Paper No. 27

FREE PERIOD OF VIBRATION OF R C.C. ELEVATED WATER TOWERS

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SYNOPSIS

In this paper a simplified formula is evolved for prediction of free period of vibration of R. C. C. elevated water towers. The formula enables to calculate the period knowing the height of staging, height of the centre of the mass, diameter of the ring beam at the top of the staging and the seismic coefficient for which the tower is intended to be designed. This will greatly simplify the aseismic design of water towers since knowing the period and assuming a reasonable value of damping coefficient the lateral force acting on the tower during an earthquake can be known with reasonable accuracy from average acceleration spectrum curves, and the tower can be designed for the lateral force so calculated.

It is also concluded that for the same magnitude of the earthquake, towers with higher staging should be designed for lower value of seismic coefficient to allow for the increased flexiblity with increase in height of the tower.

INTRODUCTION

It is very necessary that an effective water supply system be maintained during an earthquake seeing life and fire hazards generally accompanying it. It requires that the water towers be designed earthquake resistant in seismic zones. For this purpose the lateral force acting on the tower due to the expected earthquake magnitude should be accurately estimated and the tower be designed for it. The lateral force can be determined if the dynamic characteristics of the tower i.e. free period and damping coefficient are known. Therefore it would be very helpful if the free period of the tower is known before the actual design of the tower is taken up. An attempt has been made for the above purpose in this paper.

ASSUMPTION AND SCOPE

The following assumptions are made :

1. The elevated tower supporting tank is a system with a single degree of freedom with the mass concentrated at the centre of gravity of the tank.

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2. The free period T, in secs. is calculated from the following formula :

$$T=2\pi\sqrt{\delta/g}$$

where δ=the static deflection at the top of the tank under a static horizontal force equal to its own weight W acting at the centre of gravity of the tank.
g=acceleration due to gravity.

3. The staging is designed throughout of uniform strength i.e. the section of the staging at any height is provided in accordance with the forces carried by it.

4. The staging is assumed as consisting of a circular shell of uniform thickness and same ratio of longitudinal reinforcement throughout the height. The area and moment of inertia of the section is increased by increasing uniformly the diameter of the shell downward. This is clearly shown in figure 1.

5. Moment of inertia is calculated on the basis of the effective section i.e. total area of concrete section plus the transformed area of steel.

The assumptions (1) & (2) are taken from Indian Standard 1893–1962 and are reasonable.

Assumption (3) is justified both from design and economic considerations.

Assumption (4) is made for simplification of the analysis made in this paper. However, it does not disallow the use of the formula evolved for other types of stagings such as one with columns and braces since in that case, the only difference is that the section is lumped at few points instead of providing continuous along the circumference of a circle.

Assumption (5) is justified since the section will be subjected to heavy compressive load of the tank at all times and the whole section including the transformed area of steel shall be effective. This is also on the safer side from the dynamic analysis point of view.

NOTATIONS

Following are the notations used in this paper.

T=Free period of vibration of the water tower.

W=The weight of the tank when full.

δ=The static deflection at the top of the tank under a static horizontal force W.

g=Acceleration due to gravity.

H=Height of the staging.

h=Height of the centre of the mass of the tank from the top of the staging.

D_1=Diameter of the supporting ring beam at the top of the staging.

D_2=Diameter of the supporting ring beam at the bottom of the staging.

D=Diameter of the circular shell at a depth x from the top of the staging.

x=Depth of the cross section under consideration from the top of the staging.

d=Thickness of the shell for the staging.

A=Area of cross-section of the staging=πdD.

