ON THE VIBRATION BEAM AND SLAB BRIDGES

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SYNOPSIS

The beam and slab bridge is a complex structure and the calculation of its natural frequency is quite involved. A simplified procedure has been given in this paper for such a calculation. A Fourier series expansion in terms of plate-eigenfunctions has been used to express the deflection of the bridge, the bridge being considered as a plate resting on beams. It has been found that a one-term approximation gives satisfactory results and numerical work has been carried out for various bridge aspect-ratios based on a one-term approxmation. Graphs have been given to facilitate a rapid estimate of the frequency.

INTRODUCTION

The natural frequencies of a structure, especially of the graver modes, form important basic data necessary in evaluating its response under a variety of dynamic loads. These frequencies are of interest whether one wishes to determine the dynamic deflection of a bridge under the action of a moving load or its behaviour under blast loadings and strong-motion-earthquakes. An easy procedure to calculate the fundamental natural frequency, without sacrificing the accuracy of the result, would then be eminently desirable.

The dynamic behaviour of bridges has been studied extensively by various authors such as Inglis (1934) and others, but, they have confined themselves mostly to the study of Railway bridges. The natural frequencies of such bridges can be obtained easily by the theory of beam-vibrations. The situation is more complex when one considers a beam and slab bridge. The problem is essentially two dimensional and a one-dimensional approximation is not reliable unless the span is very large when compared with the width of the bridge. A rational analysis of the problem then requires, a consideration of the bridge in the light of the plate theory.

Stiffened Plate structures have been studied in the literature, most often by the orthotropic plate theory. Considering the beam and slab bridge as an orthotropic plate one arrives at a transcendental equation for the bridge frequency (Hoppmann and Huffington, 1958). This approach has been utilised by Naruoka and Yonezawa (1958) for the frequency analysis of the beam bridge. But the solution of the transcendental equation for various bridge dinamensions is quite cumbersome and is not easily amenable for engineering purposes.

In this paper the bridge has been considered as a plate resting on beams. It has been assumed that there is no restraint against slippage between the beam and the slab and that the beam offers only vertical forces of reaction against the plate. Torsional resistance of the beams and the internal damping in the bridge have been neglected. The consideration of restraint against slippage leads to very involved equations and is not amenable to a rapid calculation. The influence of restraint against slippage tends to increase the potential energy of deformation of the structure and its effect may also be considered by increasing the stiffness of the beam on the basis of the effective width concept. The question of effective width in dynamic problems has not been well understood and this consideration merits more detailed theoretical and experimental investigations.

THE PLATE-EIGEN FUNCTIONS

It is well known that the normal modes of a vibra-

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ting plate can be described in terms of known functions when its two opposite edges are simply supported. A slab bridge can be considered as a plate simply supported on two opposite edges and free at the other two (Fig. 1).

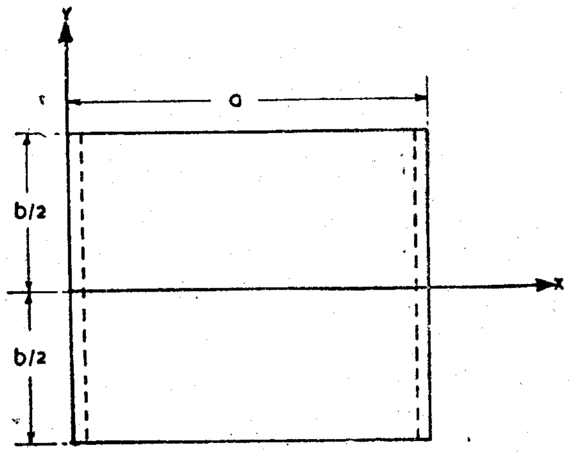

Fig. 1.

Considering the classical plate equation

$$\nabla^4 W + \frac{\rho}{D} \frac{\partial^2 W}{\partial t^2} = 0 \quad \dots (1)$$

where W is the deflection of the plate, ρ the mass of the plate per unit area, D the flexural rigidity of the plate, one can take for the m-nth mode of vibration

$$W_{mn} = Y_{mn}(y) \, \text{Sin} \frac{m\pi x}{a} \, \text{Cos} \, P_{mn} t \quad \dots (2)$$

where 'a' is the span of the slab bridge, P_{mn} the circular frequency of vibration for the m-nth mode, m and n being any two integers. Y_{mn} will now take the form,

$$Y_{mn}(y) = A_1 \, \text{Cosh} \frac{\alpha_{mn} y}{b} + A_2 \, \text{Sinh} \frac{\alpha_{mn} y}{b} + A_3 \, \text{Cos} \frac{\beta_{mn} y}{b} + A_4 \, \text{Sin} \frac{\beta_{mn} y}{b} \quad \dots (3)$$

