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EFFECT OF SYSTEM INITIAL CONDITIONS ON SEISMIC DESIGN OF LONG-PERIOD STRUCTURES

Carlos E. Ventura* and Rafael Blázquez**

*Department of Civil Engineering
The University of British Columbia, Vancouver, B.C., Canada
**Department of Civil Engineering
Universidad de Castilla-La Mancha, Ciudad Real, Spain

ABSTRACT

Initial conditions for the computation of response spectra are unknown in the case of analog earthquake records that have lost the initial portion of motion and are assumed to be zero for calculation purposes. This assumption, rigorously speaking, is only valid for high-frequency systems, such as rigid structures. Both rigid and flexible structures are sensitive to the initial conditions of the motion. The spectral values for long-period, elastic or inelastic systems tend to be particularly sensitive to the initial conditions of motion. Unconservative designs of long-period structures may result if the initial condition effects are not properly accounted for. In this paper two practical methods are developed to approximate the true "non-resting" response spectra from the conventional ones for a given set of initial conditions. As expected, these conditions clearly control the free-vibration part of the response of long-period systems, as opposed to short-period systems which are governed by the transient phase of the response.

KEYWORDS: Initial Conditions, Response Spectra, Long-Period Systems, Accelerograms

INTRODUCTION

The response spectrum method of analysis is widely used for evaluating the dynamic response of structural systems subjected to earthquake motions. In this approach, the peak response of a system due to a prescribed earthquake motion is generally computed assuming that the system is initially at rest, that is, assuming zero initial conditions. Although the significant aspects of response spectra computed assuming zero initial conditions have been extensively studied (see, for example, Veletsos and Newmark (1964)), the effects of assuming non-zero initial conditions in the spectral computations are not well understood.

Non-zero initial conditions may arise for systems already undergoing vibrations when a seismic event occurs (Trifunac and Udwadia, 1979) or from evaluation of earthquake records where a segment at the beginning of the record is not available (Pecknold and Riddell, 1978, 1979; Blázquez and Kelly, 1988; Uang and Bertero, 1990). Also, from the seismological point of view, initial conditions are very important in the near field, where the ground motion contains powerful pulses that can have very large initial velocities (M.D. Trifunac, written communication, 2006).

In many practical applications, the initial conditions are not included in the computation of response spectra, and often this can be reasonably justified. There is, however, a paucity of information on the consequences of ignoring these effects when the response spectrum method is used for the dynamic analysis of structures, especially for those with long natural periods. This research responds to this need.

1. Statement of Problem

The dynamic response of base-excited, viscously damped, single-degree-of-freedom (SDOF) system is considered here. It is assumed that the system is already vibrating at the reference time $t = 0$ when a ground acceleration, $\ddot{y}(t)$, is applied. The relative displacement and velocity of each system at $t = 0$ are known and identified as U_o and $\dot{U}_o$, respectively. A general form of the equation of motion of the SDOF system can be written as

$$m\ddot{u}(t) + c\dot{u}(t) + R(t) = -m\ddot{y}(t) \tag{1}$$

in which m is the mass, c is the coefficient of viscous damping, $R(t)$ is the restoring force (which depends on the displacement amplitude at time t), and $\dot{u}(t)$ and $\ddot{u}(t)$ are the system's relative velocity and acceleration with respect to the base motion.

For the case of elastic systems, the restoring force is expressed as $R(t) = ku(t)$, where k is the stiffness coefficient and $u(t)$ is the relative displacement of the system. In this case, Equation (1) can be expressed as

$$\ddot{u}(t) + 2\xi p \dot{u}(t) + p^2 u(t) = -\ddot{y}(t) \tag{2}$$

where $p = \sqrt{k/m} = 2\pi/T$ is the undamped circular natural frequency of the system (T is the undamped natural period), and $\xi = c/2pm = c/2\sqrt{km}$ is the fraction of critical damping.

It is of interest to evaluate the influence of U_o and $\dot{U}_o$ on the absolute value of the numerically largest values of relative displacement, SD, and absolute acceleration, SA, for moving ground-excited systems. To state the problem, the solution of the differential equation of motion of the oscillator (Equation (2)), can be expressed as follows:

$$u(t) = u_t + u_f \tag{3}$$

where u_t and u_f stand, respectively, for the forced- and free-vibration parts of the motion. The latter depends on U_o and $\dot{U}_o$, which are the system's initial conditions.

The physical existence of initial conditions in recorded earthquake motions can be explained by analyzing the way these instruments function. Unavoidably for levels of signal which fall below a prefixed threshold level $(\ddot{y}_0)$, the recorder of the instrument is not triggered and that portion of the accelerogram (for $t < t_0$) remains unrecorded. This results in unknown values of the ground and the mass motion at time $t = 0$ due to information gap. These values depend on the characteristics of the excitation and the mechanical properties of the system. Modern seismic instruments overcome this problem by incorporating in their design a pre-event buffer memory to record the ground motion for a few seconds before the trigger level is exceeded. However, the database of earthquake motions recorded without the benefit of a pre-event memory is extensive and widely used in earthquake engineering research and practice. It is, therefore, of significant interest to examine in detail the consequences of ignoring the initial state of motion of the system at the instant in which the instrument starts recording the ground motion.

Using a single cycle sinusoidal acceleration wave of period T_0 as excitation of a system of period T, Blázquez and Kelly (1988) have shown that, for short-period systems ($T \le T_0$), the free-vibration component in Equation (3) can be neglected and the effect of the initial conditions is negligible regardless of the damping level. Thus it can be reasonably assumed that

$$U_o = u(0) \approx 0 \;\; ; \;\; \dot{U}_o = \dot{u}(0) \approx 0 \tag{4}$$

In contrast, for long-period systems ($T \ge T_0$), the following asymptotic relations can be used to approximate initial conditions:

$$U_o = u(0) \approx -y_o \;\; ; \;\; \dot{U}_o = \dot{u}(0) \approx -\dot{y}_o \tag{5}$$

where y_0 and $\dot{y}_0$ are, respectively, the input displacement and the input velocity at the triggering time (t_0) of the instrument recording the excitation motion.

Since the standard method for calculating response spectra assumes zero initial conditions, it is concluded that, strictly speaking, such a procedure applies only to high-frequency systems (e.g., rigid structures) for non-zero initial conditions. For other types of systems (particularly flexible ones), the effect of initial conditions on the long-period regions of response spectra remains to be clarified.

2. Objectives of This Study

The objectives of this study are: (1) to help understand better the influence of initial conditions on the response spectra for elastic and inelastic viscously damped single-degree-of-freedom (SDOF) systems, (2) to investigate the implications of considering or neglecting the effects of the initial conditions in the response spectrum analysis, and (3) to develop a method to approximate the true response spectrum from the conventional one and a given set of initial conditions.

EVALUATION OF SEISMIC RESPONSE OF SDOF SYSTEMS

The response of a system to a prescribed excitation depends on the characteristics of the excitation, the properties of the system and the initial state of motion. If the system is either originally at rest and subjected to a transient excitation, or is subjected to prescribed initial displacement and velocity and let to vibrate in free-vibration motion only, the combined effect of both actions will represent the response of that system with non-zero initial conditions and subjected to a base excitation. Most algorithms used for evaluating the response to transient excitations can easily account for the effect of non-zero initial conditions, but it is of interest here to evaluate the responses separately.

Accordingly, the transient response, u_t, in Equation (3) can be expressed in terms of the well-known Duhamel's integral (Von Karman and Biot, 1940) as

$$u_t = -\int_0^t \ddot{y}(\mu) h(t-\mu) \mathrm{d}u \tag{6}$$

and the free-vibration response part is given by (Veletsos and Ventura, 1985)

$$u_f = U_o g(t) + \dot{U}_o h(t) \tag{7}$$

in which $g(t)$ and $h(t)$ are defined as the unit response functions of the system. These functions represent the displacement at time t produced by a unit initial displacement and a unit initial velocity, respectively, and are given by

$$g(t) = \left[\cos \bar{p}t + \left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\sin \bar{p}t\right]\exp(-\xi pt) \tag{8a}$$

and

$$h(t) = (1/\bar{p})\exp(-\xi pt)\sin \bar{p}t \tag{8b}$$

where $\bar{p} = p\sqrt{1-\xi^2}$ is the damped circular natural frequency of the system.

