Bull. Ind. Soc. Earthq. Tech., Paper No. 244, Vol. 22, No. 4, December 1985, pp-125-140

SOIL-STRUCTURE INTERACTION WITH FOUNDATION TIPPING

VINOD BATTA* AND A.R. CHANDRASEKARAN**

SYNOPSIS

Strong earthquakes could induce large overturning moments causing the base mat of the structures to temporarily lift off the ground during some cycles and results in non-linear interactive forces between the base of structure and the foundation. This study incorporates the analysis of a box type cylindrical structural system to evaluate the importance of non-linear soil-structure interaction. A parametric study involving various structural and foundation properties has been carried out incorporating both linear and non-linear behaviour of the foundation contact area. The effect of modal damping is studied and the influence of various assumptions has been brought out. It is concluded that damping values for the structures founded on softer soils considerably influence response and therefore should be carefully selected. Non-linear analysis incorporating effect of tipping should be carried out for important structures founded on softer soils, though for stiffer foundations linear analysis may be satisfactory.

Key Words : Soil-Structure Interaction, Foundation Tipping

INTRODUCTION

Earthquake forces are transmitted to a structure through its foundation. The method must commonly used by structural engineers for the analysis of buildings, assumes the structures to be firmly bonded to a rigid foundation. By contrast, real foundations provide at best a limited resistance to tension and hence the apparent tension side of a raft or footing foundation could actually lift off the ground due to large overturning moments caused by lateral forces on structures.

Under the effect of strong ground motion, the overturning moments induced may exceed the stabilizing moment of the structure due to its own weight. The interactive forces between the base of the structure and

* Presently R.A. at Concordia University, Montreal, canada

** Professor in Earthquake Engineering, University of Roorkee, Roorkee, India

foundation would be complex due to tipping action as the area of contact could vary (Kennedy, et al. 1976 and Beucke, et al. 1983).

The present study evaluates the importance of non-linear soil-structure interaction effects resulting from substantial base slab uplift occuring during a seismic excitation. Since structures like nuclear containment structures, oil storage tanks, silos etc. are of cylindrical box type, the study incorporates the analysis of a typical box type structural syetem which comprises cylindrical shell with slabs on its top and bottom.

A simplified dynamic mathematical model is utilized consisting of a conventional lumped mass structure with soil - structure interaction accounted for by translational and rotational springs, whose properties are determined by elastic half space theory, and the non-linear behavior of stiffness properties of the springs is incorporated to account for the effect of tipping Three different soil conditions (ranging from soft soil to a stiff rock site) and three different slenderness ratios of the structure are considered to take practical variations into account. An artificially generated earthquake time history was applied in a single horizontal direction for obtaining the response. Coupling between vertical and horizontal response or between response in two orthogonal horizontal directions is not considered in this study.

For each of the different combination of above parameters both linear and non-linear analyses (accounting for non-linear soil-structure interaction resulting from base slab uplift) have been performed Particular attention has been paid in this study to model damping and the effect of damping due to different assumptions has been brought out. Newmark's constant average acceleration method (Bathe, 1976) has been used for the solution of equations of motion.

The present study reveals damping needs careful attention particularly when structure is founded on softer soils. Linear analysis conservatively estimates the important behaviour of structures even under conditions of substantial uplift. Analysis of important structures founded on softer soils should invariably incorporate the non-linear effect of tipping, though the linear anlysis may be satisfactorily used to give results for stiffer foundations like that at rock sites.

ANALYSIS

In this study, for the purpose of analysis, a typical box tvpe cylindrical structural system, which comprises cylindrical shell with slabs on its top and bottom has been chosen and analysed considering soil-structure interaction effect with foundation tipping. For one set of data the structure is considered to have the following properties.

(i)	Total height of the structure	= 50.0 m
(ii)	External radius of the structure	= 25.0 m
(iii)	Thickness of top and bottom slab	= 2.5 m
(iv)	Thickness of cylindrical shell wall	= 0.6 m
(v)	Modulus of elasticity for concrete	= 3×10^6 t/m^2
(vi)	Poisson's ratio for concrete	= 0.2
(vii)	Specific weight for concrete	= 2.4 t/m^3

Analysis has also been carried out for two other sets of data. In one case, the height of the structure is increased to 75 m with radius kept the same to get increased mass and slenderness ratio of the structure. In the other case height of the structure is kept as 50 m but the radius is reduced to 10 m.

