDOF studies were conducted to illustrate the accuracy of several pushover analysis techniques in relation to the results obtained from non-linear dynamic analysis. Five building models were used, consisting of an 8-story reinforced concrete wall building used as an example in ATC-40, 3- and 9-story steel moment-resistant frames used in the SAC program, and variants in which weak stories were introduced into these frames. Only a sampling of the results obtained for these five buildings is presented here.

The 3- and 9-story steel moment frames were designed and modelled as part of the SAC joint venture. The special moment frames of the "pre-Northridge" designs, developed for Los Angeles, were modelled. A lumped plasticity beam column element (Element 02) was used to model the beams and columns in Drain-2DX. These elements extended along the beam and column centrelines, as shown in Figure 1, for the 9-story frame. Beam column joints were not modelled and rigid-end offsets were set to zero, as was done for the SAC "M1" models. P-Delta effects were considered for all building models, using dead loads in combination with 40% of the design live loads. Tributary gravity load was applied to the frames and the remainder was applied to a "dummy" column used to obtain a truss-bar approximation of P-Delta effects. Rayleigh damping was set equal to 2% of critical viscous damping at the first mode period and at a period of 0.2 seconds, as was done in the SAC program. The fundamental period of the 9-story frame was 1.03 seconds for the model with P-Delta effects incorporated.

The 8-story reinforced concrete shear wall model is based on the Escondido Village building that is described in ATC-40. Typical floor heights are 2.77 m (9′-1″), as shown in Figure 2. A two-dimensional model of the wall was developed using a beam-column fiber element (Element 15) in the Drain-2DX computer program (Powell, 1993). Rayleigh damping (proportional to mass and stiffness) was set equal to 5% of critical viscous damping in the first and fourth modes of vibration. The fundamental period was 0.71 sec, associated with the tangent stiffness of the cracked wall at a base shear equal to 60% of the effective yield strength.

The recorded ground motions were selected to represent motions that potentially could occur at a given site, characterized by NEHRP Site Class C soil conditions (as defined in FEMA (2000b)), magnitudes (M_s) between 6.6 and 7.6, and epicentral distances between 8 and 20 km. To investigate accuracy as a function of drift level, the records were each scaled to achieve a peak roof displacement of 0.5, 2, or 4% of the height for the steel frames and 0.1, 1, and 2% of the height for the concrete wall. These drift levels represent values expected for many steel frame and concrete wall buildings; the lowest value corresponds to elastic response while the other two values represent different degrees of inelastic response.

The pushover techniques consisted of quasi-first mode load vectors (consisting of first mode, inverted triangular, rectangular or uniform, and "code" load vectors, an "SRSS" load vector, and an adaptive first mode vector), as well as a modification to the Multimode Pushover Analysis (MPA) method presented by

Chopra and Goel (2002). The "code" load vector applies lateral loads in proportion to the seismic weight and height of each floor raised to a power *k*, where the exponent *k* depends on the period of the structure, in the manner described in the International Building Code (ICBO, 2000). The SRSS technique applies the lateral forces required to generate a pattern of story shears, which is determined as an SRSS combination of the story shears obtained from elastic modal responses. Three modes were included in the SRSS combination to represent at least 90% of the mass, and modal shears were determined based on an elastic response spectrum. The modified MPA method considers elastic contributions of the 2nd and 3rd modes together with potentially inelastic contributions due to the 1st mode, using an SRSS combination. This modified MPA method was used by Chopra et al. (2004) to estimate frame interstory drifts and was applied independently by Priestley and Amaris (2003) to estimate wall shears; a further modification was introduced to estimate wall moments.