The relative velocity, $\dot{u}(t)$, can be obtained by differentiation of Equation (3) with respect to time. In a stepwise numerical evaluation of the response of a SDOF system, the transient relative velocity, $\dot{u}_t$, is normally computed during the process of obtaining u_t. The free-vibration component of the relative velocity, $\dot{u}_f$, would require the evaluation of the time derivatives of the unit response functions $g(t)$ and $h(t)$, given by Equations (8a) and (8b), respectively. Then, the absolute acceleration, $\ddot{x}$, can be computed by differentiation of the relative velocity to obtain the relative acceleration, $\ddot{u}$, and then by adding the base acceleration, $\ddot{y}$, to the resulting response. Alternatively, from Equation (1) and the relation, $\ddot{x} = \ddot{y} + \ddot{u}$, one obtains (see Equation (2))

$$\ddot{x} = -(2\xi p\dot{u} + p^2 u) \tag{9}$$

This expression can be used to evaluate separately the transient and free-vibration components of the absolute acceleration. The shear force at the support of the SDOF system, V, can then be computed as $V = m\ddot{x}$.

Response Spectra for Linear Systems

The response values that are of practical interest are the relative displacement, u, the relative velocity, $\dot{u}$, and the absolute acceleration, $\ddot{x}$, of systems with the same level of damping and different values of the natural period, $T = 2\pi / p$. The numerically largest values are used to develop response spectrum plots for each of these quantities. Since the maximum response of the system may be attained after cessation of the excitation during the first half-cycle of free vibration, the analysis should be carried out over a time interval, T_d, which exceeds T_0 (the duration of the excitation) by at least one-half the natural period of the system, T, i.e., $T_d = T_0 + T/2$.

For practical structural engineering applications, true relative velocity and absolute acceleration response spectra (SV and SA, respectively) have been approximated in the past by the corresponding "pseudo" spectra, defined as follows:

$$\mathrm{PSV} = p \cdot \mathrm{SD} = \frac{2\pi}{T} \cdot \mathrm{SD} \qquad (10a)$$

$$\mathrm{PSA} = p^2 \cdot \mathrm{SD} = \left(\frac{2\pi}{T}\right)^2 \cdot \mathrm{SD} \qquad (10b)$$

where SD stands for the relative displacement response spectrum. As it is well known, PSA is directly related to the maximum force (or base shear), and PSV is related to the maximum stored energy.

For systems, for which the damping ratio is less than 5% and a quasi-linear behaviour of the system can be assumed, it has been customary to assume that the numerical values of SV and SA are about the same as the corresponding PSV and PSA, respectively. The accuracy of this assumption depends on the type of excitation, the damping and natural period of the system.

It is well known that for long-period and short-period systems, SV and PSV are not always exchangeable. Figure 1 proves this assertion for the normalized response spectra of a very simple sinusoidal wave. Two damping values are considered in this case: 0% and 10%. The single sine wave pulse has duration of T_0 = 1 sec and amplitude of 1*g*, and it is assumed that the oscillator starts from "at rest conditions". As expected, PSA and SA are the same for all periods for the undamped system and are very close to each other for the damped system. In contrast, PSV and SV are different for very short periods and long periods, the difference becoming larger as the damping increases. Beyond a certain critical period (at about T = 3.3 sec), the SV curve departs clearly from the PSV curve and approaches its asymptotic value, the maximum velocity of the ground, $|\dot{y}(t)|_{\max}$. For the same conditions, however, $\mathrm{PSV} = \frac{2\pi}{T} \cdot \mathrm{SD} \rightarrow 0$, since SD approaches its limiting value $|y(t)|_{\max}$ while T increases indefinitely (Hudson, 1979).

EFFECT OF INITIAL CONDITIONS ON SDOF RESPONSE

The state of motion of an elastic SDOF system just before the occurrence of ground motion can be characterized in terms of the initial displacement, U_o, and the initial velocity, $\dot{U}_o$. The response of the system to these initial conditions depends on its natural frequency and damping. Systems subjected to the same initial displacement and velocity, but with different natural frequency and damping, will respond differently.

The nature of the system is also important, since different responses are expected for elastic systems (such as the viscously damped system described by Equation (2)) and inelastic systems (such as the type described by Equation (1)) to the same set of initial conditions. This is shown next.

1. Elastic Systems

The peak value of the ground motion is the parameter commonly used for describing the level of ground excitation and for comparing it with the maximum response of the system. Therefore, it would be practical to relate the initial conditions of the system to this parameter. To this end, the peak ground acceleration, $\ddot{Y}$, the initial conditions, U_o and $\dot{U}_o$, and the natural frequency of the system, p, can be interrelated by the dimensionless coefficients:

$$\alpha = -p^2 U_o / \ddot{Y} \tag{11a}$$

$$\beta = -p \dot{U}_o / \ddot{Y} \tag{11b}$$

where $\ddot{Y} = |\ddot{y}(t)|_{\max}$.

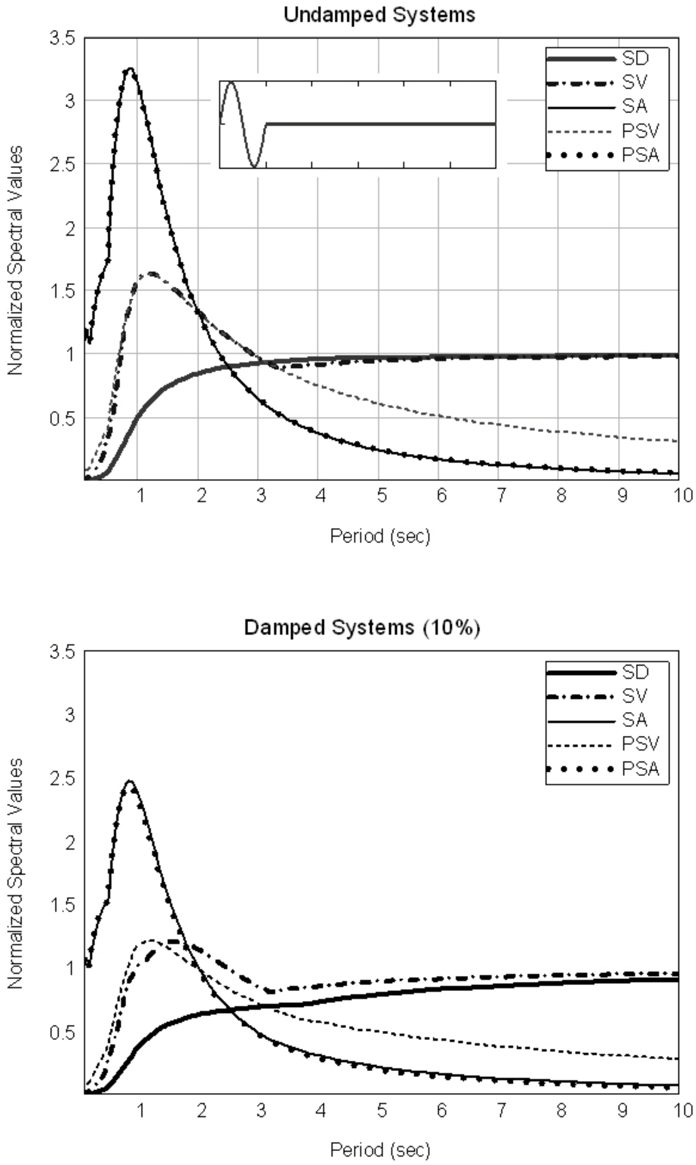

Fig. 1 Comparison of true and pseudo-response spectra for single sine acceleration pulse

The coefficients α and β represent the ratio of equivalent instantaneous accelerations of magnitude $p^2 U_o$ and $p\dot{U}_o$, respectively, to the peak base acceleration, $\ddot{Y}$. Alternatively, α can be described as the ratio of the base shear kU_0 (resulting from inducing into the undamped system a static displacement U_0) to the equivalent static force $m\ddot{Y}$, produced by the peak ground acceleration. Similarly, β can be described as the ratio of the peak base shear $mp\dot{U}_o$ (produced by an impulse $p\dot{U}_o$ applied instantaneously to the undamped system) to the equivalent static force $m\ddot{Y}$, produced by the peak ground acceleration.

