Paper No. 52

# STATIC AND DYNAMIC BEHAVIOUR OF A LATERALLY LOADED PILE

*by*

Shamsher Prakash* and S.L. Agarwal**

## SYNOPSIS

The behaviour of a vertical pile subjected to static, steady State dynamic loading both at the ground level and above and under transient loading at ground level was studied on aluminium model piles embedded in sand. The steady state dynamic load was applied through vibration table which could be excited with known amplitude and frequency of vibration. The transient load was applied by another shaking table excited at a constant frequency. The load was measured by a proving frame on which electric resistance strain gauges had been mounted. The effect of both transient and steady state loading on the displacement has been studied and compared with the static case.

## INTRODUCTION

Large lateral loads act on piles when these are used as support for retaining walls, bridge abutments, piers, fenders, dolphins and anchors for bulk-heads and water-front and off-shore structures. Dynamic loads may be caused because of wave action, impact of ships, earthquakes and machines.

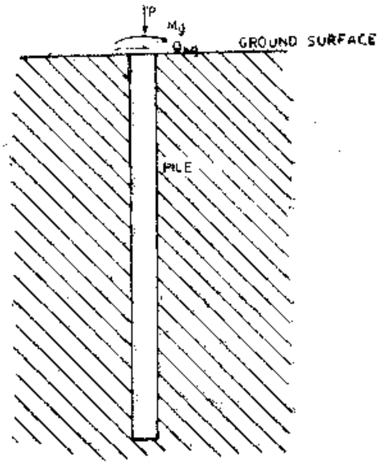
Figure 1 A illustrates the external loads applied to the pile at ground level, while in Figure 1 B the loads are applied above the ground level. P represents the vertical load, while Mg and $Q_{hg}$ represent the moment and horizontal shear respectively at the ground surface. Ms represents the moment produced by rotational restraint supplied by the structure and $Q_{hs}$ represents the horizontal shear force when applied above the ground level. Figure 2 (a) illustrates the deflected shape of the embedded portion of the pile. The pile has a modulus of elasticity E, moment of inertia I and width B. An approximate soil reaction diagram has also been shown. Figure 2 (b) shows deflections of a pile which are due to rigid body rotation and its flexural bending.

An investigation of piles subjected to lateral loads involves the study of interaction between the pile and the foundation soil. Most of the analysis of laterally loaded piles involve

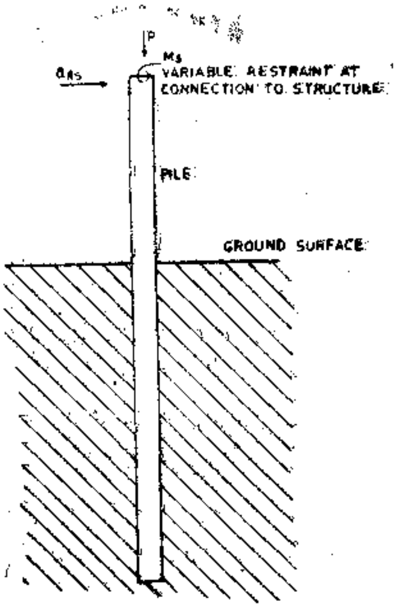
---

*Reader in Civil Engineering, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee. U P. (India).

**Lecturer in Civil Engineering, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee. U.P. (India).

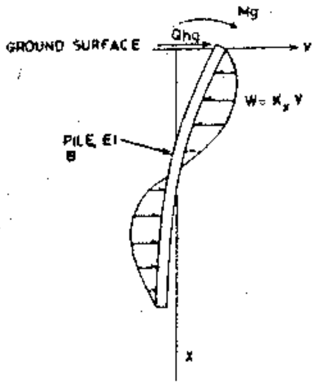


A LOADS APPLIED AT GROUND LINE

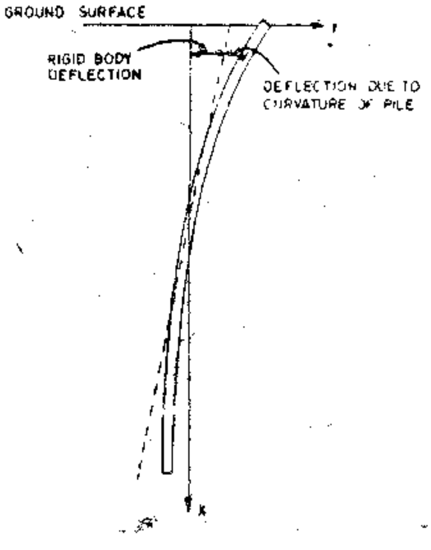


B LOADS APPLIED ABOVE GROUND LINE

Figure 1 External Loads Applied to Pile



(a) FORCE SYSTEM AND DEFLECTED SHAPE OF PILE



(b) DEFLECTION OF PILE

Figure 2 System of Forces and Deflected Shape of Pile

the concept of modulus of subgrade reaction based on Winkler's (1867) assumption, which states that soil strata may be approximated by a series of infinitely closely spaced independent and elastic springs. Spring stiffness k, khown as coefficient of subgrade reaction is defined as

$$k = \frac{w}{y} \tag{1}$$

where,

w = soil reaction at any point and

y = deflection at that point

Soil type and size of loaded area mainly influence k. Terzaghi (1955) suggested constant k for piles in over consolidated clays and k linearly varying with depth for sands,

i.e. $k_x = n_h . x$ (2)

where,

$k_x$ = value of k at depth x and

$n_h$ = constant of horizontal subgrade reaction.

The problem of laterally loaded piles under static conditions has been studied to a considerable extent by Reese and Matlock (1956), Davisson (1960), Broms (1964 b) and Aggarwal and Soneja (1965) for k varying linearly with depth and Broms (1964 & 1965) for k-constant with depth. Davisson considered effect of vertical loads also. Prakash (1962) investigated the behaviour of pile groups subjected to static lateral loads in sands. Davisson and Gill (1963) solved this problem for constant k for two layered medium. Prakash and Subramanyam (1964 a, b) studied the behaviour of battered piles under lateral loads.

According to these analyses, the factors governing the displacement of a pile are :—

(i) Type of soil,

(ii) Soil properties, e.g. density, moisture content etc.

(iii) Variation of soil modulus,

(iv) Pile properties, e.g. pile material, shape, width, length, flexural stiffness etc.

(v) Shear and moments etc acting on the pile.

(vi) Pile end conditions, free or restrained.

(vii) Method of soil and pile placement e.g. driving, jetting in the field.

(viii) Relative stiffness factor T, defined as

$$T = (EI/n_h)^{1/5} \tag{3}$$

for k linearly varying with depth. A non-dimensional coefficient,

$$Z_{max} = \frac{L_s}{T} \tag{4}$$

where, $L_s$ = Length of embedded portion of the pile,

determines the different solutions, which have been presented in the form of non-dimensional curves (Reese and Matlock 1956, Davisson 1960). A pile is considered rigid if $Z_{max} \leqslant 2$ and infinitely long if $Z_{max} \geqslant 5$.

The dynamic lateral loads may be caused because of earthquake, wave action and in machine foundations. Experimental data on actual piles is difficult to obtain because of cost considerations. Model studies have been reported by Gaul (1958), Hayashi and Miyajima (1962) and a few others. A critical review of their work has been presented elsewhere

(Prakash and Agarwal 1965). The information is far from being complete and there is considerable scope for fruitful research in this direction.

In this paper, investigation carried out on a vertical pile under static, steady state dynamic and transient loads has been reported. The lateral load was applied both at and above the ground level on a free head pile. The effect of vertical load will be included in a subsequent investigation.

The tests were performed on a model aluminium pile 15 mm in outside diameter and 2.5 mm wall thickness. The length of the pile (60 cm) was so chosen that it corresponds to a usual pile length in the field i.e. $Z_{max} \geqslant 5$. The pile was placed in a steel tank of 60 cms × 30 cms × 61 cms high and sand placed around the pile. The sand was compacted to produce homogeneous soil density. Steady state dynamic load was applied through a vibration table which could be excited with known frequency and amplitude of vibration and transient load was applied through a free vibrating table having constant frequency of oscillation. The load was measured by means of proving frame on which electric resistance strain gauges were mounted.

A comparison has been made between the behaviour of pile under dynamic and static case.

## EXPERIMENTAL SET UP

### General

The variables in the study of single vertical pile subjected to lateral loads have already been enumerated. For dynamic study, the frequency and amplitude of loading will also govern the behaviour. In this study, a single vertical aluminium pile was embedded in medium sand and static and dynamic loading had been applied both at and above the ground surface.

### Soil and Pile Properties

The sand used was medium Ranipur sand with a relative density of 0.584 during the tests (Prakash and Agarwal 1965).

Aluminium pipe section was adopted since strains and deflections at the same stress level are large as compared with those of steel. Other properties of the section were as follows :

| | | |
|---|---|---|
| (a) | Section No. 9442 (ALIND) | |
| (b) | Outside Diameter | 15 mm |
| (c) | Inside Diameter | 10 mm |
| (d) | Stiffness EI | $13.7 \times 10^4$ kg/cm$^2$ |
| | | $4.77 \times 10^4$ psi |

### Tank

The size of the tank should be large enough so that there is no interference of the walls on the behaviour of pile. At the same time it should be small enough so that volume of sand to be handled each time is not too much.

Preliminary tests under dynamic loading showed that the zones of influence may extend to a maximum of 30 times the pile width in the direction and 15 times its width perpendicular to the load. These spacings are eight to twelve and three to five times the corresponding values for static loading (Prakash 1962). A tank having dimensions of 60 cm × 30 cm × 61cm high was chosen to accommodate even small groups piles:

### Loading Device

(a) Static loading was applied by means of a string passing over a ball bearing pulley.

(b) Steady state dynamic loading was applied through a horizontal vibration table† which could be excited with amplitudes of 0.8 mm, 1.7 mm, 2.80 mm, 3.73 mm, and 4.65 mm and at frequency of 0-1000 rpm. The motion of the table does not follow a true sinusoidal path.

(c) Transient loading was applied through a wooden table 170 cm × 120 cm × 9cm thick, mounted on four m.s. rollers 6 cm in diameter and attached to a fixed support by means of a spring. The natural frequency of the table could be varied by using different springs, while the amplitude of motion could be varied by striking the table with hammer having variable fall. In order to attain an amplitude of about 5 mm, the appropriate spring was selected. The natural frequency of the table was close to 2 cps.

### Dynamic Load Measurement

The dynamic load was measured by means of a proving frame on which electric resistance strain gauges had been mounted. It was connected to the shaking table on one side and the pile on the other, by connecting rods, adaptors and clamps. The frame was designed based on the criterion laid by Casagrande and Shannon (1948). It was made of spring steel with four Rohit's (India) Electric resistance strain gauges mounted on three of the inner sides (Figure 3). The frame was calibrated both under tension and compression, with strain ($\epsilon$), deflection ($y_{pf}$) of the proving frame from its initial position and number of chart lines ($y_{pc}$) recorded on a Brush two channel oscillograph. Calibration with strain was done to check the linearity of the frame and the other calibration (Figure 4) is required for interpretation of load and displacement of pile (See next section).

The natural frequency of the frame was found to be close to 175 cps. The maximum

---

† For details, refer to Annual Report 1960-61, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee, U.P. (INDIA).