In all the above cases the structure is analysed at three different soil sites, defined as Site-A (Rock), Site-B (Moderately stiff soil), and Site-C (Soft soil) with properties as given in Table-1.

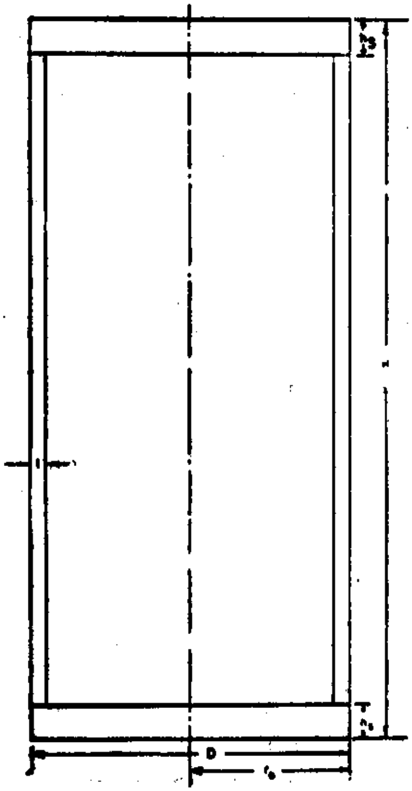
TABLE–1

Material Properties of Sites Considered

Material Property	Site-A Rock	Site-B Moderately Stiff Soil	Site-C Soft Soil
Shear wave velocity m/sec	1800	580	330
Poisson's ratio	0.30	0.35	0.40
Density t/m^3	2.50	1.80	1.60

For the dynamic analysis of these type of system the time history of the ground motion is needed as any other method of specifying earthquake effect in the form of seismic coeffiecient or spectra will not do. The shape of the spectra given in IS:-1893-1975 has been chosen as a model and an artificial time history that matched the 5% damping curve of IS Code has been used. However the amplitude of ground motion has been varied and the two peak values considered in the study are 0.1g and 0.2g keeping the same waveform of ground motion. It is to be pointed out that these values are more severe than suggested by IS Code and are more likely during actual earthquake environment in severe zones.

A simplified mathematical model of the structure consisting of a conventional finite beam elements is utilized. A schematic view of the structure and its model together with the node and beam numbers used is shown in Fig. 1 & 2 The physical properties of the structural model are given in Table 2. Each mass point has both translational and rotational degree of freedom.



t=Thickness of the Walls, h_s=Thickness of Top and Bottom Slab, h = Overall Height of structure r_o= Radius of base Slab, D=Diameter of base slab

Fig. 1. General View of Structure

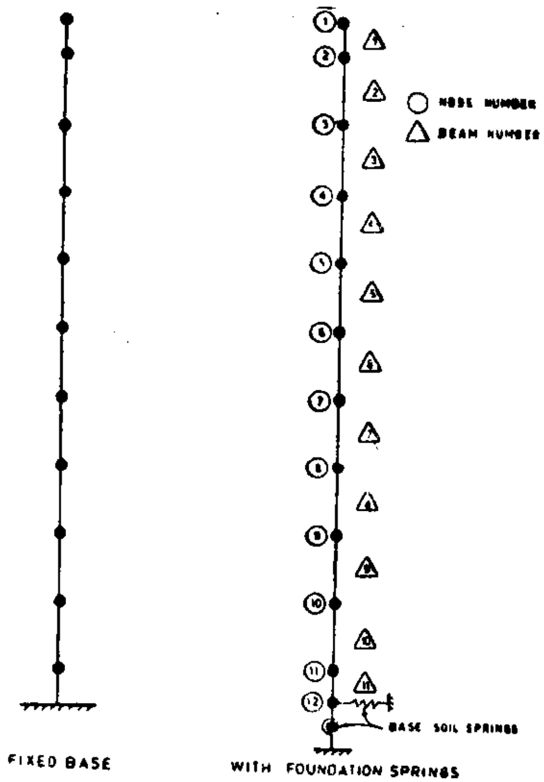


Fig. 2. Structural Model

TABLE—2

Structural Model Member Properties

Member No	Material No	Area m^2	Effective Area in Shear m^2	Moment of Inertia m^4
1	1	1963.5	981.7	306796
2-10	1	93.1	46.6	28409
11	1	1963.5	981.7	306796