where b is the width of the slab bridge,

$$\alpha_{mn} = b \sqrt{\sqrt{\frac{\rho p^2_{mn}}{D}} + \frac{m^2\pi^2}{a^2}} \quad \dots (4a)$$

$$\beta_{mn} = b \sqrt{\sqrt{\frac{\rho p^2_{mn}}{D}} - \frac{m^2\pi^2}{a^2}} \quad \dots (4b)$$

and $$\alpha^2_{mn} - \beta^2_{mn} = 2m^2\pi^2 \frac{b^2}{a^2} \quad \dots (4c)$$

The free edge boundary conditions at $y = \pm \frac{b}{2}$ may be expressed as

$$\left. \frac{d^2 Y_{mn}}{dy^2} - \upsilon \frac{m^2\pi^2}{a^2} Y_{mn} \right|_{y=\pm\frac{b}{2}} = 0 \quad \dots (5a)$$

$$\left. \frac{d^3 Y_{mn}}{dy^3} - (2-\upsilon) \frac{m^2\pi^2}{a^2} \frac{dY_{mn}}{dy} \right|_{y=\pm\frac{b}{2}} = 0 \quad (5b)$$

where υ is the Poisson's ratio.

Considering only modes symmetric in y we obtain the frequency equation to satisfy these boundary conditions for the slab bridge as

$$\tan \frac{\beta_{mn}}{2} = - \frac{\alpha_{mn}}{\beta_{mn}} \left(\frac{\beta^2_{mn} + \upsilon m^2\pi^2 \frac{b^2}{a^2}}{\beta^2_{mn} + (2-\upsilon) m^2\pi^2 \frac{b^2}{a^2}} \right)^2 \times \tanh \frac{\alpha_{mn}}{2} \quad \dots (6)$$

This equation has to be solved together with (4c) to obtain the frequency parameters α_{mn} and β_{mn}. These equations will have an infinite number of roots for each value of m. The eigenfunction corresponding to the nth root may now be taken as

$$Y_{mn}(y) = \frac{\text{Cosh} \frac{\alpha_{mn} y}{b}}{\text{Cosh} \frac{\alpha_{mn}}{2}} + \frac{\alpha^2_{mn} - \upsilon m^2\pi^2 \frac{b^2}{a^2}}{\beta^2_{mn} + \upsilon m^2\pi^2 \frac{b^2}{a^2}} \times \frac{\text{Cos} \frac{\beta_{mn} y}{b}}{\text{Cos} \frac{\beta_{mn}}{2}} \quad \dots (7)$$

The first roots of the equations corresponding to the fundamental mode of vibration have been obtained by an iteration procedure given in Appendix I. The values of the roots are given in Table 1 for $\upsilon=0$ and $\upsilon=0.2$.

It can be shown that integrals of the type

$$\int_{-b/2}^{+b/2} \int_0^a Y_{mn} Y_{ij} \, \text{Sin} \frac{m\pi x}{a} \, \text{Sin} \frac{i\pi x}{a} \, dx \, dy = 0 \quad \dots (8)$$

whenever $i \neq m$ or $j \neq n$. It may thus be seen that the functions $Y_{mn} \, \text{Sin} \frac{m\pi x}{a}$ form an orthogonal set in the domain $x = 0$ to a and $y = -\frac{b}{2}$ to $+\frac{b}{2}$.

This property of the plate-eigenfunctions allows a Fourier series expension of an arbitrary function in terms of these functions. A knowledge of the value of the integral

$$\frac{1}{ab}\int_{-b/2}^{+b/2}\int_{0}^{a} Y^2_{mn}\ \mathrm{Sin}^2\frac{m\pi x}{a}\,dx\,dy = K_{mn} \quad \ldots \quad (9)$$

is essential for such a Fourier series expansion. The values of this integral have been given in Table 2 for $m = 1$ and $n = 1$

ANALYSIS

A typical beam and slab bridge with k beams disposed symmetrically about the x–axis, the free edges being parallel to the x-axis (Fig. 2) may now be considered.