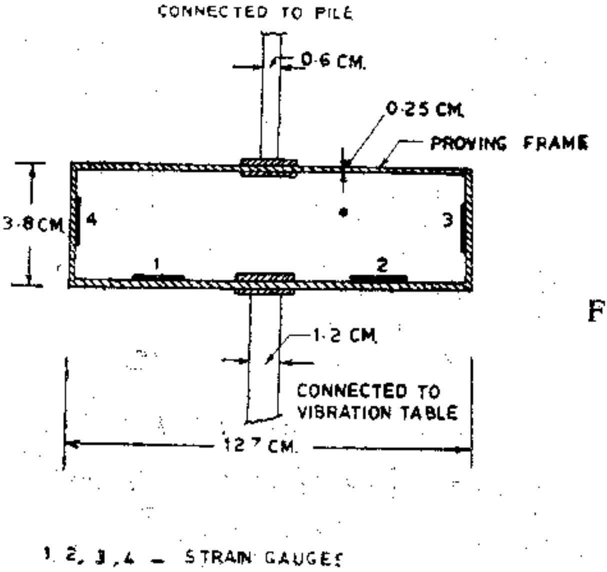


Figure 3 Proving Frame

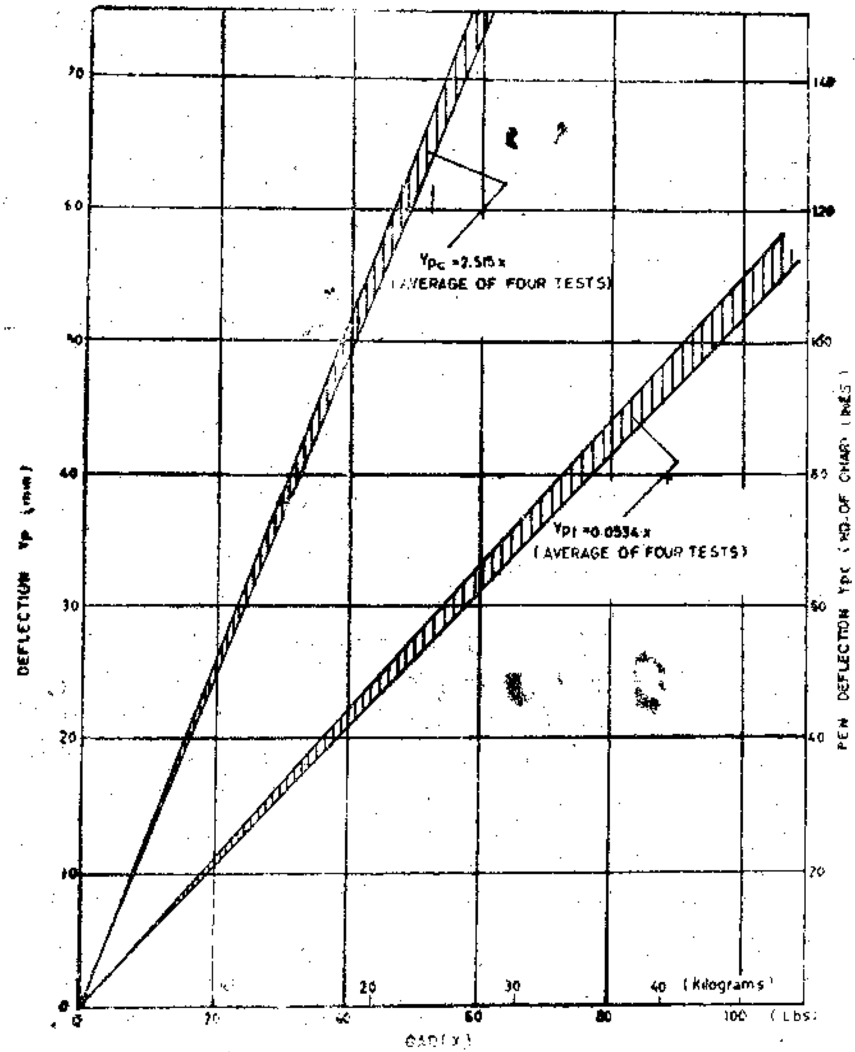


Figure 4
Calibration of Proving Frame

frequency of loading was 10 cps. Therefore, the natural frequency of the proving frame was about 17.5 times that of load application against 20 recommended.

**Measurement of Pile Displacement.**

(a) Static Test—

The displacement of the pile was measured by a dial gauge abutting against a flat connected to the pile at the ground level.

(b) Steady State Dynamic Test—

Displacement of the pile was obtained by subtracting the deflection of the proving frame ($y_{pf}$) from the displacement of the vibration table ($y_t$), Figure 5, when the load was applied at G.L. The displacement of the table motion was set at a known value. The deflection of the proving frame was determined with the help of Figure 4. In case the load was applied above the ground level, a relationship between displacement of pile ($y_l$) at load level and displacement of pile ($y_g$) at ground level was established (See Test Procedure). Figure 6 shows the assembly diagram of tank and pile.

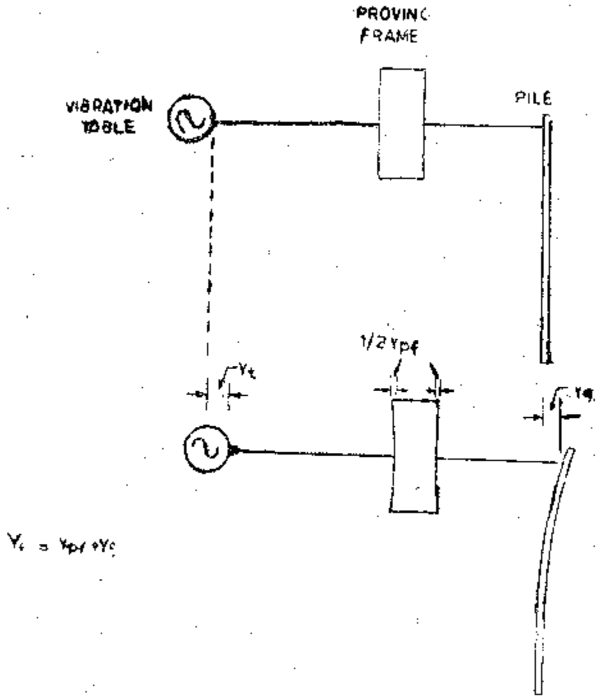


Figure 5 Method of Determining Pile Displacement ($y_g$)

(c) Transient Test—

The displacement of the vibration table was measured by a CEC vibration pick-up and read on a vibration meter. The natural frequency of the table was 2 cps. As the vibration meter was calibrated from 5 to 1000 cps, the displacement pick-up was calibrated on a small shaking table at frequency of 2 cps (Agarwal 1964).

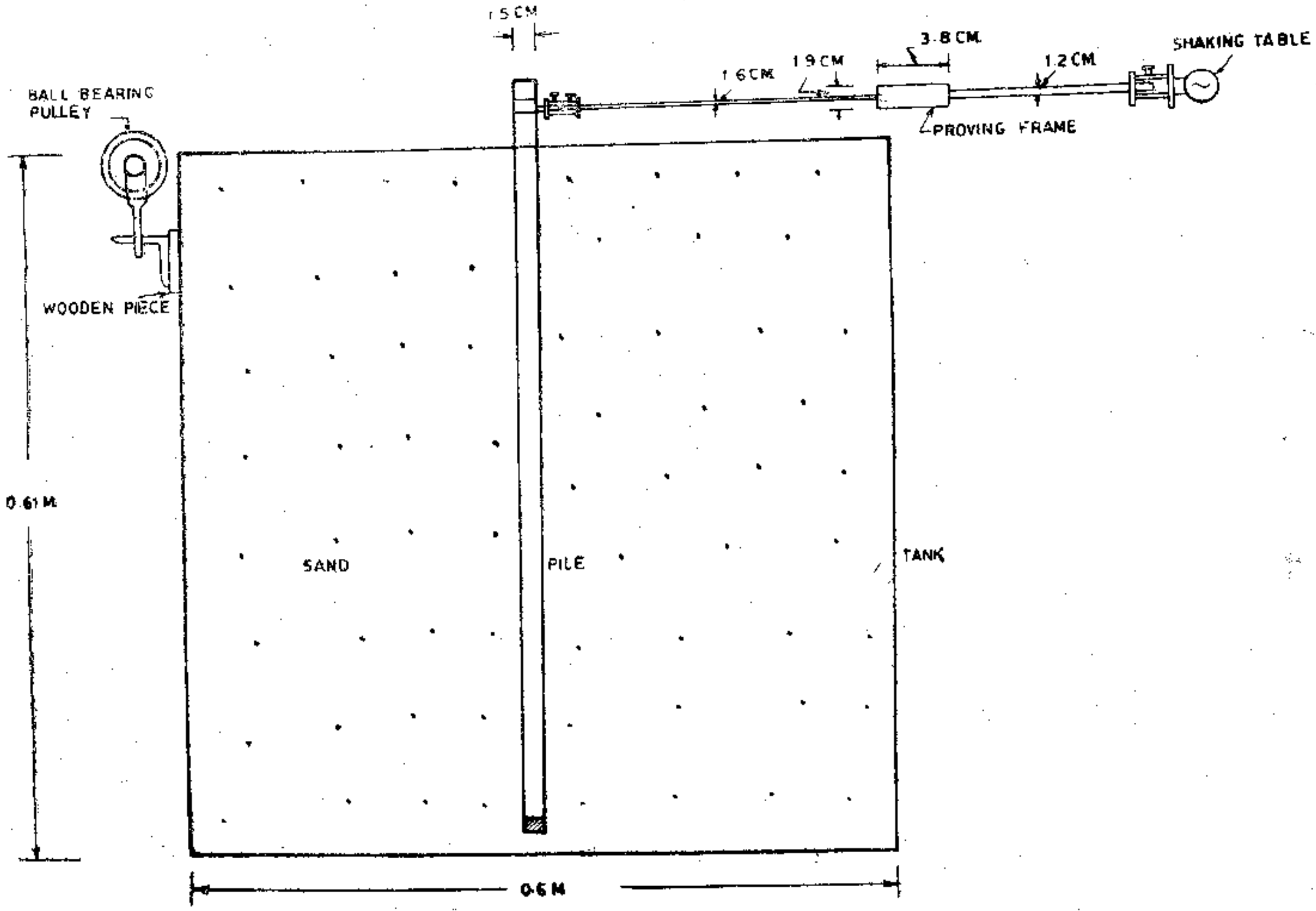


Figure 6 Pile Tank Assembly with Necessary Connections

## TEST PROCEDURE

### Placement of Sand and Pile

The Sand was placed in 10 cm thick layers and each layer was compacted 300 times with the standard Proctor's hammer. The pile was held in position when the sand was being placed.

### Test Procedure—

(a) Static Loading—

Before dynamically loading, the pile was first tested under static loading. Two cases are of interest.

(i) Load applied at Ground Level—The displacement was measured by a dial gauge. Each load increment was allowed to stand for five minutes before deflection was recorded. The procedure was continued upto 40 lbs load.

(ii) Load applied above Ground Level—Displacement was measured both at ground level and at load level, by two dial gauges.

In order to establish a relationship between displacement at load level ($y_l$) and displacement at ground level ($y_g$), a plot was made between $y_l$ and $y_g$ obtained during all the static tests of each series. A straight line was drawn which established the relationship between $y_l$ and $y_g$. Similar relationships were established for all Test Series 2 to 5 which are as follows :—

| Load Position | $y_l/y_g$ |
|---|---|
| G.L. | 1.00 |
| 2.0 cm above G.L. | 1.15 |
| 3.0 cm above G.L. | 1.20 |
| 4.0 cm above G.L. | 1.35 |
| 5.0 cm above G.L. | 1.54 |

The ratio of $y_l/y_g$ was used to determine $y_g$ from $y_l$, values obtained directly in dynamic tests.