Material No	Material	E (t/m^2)	ν
1	Reinforced Concrete	3×10^6	0.2

Soil-Structure Interaction

Soil-structure interaction is most often modelled either by a lumped parameter finite beam element model or by finite element model. The finite beam element approach is substantially simpler and less costly, and enables a much greater variety of parameteric studies to be performed for a given time and cost constraint. Furthermore, for conditions where the soil can be modelled as an elastic halfspace the soil-spring approach is often considered to be sufficiently reliable model Extensive analytical and experimental studies are now available for modelling the foundation soil in the lumped parameter approach. As a result, the lumped parameter approach has been used extensively to model the soil-structure interaction in this study.

The elastic base spring properties K_x and K_ϕ are calculated based upon spring constants for circular footing resting on elastic half-space (Richart et al., 1970). These are :

$$K_x = [\,32\,(1-\nu)\,G\,r_o\,]/[7-8\nu] \tag{1}$$

$$K_\phi = [\,8\,G\,r_o^3]/3\,(1-\nu) \tag{2}$$

where,

G = Shear modulus of soil

r_o = Radius of circular footing

ν = Poission's ratio

The base slab of box type cylindrical structure with its vertical perimeter shell walls is assumed to be rigid.

Non-Linear Soil-structure Interaction

Tipping Criterion

Tipping occurs - by definition in this study - when and where the normal compressive stress under the base mat drops to zero. Tensile stresses are not permitted between the base mat and the soil. Thus, with sufficient overturning moment there is a tendency for a portion of the base mat to lift off the underlying supporting media. Figure 3 illustrates the behaviour of base mat under the assumption that no uplift tension can develop at the soil-slab interface. At the bottom of the slab, the soil-slab interface is subjected to a vertical load P_b, an overturning moment M_b and the lateral shear V_b. The overturning moment M_b and the lateral shear V_b are the values associated with soil springs K_ϕ and K_x respectively and represent the moment and shear which must be transmitted to the soil alongwith the vertical load through the base slab.

Now for a given base moment and vertical force, the contact length can be obtained from equilibrium condition if a stress distrbution under the base mat is assumed. In this study, it is assumed that the interface pressure varies linearly across the contact length of the slab as shown in Fig. 3 Based on this assumption tipping will occur when,