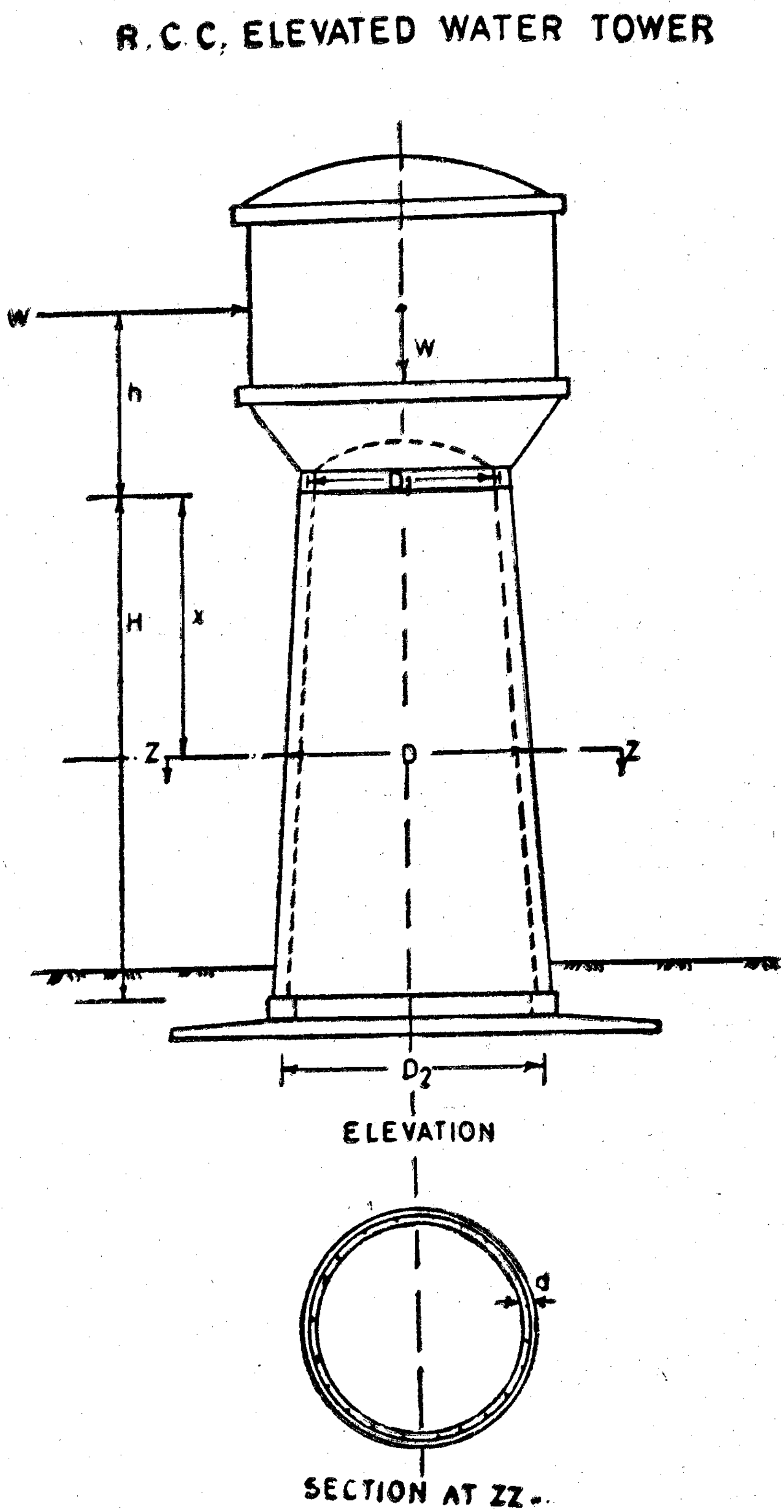
R.C.C. ELEVATED WATER TOWER
W
W
h
D1
H
x
Z
D
Z
D2
ELEVATION
d
SECTION AT ZZ.

Figure 1

A_t=Area of the longitudinal reinforcement.
$p=A_t/A$
m=Modular ratio
d_e=effective thickness of the shell$=d\{I+(m-I)p\}$
A_e=effective area of x-section$=\pi d_e D$
I_e=Effective moment of inertia$=\pi d_e D^3/8$
c=maximum allowable compressive stress in concrete.
α_h=Design seismic coefficient.
E=modulus of elasticity of concrete.
n=Dimensionless factor=H/h
a=Dimensionless factor$=h/D_1$
b=dimensionless factor$=D_2/D_1$
k=Dimensionless coefficient depending upon α_h
K=Dimensionless coefficient, a function of α_h, n and a
M=Bending moment at any cross-section.
N=Multiplying factor depending on the magnitude of the earthquake.
$\triangle$=Maximum dynamic deflection of the tower.