(b) Steady State Dynamic Loading—

The pile was connected to the steady state vibrating table as shown in Figure 7. The table amplitude of 0 8 mm (1.60 mm peak to peak) was adjusted. The lead wires of the strain gauges No. 2 and 4 were connected to the universal amplifier, which was in turn connected to the Brush oscillograph. The table was then excited with frequency increasing from 0–10 cps, and record of load obtained at almost equal increments of frequency. A record at the same frequency was obtained while decreasing the frequency from 10–0 cps.

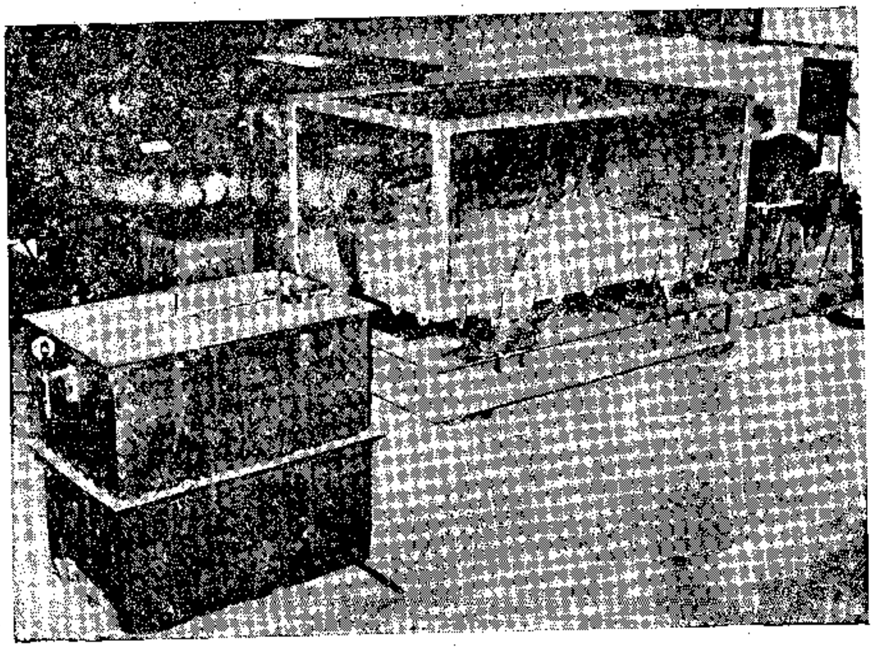


Figure 7 Pile Connected to Steady State Vibration Table

The pile was then tested under static loading. After this test, the table amplitude was set at 1.77 mm and dynamic test performed. Thus one test series consisted of the following tests :—

(i) Static test.
(ii) Dynamic test with table amplitude 0;8 mm.
(iii) Static test.
(iv) Dynamic test with table amplitude 1.77 mm.
(v) Static test.
(vi) Dynamic test with table amplitude 2.8 mm.
(vii) Static test.
(viii) Dynamic test with table amplitude 3.73 mm.
(ix) Static test.
(x) Dynamic test with table amplitude 4.65 mm.
(xi) Static test.

Every static test took about 1 hour and dynamic test about 10 minutes. When the load was applied above the ground level, the same procedure was followed.

(c) Transient Loading—

The pile was connected to the transient vibration table, Figure 8. The displacement pick-up and strain gauges of proving frame were connected to suitable recording devices. The table was first struck gently with a hammer and displacements and loads recorded. Subsequently, it was struck with increasing forces and the corresponding deflections and loads

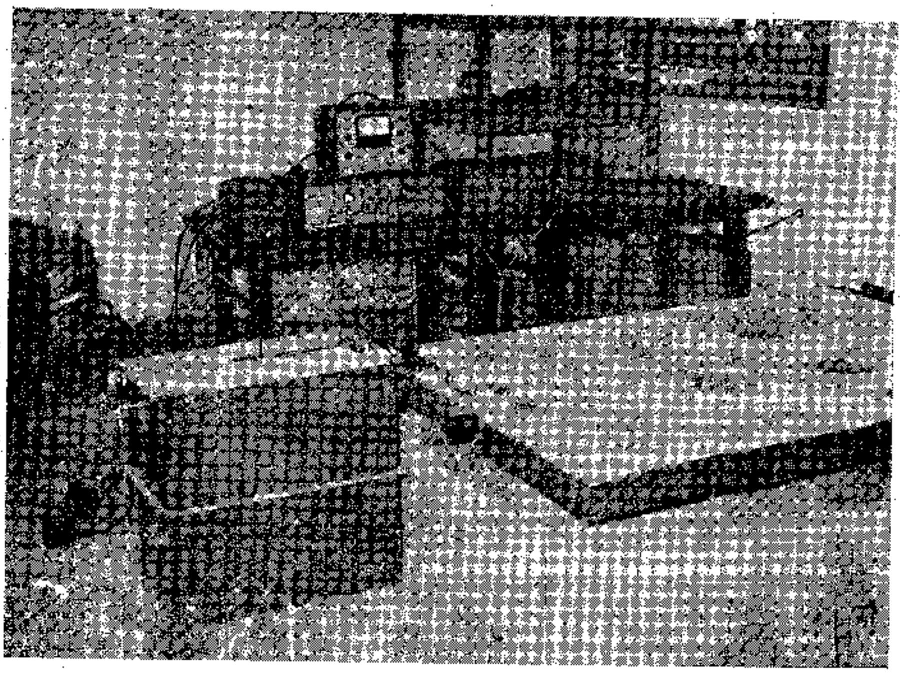


Figure 8 Pile Connected to Transient Vibration Table

recorded. The maximum load applied was about 45 lbs. After the transient test, a static test was also performed. Thus one series of test under transient load consisted of the following :—

(a) Static test in the beginning.
(b) Test under transient load.
(c) Static test at the end.

### Tests Performed—

Tests performed included :—

(a) Test Series No. 1 Load applied at the ground level.
Test Series No. 2 Load applied at 2.0 cm above the ground level.
Test Series No. 3 Load applied at 3.0 cm above the ground level.
Test Series No. 4 Load applied at 4.0 cm above the ground level.
Test Series No. 5 Load applied at 5.0 cm above the ground level.

Test Series No. 1 and 5 were repeated to check the reproducibility of the data.

(b) Transient Loading—

Test Series No. 6—Load applied at the ground level.

### Depression Around the Pile

During Test Series No. 1 a depression in the soil in the immediate vicinity of the pile was observed at table amplitude of 1.77 mm. Similar depressions were observed at higher table amplitudes, their dimensions being as follows :—

| | Size of Depression | |
|---|---|---|
| Table amplitude ($y_t$) mm | Diameter cm | Depth cm |
| 1.77 | 4.0 | 3.0 |
| 2.80 | 5.0 | 3.5 |
| 3.73 | 6.0 | 4.0 |
| 4.65 | 7.0 | 4.5 |

### Soil Column Vibrating with the Pile

In Test Series No 1 to 5, a soil column could be seen vibrating with the pile. The dimensions of this column (Figure 9) elliptical in section were 39 0 cm. in the direction of loading and 21.0 cm. perpendicular to it in Test Series No. 5 when the table amplitude was 4 65 mm.

### Natural Frequency of Soil-Pile System

The pile while embedded in sand was disconnected from the vibrating table and a CEC vibration (displacement) pick up was connected rigidly to the pile. To determine the natural frequency of the system, the pile was struck gently with a rod and the record of the pick-up signal indicated a frequency of about 50 cps.

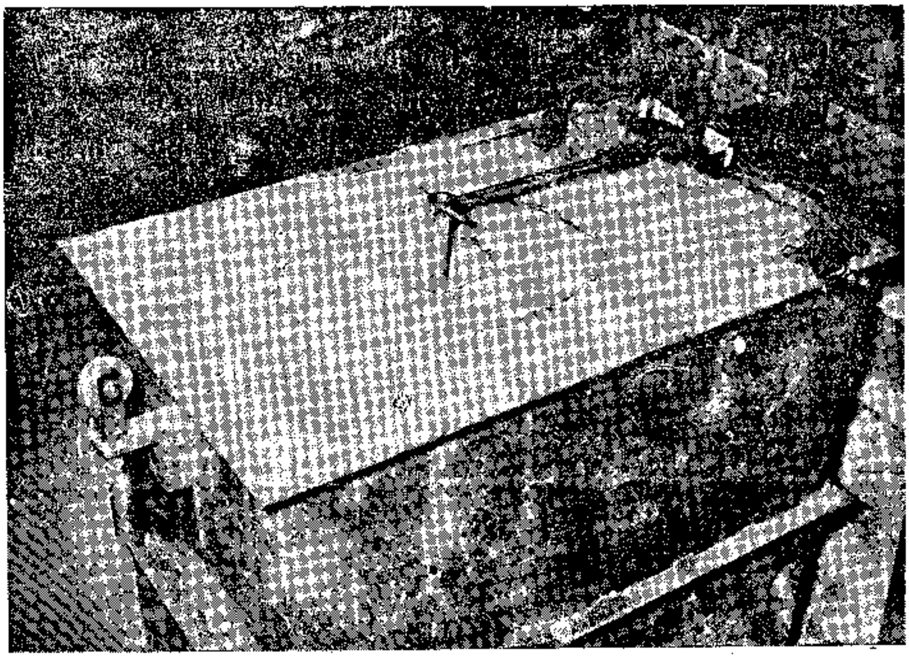


Figure 9 Zone of Influence for Pile Under Dymamic Lateral Loads

## RESULTS OBTAINED

### Static Tests—

Figure 10 illustrates Load-Displacement at ground level ($y_g$) curves for all the six static tests of Test series No. 1, when the load was applied at ground level. Figure 11 is a similar diagram for test series No. 5, (Load applied at 5.0 cms above the ground level). Figures for Tests No. 2, 3 and 4 have not been included for want of space.

### Steady State Dynamic Tests

(a) Load applied at G. L.