The sensitivity of the response to various combinations of α and β can be evaluated for SDOF systems subjected to a single cycle of sinusoidal base excitation of the form $\ddot{y}(t) = \ddot{Y}\sin(2\pi t / T_o)$, in which T_0 is the duration of the cycle.

Veletsos and Newmark (1964) have demonstrated that, for base excitations of the complexity of earthquake records, the response spectra are similar to those for simple excitations, and that their salient features can be reasonably identified from the spectra for single pulses, provided the gross characteristics of the ground acceleration, velocity and displacement are known. To this end, normalized displacement, SD/Y, velocity, $\mathrm{SV}/\dot{Y}$, and absolute acceleration, $\mathrm{SA}/\ddot{Y}$, response spectra of undamped systems were computed for various combinations of α and β (Ventura and Blázquez, 1990, 1992). The results are shown in Figures 2–4, where the spectra are plotted as a function of the ratio T/T_0, and the spectral amplitudes are normalized with respect to the corresponding peak value of the ground motion. To complete the picture, 5%-damped PSV spectra normalized with respect to the peak base velocity are drawn on log-log scale in Figure 5(a) and are complemented with the corresponding SA values drawn in linear scale and normalized with respect to the peak base acceleration (Figure 5(b)).

It is clear from Figure 3 (relative displacement spectra) that non-zero initial conditions, when expressed in terms of α and β, have a greater effect on the response of very flexible systems, $T >> T_0$, than on the response of less flexible systems. This effect is not very significant for rigid systems (that are more sensitive to high-frequency inputs) nor is it significant at the resonant period. The response for flexible systems can be reliably predicted via reasoning as follows. It has been demonstrated by Veletsos and Newmark (1964) that for a truly undamped, very flexible system with $\alpha = \beta = 0$, the relative displacement is opposite and nearly equal to the base displacement, i.e., $u \approx -y$. By disregarding the sign of the peak response, this result is shown by the solid line in Figure 2, where the normalized displacement converges to 1 for large values of T/T_0.

If the values of α and β differ from zero, one can derive from Equations (11a) and (11b) the following expressions:

$$U_o = -\ddot{Y}\alpha / p^2 = -Y\alpha(T/T_o)^2 \tag{12a}$$

and

$$\dot{U}_o = -\ddot{Y}\beta / p = -\dot{Y}\beta(T/T_o) \tag{12b}$$

From these expressions it can be seen that, for fixed values of α and β, U_o and $\dot{U}_o$ are proportional to $(T/T_0)^2$ and (T/T_0), respectively. For very flexible systems the ratio (T/T_0) is large, and U_0 and $\dot{U}_0$ are much larger than the peak value of the base displacement, Y, and the peak value of the base velocity, $\dot{Y}$, respectively. Since the maximum response of the transient part, u_t in Equation (3), approaches Y, it becomes clear that the response is dominated by the free-vibration part, u_f. This is the reason why the broken lines in Figure 2 exhibit a parabolic shape for large values of T/T_0. This effect is most significant for the systems where α and β are both different from zero.

The behaviour of the relative velocity spectra is quite similar to the displacement spectra. It can be seen from Figure 3 that non-zero initial conditions affect a wide range of systems. Again, the effect is not very significant for rigid systems, but the response at the resonant period is affected in some cases. The response for flexible systems can also be predicted in a manner similar to that discussed above. For the systems initially at rest, the response is essentially equal and opposite to the base velocity, i.e., $\dot{u}_t \approx -y$, as shown by the solid lines in Figure 3, where the normalized velocity converges to 1 for large values of T/T_0. For values of α and β different from zero, Equations (12a) and (12b) apply, and the response for flexible systems is controlled by the derivative of the free-vibration part of Equation (3). In this case the broken lines in Figure 3 show a linear shape for large values of T/T_0.

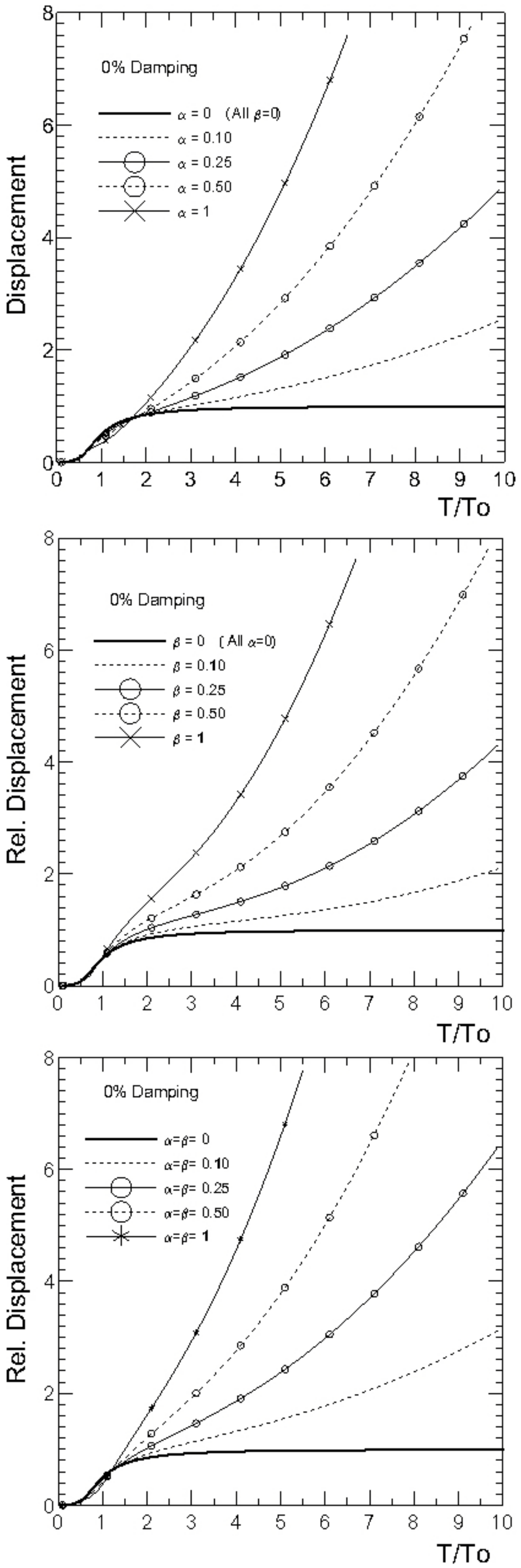

Fig. 2 Effect of initial conditions on the relative displacement spectra of undamped SDOF systems subjected to sinusoidal base acceleration