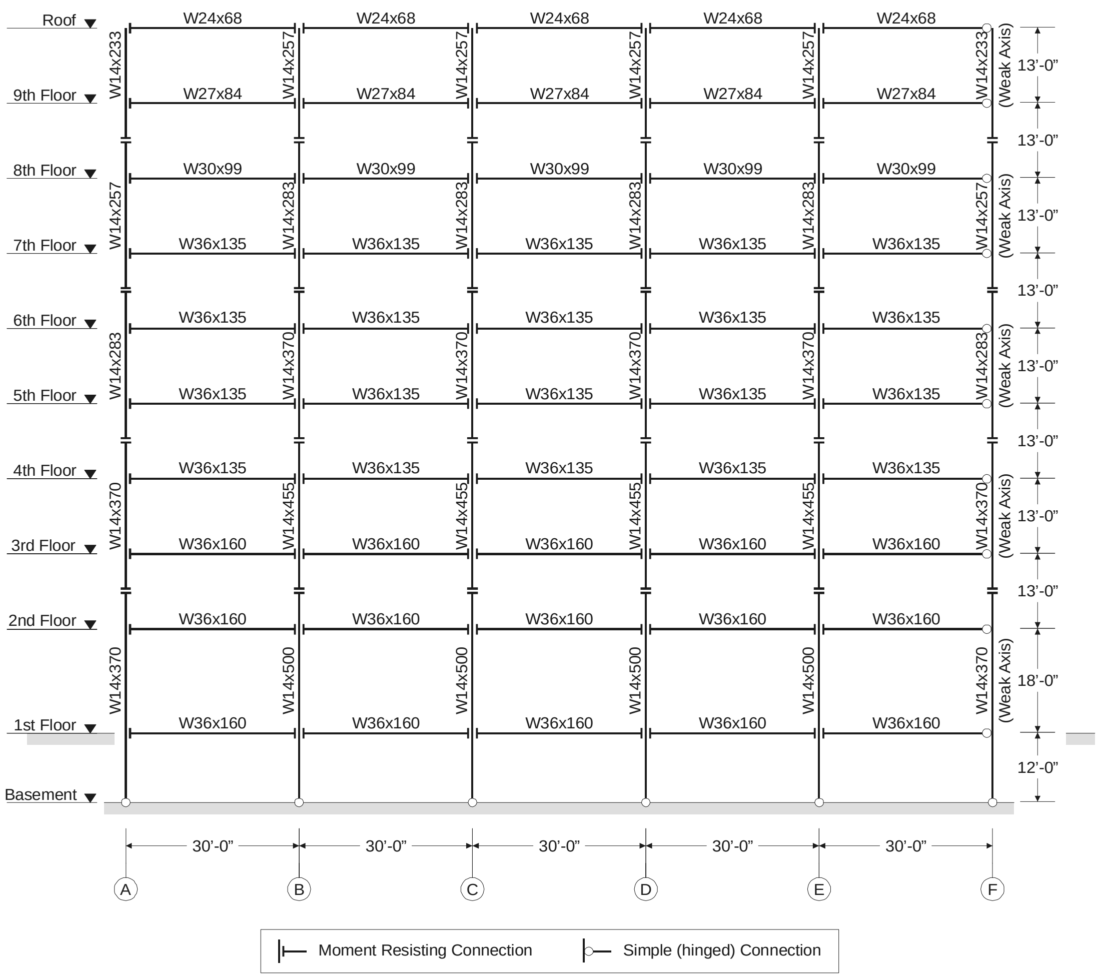
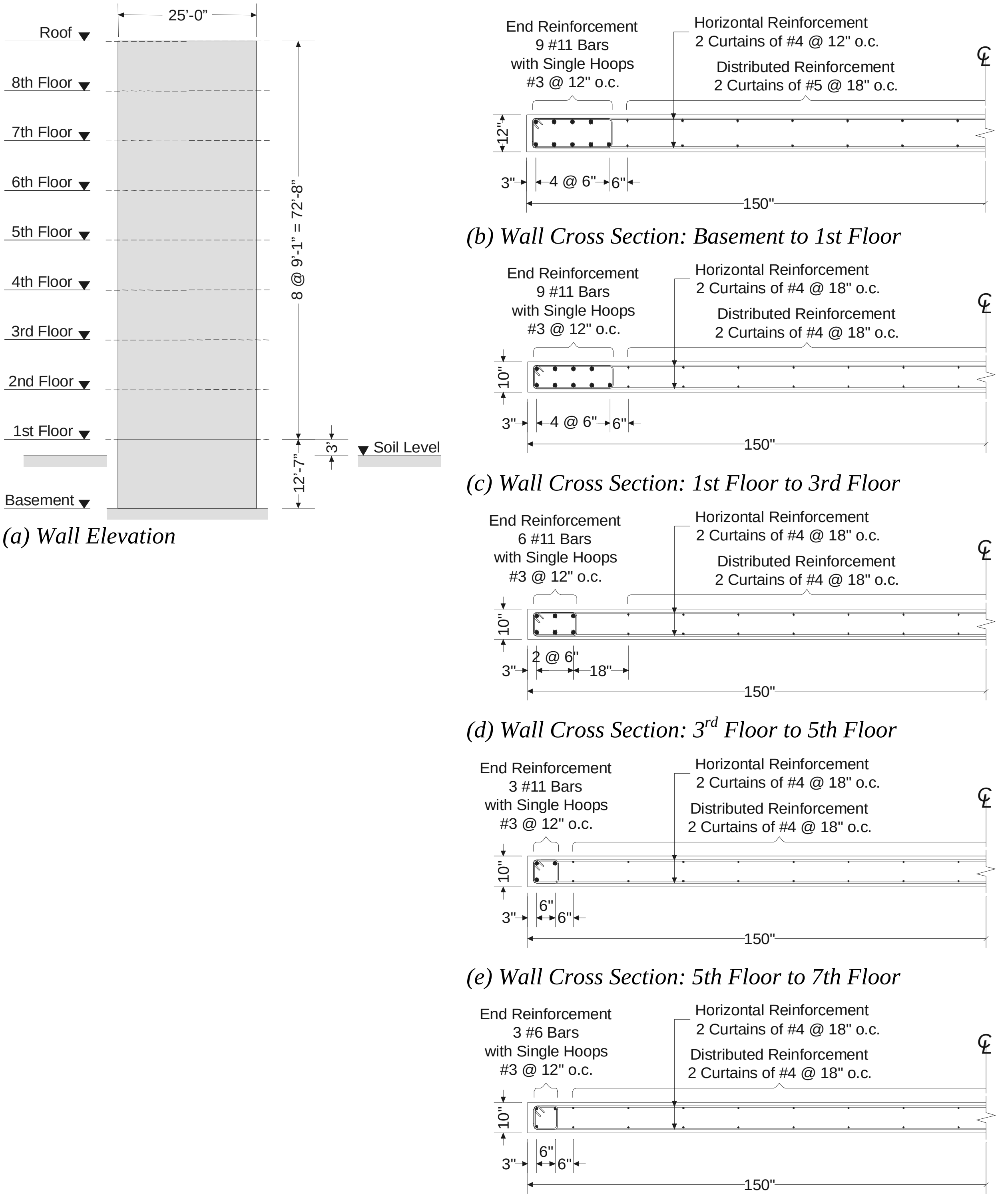


Fig. 1 Elevation view of 9-story frame

The quasi-first mode load vectors were applied until the roof displacement was equal to the pre-determined target displacement, in order to identify differences due to the choice of load vector. Response quantities (e.g. interstory drifts, story shears, and overturning moments) were determined at this displacement for the quasi-first mode load vectors. Higher mode contributions were determined based on the mean of the elastic spectra associated with the ground motions for the particular building and drift level, since the ground motions had been scaled individually to obtain the same predetermined peak roof displacement in the non-linear dynamic analyses. Some results are illustrated in the following paragraphs.

Figure 3 compares estimates of the interstory drifts (Figure 3(a)) and story shears (Figure 3(b)) made using various pushover methods with the range of values computed by non-linear dynamic analysis, for the SAC 9-story steel frame, at a roof drift of 4% of the height of the building. The bar symbol at each floor (or story) indicates the minimum, maximum, mean, and mean plus and minus one standard deviation results obtained from the 11 dynamic analyses; the "+" indicates the median value. Higher modes are reflected in the results obtained from the non-linear dynamic analysis of the yielding system, but are absent from the quasi-first mode pushover results. The more complex multiple mode calculation is often an improvement over the quasi-first mode estimates, but the estimates obtained by this approach were not consistently reliable, with significant errors developing for some cases.



(a) Wall Elevation

(b) Wall Cross Section: Basement to 1st Floor

(c) Wall Cross Section: 1st Floor to 3rd Floor

(d) Wall Cross Section: 3rd Floor to 5th Floor

(e) Wall Cross Section: 5th Floor to 7th Floor

(f) Wall Cross Section: 7th Floor to Roof

Fig. 2 Elevation view and sections of 8-story wall

Figure 4 compares estimates of story shears (Figure 4(a)) and overturning moments (Figure 4(b)) made using various pushover methods with the range of values computed by non-linear dynamic analysis, for the 8-story reinforced concrete wall building, for the roof displacement equal to a value of 2% of the height of the building. Again, the contribution of higher "modes" (also described as "MDOF effects" for non-linear systems) causes the dynamic peaks to be systematically higher than the quasi-first mode pushover estimates.