Figure 12 is a plot of frequency and load at constant table amplitude of 0.8 mm, 1.77 mm, 2.80 mm, 3.73 mm and 4.65 mm respectively. The full lines and points indicate that the frequency is increasing while the dotted lines stand for decreasing frequency. Figure 13 contains Load Displacement ($y_g$) curves with frequency of oscillation of 2, 4, 6, 8 and 10 cps along with similar curves for static tests in the beginning and at the end. At any frequency, the load was determined from Figure 12 and deformation of the proving frame was determined from the calibration curve, Figure 4. The difference of the two gave displacement at G.L. i.e.

$$y_g = y_t - y_{pf} \qquad (5)$$

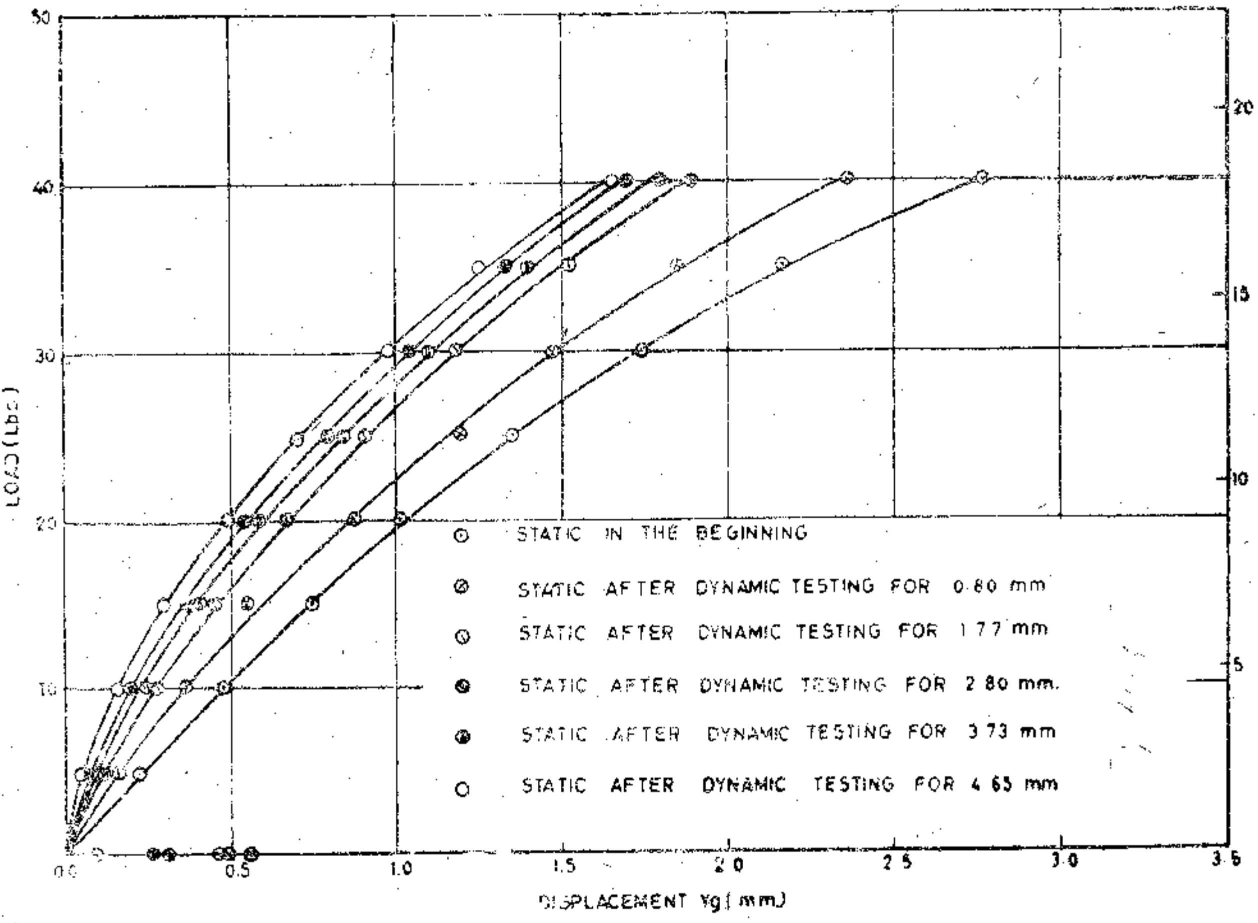


Figure 10 Load v/s Displacement ($y_g$) Under Static Loads with Load at G.L.

(b) Load Applied above G. L.

The data for Test series No. 2, 3, 4 and 5 were analysed in a similar manner except for the pile displacement at the ground level which was obtained as follows :—

Displacement ($y_l$) at load level during dynamic loading was obtained by subtracting the deflection of the proving frame ($y_{pf}$) from the table displacement ($y_t$). Pile displacement at ground level ($y_g$) was then obtained by dividing $y_l$ by the corresponding ratio of $y_l/y_g$, which had already been established from static tests. Load displacement for Test Series No. 5 have been plotted in Figure 14. Similar curves were obtained for Test Series No. 2, 3 and 4.

**Transient Loading—**

Figure 15 illustrates the specimen of transient load record as obtained on the oscillograph for the 2nd, 10th and 11th blows respectively. Peak to peak amplitude of the first cycle has been analysed. The corresponding displacement was obtained from measurements of the table motion and compression of the spring at any particular measured load. The frequency of oscillation was governed by the spring used and it equalled 2 cps. The load displacement curve for transient loading is shown in Figure 16. Curves for static tests in the beginning and at the end of the transient test have also been shown.

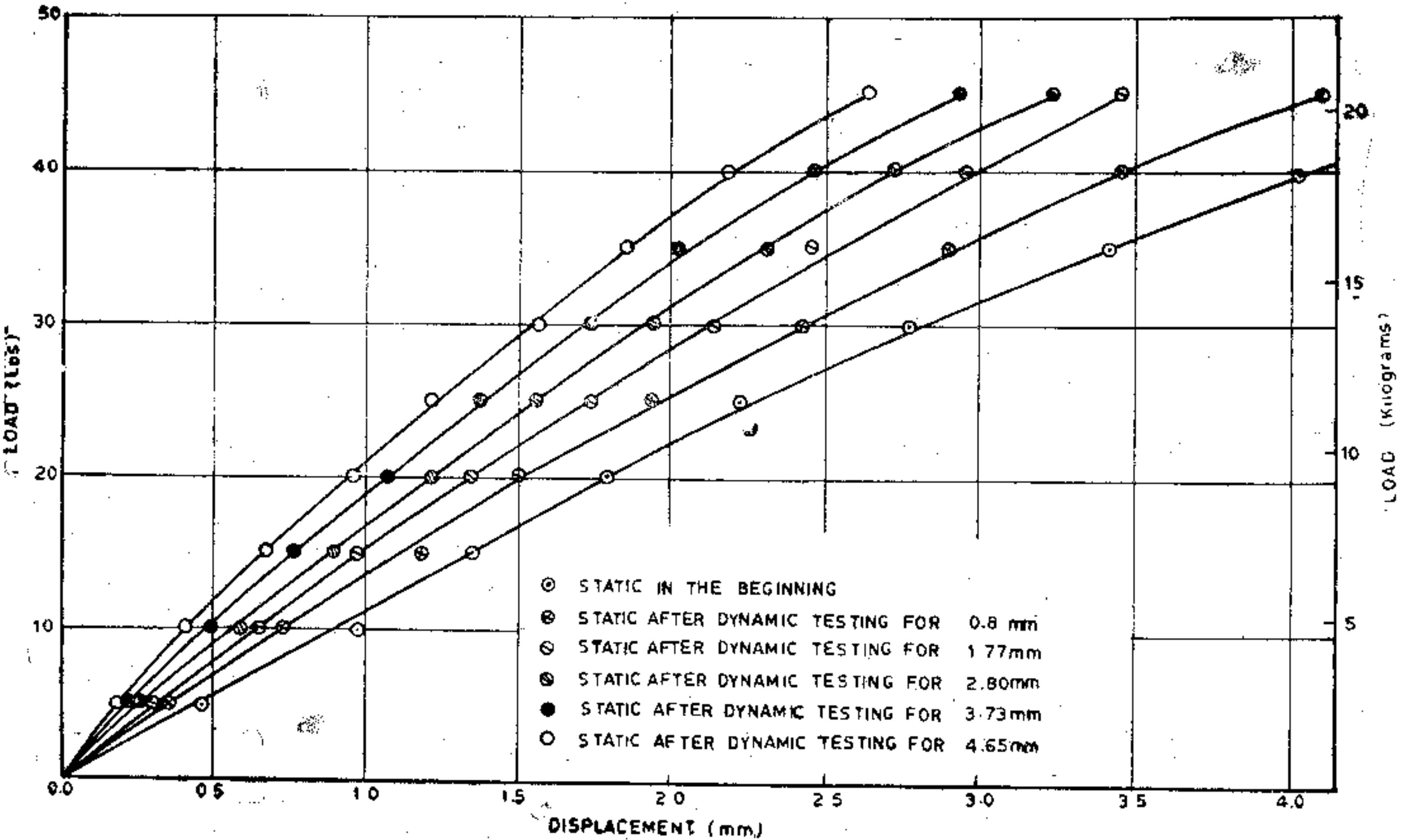


Figure 11 Load Displacement Curve Under Static Loads (With Load 5 cm Above G.L.)

**Estimation of $n_h$**

Reese and Matlock (1956) presented non-dimensional solutions for deflection of a vertical flexible pile embedded in sand according to which

$$y_g = A_y . \frac{Q_{hg} T^3}{EI} + B_y . \frac{M_g . T^2}{EI} \qquad (6)$$

where,

$y_g$ = deflection at ground level.
$Q_{hg}$ = Horizontal shear at ground level.
$M_g$ = Moment at ground level.
T = Relative stiffness factor $= 5\ (EI/n_h)^{1/5}$
EI = Flexural stiffness of pile.
$A_y$ = Non-dimensional deflection coefficient corresponding to shear and
$B_y$ = Non-dimensional deflection coefficient corresponding to moment.

Values of $n_h$ can be computed by solving the above equation for T. For a load of 20 lbs applied at the ground level in a static test and in Test Series No. 1, 2, 5 and 6, the following values were obtained:—

TABLE—1

SOIL MODULUS VALUES

|  | $n_h$. (lbs/in³) |  |  |  |
|---|---|---|---|---|
| Frequency | Load at G.L. | Load at 2.0 cms above G.L. | Load at 5.0 cm above G.L. | Remarks |
| 0 | 103 | 76 | 68 | Static Test |
| 2 | 190 | 145 | 132 | Steady State Test |
| 4 | 215 | 170 | 145 | Steady State Test |
| 6 | 226 | 203 | 162 | Steady State Test |
| 8 | 262 | 240 | 170 | Steady State Test |
| 10 | 287 | 279 | 191 | Steady State Test |
| 2 | 250 | — | — | Transient Test |

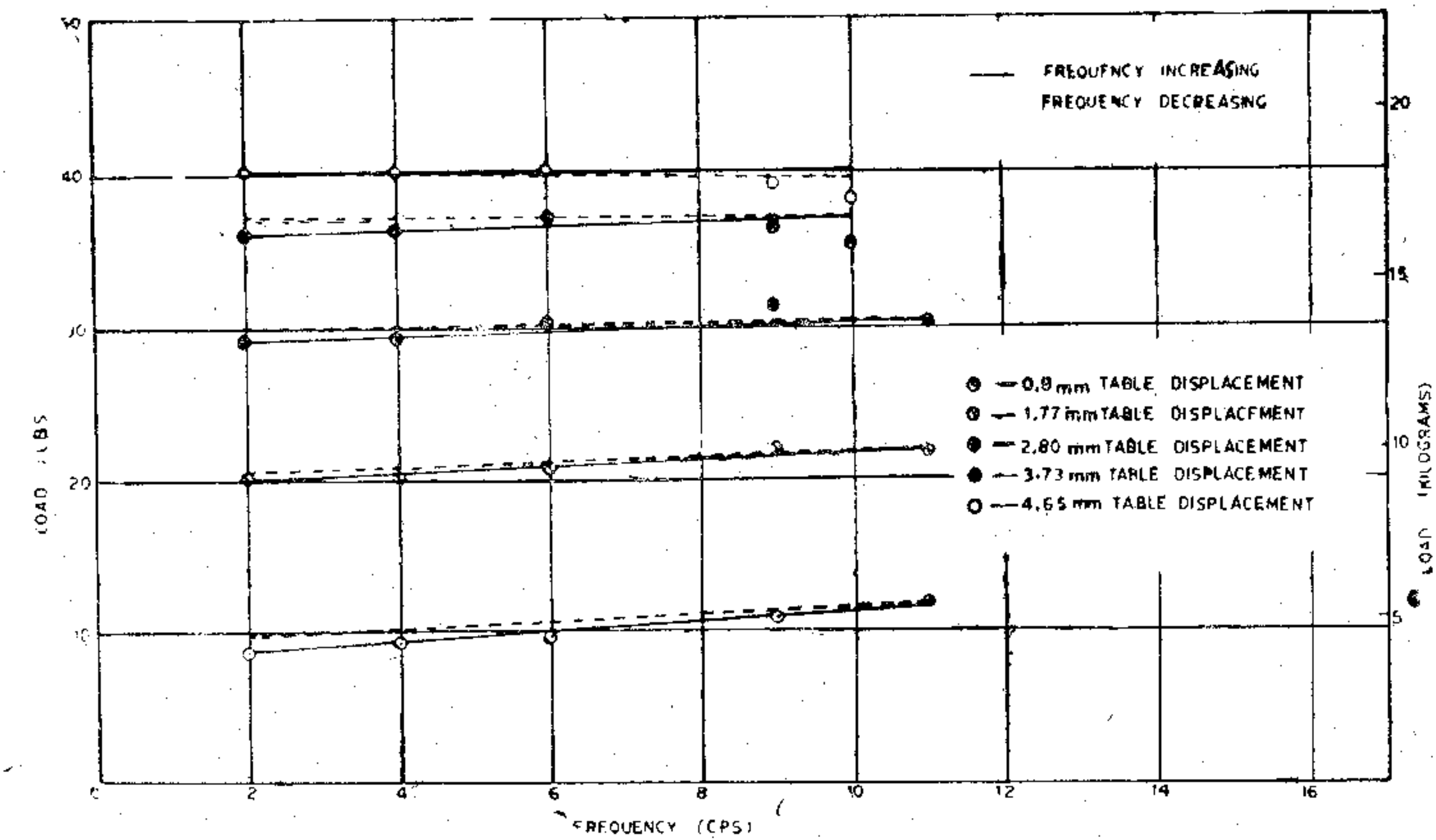


Figure 12 Load Frequency Curve (With Load at G.L.)