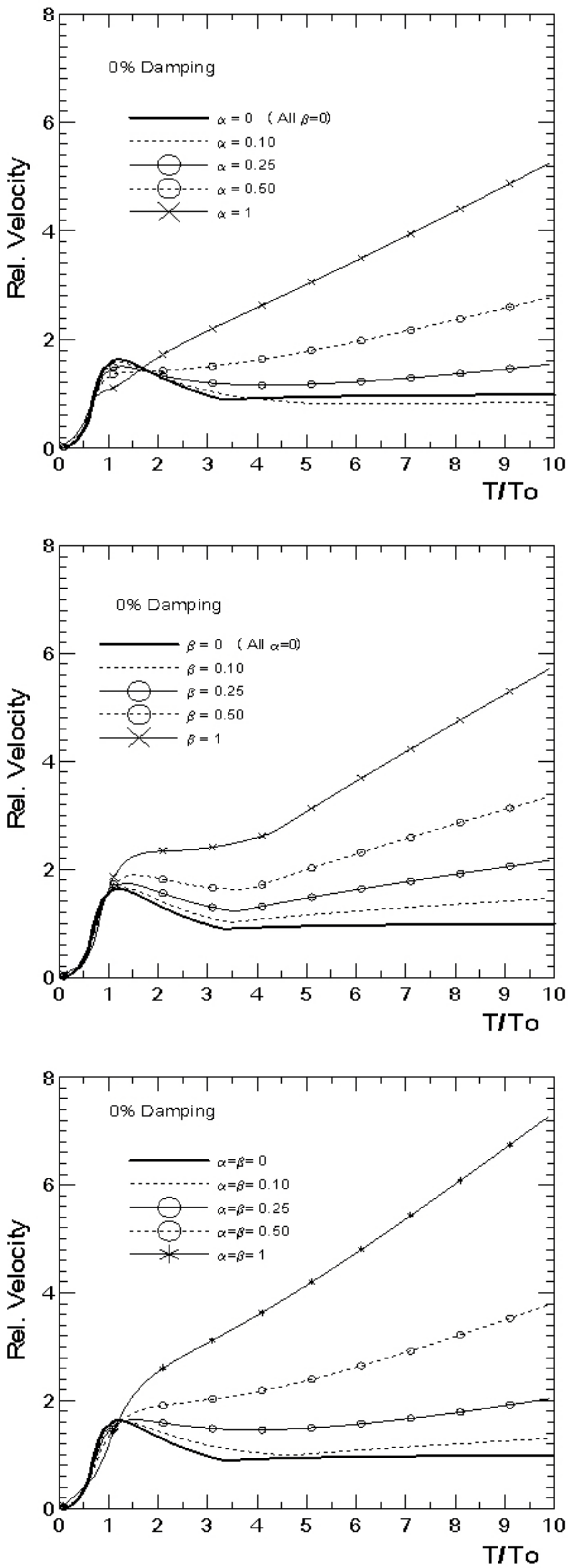

Fig. 3 Effect of initial conditions on the relative velocity spectra of undamped SDOF systems subjected to sinusoidal base acceleration

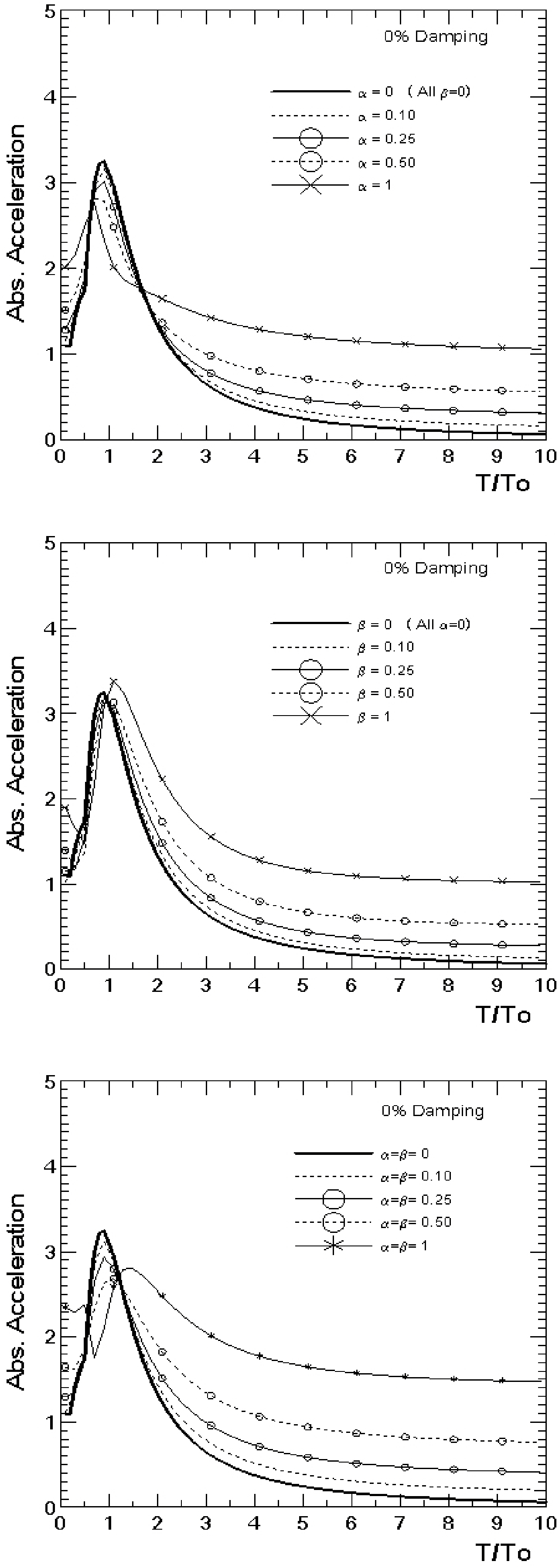

Fig. 4 Effect of initial conditions on the absolute acceleration spectra of undamped SDOF systems subjected to sinusoidal base acceleration