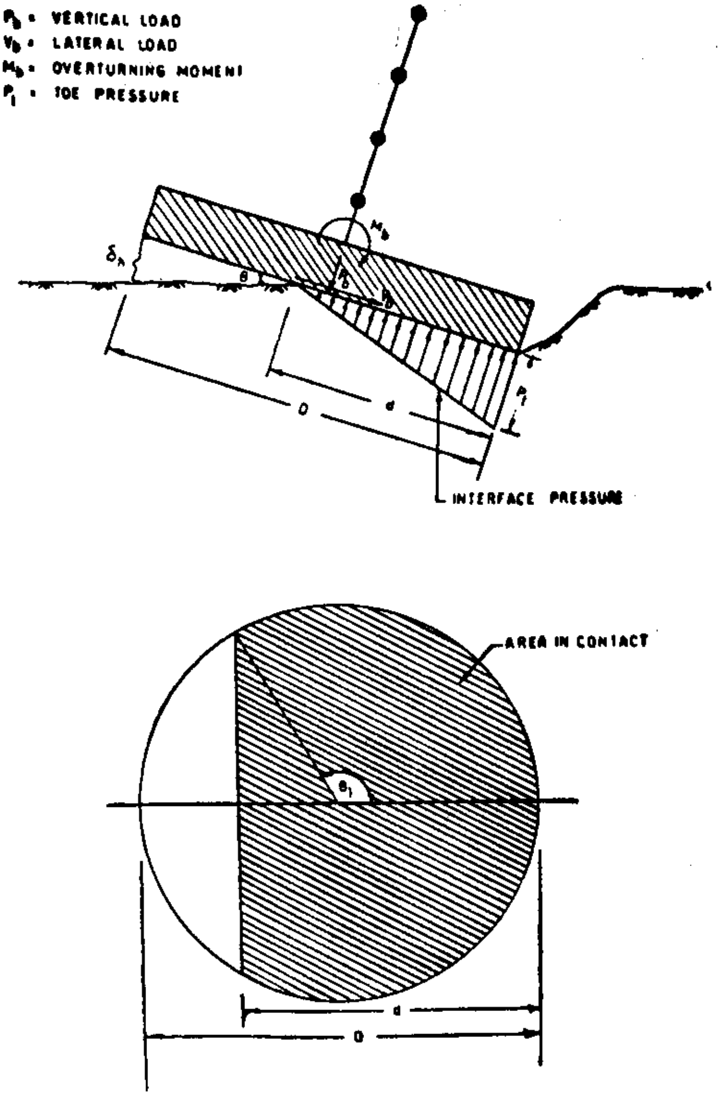


Fig. 3 Base Slab Rocking Showing Uplift

$$|M_b| \geq M_u$$

where,

M_u = Uplift moment

For a circular base slab, M_u is given by

$$M_u = P_b D / 8 \tag{3}$$

where, D = Diameter of base slab.

Prior to uplift initiation ($|M_b| \leq M_u$), the entire slab remains in contact i.e. d = D; the base spring stiffnesses are given by (1) & (2); and toe pressure is given by

$$P_t = 4P_b / D^2 (1+8M_b/D) \quad (\text{For } |M_b| \leq M_u) \tag{4}$$

where terms are already defined.

Assuming no interface tensile capacity, once the absolute value of M_b exceeds M_u, only a portion of base slab remains in contact; the toe pressures rise; and the base spring stiffenesses soften. For a circular base amount of slab in contect is given by

$$d/D = 4/3 - M_b/3M_u \quad (|M_b| \geq M_u) \tag{5}$$

where, d = Width of contact area after uplift.

Referring Fig (3), 'd' can be written as

$$d = r_o (1 - \cos \theta_1) \tag{6}$$

where, r_o = Total radius of base slab and θ_1 is as shown in the figure.

If 'A' is total area of contact after uplifting ($|M_b| > M_u$) from Fig (3), can be written as

$$A = r_o^2 d\theta_1 - r_o^2 \sin \theta_1 \cos \theta_1$$
$$= r_o^2 (\theta_1 - \sin 2\theta_1/2) \tag{7}$$

The moment of inertia of the contact area about the axis perpendicular to the direction along which the contact length is measured, derived from fundamental principle comes out to be

$$I = r^4_o [(1/4) (\theta_1 + \sin 2\theta_1/2) - (5/36) \cos^2 \theta_1 \sin 2\theta_1 + (\sin^2\theta_1/(\theta_1 - \sin 2\theta_1/2) [\sin \theta_1 (I + \cos^2 \theta_1)]]$$

where r_o and θ_1 are as defined earlier.

The toe pressure P_t is given by

$$P_t = Pb/A + M_b y/I$$

where terms have usual notations.

The heel uplift 'δ_h' as shown in Fig. (3) would then be written as

$$\delta_h = D (I - d/D)\theta_1$$

Once the base slab uplifts, base stiffenesses soften because less area remains in contact. The foundation spring constants can than be determined by substituting the radius of equivalent circular contact area in the expressions for K_x and K_ϕ. The equivalent radius r_x to be used for calculating the translational foundation spring stiffness K_x is given by

$$\pi r_x^2 = A \quad \text{or } r_x = (A/\pi)^{1/2} \tag{11}$$

Similarly equivalent radius r_ϕ, to be used for evaluating the equivalent rotational foundation spring stiffness will be given by

$$(\pi/4)\, r_\phi^4 = I \qquad \text{or } r_\phi = (4I/\pi)^{1/4} \tag{12}$$

Thus, the non-linear behaviour of tipping system is accounted for by calculating the foundation spring constants at each time step, depending upon the actual area of contact at that time step. It was assumed that during a particular time step, the contact area does not change substantially and thus the foundation spring constants as calculated at the beginning of each time step have been used during that time increment.