METHOD AND CALCULATION

The cross-section of the staging at any depth x is subjected to a direct load W and a bending moment M given by,

$$M=\alpha_h W(x+h) \tag{1}$$

The max^m compressive stress in concrete is given by

$$c=\frac{W}{A_e}+\frac{M}{I_e}\frac{D}{2} \tag{2}$$

or

$$c=\frac{W}{\pi d_e D}+\frac{\alpha_h W(x+h)}{\pi d_e D^3/8}\cdot\frac{D}{2} \tag{3}$$

$$=\frac{W}{\pi d_e D}\left[1+\alpha_h(x+h)\cdot\frac{4}{D}\right] \tag{4}$$

For cross-section at the top of the staging

$$x=0,\ D=D_1$$

$$\therefore\quad c=\frac{W}{\pi d_e D_1}\left[1+\alpha_h(0+h)\cdot\frac{4}{D_1}\right]$$

or

$$de=\frac{W}{\pi c D_1}\left[1+4\alpha_h\frac{h}{D_1}\right] \tag{5}$$

For the cross-section at the bottom of the staging

$$x=H,\ D=D_2=bD_1$$

$$c=\frac{W}{\pi d_e D_2}\left[1+\alpha_h(H+h)\frac{4}{D_2}\right]$$

From which

$$D_2=\frac{W}{2\pi d_e c}\left[1+\sqrt{1+\frac{16c\pi d_e}{W}\alpha_h(H+h)}\right] \tag{6}$$

Putting the value of d_e from (5) into (6), we get

$$D_2=\frac{D_1}{z\left(1+4\alpha_h\frac{h}{D_1}\right)}\left[1+\sqrt{1+\frac{16\alpha_h(H+h)}{D_1}\left(1+\frac{4\alpha_h h}{D_1}\right)}\right] \tag{7}$$

Now putting

$$\frac{D_2}{D_1}=b\,;\ \frac{h}{D_1}=a\,,\ \frac{H}{h}=n$$

$$\frac{H}{D_1}=\frac{H}{h}\times\frac{h}{D_1}=na$$

We get from equation (7)

$$b=\frac{1}{2(1+4\alpha_h a)}\left[1+\sqrt{1+16\alpha_h a(1+n)(1+4\alpha_h a)}\right] \tag{8}$$

Increasing the diameter of the shell uniformly from D_1 to D_2 downward

$$D=D_1+\frac{D_2-D_1}{H}x=D_1\left[1+\frac{(b-1)x}{H}\right] \tag{9}$$

The deflection δ at the centre of the mass due to a static horizontal load W is given by applying moment area method as follows

$$\delta=\int_0^H\frac{W(x+h)^2}{E\pi d_e D^3/8}dx \tag{10}$$

Putting the value of d_e from (5) and value of D from (9) into equation (10), we get on integrating,

$$\delta=\frac{8cha^2}{E(1+4\alpha_h a)}\left[\frac{2n}{b-1}\left\{1-\frac{(1+n)^2}{b^2}\right\}+\left(\frac{2n}{b-1}\right)^2\left\{1-\frac{1+n}{b}\right\}+\frac{4n^3}{(b-1)^3}\log_e b\right] \tag{11}$$