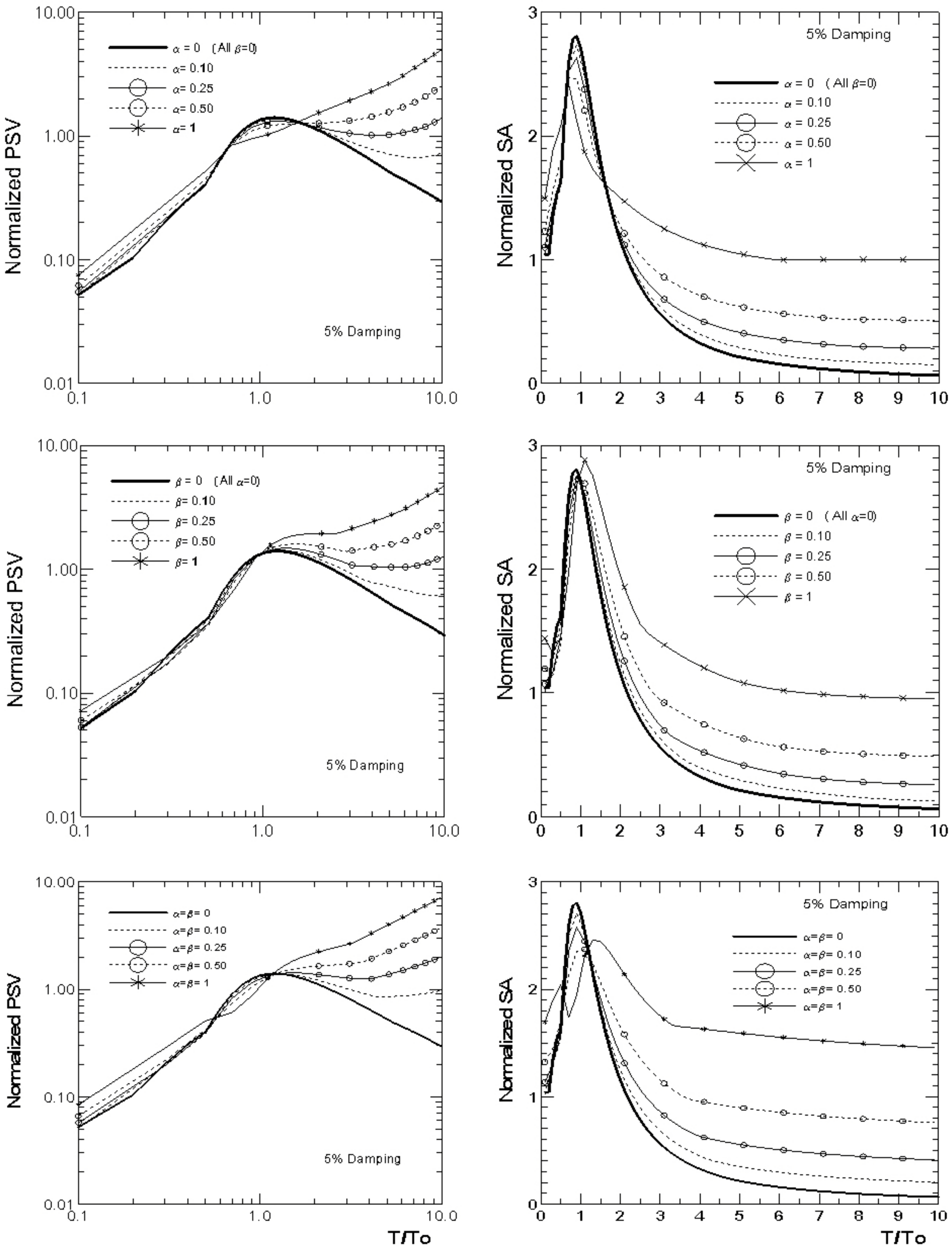

Fig. 5 Influence of initial displacement (α) and initial velocity (β) on pseudo-velocity (PSV) and absolute acceleration (SA) response spectra for elastic systems subjected to sinusoidal base excitation

The effect of initial conditions on the absolute acceleration spectra for undamped and damped systems is visualized in Figures 4 and 5(b), respectively. It can be seen that non-zero initial conditions affect the response of systems over the whole range of values of T/T_0, but most significantly for systems in the long-period range. This is expected, since $\ddot{x}$ is a function of u and $\dot{u}$ as shown in Equation (9), and for undamped systems, $\ddot{x} = -p^2 u$. As T becomes larger, p decreases, cancelling the multiplying effect of $(T/T_0)^2$ in the displacement, and $\ddot{x}$ becomes proportional to α and β. It follows that the peak value of $\ddot{x}$ of undamped systems is proportional to $\ddot{Y}\sqrt{\alpha^2 + \beta^2}$. For values of α and β different from zero the effects at the resonant period are very significant, since the amplitude of the response is less than that for a system initially at rest. Damping simply reduces the amplitude of motion, as a direct comparison of Figures 4 and 5(b) clearly shows.

Figure 5 evidences that the effect of initial conditions on the response of rigid systems is quite negligible, since they are more sensitive to the high-frequency components of the input motion than to the initial state of the system.

For more complex excitations the same trends on the variation of the response spectra can be expected. The response spectra for the 1940 El Centro earthquake record (north-south component) are shown in Figure 6, and the spectra for the 1985 Mexico earthquake recorded at the SCT station (east-west component) are shown in Figure 7. The El Centro earthquake record was selected for this study because it produces significant responses over a wide band of system periods, while the Mexico earthquake record was selected because it produces significant responses for a narrow range of periods, mostly for periods around 1.5 to 3 sec. Figures 6(a) and 7(a) show the 5%-damped PSV elastic spectra, and Figures 6(b) and 7(b) show the SA elastic spectra, both for different values of α (with $\beta = 0$). These spectra represent the peak values of pseudo-velocity and absolute acceleration during the duration of the excitation only.

As in the case for sinusoidal base motion, the long-period regions of the PSV and SA spectra are more sensitive to the effects of the initial conditions than the other regions of the spectra. The SA spectra also show that the base shear for a long-period system with non-zero initial conditions is larger than that for a system initially at rest. This may lead to unconservative designs of long-period structures if the effects of initial conditions are not properly accounted for.

2. Inelastic Systems

Two important parameters that usually characterize the response of inelastic systems are the yield displacement, U_y, and the associated force level, R_y, that produces this displacement. To obtain the dynamic response of an inelastic system for which its force-deformation characteristics are known, Equation (1) can be solved directly. However, it is more desirable to express this equation in a normalized form such that the specific parameters that influence the response can be more readily identified, as in the case of elastic systems (Mahin and Lin, 1983). A possible normalized version of Equation (1) can be written as

$$\ddot{\mu}(t) + 2\xi p \dot{\mu}(t) + p^2 \rho(t) = -A\ddot{y}(t) \tag{13}$$

in which $\mu(t) = u(t)/U_y$ is defined as the displacement ductility and its peak value, U_y, is referred to as the ductility factor (Clough and Penzien, 2003), $\rho(t) = R(t)/R_y$ and $A = p^2/\eta\ddot{Y}$. The dimensionless parameter $\eta = R_y/m\ddot{Y}$ represents the yield strength relative to the peak inertia force of the system. Note that for truly undamped elastic systems, the peak inertia force is equal and opposite to the peak value of the base shear. So the strength index is directly related to the peak base shear of the undamped elastic version of the system.

Equation (13) provides an efficient way to evaluate $\mu(t)$ for all systems that have the same elastic natural frequency, the same hysteretic characteristics, the same strength index, and are subjected to ground motions having the same time histories.

For systems with hysteresis diagrams that exhibit an initial elastic behaviour, like the elasto-plastic systems, R_y and U_y can be related by $R_y = kU_y$, and the displacement ductility can be expressed directly in terms of η as

$$\mu(t) = p^2 u(t) / \eta \ddot{Y} \tag{14}$$

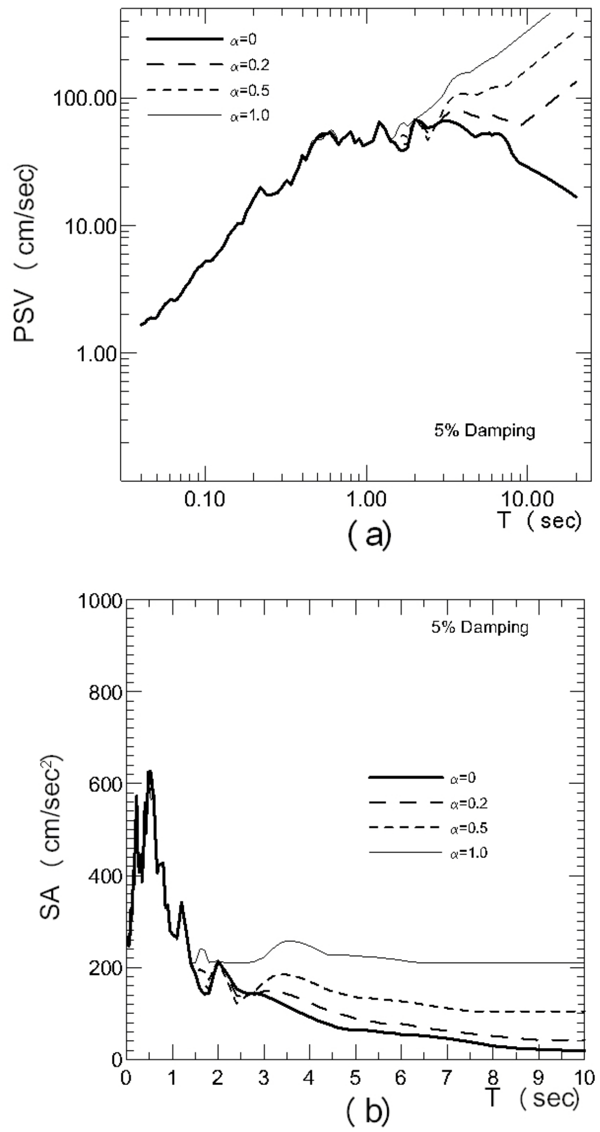

Fig. 6 Influence of initial displacement (α) on pseudo-velocity (PSV) and absolute acceleration (SA) response spectra for elastic systems subjected to El Centro 1940 earthquake, N-S component