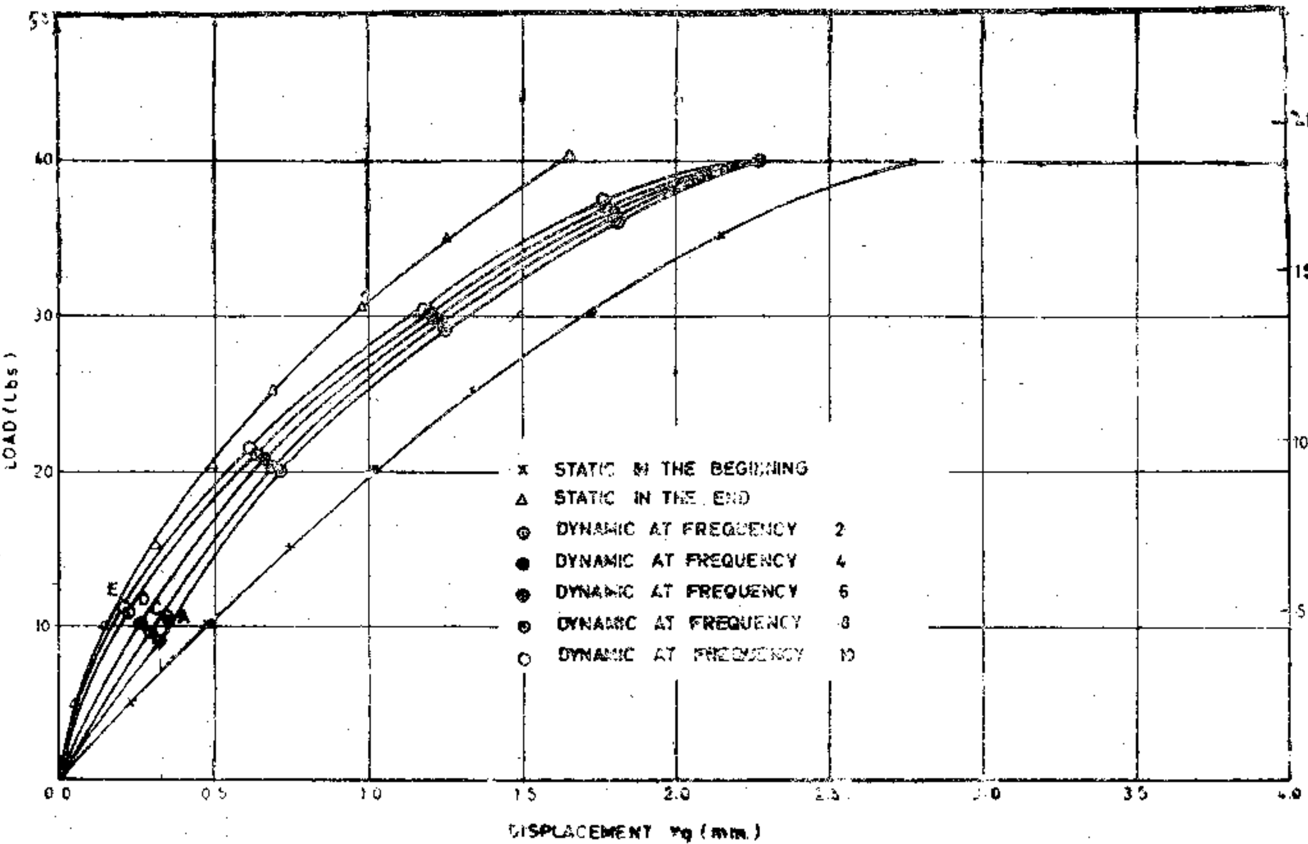


Figure 13 Load Displacement Curves with Loads at G.L. (Under Steady State Vibrations)

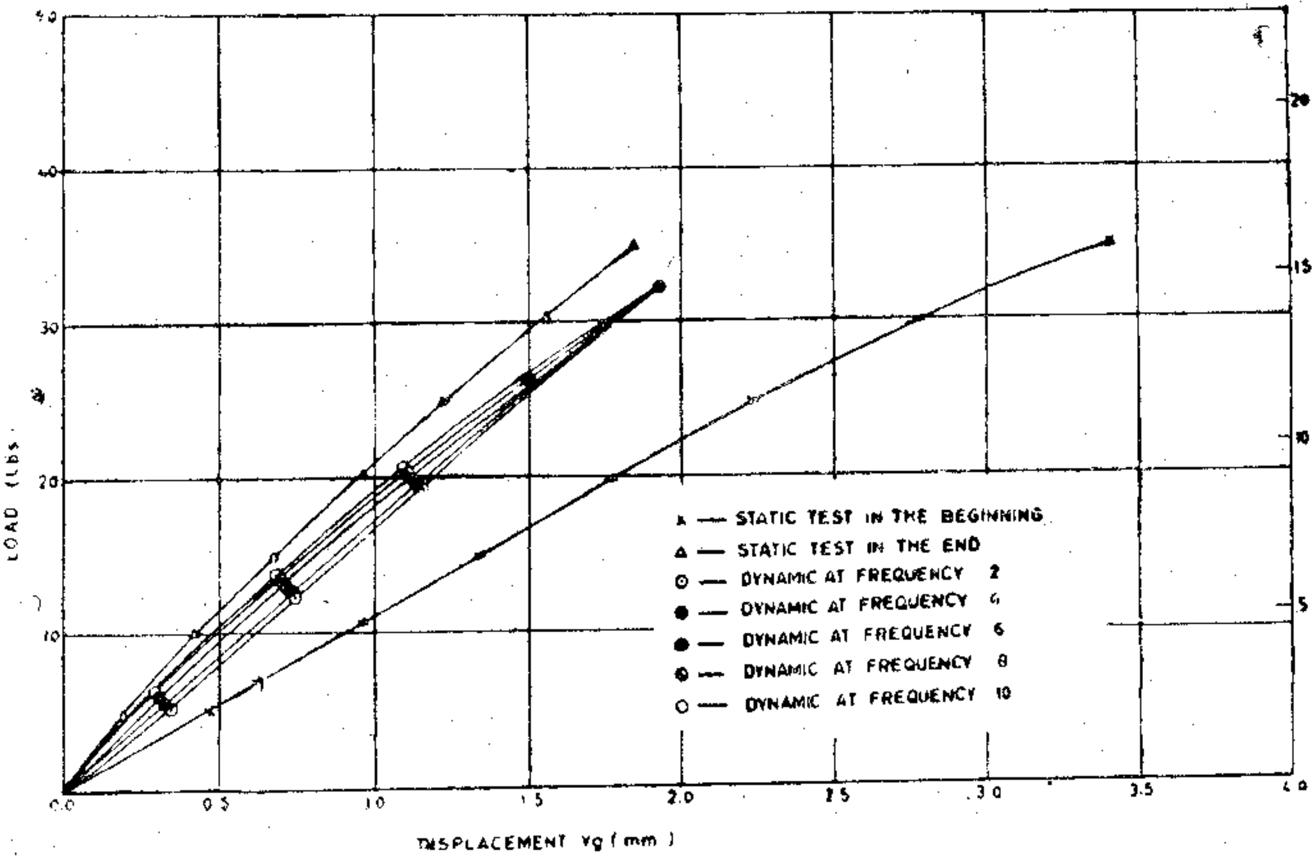


Figure 14 Load Displacement Curve with Load 5 cm High (Under Dynamic Load)

## DISCUSSION OF RESULTS

### Steady State Dynamic Loading—

It would appear from Figures 13 and 14 that the pile becomes stiff as frequency increases. Also it shows smaller displacement under a dynamic load than under a static load initially. It is, however, important to note that when the pile is vibrated at a fixed table