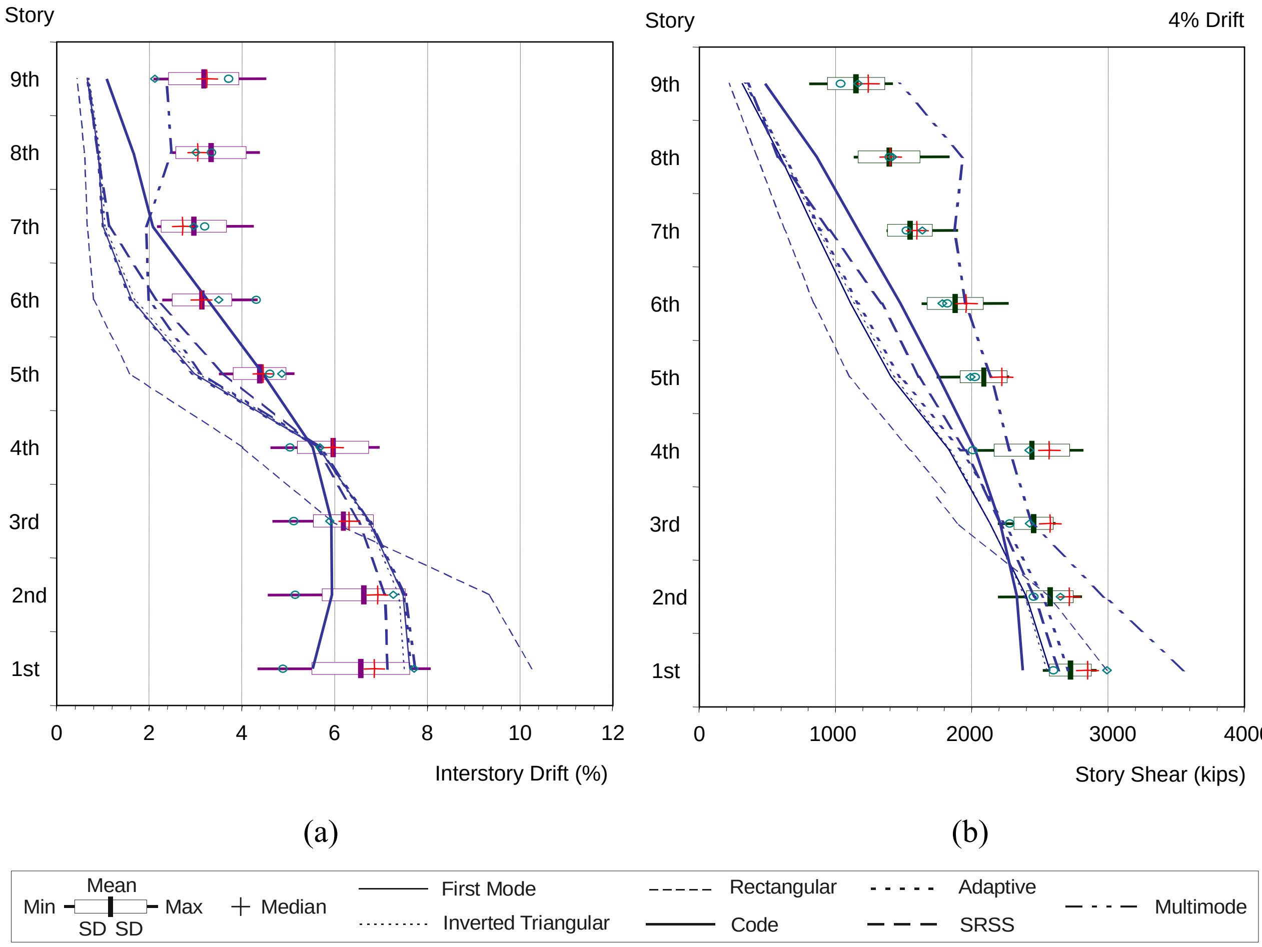


Fig. 3 Comparison of NSP estimates and values computed by non-linear dynamic analysis using 11 ground motion records scaled to achieve a roof drift of 4%, for the 9-story steel frame building: (a) interstory drifts, and (b) story shears (Note: 1 kip = 4.448 kN.)

Also shown in Figures 3 and 4 (with circles and triangles) are peak values obtained using two additional ground motions, each scaled to achieve the same predetermined roof drift in the non-linear dynamic analyses. It can be observed that these results are consistent with the results from the 11 ground motions. Hence, it is observed that for many structures, a single non-linear dynamic analysis provides results of higher fidelity than those obtained with pushover analyses. This observation is the basis of the Scaled NDP described below.

THE SCALED NDP

1. Description of the Method

Step 1 Given a spectrum representative of the site hazard of interest, estimate the peak displacement of the roof (or more generally, a "control point") using the displacement coefficient or capacity spectrum approach. Improved methods developed by the ATC-55 project may be used for this purpose.

Step 2 Select n ground motion records that reflect the characteristics of the hazard (e.g. magnitude, distance, site class) and for each record, conduct a non-linear dynamic analysis, with the record scaled iteratively until the peak displacement of the control point is equal to the estimate determined in Step 1. Extract peak values of the response quantities of interest from the results of each analysis and compute the sample mean, $\bar{x}_n$, of each peak quantity of interest. At least three analyses ($n \geq 3$) are suggested.

Step 3 Although the sample mean is the best estimate of the true mean, sampling error may be present due to the limited number of observations of each quantity. Furthermore, estimates of response quantities may be desired at the mean plus κ standard deviation level and at a desired level of confidence. Thus, the estimate at the mean plus κ standard deviation level may be determined at a desired level of confidence by multiplying the sample mean, $\bar{x}_n$, by $c(1+\kappa\mathrm{COV})$. As shown in the Appendix, the quantity $c(1+\kappa\mathrm{COV})\bar{x}_n$ exceeds the true mean plus κ standard deviation level with confidence level α if the response quantities are normally distributed. In the preceding, c is given by

$$c = \frac{1}{1-\left(\Phi^{-1}(\alpha)\right)\dfrac{\mathrm{COV}}{\sqrt{n}}} \tag{3}$$

where $\Phi^{-1}(\alpha)$ is the value for which the cumulative standard normal probability distribution is associated with a confidence level α. Equation (3) simplifies to $c = 1$ for $\alpha = 50\%$. For $\alpha = 90\%$, Equation (3) simplifies to