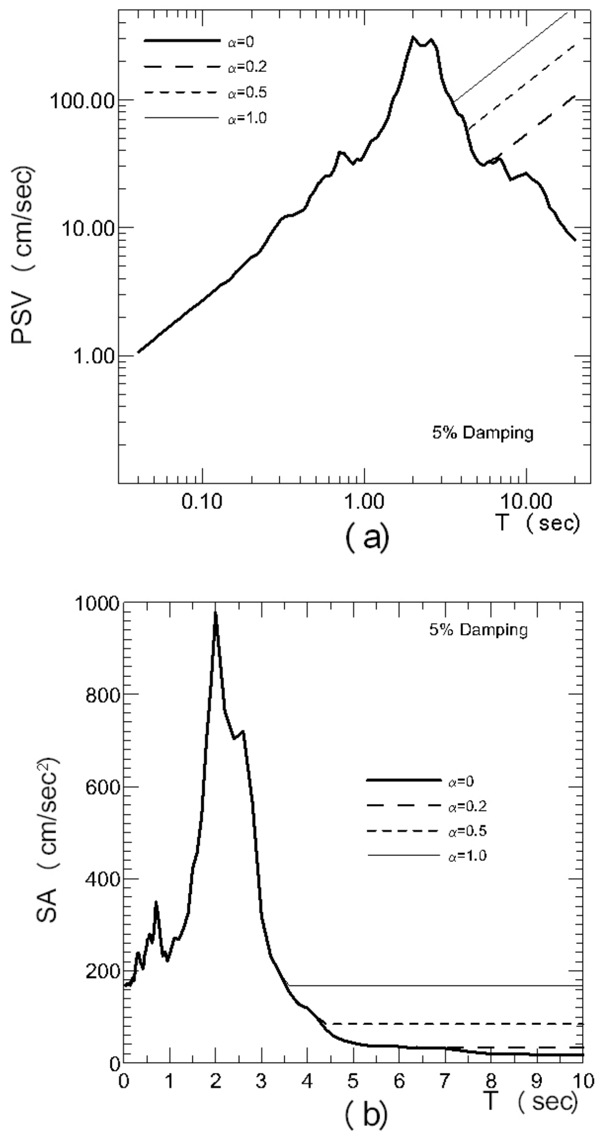

Fig. 7 Influence of initial displacement (α) on pseudo-velocity (PSV) and absolute acceleration (SA) response spectra for elastic systems subjected to Mexico 1985 earthquake, E-W component

In a response spectrum analysis, for a given value of η, the peak value of $u(t)$ can be obtained from the corresponding response spectrum and the ductility factor can be readily computed from Equation (14). To illustrate the sensitivity of the response of elasto-plastic systems to variations in η, the maximum responses due to El Centro and Mexico records were computed using a modified version of the computer program as described in Mahin and Lin (1983).

The maximum responses for systems initially at rest ($\alpha = \beta = 0$) are shown in Figures 8(a) and 9(a), where they are presented as pseudo-velocity spectral values using the natural period of the elastic portion of the hysteresis diagram as the reference period. The corresponding absolute acceleration spectra are shown in Figures 8(b) and 9(b). For reference, the associated elastic spectra are also included in the figures and are used as a basis of comparison between the elastic and inelastic responses.

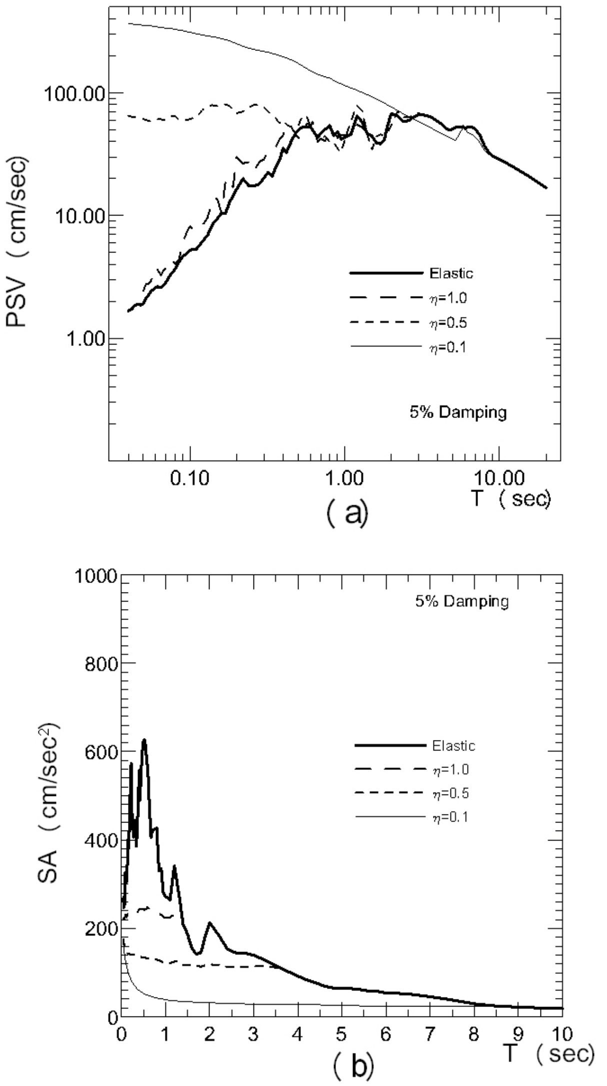

Fig. 8 El Centro 1940 earthquake: pseudo-velocity (PSV) and absolute acceleration (SA) response spectra for elastic and elasto-plastic systems initially at rest

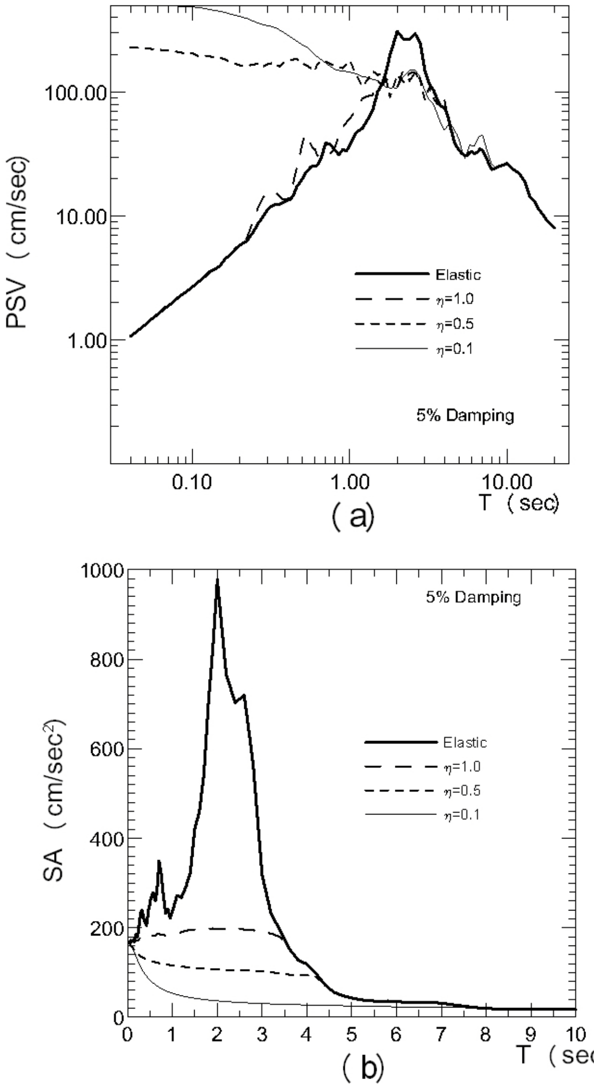

Fig. 9 Mexico 1985 earthquake: pseudo-velocity (PSV) and absolute acceleration (SA) response spectra for elastic and elasto-plastic systems initially at rest

These figures show that, for $\eta > 0.5$, the responses of systems with periods less than 4 sec are generally more sensitive to variations of η than those for very flexible systems. As the value of η increases, the response approaches that of the truly elastic systems, and the results become less sensitive to variations of η. For small values of η (η = 0.1) a wider range of systems are affected, since low values of η correspond to the systems that have a low equivalent value of the maximum elastic base shear force and, therefore, are expected to undergo large excursions beyond the yield displacement, U_y.