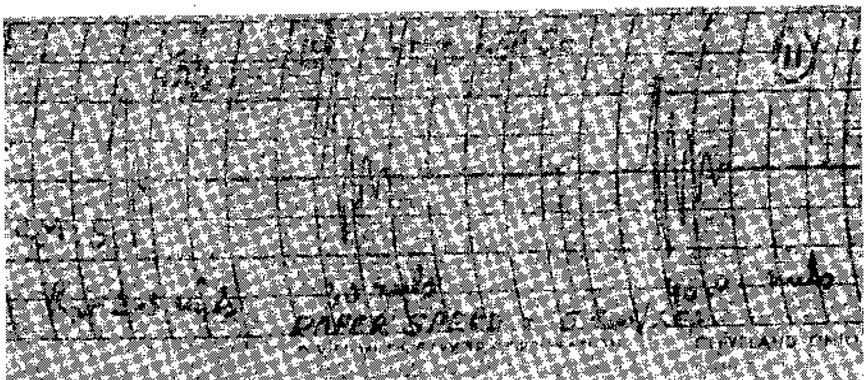


FIG.15_RECOR OF DYNAMIC LOADS UNDER TRANSIENT LOADING

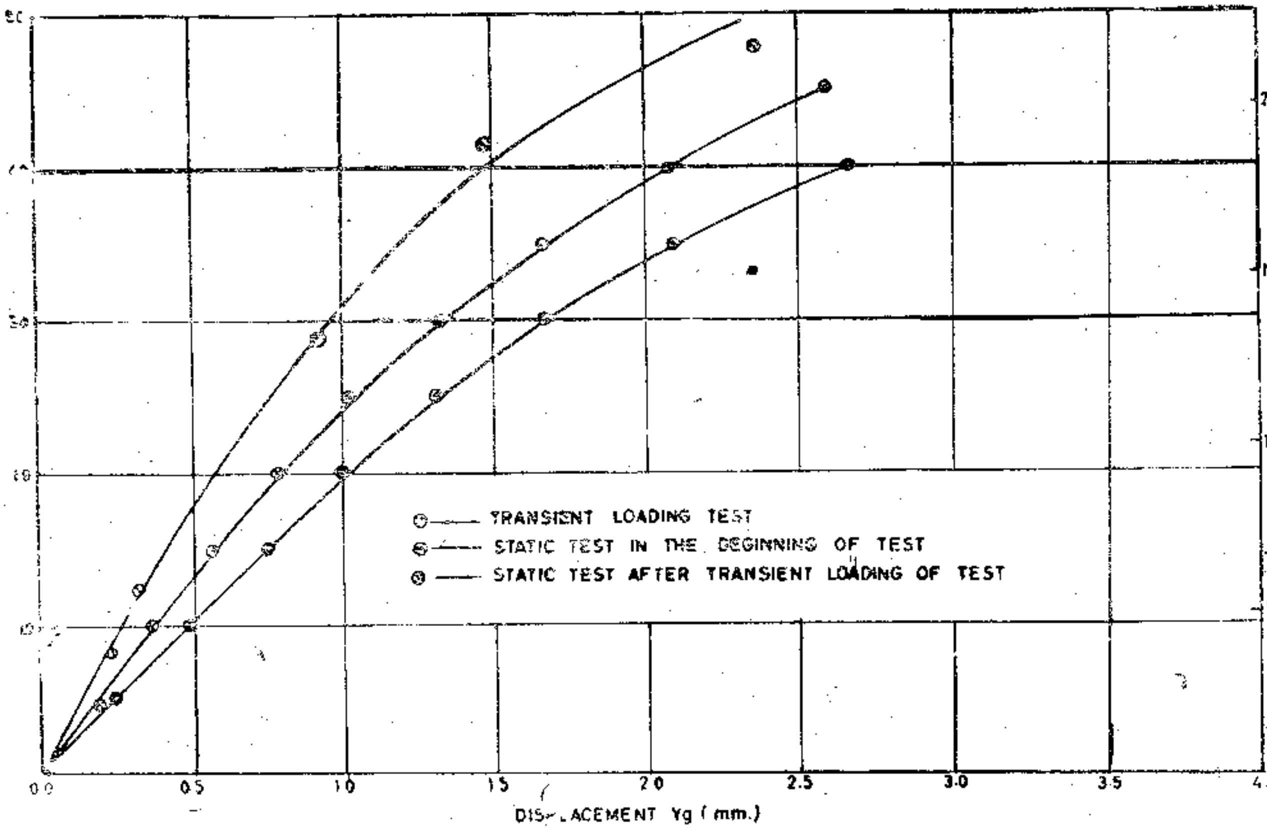


Figure 16 Load VS Displacement Curve Under Transient Loading (Loan at G.L.)

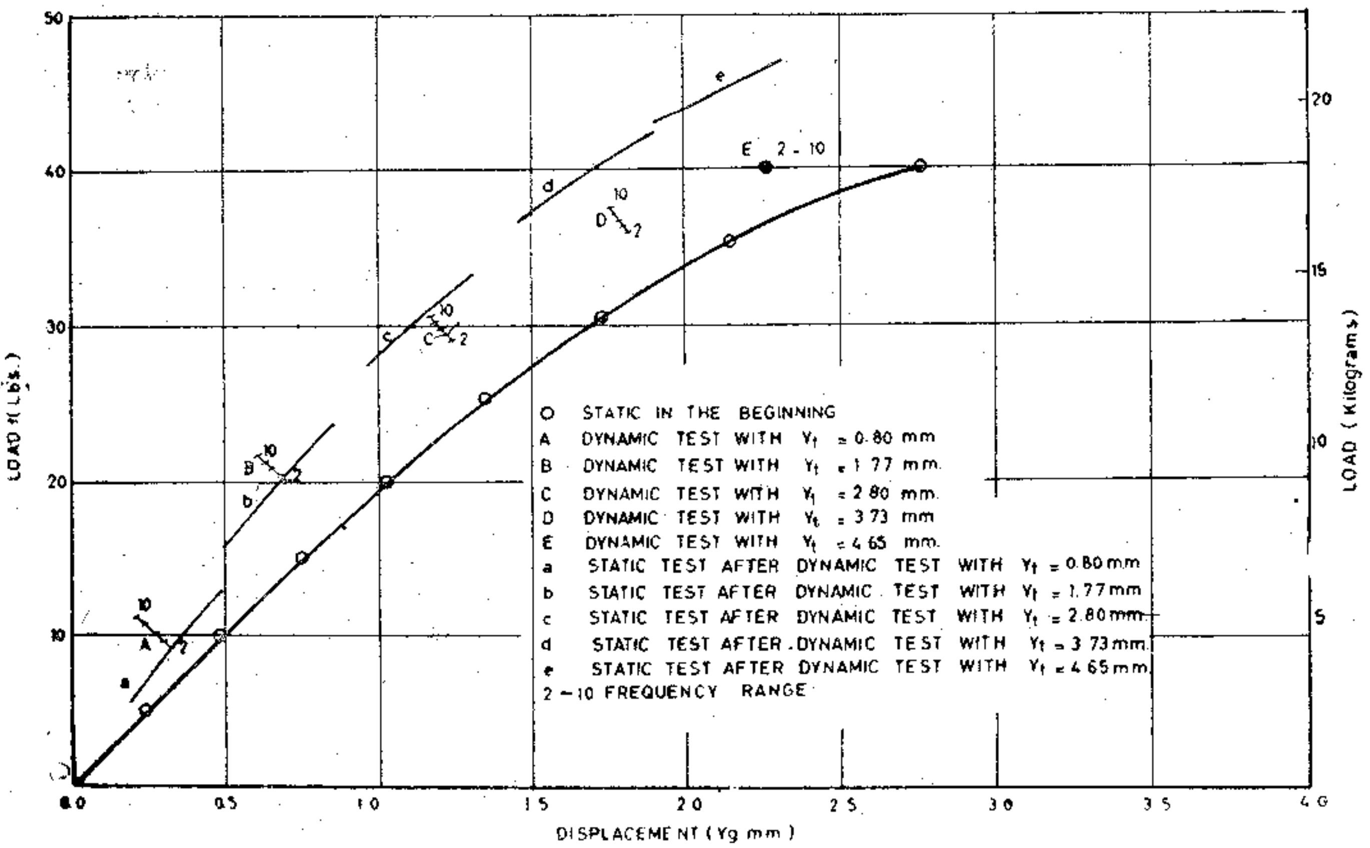


Figure 17 Load Displacement curves Under Static and and Dynamic Loads with Load at G.L.

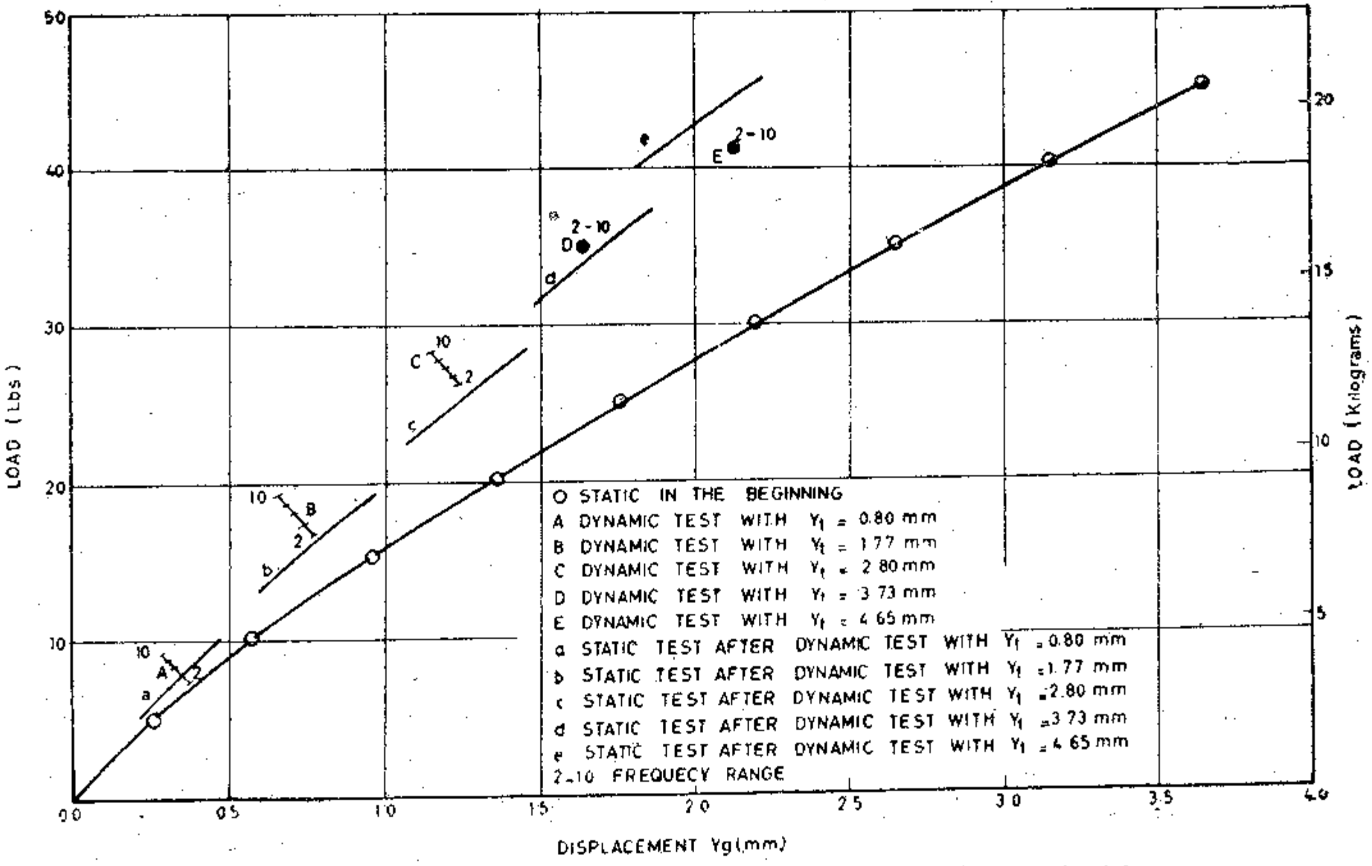


Figure 18 Load Displacement Curves Under Static and Dynamic Load with Load 2.0 cm High

displacement the load varies, usually increasing with increase in frequency (Figure 12). The charge in load is a manifestation of both the rate of loading and the increase in density of sand due to compaction.