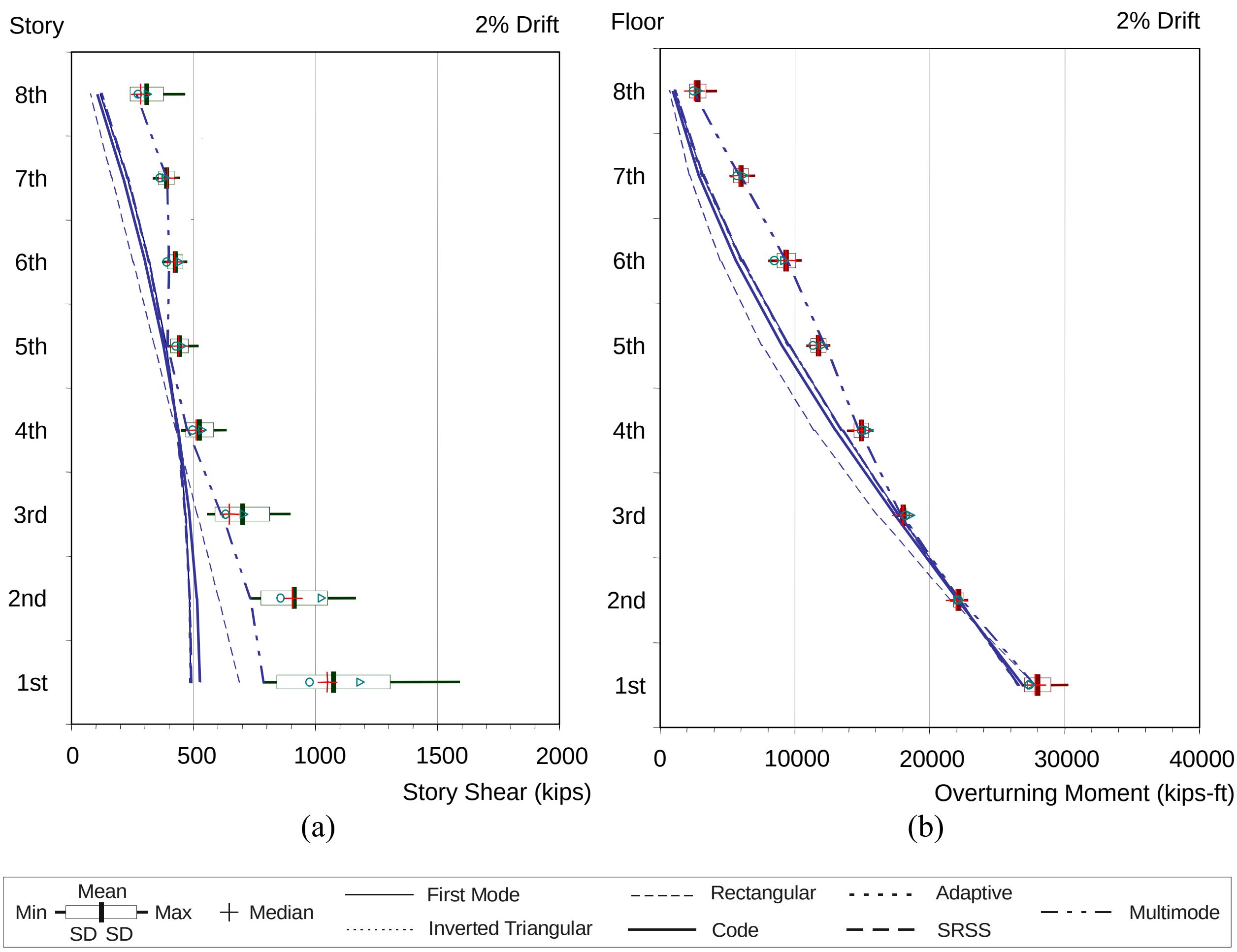


Fig. 4 Comparison of NSP estimates and values computed by non-linear dynamic analysis using 11 ground motion records scaled to achieve a roof drift of 2%, for the 8-story reinforced concrete wall building: (a) story shears, and (b) overturning moments (Note: 1 kip = 4.448 kN and 1 kip-ft = 1.356 kN-m.)

$$c = \frac{1}{1 - 1.28\frac{\text{COV}}{\sqrt{n}}} \tag{4}$$

The coefficient of variation (COV) may be estimated as the sample COV for large samples (perhaps for $n \geq 7$). For smaller sample sizes, it is suggested that the COV be assumed equal to a baseline value of perhaps 0.25 to 0.30, based on the results of the MDOF studies. The term κ assumes a value of zero where estimates of the true mean are sought.

Thus, using Equation (4), there is a 90% probability that $c(1+\kappa\text{COV})\overline{x}_n$ exceeds the true mean plus κ standard deviations, assuming that the response quantities are normally distributed.

Values of c, computed using Equation (4), are provided in Table 1. Table 1 can be used to indicate the number of analyses to run—that is, the point where additional analytical data is of negligible benefit. The derivation of Equation (3) is presented in the Appendix.

Table 1: Values of *c* at the 90% Confidence Level

	Coefficient of Variation (COV)									
n	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
3	1.04	1.08	1.12	1.17	1.23	1.29	1.35	1.42	1.50	1.59
5	1.03	1.06	1.09	1.13	1.17	1.21	1.25	1.30	1.35	1.40
7	1.02	1.05	1.08	1.11	1.14	1.17	1.20	1.24	1.28	1.32
10	1.02	1.04	1.06	1.09	1.11	1.14	1.17	1.19	1.22	1.25
20	1.01	1.03	1.04	1.06	1.08	1.09	1.11	1.13	1.15	1.17
100	1.01	1.01	1.02	1.03	1.03	1.04	1.05	1.05	1.06	1.07

The non-linear static analysis of Step 1 typically requires the greatest effort, principally with regard to the preparation of the non-linear model of the structure. Once the non-linear model has been developed, the dynamic analyses of Step 2 are relatively easy to run. Software such as OpenSees, RAM-Perform, SAP-2000, RUAUMOKO, and Drain-2DX may be used. The statistical estimates of Step 3 are straightforward and easily made. Consequently, the Scaled NDP requires little effort beyond the determination of the NSP displacement estimate.

2. Illustration of the Method

It is anticipated that the NSP will be used in preliminary design to determine the strength and stiffness required for the structure to satisfy global performance criteria. Once the proportions of the structural members have been established, the Scaled NDP may be used to assess or characterize the performance of the structure, or to determine some quantities required for design, such as the forces in brittle members that are intended to remain elastic. Two examples from the ATC-55 analyses are used to illustrate the method.

Interstory Drift Estimate: The sample mean of the peak values of interstory drift at the lowest story of the 9-story frame at a predetermined roof drift of 4% is $\overline{x}_n$ = 6.5% (Figure 3(a)). The true COV is estimated from the 11 peak dynamic responses to be 0.16. For this COV, Equation (4) results in c = 1.05. The true mean value of peak interstory drift is estimated to not exceed $c\overline{x}_n$ = 1.05(6.5%) = 6.8% at the 90% confidence level. That is, there is a 90% probability that the true mean peak interstory drift at the lowest story is less than 6.8% at the hazard level that produces a roof drift of 4%.