Introducing initial conditions into the computation of the PSV spectra for elasto-plastic systems has the effect shown in Figures 10(a) and 11(a). The effect on the SA spectra is shown in Figures 10(b) and 11(b). In these cases, including only an initial displacement $(\alpha = 0.5,\ \beta = 0)$ in the computation of the inelastic response of systems to the El Centro and Mexico records completely alters the shape of the spectrum in the long-period region when compared to that of the inelastic systems with zero initial conditions. Including the initial velocity effects will further alter the shape of the spectrum. The middleand low-period regions of the spectrum remain more sensitive to variations of the strength over inertia force index (η) than to variations of the initial conditions (α). For small values of η the SA values are insensitive to the effects of initial conditions.

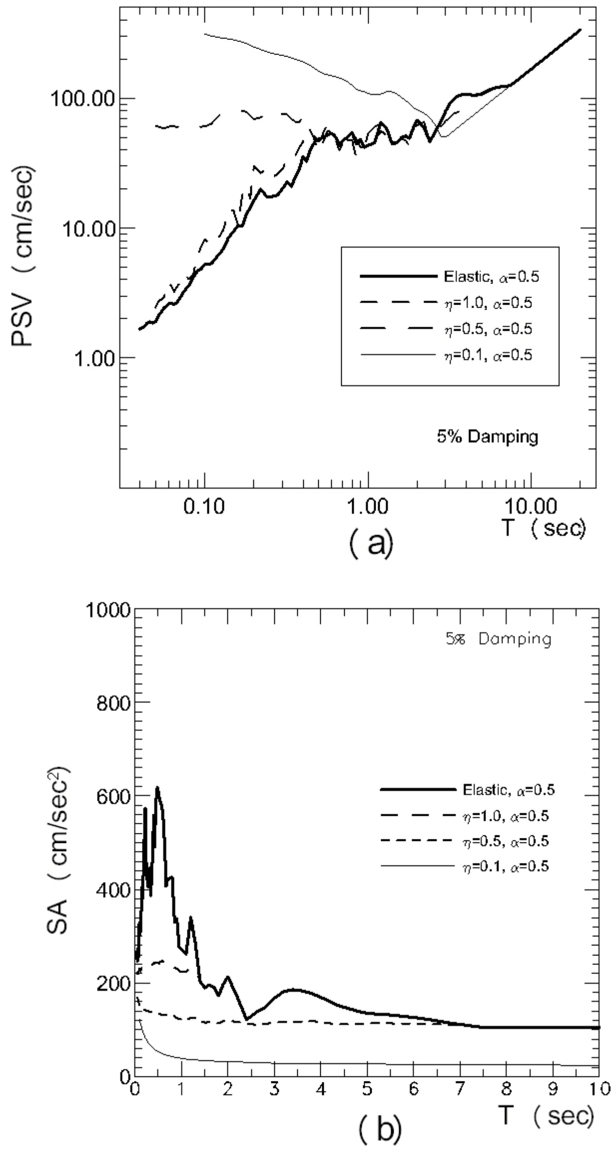

Fig. 10 Influence of initial displacement (α) on pseudo-velocity (PSV) and absolute acceleration (SA) response spectra for elasto-plastic systems subjected to El Centro 1940 earthquake

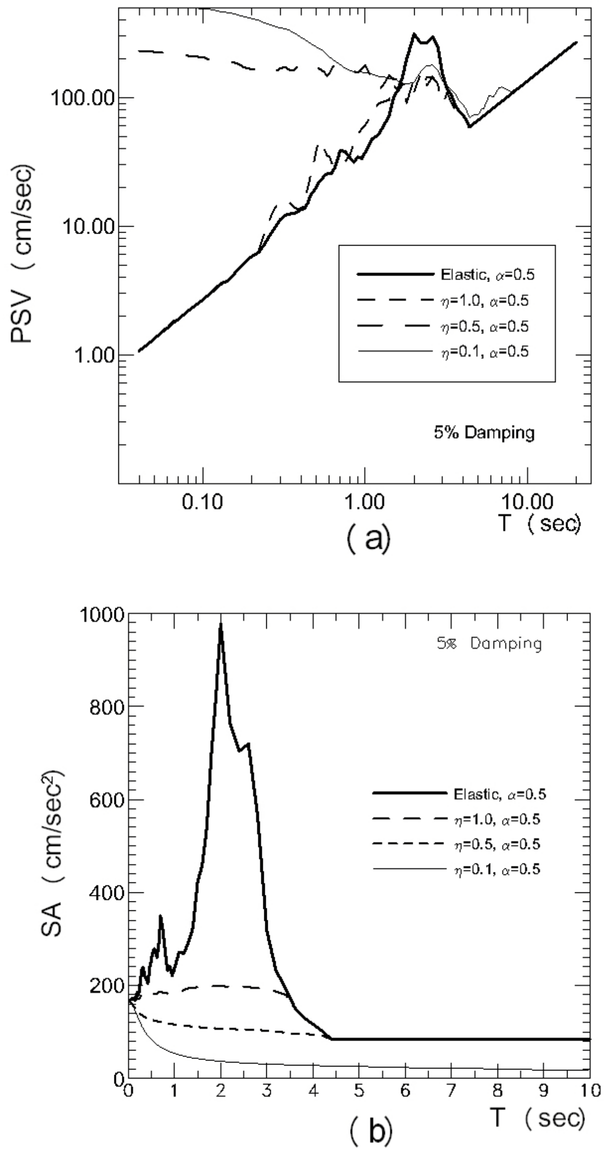

Fig. 11 Influence of initial displacement (α) on pseudo-velocity (PSV) and absolute acceleration (SA) response spectra for elasto-plastic systems subjected to Mexico 1985 earthquake

APPROXIMATE CORRECTION PROCEDURES TO ACCOUNT FOR INITIAL CONDITIONS (ELASTIC SYSTEMS)

The maximum response of long-period systems is controlled by the free-vibration part of Equation (3) as discussed earlier. The response for other systems is a combination of the transient and free-vibration parts. If the response spectrum for a system initially at rest is readily available, and if one assumes that the initial conditions can be determined or are given, then it is of interest to develop a method to approximate the true response spectrum using the information provided.

Let U_f be the maximum free-vibration response of a system subjected to initial displacement U_0 and initial velocity $\dot{U}_o$, and let U_t be the maximum relative displacement of a SDOF system initially at rest. The value of U_t can be obtained from the available response spectrum and U_f can be obtained by maximizing Equation (7). The latter operation leads to a closed-form expression for U_f that can be readily used, instead of performing a time-domain analysis, to find the maximum response value (Veletsos and Ventura, 1984). Two methods are investigated here: (a) the Sum of Absolute Values (SAV) Method (Biot, 1941); and (b) the Square Root of the Sum of the Squares (SRSS) Method (Rosenblueth, 1951). In the SAV method, the true response, U, is approximated by a linear combination of the individual absolute maximum values, i.e.,

$$U \approx |U_t| + |U_f| \tag{15}$$

in which the pairs of vertical bars denote absolute value of the quantity enclosed. For the proposed approximation using the SRSS method, U is computed as

$$U \approx \left(U_t^2 + U_f^2\right)^{1/2} \tag{16}$$

Equations (15) and (16) are also applicable for the computation of the relative velocity and absolute acceleration response spectra. The response spectra for systems with 5% damping and approximated by the SAV method are shown in Figure 12 where they are compared with the true spectra. The approximation leads to excellent results for relative displacement, conservative results for relative velocity, and very conservative results for absolute acceleration values of rigid and intermediate systems. However, the results for flexible-period systems are in good agreement. The results using the SRSS method are shown in Figure 13. In this case the relative displacement and velocity values are less than the true values, but the absolute acceleration values for rigid and intermediate systems are closer to the true values than those obtained by the SAV method. For flexible systems, the results underestimate the exact ones. It can be concluded that, for the type of excitation considered, the SAV method leads to better results, in general, except for the computation of maximum absolute acceleration of rigid and intermediate systems where the SRSS method leads to better results.