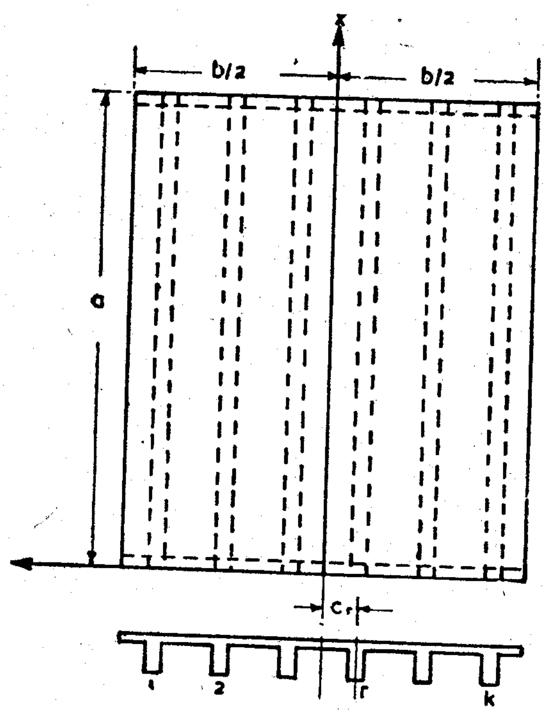

Fig. 2.

The analysis of free vibration of this structure will now be based on the following assumptions.

(i) There is no restraint against slippage between the beams and the slab,

(ii) The torsion of the beams may be neglected and

(iii) The internal damping in the bridge may be neglected.

Though the first assumption is rarely realised in practice, the inaccuracies arising from this assumption may be compensated for by suitably increasing the stiffness of the beam based on the effective width concept. Such an engineering approach to simplification is necessary since the equations would otherwise become very complicated. The second assumption is besed on the fact that the rotations of plate cross-sections will be small in the first mode of vibration.

The motion of the slab may be described by the equation

$$D\nabla^4 W - \rho p^2 W = -\sum_{r=1}^{k} \delta(c_r)\, f_r(x) \quad \ldots \quad (10)$$

after separating the time variable, where $y = c_r$ denotes the location of the r^{th} beam, $f_r(x)$ is the amplitude of the vertical force of interaction between the r^{th} beam and the slab and $\delta(c_r)$ is the Dirac-delta function at $y = c_r$. The deflections of the beams are given by the equations

$$EI\frac{d^4\overline{W}_r}{dx^4} - \gamma\, p^2\, \overline{W}_r = f_r(x) \quad \ldots \quad (11)$$

$$r = 1, 2, \ldots k$$

where EI is the stiffness of any beam, γ is the mass per unit length of any beam and $\overline{W}_r(x) = W(x, c_r)$.

W is now expanded in terms of the plate eigenfunctions discussed previously and one can write

$$W = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\, Y_{mn}\, \mathrm{Sin}\frac{m\pi x}{a} \quad \ldots \quad (12)$$

It may be noted that the functions used in the above expansion satisfy all the boundary conditions exactly. The coefficients A_{mn} now remain to be chosen so as to satisfy the equations (10) and (11). Expanding

$\sum_{r=1}^{k} \delta(c_r)\, f_r(x)$ by another Fourier series,

$$\sum_{r=1}^{k} \delta(c_r)\, f_r(x) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\, Y_{mn}\, \mathrm{Sin}\frac{m\pi x}{a} \quad (13)$$

where

$$a_{mn} = \frac{1}{K_{mn}\, ab}\sum_{r=1}^{k} Y_{mn}(c_r)\int_{0}^{a} f_r(x)\, \mathrm{Sin}\frac{m\pi x}{a}\, dx \quad (14)$$

Using (13) and (12) in (10),

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\frac{\rho p^2_{mn}}{D} Y_{mn}\,\text{Sin}\,\frac{m\pi x}{a} - \frac{\rho p^2}{D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} Y_{mn}\,\text{Sin}\,\frac{m\pi x}{a} = -\frac{1}{D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} Y_{mn}\,\text{Sin}\,\frac{m\pi x}{a} \quad \ldots (15)$$

where p_{mn} is the circular frequency for the $m-n^{th}$ mode of vibration of the slab without beams.

Collecting the coefficient of each $Y_{mn}\,\text{Sin}\frac{m\pi x}{a}$ in (15) and using (14)

$$A_{mn}\frac{\rho}{D}(p^2_{mn}-p^2) = -\frac{1}{K_{mn}\,Dab}\sum_{r=1}^{k} Y_{mn}(c_r)\int_0^a f_r(x)\,\text{Sin}\,\frac{m\pi x}{a}\,dx \quad (16)$$

Since $\overline{W}_r = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} Y_{mn}(c_r)\,\text{Sin}\frac{m\pi x}{a}$, equation (11) becomes, $f_r(x)$

$$= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} Y_{mn}(c_r)\times\left(EI\frac{m^4\pi^4}{a^4} - \gamma p^2\right)\text{Sin}\frac{m\pi x}{a} \quad \ldots (17)$$