Story Shear Estimate: The sample mean of the peak story shears at the lowest story of the 8-story wall at a predetermined roof drift of 2% is $\overline{x}_n$ = 4.76 MN (1070 kips, in Figure 4(a)). To guard against the potential for shear failure, an "upper bound" limit on shear demands is desired. Based on the 11 analyses, the true COV of the story shears is estimated to be 0.22. Using Equation (4), c = 1.09. Therefore, there is a 90% probability that the true mean plus one standard deviation peak story shear is less than $(1 + \kappa\text{COV})\,c\bar{x}_n = (1 + 0.22)(1.09)(4.76\text{ MN}) = 6.33\text{ MN}$ (1420 kips), for the hazard that produces a roof drift of 2%.

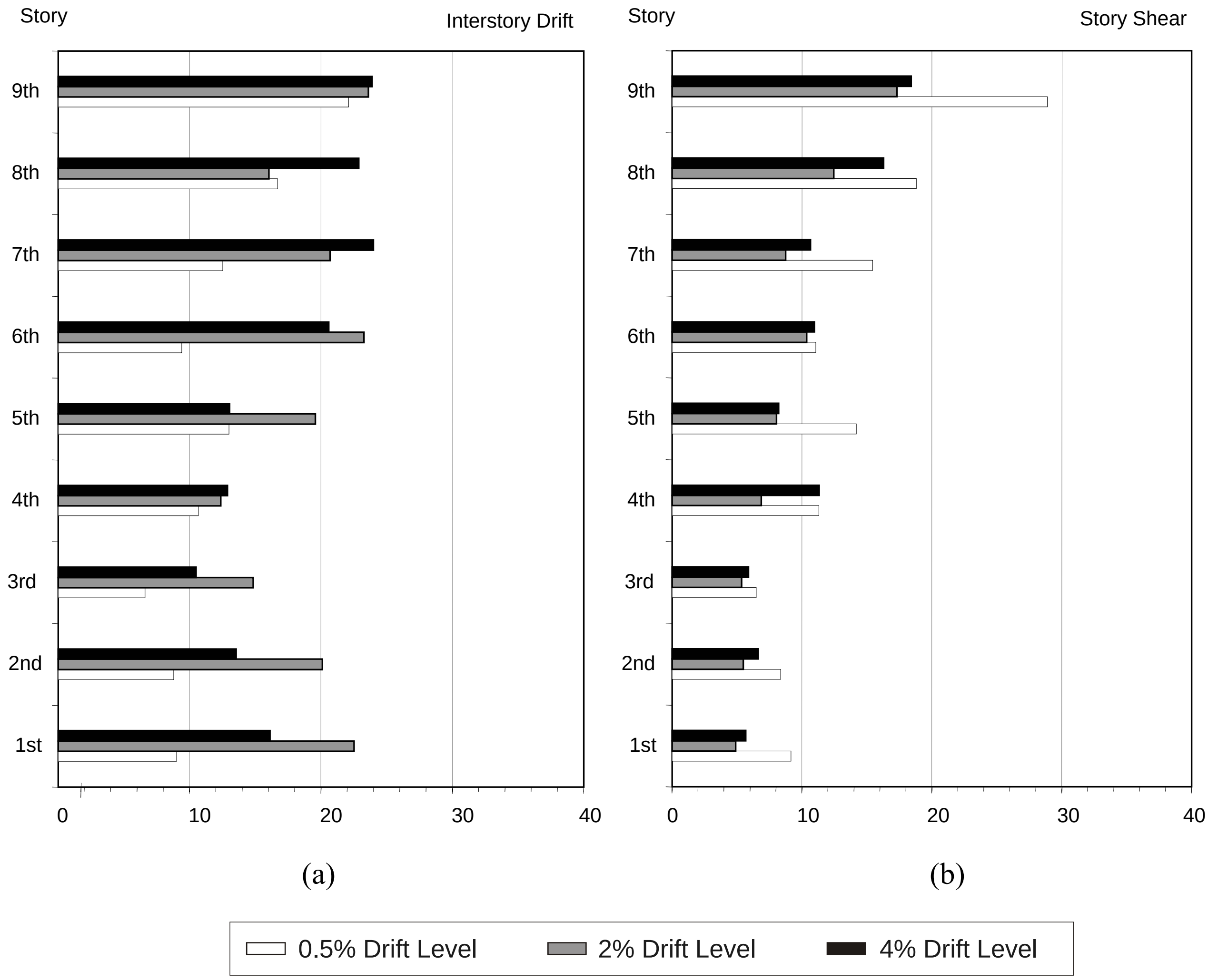


Fig. 5 COVs of (a) interstory drifts, and (b) story shears determined by non-linear dynamic analysis using 11 ground motion records scaled to achieve roof drift of 0.5, 2, and 4%, for the 9-story steel frame building

3. Observed Coefficients of Variation

The coefficients of variation (COVs) of the response quantities, determined in the MDOF studies, were examined for each response quantity at each floor or story for each of the five building models, for each of the three predetermined drift levels. In general, the COVs differ for each response quantity and are highest at the upper stories and near the base of each model. The COVs of several response quantities, determined for two of the building models, are plotted in Figures 5 and 6, at each of the three drift levels. In general, the COVs are highest at the upper stories and near the base of each model, and differ for each response quantity. The COVs for floor displacements diminish to zero at the top, due to the methodology employed in the study. Approximate upper bounds to the COVs are summarized in Table 2, where "approximate" indicates that the limit was exceeded by a small amount at a limited number of locations. As a preliminary step, one may use a COV of 0.25 to 0.30 for all quantities in cases where a sufficient number of analyses are not available for establishing a better estimate of the true COV.

4. Dependence of Sample Mean and COV on Sample Size

Data generated in the ATC-55 studies was re-interpreted in order to observe the influence of n on the sample mean and sample COV. Three sequences of the eleven ground motions, used in the original analyses, were randomly selected and statistics on the peak response quantities (displacement, interstory drift, story shear, and overturning moment) were computed for the first n records of each sequence, for $2 \le n \le 11$. Results are presented in Figure 7 for selected locations in the 9-story steel frame and in Figure 8 for selected locations in the 8-story reinforced concrete wall. The figures illustrate a reduction in scatter as n increases, although sampling error must be assumed to be present even for $n = 11$. One may interpret the figures as supporting the use of $n \ge 3$ for determination of $\overline{x}_n$ and $n \ge 7$ for determination of the COV.