CONCLUSIONS

- For long-period (flexible) systems, the effect of non-zero initial conditions in the motion is a magnification of the computed spectral responses (displacement, velocity and acceleration), regardless of the damping level of the system.
- For rigid systems the above effect is negligible, since they are more sensitive to the high-frequency components of the input motion than to the initial state of the system.
- The spectral values in the long-period range are dominated by the free-vibration part of the response of the system.
- The base shear for a long-period elastic system with non-zero initial conditions may be significantly larger than that for a system initially at rest, leading to an unconservative design if this effect is not properly accounted for.
- Omitting initial conditions makes the pseudo-velocity spectrum deviate from zero at very long periods and asymptotically approach the initial absolute velocity of the system (equal to the initial excitation velocity).
- For non-resting elastic systems, the SAV method (adding the peak absolute value of free-vibration response to the standard response spectrum) leads to an acceptable degree of approximation of all spectral values in the long-period range.
- Response spectra of inelastic systems are influenced by the initial conditions but are also sensitive to the value of the system's yield strength index (parameter η). For high η values (η > 0.5), the sensitivity of very flexible systems to variations in η decreases and the response approaches that of the truly elastic systems.

- Although it has been proved that initial conditions of the motion of the system are a crucial factor in the computation of response spectra, these conditions have a lesser influence on modern digital recording systems, since those incorporate a buffer memory that allows convenient retrieval of the initial portion of the acceleration record.

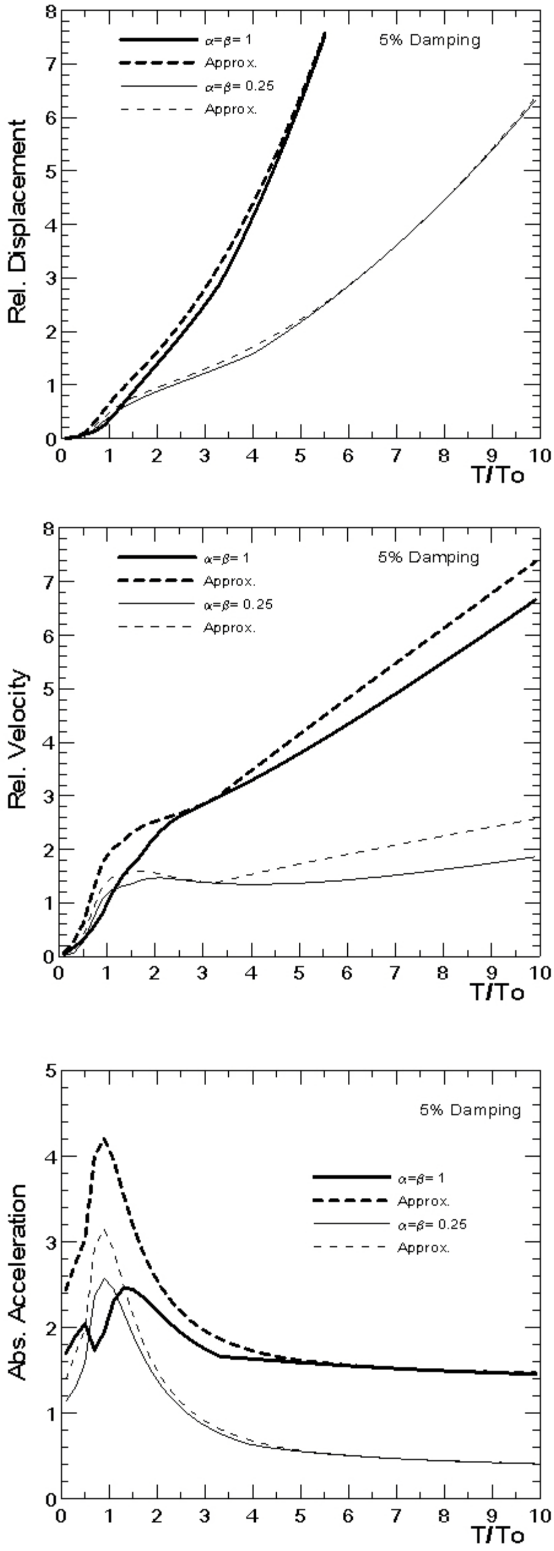

Fig. 12 Comparison of exact and approximate normalized response spectra computed by the proposed SAV method: sinusoidal excitation

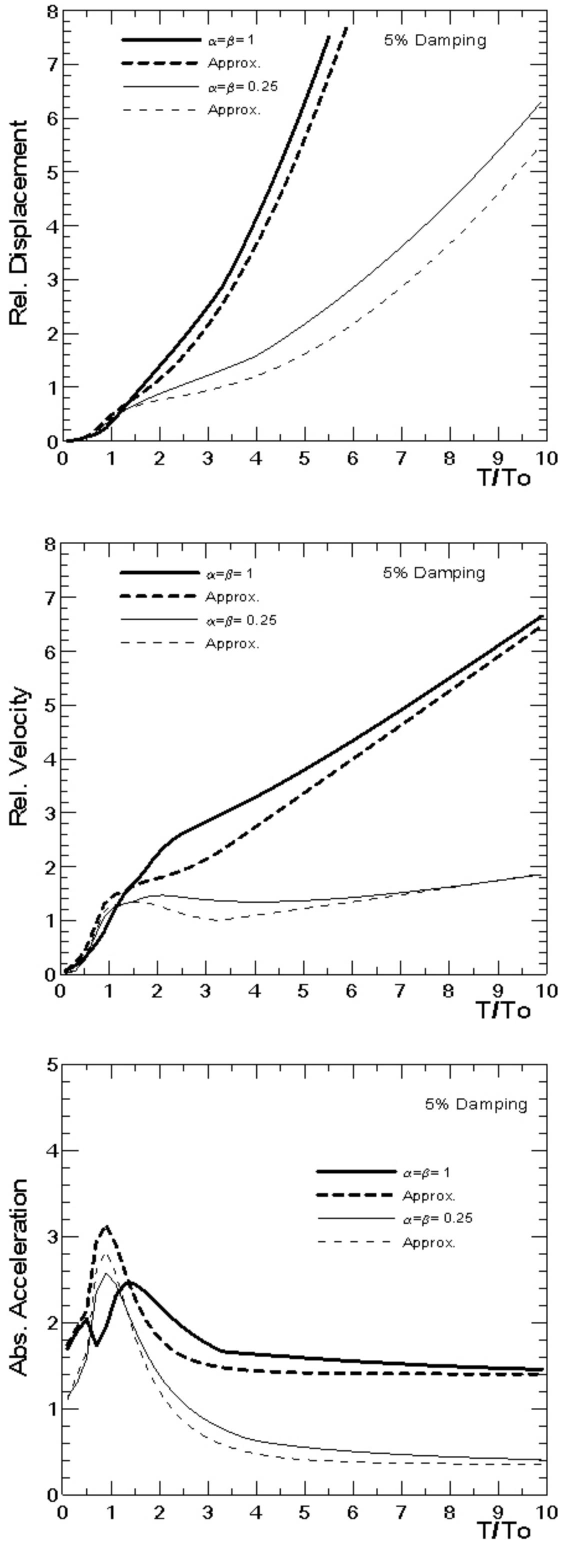

Fig. 13 Comparison of exact and approximate normalized response spectra computed by the proposed SRSS method: sinusoidal excitation

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