Equation of Motion

Because of the non-linear behaviour it is necessary that a time-history dynamic analysis be performed instead of a response spectral type of analysis. The basic equation of equilibrium for n-node system, having two degrees of freedom per node and subjected to horizontal ground acceleration can be written as

$$[M]\{\ddot{z}\} + [C]\{\dot{z}\} + [K]\{z\} = -[M]\{I_x\}\ddot{y} \tag{13}$$

where,

[M], [C] and [K] are the mass, damping and system stiffness matrices

$\{z\}$ is the relative displacement vector

$\ddot{y}$ is the ground acceleration

I_x is a column vector with unit values corresponding to horizontal direction of excitation and zero value elsewhere.

Damping

Since a soil-structure system involves more than one material and since the damping values in soil are relatively larger than the structural damping values, a single damping value has been assigned to each mode which reflects both low damping of the structure and high damping of the soil. This has been accomplished by adopting a criterion of weighted damping based on strain energy stored in various materials. The weighted modal damping of the system is given by

$$\zeta_k = \frac{\sum_{i=1}^{n} \zeta_i E_i^{(r)}}{\sum_{i=1}^{n} E_i^{(r)}}$$

where, ζ_i is the damping in material 'i' having strain energy $E_i^{(r)}$ in r^{th} mode. For this study, the following damping values have been used (Chandrasekaran and Singhal, 1983)

Reinforced concrete = 7 %
Soil in rocking = 7 %
Soil in translation = 20%

Having obtained the weighted model damping, the damping matrix [C] can be expressed as a linear combination of mass [M] and stiffness [K] matrices.

Thus,

$$[C] = \alpha\,[M] + \beta\,[K] \tag{15}$$

where α and β are multipliers and can be related to damping as

$$\zeta_r = \alpha / 2p_r + \beta p_r/2 \tag{16}$$

where p_r is the circular natural frequency in r^{th} mode

Solution

Since in the present study the non-linear equations of motion are formulated at each time step, by change the stiffness matrix, method of their solution has obviously to be direct integration using numerical step-by-step procedure. Here Newmark's constant average acceleration technique has been used. In this method, in order to obtain the solution at time $t + \Delta t$, equilibrium equations are considered at time $t + \Delta t$ itself, i.e.

$$\underline{M}\,\ddot{\underline{z}}_{t+\Delta t} + \underline{C}\,\dot{\underline{z}}_{t+\Delta t} + \underline{K}\,\underline{z}_{t+\Delta t} = \underline{R}_{t+\Delta t} \tag{17}$$

This equation after necessary transformation yields

$$\underline{K}^*\,\underline{z}_{t+\Delta t} = \underline{R}^*_{t+\Delta t} \tag{18}$$

where,

$\underline{K}^* = [(4/\Delta t^2)\,\underline{M} + (2/\Delta t)\,\underline{C} + \underline{K}\,]$ = Effective stiffness matrix

$\underline{R}^*_{t+\Delta t} = \underline{R}_{t+\Delta t} + \underline{M}\,[(4/\Delta t^2)\,(\underline{z}_t + \dot{\underline{z}}_t \Delta t) + \ddot{\underline{z}}_t] +$
$\underline{C}[(2/\Delta t)\,\underline{z}_t + \dot{\underline{z}}_t]$ = Effective load vector.

Since in the above equation all other quantities are known, $\underline{z}_{t+\Delta t}$ can be directly obtained. However, it was observed that such a direct solution leads to large numerical instabilities in this case, probably due to quantitatively large character of stiffness matrix elements. This difficulty is overcome by substituting.

$$\underline{z}_{t+\Delta t} = \underline{z}_t + \Delta\, \underline{z}_{t+\Delta t} \tag{19}$$

and solving the above equation for $\Delta \underline{z}_{t+\Delta t}$ from which $\underline{z}_{t+\Delta t}$ can be obtained. Time step size used is 0.01 sec.

RESULTS AND DISCUSSIONS

Table (3) present the results of the various analyses. In the linear analysis the initial linear soil-structure interaction stiffnesses are used throughout the analysis. In the non-linear analysis, on the other hand, stiffness of the springs depending upon the actual area of contact of the base slab (whenever base slab is in a state of uplift) at the instant of time are used.

The tension bearing capacity of the soil-structure interface is assumed to be zero at all sites considered. Also, stresses generated in the soil and the structure due to impact have been assumed to be negligible.