Substituting (17) in (16)

$$A_{mn}\frac{\rho}{D}(p^2_{mn}-p^2) = -\frac{1}{K_{mn}\,ab}\sum_{r=1}^{k} Y_{mn}(c_r)\int_0^a \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} A_{ij} Y_{ij}(c_r)\,\text{Sin}\,\frac{m\pi x}{a}\,\text{Sin}\frac{i\pi x}{a}\left(\frac{EI}{D}\,\frac{i^4\pi^4}{a^4} - \frac{\gamma p^2}{D}\right)dx \quad \ldots (18)$$

Since $\int_0^a \text{Sin}\,\frac{i\pi x}{a}\,\text{Sin}\,\frac{m\pi x}{a}\,dx = 0$ when $i \neq m$

$= \frac{a}{2}$ when $i = m$,

(18) reduces to

$$A_{mn}\frac{\rho b^4}{D}(p^2_{mn}-p^2) = -\frac{1}{2K_{mn}}\sum_{r=1}^{k}\sum_{j=1}^{\infty} A_{mj} Y_{mn}(c_r)\, y_{mj}(c_r)\left(\frac{EI}{Db}\,\frac{m^4\pi^4 b^4}{a^4} - \frac{\gamma}{\rho b}\times\frac{\rho p^2 b^4}{D}\right)$$

$$n = 1, 2, \ldots \quad \ldots (19)$$

Here one has an infinite set of homogeneous equations in A_{mj} for each value of m.

Putting $\lambda_{mn} = \frac{\rho p^2_{mn} b^4}{D}$, $\lambda = \frac{\rho p^2 b^4}{D}$, $K_1 = \frac{EI}{Db}$ and $K_2 = \frac{\gamma}{\rho b}$, the equation may be written as

$$A_{mn}(\lambda_{mn}-\lambda) = -\frac{1}{2K_{mn}}\sum_{r=1}^{k}\sum_{j=1}^{\infty} A_{mj} Y_{mj}(c_r)\, Y_{mn}(c_r)\times\left(K_1\frac{m^4\pi^4 b^4}{a^4} - K_2\lambda\right)$$

$$n = 1, 2, \ldots \quad \ldots (20)$$

For non trivial solutions of A_{mj} the determinant of the coefficients of A_{mj} should vanish. The roots of this determinant give the values of λ, the frequency parameter, for various modes.

CONVERGENCE OF SOLUTION

For purposes of numerical analysis it is necessary to consider a finite number of terms in the series, the number of terms depending upon the rapidity of the convergence of the solution. A typical case has been examined by taking $\frac{a}{b} = 2.0$ and considering a three-beam bridge such that $c_1 = \frac{3b}{8}$, $c_2 = 0$, and $c_3 = -\frac{3b}{8}$. Putting $m = 1$, one obtains modes with no nodal lines in the x direction. Taking three terms in the series corresponding to A_{11}, A_{12} and A_{13} a third order determinant is obtained: The determinant is given below putting $K_1 = 2.0$ and $K_2 = 0.2$.

$$\begin{vmatrix} 749.0910 - 1.58871\lambda & -29.6167 + 0.48647\lambda & -4.3230 + 0.07100\lambda \\ -34.6162 + 0.56859\lambda & 15488.7 - 1.50991\lambda & -8.1135 + 0.13326\lambda \\ -5.1452 + 0.08451\lambda & -8.2618 + 0.1357\lambda & 91053.3 - 2.04541\lambda \end{vmatrix} = 0$$

The values of $\sqrt{\lambda}$ for the first mode corresponding to a one-term approximation, two-term approximation and three-term approximation have been determined and they are,

No. of terms	1	2	3
$\sqrt{\lambda}$	21.714	21.669	21.669

It may be noticed that the convergence is very rapid and that a one-term approximation is sufficiently accurate for engineering purposes. The rapidity of convergence stems from the fact that the diagonal terms are very large when compared when the others, and they increase very rapidly as the order of the diagonal term is increased. The same situation prevails in other determinants corresponding to various aspect-ratios of the bridge. The results of one-term approximation may therefore be used for a practical calculation of the bridge frequency.