For the same table displacement ($y_t$), at greater load, the deflection of the proving frame increases resulting in smaller displacement of the pile (Equation 5). In order to study this phenomenon, Figures 17, 18, 19, 20 and 21 were plotted for the load at the ground level, 2 cms, 3 cms, 4 cms and 5 cms above the ground level respectively. In these figures A, B, C, D, E represent load versus displacements of pile for table displacements of 0.8 mm, 1.77 mm. 2.80 mm, 3.73 mm and 4.65 mm respectively. The frequency interval plotted is 2 to 10 cps. Corresponding static test data after each dynamic test is shown by curves a, b, c, d and e respectively. In Figure 17, these curves are displaced up from the initial load-displacement curve, because vibration of pile causes compaction. The difference between b and a is larger than that between c and b, and the difference between e and d is very small. This indicates that compaction of the soil has taken place to a large degree first and with increase in amplitude of vibration, further compaction is small. But when the load is applied above the ground level, (Figures 18, 19, 20 and 21), the soil continues to get compacted so that curve

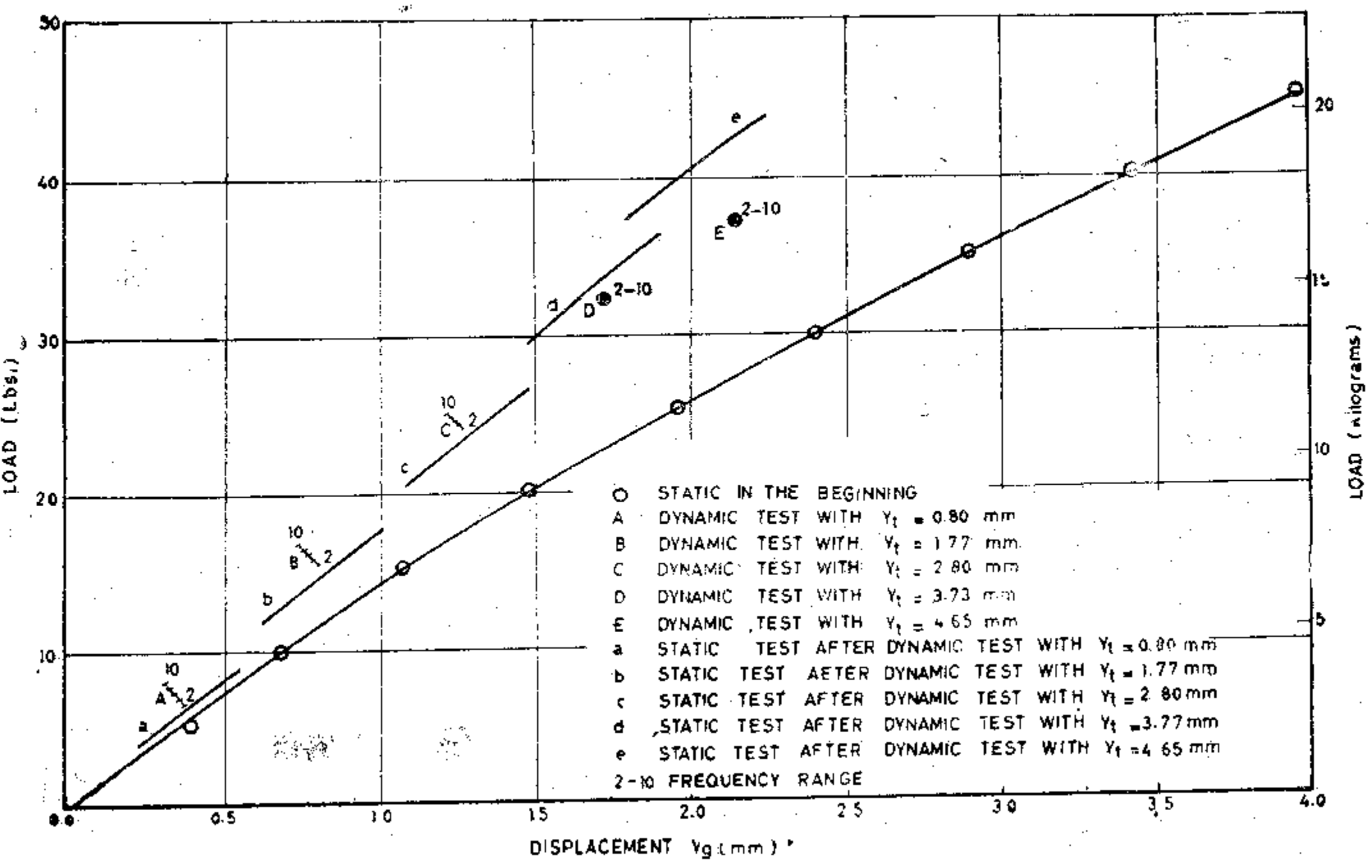


Figure 19 Load Displacement Curves Under Static and Dynamic Loads with Load 3.0 cms High

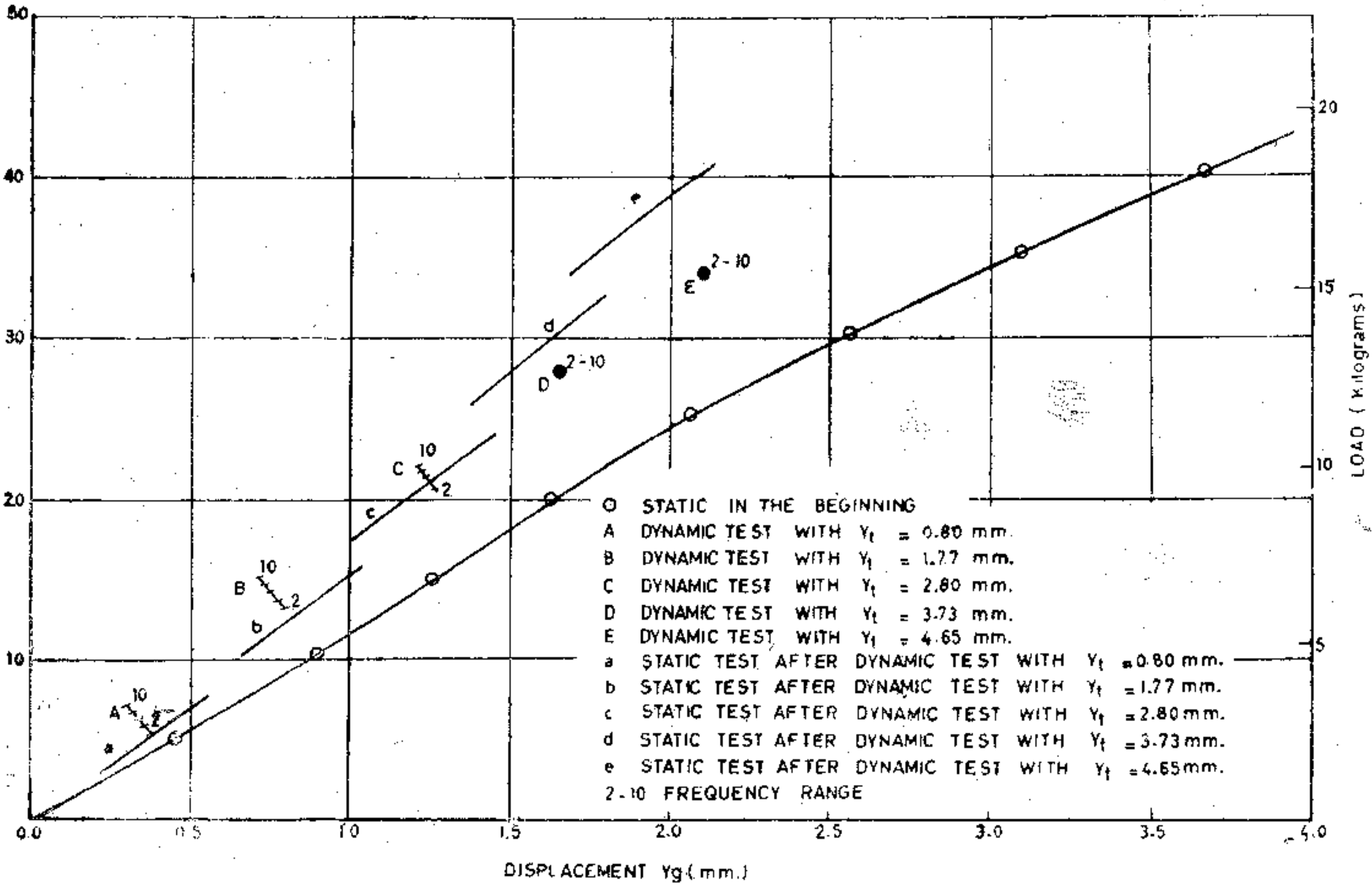


Figure 20 Load Displacement Cusves Under Static and Dynamic Loads with Load 4.0 cm High

e is displaced from d almost as much as b from a. Despite this apparent variation in the compactive behaviour of the soil, the following dynamic behaviour is exhibited in all the curves.

Curve a and b are lower than curves A and B respectively indicating that effect of compaction during vibration is more predominant. Curves c, d, e are higher than C, D, E respectively for load at ground level (Figure 17), e is higher than E for load at 2.0 cm above the G.L. in Figures 18, d and e are higher than D and E for load at 3, 4 and 5 cms above the G.L. (Figures 19, 20 and 21 respectively). The trend is that load displacement curves in static case are above the load displacement in the dynamic case after some compaction of the soil has taken place. This indicates that the effect of rate of loading is predominent. The static strength immediately after the dynamic test is greater than the corresponding dynamic strength in cases cited above. Similar explanation can be advanced if comparison is sought between load-displacement curves in the dynamic test and static test before such a test. These findings are only qualitative in nature.

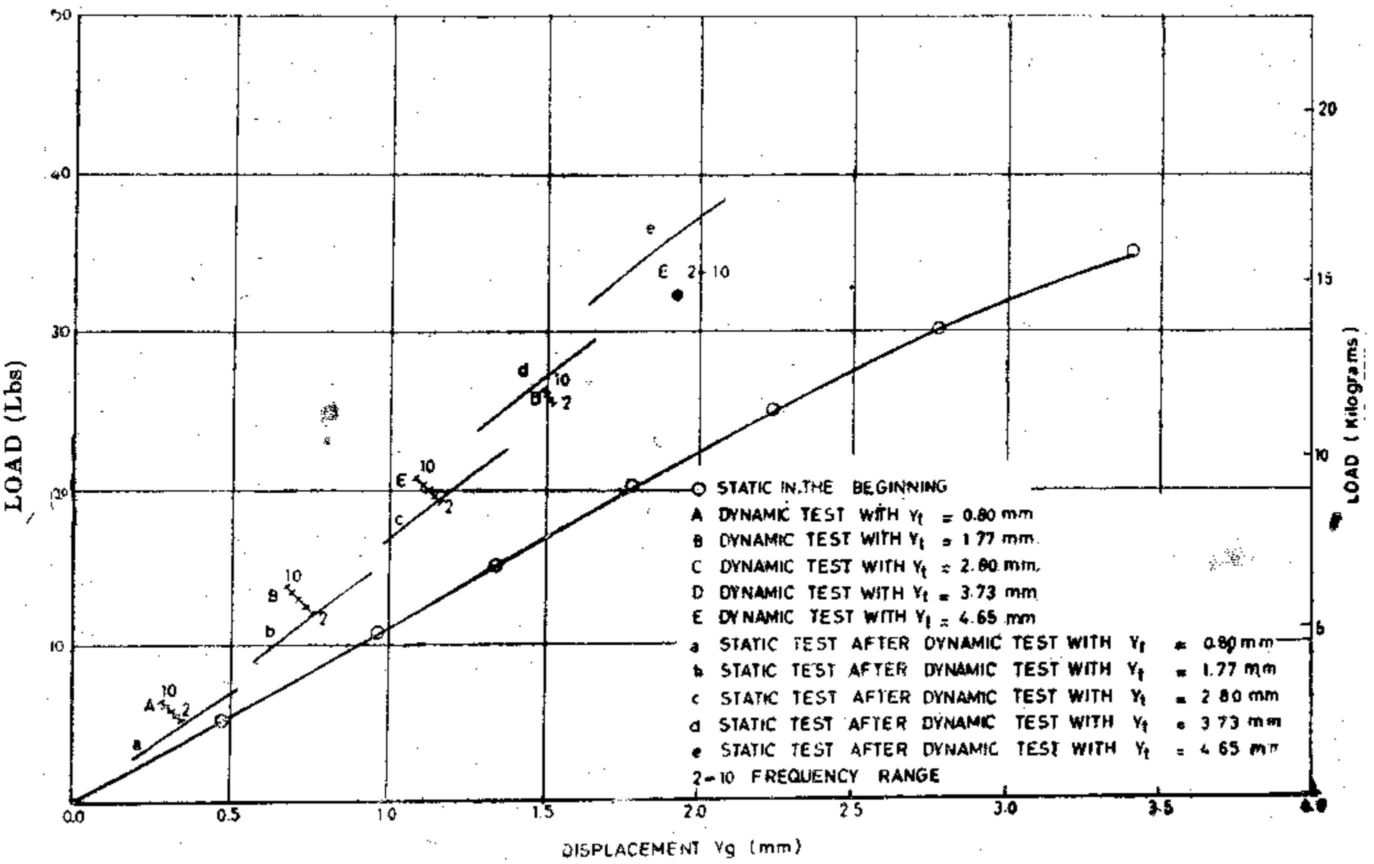


Figure 21 Load Displacement Curves Under Static and Dynamic Loads with Load 5.0 cm High

Another interesting point was that in many cases of dynamic test, only one point was obtained, E in Figures 17 and 21, D and E in Figures 18, 19 and 20. This indicates that the effect of compaction during vibrations is negligible. Thus, it would appear that the effect of dynamic loading is to make the pile soil system less stiff while the change in frequency of loading has no effect, within the range of frequencies of these tests.