The study indicates that the phenomenon of foundation tipping bec mes more pronounced in softer as compared to a rock. All the tables (3) show that the softer the soil the greater the uplift of the base mat due to lateral earthquake load.

TABLE—3 (a)

H=50.0 m, R=25.0 m, Maximum Acceleration=0.1g

	SITE-A		SITE-B		SITE-C	
	L	NL	L	NL	L	NL
Max. Base Overturning Mom. M_b in t-m$\times 10^5$	2.10	2.10	2.73	2.55	5.94	4.55
Max Base Rotation 'θ' Rad. $\times 10^{-5}$	0.39	0.39	6 70	7.81	48.70	94.00
Min Slab Contact Ratio d/D	1.00	1.00	0.89	0 96	0 89	0 61
Max. Heel Uplift Height (cm)	0.00	0.00	0 03	0 01	1.48	1 29
Max. Toe Pressure P_t (t/m²)	34.84	34 84	36.13	34.58	94.63	53.54

It is also observed that a slender structure has a greater tendency to uplift than a squatty structure. Looking at the tables we find that the structure with a slenderness ratio 2 shows no uplift at rocky site with 0.1 g earthquake [3 (a)], but at the same site the structure with slenderness ratio 3 loses about 5% of contact [3 (a)], and that with slenderness ratio 5 more than 30% [3 (e)].

TABLE 3 (b)

H = 50.0 m, R = 25.0 m, Maximum Acceleration = 0 2g

	SITE-A		SITE-B		SITE-C	
	L	NL	L	NL	L	NL
Max. Base Overturning Mom. M_b in t-m x 10^{-5}	4.20	4 09	5.46	4.80	5.84	5.12
Max. Base Rotation 'θ' Rad. x 10^{-5}	0.79	2.43	13.40	24.50	47.90	104.00
Min. Slab Contact Ratio d/D	0.66	0.68	0.46	0.57	0.40	0 51
Max. Heel Uplift Height (cm)	0.01	0.03	0.34	0.48	1.42	1.77
Max. Toe Pressure P_t (t/m²)	48.35	48 97	75 03	57.96	89.79	65.19

TABLE 3 (c)

H = 75.0 m, R = 25.0 m, Maximum Acceleration = 0.1g

	SITE-A		SITE-B		SITE-C	
	L	NL	L	NL	L	NL
Max Base Overtuning Mom M_b in t-m x 10^{5}	2.80	2 74	3 03	3.00	5.41	5.25
Max. Base Rotation 'θ' Rad. x 10^{-5}	0 52	0 58	7 48	8 20	44 30	76.90
Min. Slab Contact Ratio d/D	0.95	0 96	0 92	0.92	0 59	0 62
Max. Heel Uplift Height (cm)	0 01	0.01	0.02	0 03	0 89	1 31
Max. Toe Pressure P_t (t/m²)	40 66	40.48	41 46	41.35	64.14	61 51

TABLE 3 (d)
H = 75.0 m, R = 25.0 m, Maximum Acceleration = 0.2g

	SITE-A		SITE-B		SITE-C	
	L	NL	L	NL	L	NL
Max. Base Overturning Mom. M_b in t-m x 10^{-3}	5.61	5.56	6.06	5.27	7.03	5.54
Max. Base Rotation 'θ' Rad. x 10^{-5}	1.05	2.50	15.00	24.00	57.00	104.00
Min. Slab Contact Ratio d/D	0.56	0.57	0.50	0.61	0.37	0.57
Max. Heel Uplift Height h (cm)	0.02	0.05	0.36	0.38	1.80	1.97
Max. Toe Pressure P_t (t/m²)	67.90	66.91	78.26	61.76	116.22	66.48

Comparison between linear and non-linear results show that in almost all the cases, the non-linear analysis predicts lower base overturning moments and higher base rotations than predicted by linear analysis. This difference is found to increase with the softness of soil, e.g. in one particular case the ratio of non-linear to linear base overturning moment is observed to be 0.98 at rocky site, 0.85 at moderately stiff soil site and 0.78 at soft soil site. Also, the non-linear analysis predicts a preater portion of the slab remaining in contact with the supporting soil than linear analysis. Difference between linear and non-linear results increases with the slenderness ratio of the structure and softness of the soil [Tables 3 (a) & (b)] Thus, the portion of the slab which might uplift is never as great as one would predict based upon linear analysis, which again proves to be conservative