ONE-TERM APPROXIMATION

The frequency analysis now becomes very much simplified since no determinantal equation need be solved. As the frequency of the first symmetric mode is the most important data, numerical work has been carried out only for this mode. Putting $m=1$ and taking the first term in the series corresponding to A_{11} the frequency equation becomes

$$2K_{11}\ \lambda_{11}+K_1\ \frac{\pi^4 b^4}{a^4}\sum_{r=1}^{k} Y^2_{11}\ (c_r) = [2K_{11}+K_2 \sum_{r=1}^{k} Y^2_{11}\ (c_r)]\ \lambda \qquad \ldots\ (21)$$

The calculation of λ is now quite straight forward once the values of K_{11}, λ_{11}, α_{11} and β_{11} for each $\frac{a}{b}$ ratio are known. The values of α_{11}, β_{11}, and K_{11} are to be found in Tables 1 and 2. The values of λ_{11} are given in Table 3.

The frequency parameters for various bridge dimensions have been presented in the graphs (Figs. 3 to 14)* both for $\upsilon=0$ and $\upsilon=0.2$ to facilitate rapid frequency determination. The variation of $\sqrt{\lambda}$ with respect to K_1 is very nearly a straight line in the range of values of K_1 chosen and the graphs have been drawn as such. The graphs cannot be extended much beyond the value of $K_1 = 10$, since then the variation will not be linear, and also the accuracy of the one-term approximation is affected for large values of K_1. Nonetheless, the approximation gives satisfactory results in the range of bridge dimensions met with in practice.

CONCLUSIONS

The procedure outlined in the previous sections provides a rapid way of calculating the bridge frequency unlike the one using the orthotropic plate theory. The orthotropic plate theory necessitates the solution of a cumbersome transcendental equation for every frequency calculation. The present procedure owes its simplicity to the fact that the bridge deflection is expanded in terms of the plate-eigenfunctions and also to the orthogonal properties of these functions.

It may be noticed from the graphs that in general, except for $\frac{a}{b}= 1.5$, the bridge frequency is less than the frequency of the corresponding beamless slab, in the range of values of K_1 and K_2 considered. This indicates that the influence of the mass of the beams dominates over the influence of the stiffness of the beams in the above range.

The graphs also clearly demonstrate the influence of Poisson's ratio on the bridge frequency, which arises due to the presence of free edges. A decrease in Poisson's ratio is attended by an increase in the frequency. This is in conformity with the well known theorem due to Rayleigh that the introduction of constraints raises the frequency of a system. One may also notice that as the span/width ratio increases the Poisson's ratio effect is less pronounced. For span/width ratios beyond 3.0 the neglect of Poisson's ratio introduces only negligible errors. This mey be considered as being due to the predominently one-dimensional action of the bridge at large span/width ratios.

It is necessary to remark here that the assumption of absence of restraint against slippage leads to frequencies lower than the actual, since this absence of restraint is not realized in practice. This again follows from the well known theorem due to Rayleight (1945). The consideration of this restraint in the problem would render the equations sufficiently complicated as to make the numerical labour prohibitive. As mentioned earlier in the introduction this difficulty may be circumvented by suitably calculating the stiffness of the beam by taking a portion of the plate to act with the beam. This altered stiffness may then be used in the procedure developed in this paper to calculate the frequency. A similar problem has been encountered in the treatment of buckling of plate-beam systems (Timoshenko and

*Figures given at the end of this paper.

Gere. 1961). It is needless to emphasize that the width of the plate to be included in the stiffness calculation can only be determined by detailed theoretical and experimental investigations.

Table 1

FREQUENCY PARAMETERS FOR THE SLAB

$\frac{a}{b}$	$v=0$		$v=0.2$	
	α_{11}	β_{11}	α_{11}	β_{11}
1.5	5.9283	5.1353	5.8439	5.0376
2.0	5.4609	4.9886	5.4013	4.9233
2.5	5.2188	4.9069	5.1757	4.8610
3.0	5.0784	4.8576	5.0463	4.8241
3.5	4.9902	4.8261	4.9655	4.8005
4.0	4.9315	4.8049	4.9120	4.7848

Table 2

VALUES OF K_{11}

$\frac{a}{b}$	$v=0$	$v=0.2$
1.5	0.71599	0.67541
2.0	0.62231	0.60131
2.5	0.57881	0.56587
3.0	0.55501	0.54619
3.5	0.54056	0.53415
4.0	0.53113	0.52626

Table 3

VALVES OF λ_{11}

$\frac{a}{b}$	$v=0$	$v=0.2$
1.5	946.056	885.912
2.0	748.229	713.235
2.5	658.269	635.473
3.0	609.754	593 825
3.5	580.649	568.846
4.0	561.850	552.767