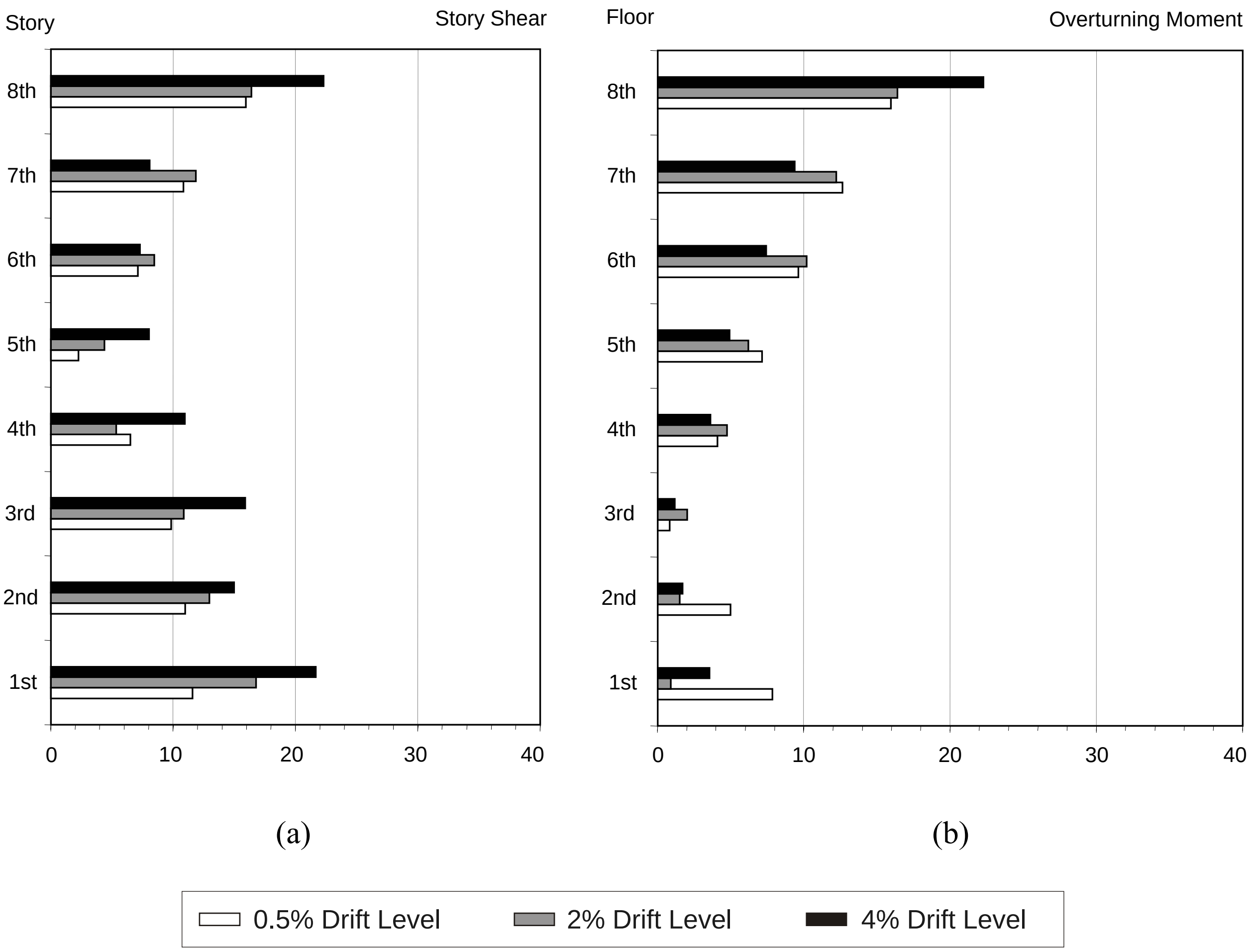


Fig. 6 COVs of (a) story shears, and (b) overturning moments determined by non-linear dynamic analysis using 11 ground motion records scaled to achieve roof drift of 0.2, 1, and 2%, for the 8-story reinforced concrete wall building

Table 2: Approximate Upper Bounds to the COVs over the Height of Each Building Model

Building Model	Interstory Drift	Story Shear	Overturning Moment
3-story steel frame	0.15	0.15	0.15
3-story steel frame (weak story)	0.20	0.15	0.15
8-story reinforced concrete wall	0.10	0.20	0.15
9-story steel frame	0.20	0.20	0.20
9-story steel frame (weak story)	0.30	0.25	0.25