TABLE 3 (e)
H = 50.0 m, R = 10.0 m, Maximum Acceleration = 0 lg

	SITE-A		SITE-B		SITE-C	
	L	NL	LN	L	LN	L
Max. Base Overtuning Mom. M_b in t-m × 10^{5}	0.40	0.37	0.38	0.30	0.38	0.41
Max. Base Rotation 'θ' Rad. × 10^{-5}	1.17	2.18	14.80	16.50	49.00	60.00
Min. Slab Contact Ratio d/D	0.63	0.68	0.66	0.80	0.67	0.61
Max. Heel Uplift Height (cm)	0.01	0.02	0.09	0.06	0.32	0.42
Max Toe Pressure P_t (t/m²)	72.76	67.59	69.19	56.69	68.97	76.30

TABLE 3 (f)

H=50.0 m, R=10 0m, Maximum Acceleration=0.2g

	SITE-A		SITE-B		SITE-C	
	L	NL	L	NL	L	NL
Max Base Overturing Mom M in t-m $\times 10^5$	0 48	0.54	0.52	0.36	0.55	0 46
Max. Base Rotation 'θ' Rad. $\times 10^{-5}$	1.40	5.90	20.00	30.50	71.60	77.00
Min Slab Contact Ratio d.D	0.50	0 38	0.42	0.70	0.36	0.52
Max. Heel Uplift Height (cm)	0.01	0.04	0.23	0 16	0 90	0.71
Max. Toe Pressure P_t (t/m²)	97 53	134 50	121.48	64.83	147 80	92.07

Non-linear analysis always predicts greater uplift height than linear linear analysis for site-A. However, for softer soils linear analysis generally predicts greater uplift heights Moreover the difference grows with slenderness ratio

Also, the linear analysis almost always predicts significantly greater maximum toe pressures than non-linear analysis, which accounts for softening influence of base uplift. The difference is of the order of 60% in some cases.

Figures 4 (a) and 4 (b) show the variation of contact ratio and uplift height with time for a squatty structure at site-C. It is seen that out of total of 27 instances of uplift only 7 or 8 are significant and also that the duration of each uplift is very short. Figures 5 (a) & (b) show similar plots for a rocky site and the exhibit too insignificant uplift.