Now period T is given by

$$T=2\pi\sqrt{\frac{\delta}{g}}$$

$$T=2\pi\sqrt{\frac{8cha^2}{E(1+4\alpha_h a)g}\left[\frac{2n}{b-1}\left\{1-\left(\frac{1+n}{b}\right)^2\right\}+\left(\frac{2n}{b-1}\right)^2\left\{1-\frac{1+n}{b}\right\}+\frac{4n^3}{(b-1)^3}\log_e b\right]}$$

or

$$T=2\pi\sqrt{\frac{8ch}{Eg}}\times K \tag{12}$$

where,

$$K=\sqrt{\frac{a^2}{(1+4\alpha_h a)}\left[\frac{2n}{b-1}\left\{1-\left(\frac{1+n}{b}\right)^2\right\}+\frac{4n^2}{(b-1)^2}\left(1-\frac{1+n}{b}\right)+\frac{4n^3}{(b-1)^3}\log_e b\right]} \tag{13}$$

and is dimension less

Substituting values of b from equation (8)

$$K=f(a, n, \alpha_h) \tag{14}$$

On actual numerical analysis it is found that values of K as given by (13) are also approximately given by the following simple equation for values of a=0.5 to 1.5 and n=1 to 10.

$$\therefore \quad K=(kn+1)a+(0.6n-0.3) \tag{15}$$

Where k depends upon seismic coefficient α_h and is given by Table No. 1 below

TABLE No. 1

α_h=0.05	0.075	0.10	0.125	0.15
k=1.23	1.0	0.84	0.7	0.6

Taking an average value of k=0.9 for all values of α_h we get

$$K=(0.9n+1)a+(0.6n-0.3) \tag{16}$$

Therefore from equations (12) and (16)

$$T=2\pi\sqrt{\frac{8ch}{Eg}}\times\left[(0.9n+1)a+(0.6n-0.3)\right] \tag{17}$$

Now using 1:2:4 concrete mix with maximum allowable stress,

c=1000 psi (including 33% increase in stresses)

$E=1.67\times10^6$ psi

$g=32.0$ ft/sec^2

The period is given by

$$T=0.077[(0.9n+1)a+(0.6n-0.3)]\sqrt{h} \tag{18}$$

where h is in feet.

or

$$T=0.14[(0.9n+1)a+(0.6n-0.3)]\sqrt{h} \tag{19}$$

where h is in meters.

EXAMPLE

For a 10^5 gallon capacity Intze tank as shown in Fig. 1, say

$h=17'$; $D_1=20'$

$a=\frac{17}{20}=0.85$

Period T is given by from equation (18) on substituting the values of h and a

$$T=0.318(1.365n+0.55) \tag{20}$$

In Table No. 2 are given the values of T, average acceleration S_a for 5% damping, seismic coefficient α_h for Toft California Earthquake with N=1.6, Max^m dynamic reflection $\triangle$ due to the above earthquake and values of $\triangle/H$ for heights of the tower from 25 ft. to 100 ft.

CONCLUSIONS

The following conclusions can be drawn.

1. The free period of vibration of R.C.C. water tower can be found out from equations (17), (18) or (19).
2. For the same magnitude of the earthquake, the water towers with higher staging should be designed for lower seismic coefficient to allow for the inereased flexiblity with increase in height of the staging.
3. The maximum dynamic deflection due to the same earthquake in water towers of different heights bears almost the same ratio with height of the staging and is found to be with in permissible limit.

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TABLE No. 2

Height H in ft. of the staging	25	50	75	100
$n=\frac{H}{h}=\frac{H}{17}$	1.5	3.0	4.5	6
Free period in sec. $T=0.318\ (1.365n+0.55)$	0.83	1.48	2.13	2.8
Average acceleration cm/sec² For 5% damping (S_a) From spectrum curve	117.5	71	59	47
$\frac{S_a}{g}$	0.12	0.072	0.06	0.048
For Taft., California Earthquake with N=1.6 Seismic coeff. $\alpha_h = \frac{S_a}{g} \times 1.6$	0.19	0.115	0.096	0.072
Maximum dynamic deflection $\Delta=\alpha_h \times g \times \left(\frac{T}{2\pi}\right)^2$ in ft.	0.1065	0.205	0.353	0.456
$\frac{\Delta}{H}$	$\frac{1}{235}$	$\frac{1}{244}$	$\frac{1}{212}$	$\frac{1}{219}$