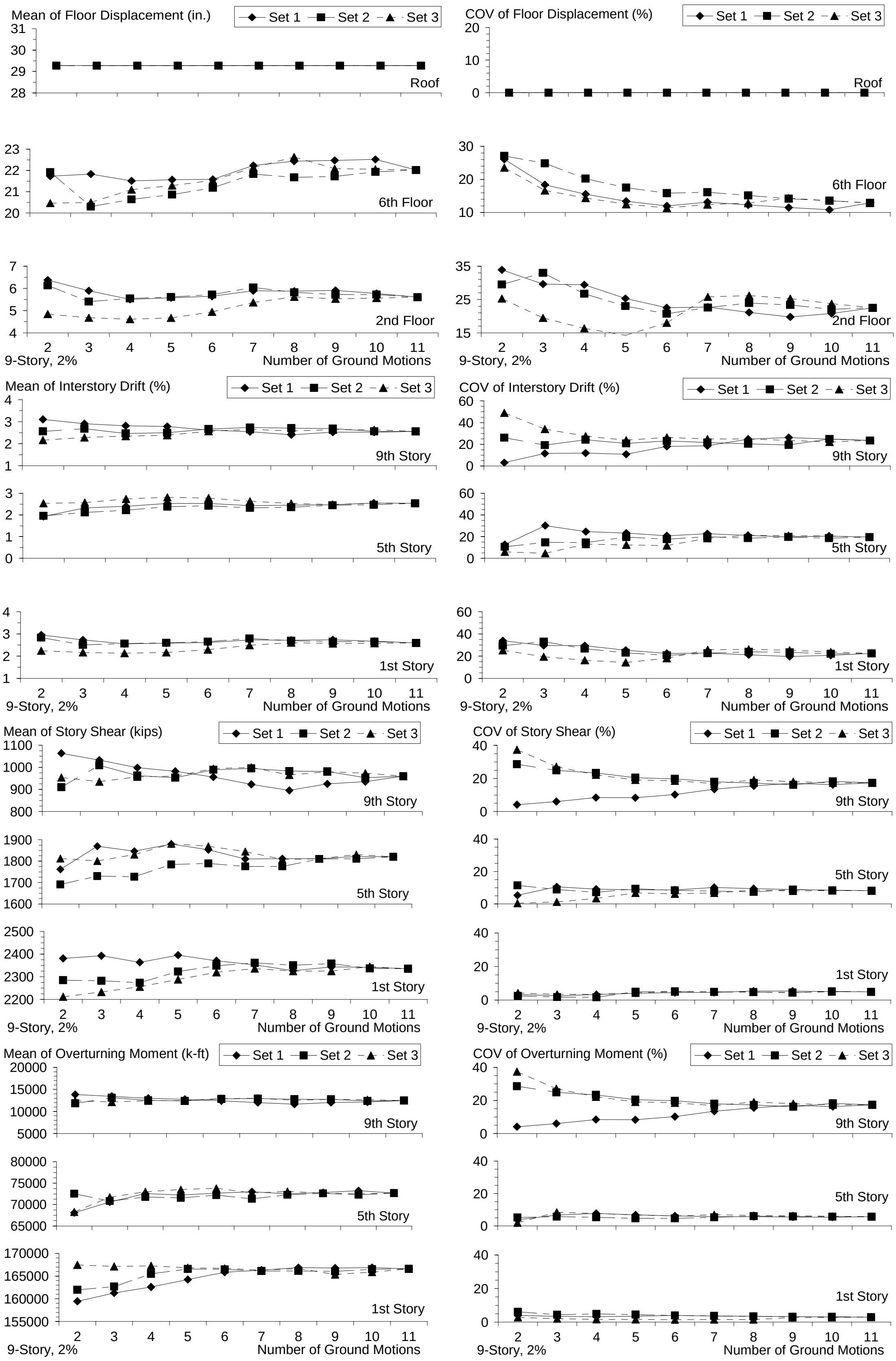


Fig. 7 Means and COVs as functions of *n* for the 9-story frame at 2% drift

DISCUSSION

Most analysis methods, used for design, produce single-valued (deterministic) estimates of design quantities, despite the widely-recognized fact that there is substantial uncertainty and variability in seismic demands. Although non-linear dynamic analysis is generally considered to be the most accurate of the available analysis methods, the substantial variability in demands determined using current techniques for scaling ground motions (based on elastic spectral ordinates) has been cumbersome for design. Alternative analytical techniques involve simplifying assumptions (such as the neglect of higher modes, assumptions of linearity, or the assumptions of independence of each potentially inelastic "modal" response), resulting in design quantities that may bear little relation to the demands expected in the design event. In contrast, the Scaled NDP inherently reflects the interaction of higher "modes", accounts for capacity limits (associated with mechanism development) on demands, and accounts for the irregular variation in dispersions, which appear to vary with the response quantity of interest, its location within the structure, and the degree of inelasticity (or drift) that develops.

The higher modes have a small contribution to the roof displacement relative to that associated with the first mode equivalent SDOF system. In effect, the Scaled NDP removes the dispersion associated with the inelastic response of a SDOF oscillator, while preserving the dispersion associated with higher mode response on the non-linear response of the structure. This should allow a reduction in the number of analyses (n) that must be run to obtain statistically relevant response data relative to that required if the records are scaled based on elastic spectral ordinates. The emphasis on the estimated peak roof displacement effectively allows the seismic hazard to be expressed in terms of roof displacement, rather than in terms of the spectral displacement (or spectral pseudo-acceleration).

Various researchers, including the authors, have already developed relatively simple procedures for performance-based design based on "equivalent" SDOF systems. Thus, it is feasible to develop a preliminary design based on global performance criteria using an NSP and to use the Scaled NDP to characterize the performance of the design and to determine additional quantities needed for the design, such as the forces to be sustained by brittle members that must remain elastic.

The Scaled NDP is a relatively new procedure. Refinements and improvements potentially may be made in the areas of (1) characterization and selection of site specific ground motions, (2) determination of the confidence levels (α) and numbers of standard deviations above the mean (κ) that should be used for various response quantities, (3) establishment of minimum number of analyses required for estimation of the mean and COV, and (4) improvement of precision of the NSP estimates of peak roof displacement. As with other analysis procedures, the accuracy of the results depends on the fidelity of the structural model. Unlike linear procedures and non-linear static procedures, the Scaled NDP requires that the cyclic behavior of the components be defined to an acceptable degree of accuracy.

CONCLUDING REMARKS

Results obtained in the ATC-55 studies illustrate that substantial errors can occur when estimating response quantities such as interstory drift, story shear, and overturning moments using various load vectors that have been proposed for the NSP. These errors are attributed to the presence of significant contributions from the higher "modes", also termed MDOF effects. In many cases, a single non-linear dynamic analysis provided a better estimate of these response quantities than could be obtained with a non-linear static analysis. Based on this observation, a method known as the Scaled Non-linear Dynamic Procedure was formalized. Two examples illustrated the application of the method.

The Scaled NDP makes use of existing NSPs for estimating peak displacement response, and inherently accounts for higher mode contributions and capacity limits associated with inelastic behavior. Each Scaled NDP analysis may be considered to be an application of a dynamic load vector (rather than a static load vector) to reach a target displacement. Each ground motion thus represents a different dynamic load vector. The Scaled NDP is easy to implement in practice because relatively little effort is required beyond that required for the non-linear static analysis.

It is feasible to use methods based on NSPs for preliminary determination of the strength and stiffness required for the structure to satisfy global performance objectives in the context of performance-based design. Various proposals for this have already been made by the authors and others. These preliminary requirements may be used to develop the detailed design of the structure. The Scaled NDP may then be used to establish the values of forces that must be resisted elastically (e.g. wall shears) to ensure that the intended mechanism develops and to evaluate the expected performance of the design (e.g. interstory drifts).

Because elastic spectra are the basis for the peak displacement estimates of the NSP, the Scaled NDP makes use of the substantial effort that already has gone into the development of site-specific descriptions of hazard in many countries, represented as elastic spectra at specified probabilities of exceedance. The simplicity of the method and the reliance on spectral descriptions of hazard makes the Scaled NDP amenable to specification in codes for the design of buildings.