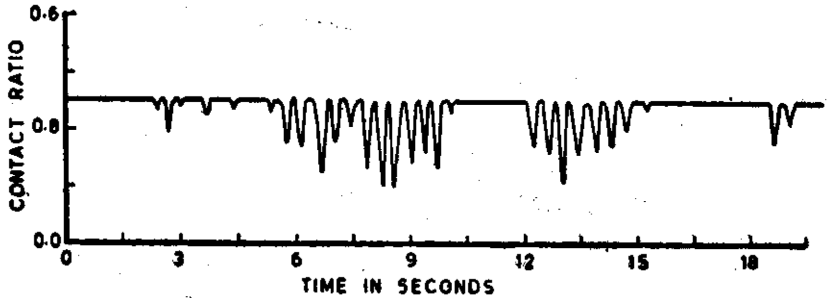


Fig. 4 a. Contact Ratio Variation–Soft Soil

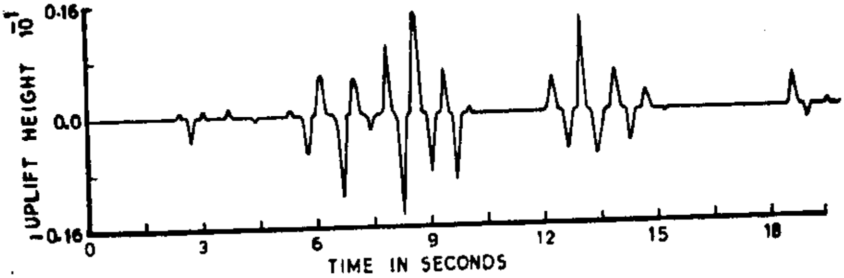


Fig. 4 b. Uplift Height Magnitude-Soft Soil

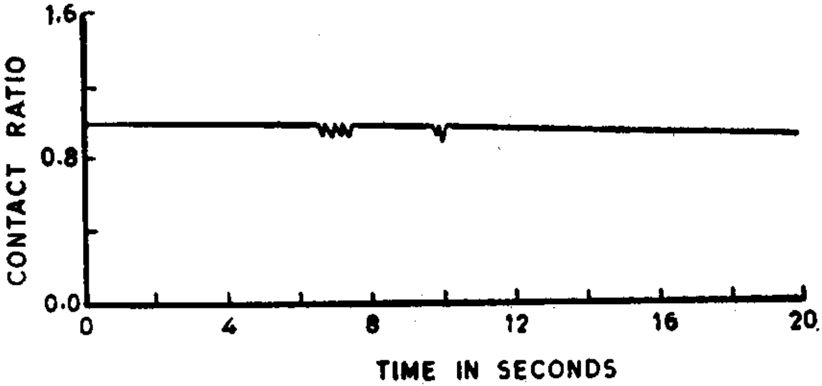


Fig 5 a. Contact Ratio Variation for Non Linear 0.1 G Stiff Soil

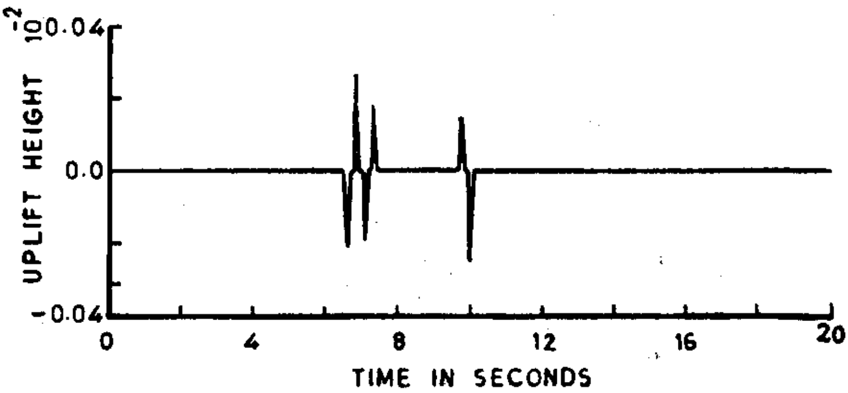


Fig 5 b. Uplift Height Magnitude for Non Linear 0 1 G Stiff Soil

CONCLUSIONS

1. Slender structures considered to be resting on ground surface are more liable to uplift under strong horizontal excitation than squatty structures.

2. Softer the foundation soil, the greater is the tendency of structure to uplift

3. Damping of soil-structure system, evaluated using a realistic approach should be used in the analysis, instead of using any arbitrary value, particularly if the structure is founded in soft-soil.

4. Linear analysis can be used to conservatively estimate the important behaviour of base slab even if substantial portion of base slab loses contact from supporting soil.

5. Toe pressure calculation based upon the linear analysis appear to always be greater than those based upon non-linear analysis Thus, ~~if toe pressure from linear~~ analysis is not excessive, non-linear analysis need not be performed even under condition of substantial uplift.

6. Heel uplift height appears to remain small even when the structure is founded in very soft soil.

7. Tipping is a short transient phenomenon.

REFERENCES

1. Bathe, K.J. and Wilson, E.L. (1976), "Numerical Methods in Finite Element Analysis", Prentic Hall of India Pvt. Ltd.

2. Beucke, K. et al. (1983), "Non-Linear Soil-Structure Interaction with Base Uplift", 7th Structural Mechanics in Reactor Technology, Vol. K (b).

3. Chandrasekaran, A.R. and Singhal, N C. (1983), "Dynamic Analysis of Embedded Box type Cylindrical Structures", Paper III-43, Proc. I.G.C. '83, IIT Madras.

4. Kennedy, R.P. et al. (1976), "Effect of Non-Linear Soil-Structure Interaction Due to Base Slab Uplift on the Seismic Response of a HTGR", Nuclear Engg. and Design, Vol 38.

5. Richart, F.E. et al. (1970), "Vibration of Soil and Foundation", Prentice Hall, N.J., U.S.A.