APPENDIX I

(i) Iteration Procedure to obtain α_{mn} and β_{mn}

The frequency may be written as

$$\tan\frac{\beta_{mn}}{2} = -\frac{\alpha_{mn}}{\beta_{mn}}\left(\frac{\beta^2{}_{mn}+v\,\frac{m^2\pi^2b^2}{a^2}}{\beta^2{}_{mn}+(2-v)\,\frac{m^2\pi^2b^2}{a^2}}\right)^2 \times \tanh\frac{\alpha_{mn}}{2}$$

First approximations to the values of α_{mn} and β_{mn} for various values of $\frac{b}{a}$ ratio have been given by Thein Wah (1961) using a graphical procedure. These values have been used as the initial values for the iteration. Substituting an initial value in the right hand side of the above equation, a new value of β_{mn} was obtained by equating $\tan\frac{\beta_{mn}}{2}$ to the calculated right hand side. Using this second approximation to β_{mn}, a second approximation to α_{mn} was obtained by the relation $\alpha^2{}_{mn} = \beta^2{}_{mn}+2m^2\pi^2\frac{b^2}{a^2}$. The procedure was then repeated using these second approximations. The iterations were repeated until the differences between successive approximations were negligible. The results of such an iteration are presented in Table 1.

(ii) Expression for K_{mn}

Substituting the expresson for Y_{mn} in the relation

$$K_{mn} = \frac{1}{ab}\int_{-b/2}^{+b/2}\int_{0}^{a} Y^2{}_{mn}\,\sin^2\frac{m\pi x}{a}\,dx\,dy$$

$$= \frac{1}{2b}\int_{-b/2}^{+b/2} Y^2{}_{mn}\,dy$$

one obtains,

$$K_{mn} = \frac{1}{4\,\cosh^2\frac{\alpha_{mn}}{2}} + \frac{\tanh\frac{\alpha_{mn}}{2}}{2\alpha_{mn}} + \frac{(\alpha^2{}_{mn}-vm^2\pi^2\frac{b^2}{a^2})^2}{(\beta^2{}_{mn}+vm^2\pi^2\frac{b^2}{a^2})^2} \times \left(\frac{1}{4\,\cos^2\frac{\beta_{mn}}{2}} + \frac{\tan\frac{\beta_{mn}}{2}}{2\beta_{mn}}\right) + \frac{2}{(\alpha^2{}_{mn}+\beta^2{}_{mn})} \times \frac{(\alpha^2{}_{mn}-vm^2\pi^2\frac{b^2}{a^2})}{(\beta^2{}_{mn}+vm^2\pi^2\frac{b^2}{a^2})}\left(\alpha_{mn}\tanh\frac{\alpha_{mn}}{2}+\beta_{mn}\tan\frac{\beta_{mn}}{2}\right)$$

The values of K_{11} are given in Table 2.

APPENDIX II

NOTATION

a	Span of the bridge
b	Width of the bridge
c_r	y-co-ordinate of the r^{th} beam
D	Flexural rigidity of the slab
EI	Flexural rigidity of any beam
K_{mn}	A definite integral depending on m and n
k	No. of beams
K_1	$\frac{EI}{Db}$
K_2	$\frac{\gamma}{\rho b}$
$f_r(x)$	Amplitude of the reaction of the r^{th} beam against the slab
m,n,i,j	Integers
P_{mn}	Circular frequency of the slab alone for the m-n^{th} mode
p	Circular frequency of the beam and slab bridge
W	Deflection of the bridge
$\overline{W}_r$	$W(x, c_r)$ Deflection of the r^{th} beam
W_{mn}	Deflection of a slab bridge in the m$-n^{th}$ mode of vibration
α_{mn}	Frequency perameter for the slab
β_{mn}	Frequency parameter for the slab
γ	Mass of beam per unit length
$\delta(c_r)$	Dirac-delta function in one dimension at $y = c_r$
λ_{mn}	$\frac{\rho p^2_{mn} b^4}{D}$ Frequency parameter of the slab
λ	$\frac{\rho p^2 b^4}{D}$ Frequency parameter of the beam and slab
υ	Poisson's ratio
ρ	Mass of slab per unit area.