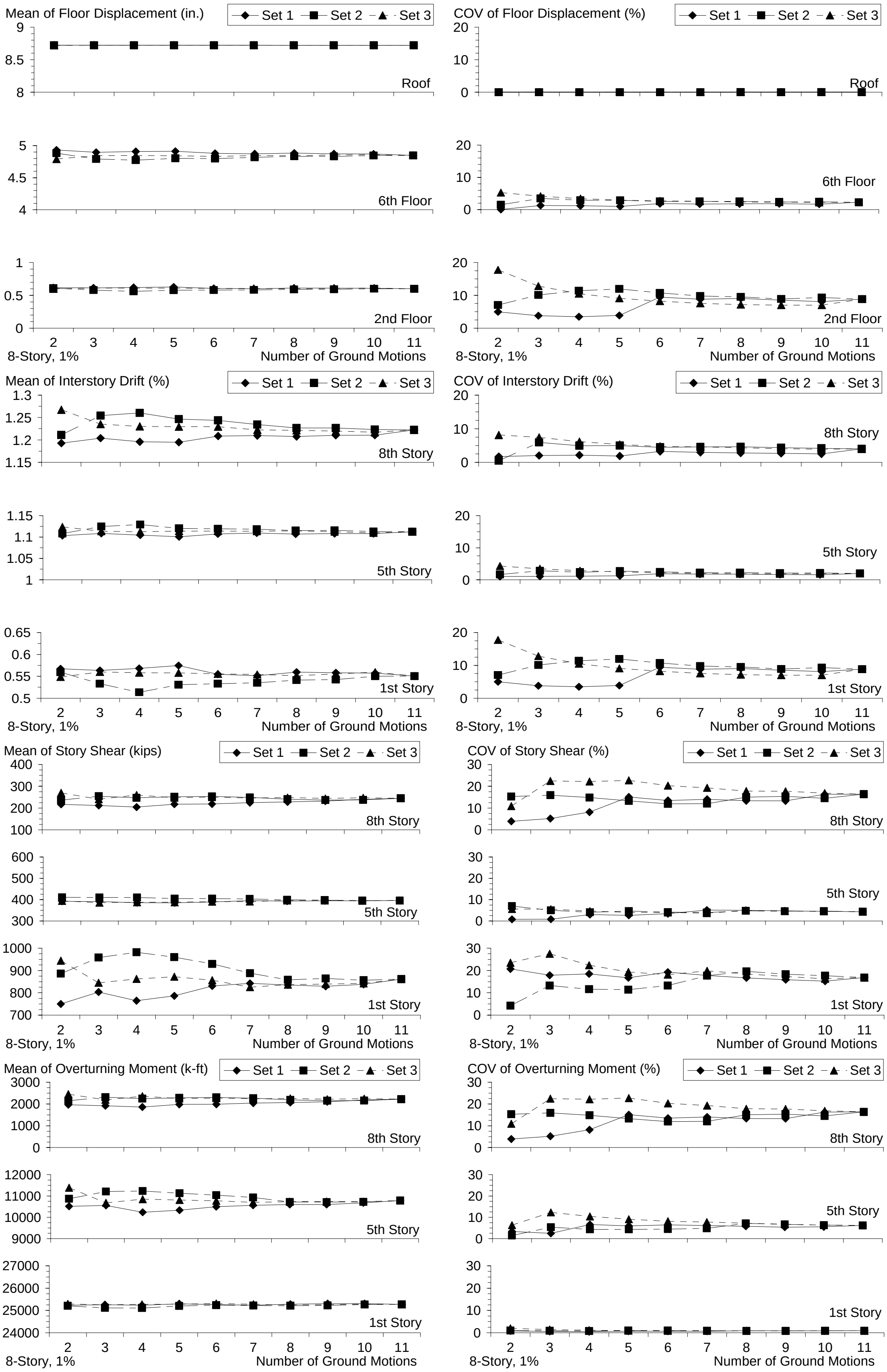


Fig. 8 Means and COVs as functions of *n* for the 8-story wall building at 1% drift

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APPENDIX: STATISTICAL DERIVATION

The problem is formulated as follows: a response quantity X has peak values x_1, x_2,…, x_n in n dynamic analyses of a structure. The mean of the n responses is $\overline{x}_n$. The responses are assumed to be normally distributed with true mean μ and standard deviation σ_x. What is the scale factor c' such that $c'\overline{x}_n$ exceeds the true mean plus κ standard deviations ($\mu + \kappa\sigma_x$) with a specified level of confidence α?

X is normally distributed with true mean μ and standard deviation σ_x, i.e. $X \sim N(\mu, \sigma_x)$. The sample mean of X is given by $\overline{x}_n$ where

$$\overline{x}_n \sim N\left(\mu, \frac{\sigma_x}{\sqrt{n}}\right) \tag{A.1}$$

We are interested in c' such that the probability that $c'\overline{x}_n \geq \mu + \kappa\sigma_x$ is α, which can be stated as $P\left[c'\overline{x}_n \geq \mu + \kappa\sigma_x\right] = \alpha$. This is equivalent to $P\left[\overline{x}_n \geq \frac{\mu + \kappa\sigma}{c'}\right] = \alpha$.

Since $\overline{x}_n$ is normally distributed,

$$P\left[\overline{x}_n \geq \frac{\mu + \kappa\sigma_x}{c'}\right] = 1 - P\left[\overline{x}_n < \frac{\mu + \kappa\sigma_x}{c'}\right] = 1 - \Phi\left(\frac{\frac{\mu + \kappa\sigma_x}{c'} - \mu}{\sigma_x / \sqrt{n}}\right) = \alpha \tag{A.2}$$

where Φ is the cumulative standard normal distribution. This can be restated as

$$\Phi\left(\frac{\frac{\mu + \kappa\sigma_x}{c'} - \mu}{\sigma_x / \sqrt{n}}\right) = 1 - \alpha \tag{A.3}$$

or, equivalently as

$$\Phi\left(\frac{\mu - \frac{\mu + \kappa\sigma_x}{c'}}{\sigma_x / \sqrt{n}}\right) = \Phi\left(\frac{1 - \frac{1 + \kappa\mathrm{COV}}{c'}}{\mathrm{COV} / \sqrt{n}}\right) = \alpha \tag{A.4}$$

where COV is the coefficient of variation. Simplification leads to

$$c' = \frac{1 + \kappa\mathrm{COV}}{1 - \left(\Phi^{-1}(\alpha)\right)\frac{\mathrm{COV}}{\sqrt{n}}} \tag{A.5}$$

Equation (A.5) can be further simplified by setting $c' = c(1 + \kappa\mathrm{COV})$, resulting in Equation (3). For a confidence level, α, of 90%, $\Phi^{-1}(\alpha) = 1.28$, resulting in

$$c = \frac{1}{1 - 1.28\dfrac{\mathrm{COV}}{\sqrt{n}}} \tag{A.6}$$

(given previously as Equation (4)). Thus, using Equation (A.6), there is a 90% level of confidence that quantity $c(1+\kappa\mathrm{COV})\bar{x}_n$ exceeds the true mean plus κ standard deviations.

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