**Transient Loading—**

Figure 16 is a plot of load-displacement at ground level under the transient test at a frequency of 2 cps. Plots of static test before and after the transient test have also been shown. It will be observed that the soil gets compacted, the plot of the static test after the transient test being higher than the one before the transient test. But the displacements in transient test are smallest of all the tests.

**Comparison of Static, Steady State and Transient Loading—**

Figure 22 is a plot of load displacement for the initial static case and steady state and transient loading, both at 2 cps applied at G.L. The displacements in transient test are the

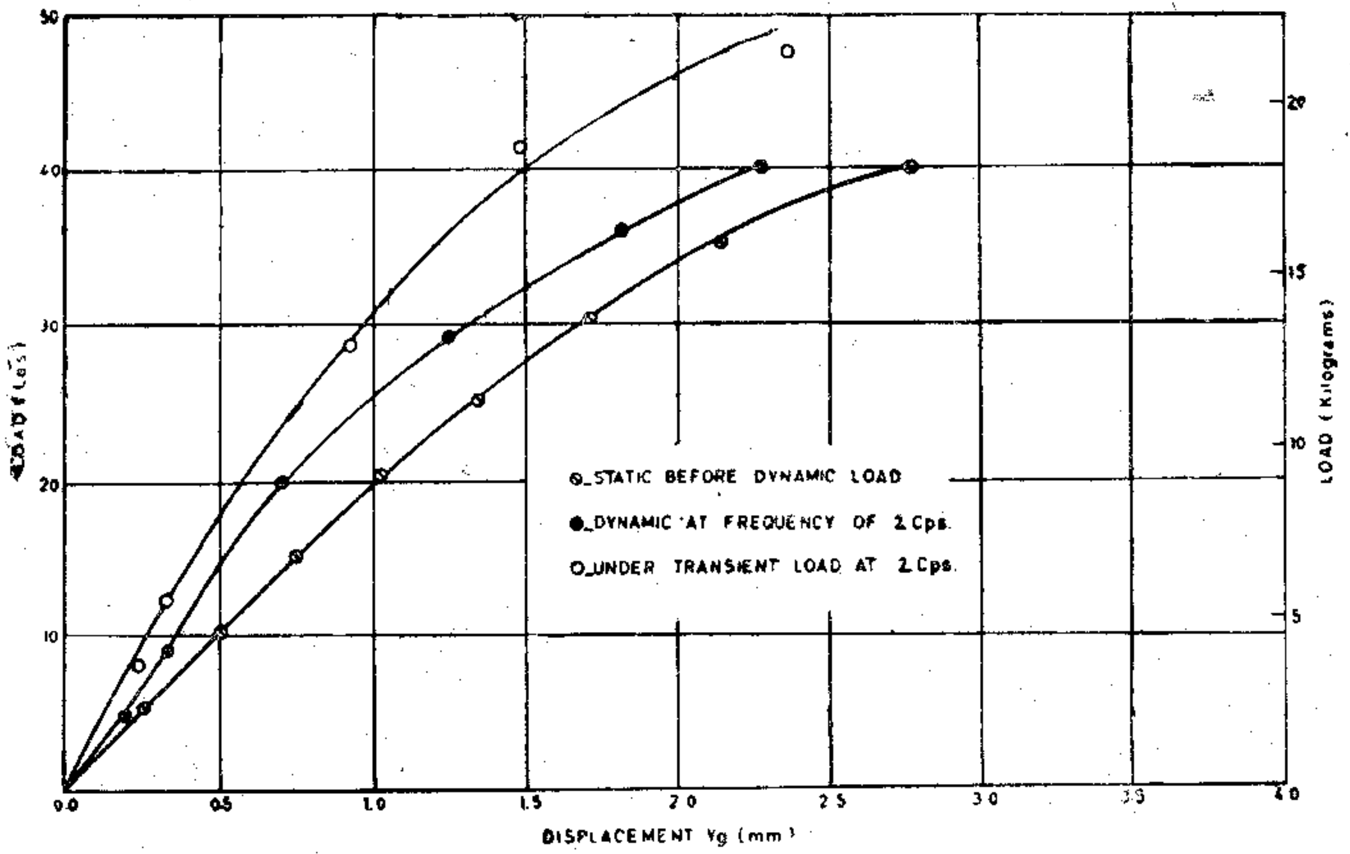


Figure 22 Load Displacement Curves Under Different Kinds of Loads

smallest, while under steady state loading, these are smaller than the initial displacements under a corresponding load.

## Soil Modulus and Damping

The values of the soil modulus are shown in Table 1.

The natural frequency record of the soil-pile system afforded an opportunity to calculate the damping of the system, which was about 3 percent.

## CONCLUSIONS

1. Under steady state dynamic loading, the soil around the pile gets compacted. In initial stages, the effect of compaction is more predominent.

2. The initial static strength of the pile is less than the dynamic strength. After the soil gets sufficiently compacted the static strength is greater than the dynamic strength.

3. Under transient loads, the soil around the pile also gets compacted and the static strength obtained before and after transient testing is less than the transient strength.

4. The transient strength of a pile is greater than the steady state dynamic strength which in turn is greater than the initial static strength.

5. The zone of influence of a dynamically loaded pile extends to a considerably greater distance then that for a statically loaded pile.

6, The damping of soil-pile system is about 3 percent.

## ACKNOWLEDGEMENTS

The investigation was carried out in the School of Research and Training in Earthquake Engineering. Mr. H.C. Dhiman assisted in fabrication of the test set up and Mr.A.P.Sharma assisted in recording the test data. The co-operation of all the staff and encouragement of the Director are very much appreciated.

## REFERENCES

Aggarwal, S.L, (1964) "Static and Dynamic Behaviour of a Vertical Pile Subjected to Lateral Loads"- Master of Engineering Thesis, University of Roorkee, Roorkee 1964.

Aggarwal, S.L. and M.R. Soneja (1965) "Lateral Load Tests on Piles", Journal of Central Board of Irrigation and Power, Jan. 1965. Vol. 22, No. 1, pp 43–55.

"ANNUAL REPORT", School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee. U.P. (INDIA) 1960–61.

Broms, B.B. (1964 a) "Lateral Resistance of Piles in Cohesive Soils", Proc. ASCE Vol 90 No. SM 2

Broms, B.B. (1964 b). "Lateral Resistance of Piles in Cohesionless Soils", Proc. ASCE Vol. 90 No. SM 3.

Broms, B.B. (1965), "Design of Laterally Loaded Piles", Proc. ASCE, SM3 May 1965, P. 93.

Casagrande, A. and W.L. Shanon (1948). "Research on Stress–Deformation and Strength Characteristics of Soils and Soil Rocks Under Transient Loading". Soil Mechanics Series No. 31, Harvard University.

Davisson, M T. (1960) "Behaviour of Flexible Vertical Piles Subjected to Moment, Shear and Axial Load". Ph.D. Thesis, University of Illinois.

Davisson, M.T. and H L. Gill (1963) "Laterally Loaded Piles in a Layered Soil System". Proc. of Am. Soc. of Civil Engineers SM 5.

Gaul, R.D., (1958) "Model Study of a Dynamically Laterally Loaded Pile" Proc. Am. Soc. of Civil Engrs. SM 1.

Hayashi, S. and N. Miyajima (1962) "Dynamic Lateral Load Tests on Steel H–Piles". Proc. Japan National Symposium on Earthquake Engineering, Tokyo. Japan.

Prakash, S. (1962) "Behaviour of Pile Groups Subjected to Lateral Loads" Ph.D. Thesis, University of Illinois.

Prakash, S. and S L. Aggarwal (1965) "Study of a Vertical Pile Under Dynamic Lateral Load" Proc. Third World Conference on Earthquake Engineering, Auckland, New Zealand.

Prakash, S. and G. Subramanyam (1964) "Load Carrying Capacity of Battered Piles", University of Roorkee Research Journal, September 1964.

Prakash, S. and G. Subramanyam (1964b) "Behaviour of Battered Piles Under Lateral

Loads", Journal of Ind. Nat. Soc. of soil Mechs. and Found. Engg., Vol. 4 No. 2 April 1965, pp 197–196.

Reese, L.C. and H. Matlock (1956) "Non-dimensional Solutions for Laterally Loaded Piles with Soil Modulus Assumed Proportional to Depth". Proc. Eighth Texas Conf. on Soil Mechanics and Foundation Engineering, Austin.

Terzaghi, K. (1955) "Evaluation of Coefficients of Subgrade Reaction". Geotechnique, Vol. 5, pp 297–326.

Winkler, E. (1867) "Die Lehre Von Elastigatat UnD Festigkeit" Prague, p. 182.

## APPENDIX A—NOTATIONS

The units are in Force (F), Length (L) and Time (T)

| Symbol | | | Units |
|---|---|---|---|
| $A_y$ | — | Non-dimensional deflection coefficient corresponding to horizontal shear | |
| B | — | Width of pile | L |
| $B_y$ | — | Non-dimensional deflection coefficient corresponding to Moment | |
| D | — | Diameter of pile | L |
| E | — | Modulus of Elasticity of pile | $FL^{-2}$ |
| EI | — | Flexural Stiffness of Pile | $FL^2$ |
| f | — | Frequency | $T^{-1}$ |
| I | — | Moment of intertia | $L^4$ |
| k | — | Modulus of subgrade Reaction | $FL^{-3}$ |
| $k_x$ | — | Value of k at any depth x | $FL^{-2}$ |
| $L_s$ | — | Embedded length of pile | L |
| M | — | Moment | FL |
| $M_g$ | — | Moment at Ground level | FL |
| $M_s$ | — | Moment produced by rotational restraint | FL |
| n | — | Empirical coefficient | |
| $n_h$ | — | Constant of modulus of subgrade reaction | $FL^{-3}$ |
| $Q_{hg}$ | — | Horizontal shear at G.L | F |
| T | — | Relative stiffness factor $(EI/n_h)^{1/5}$ | L |
| $y_g$ | — | Displacement of pile at G.L | L |
| $y_l$ | — | Displacement of pile at load level | L |
| $y_{pe}$ | — | Number of chart lines deflected in pen recorder according to calibration | |
| $y_{pf}$ | — | Deflection of proving frame | L |
| $y_t$ | — | Displacement of shaking table | L |
| $Z_{max}$ | — | Non-dimensional depth coefficient | |
| | — | L/T | |
| $\varepsilon$ | — | Strain | |