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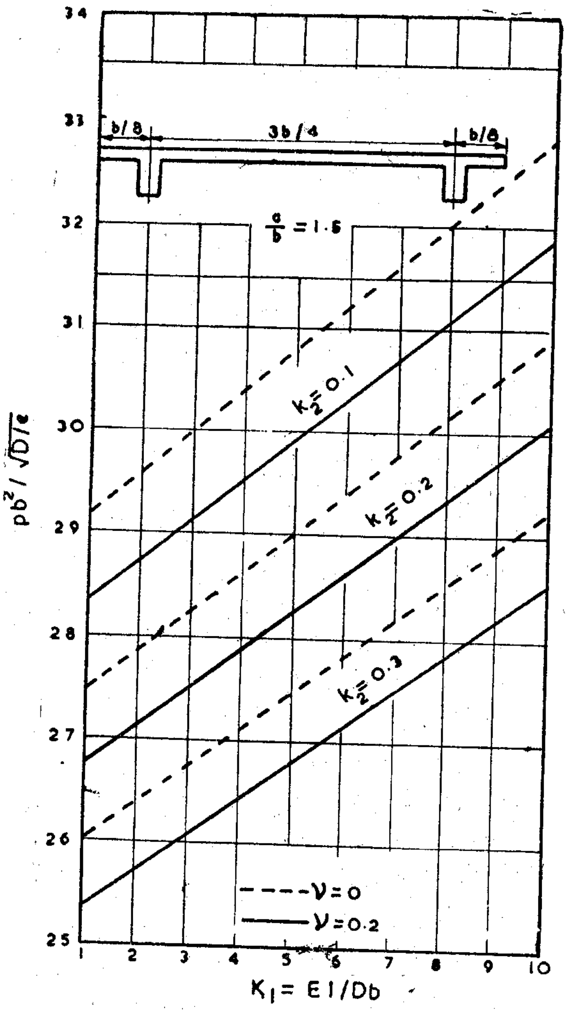

Fig. 3.

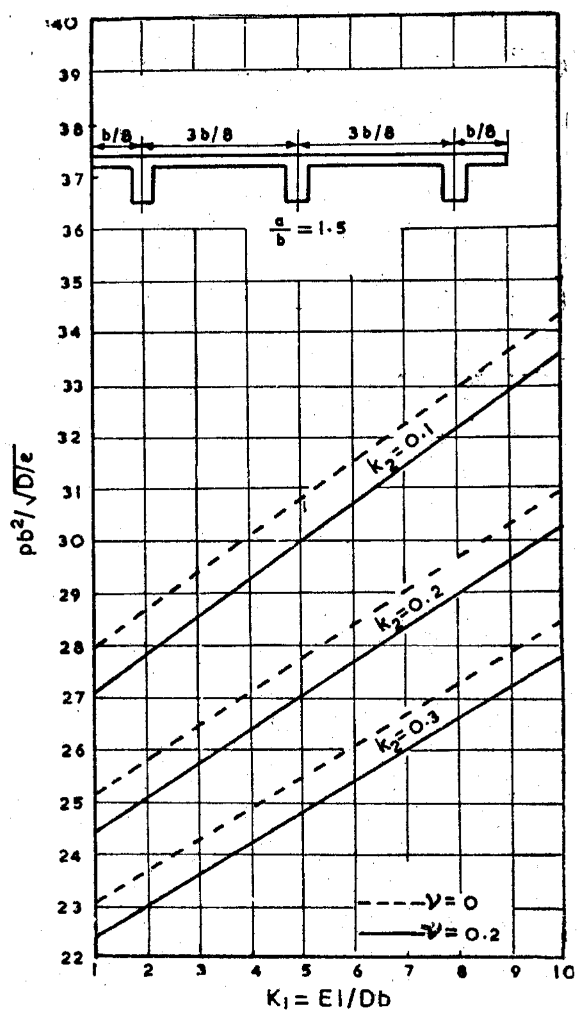

Fig. 4.

Values of $pb^2\sqrt{D/e}$ for a bridge with a/b=1.5 for the first symmetric mode.

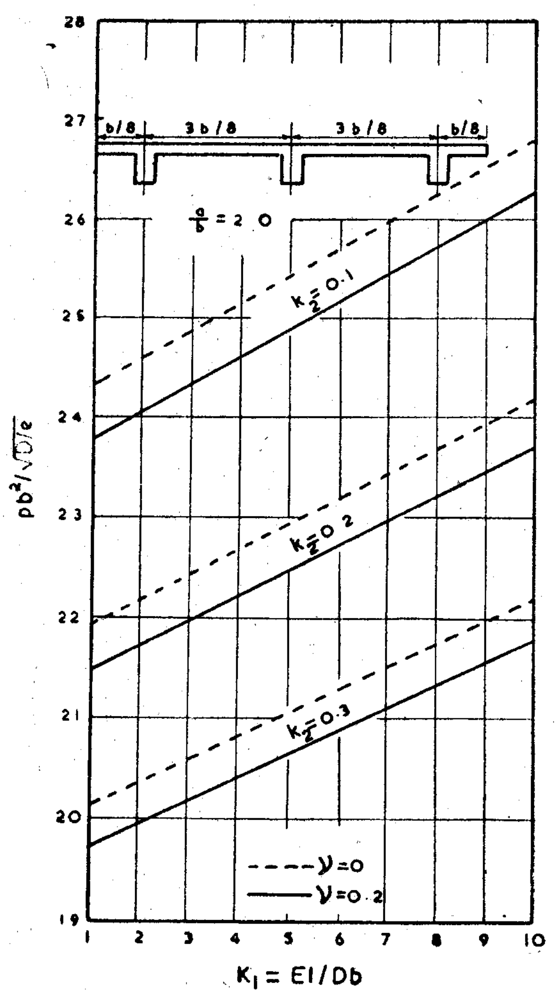

Fig. 5.

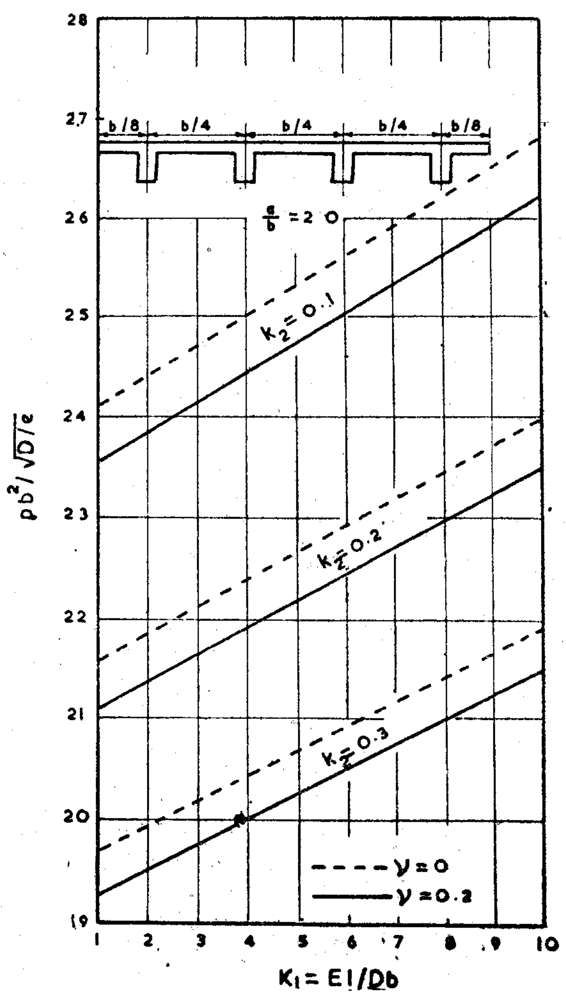

Fig. 6.

Values of $pb^2\sqrt{D/e}$ for a bridge with $a/b=2.0$ for the first symmetric mode.

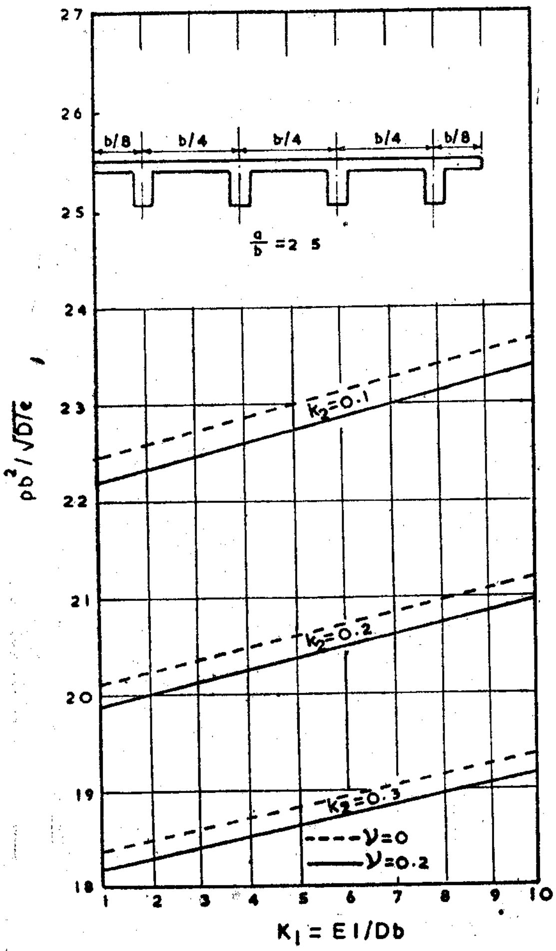

Fig. 7.

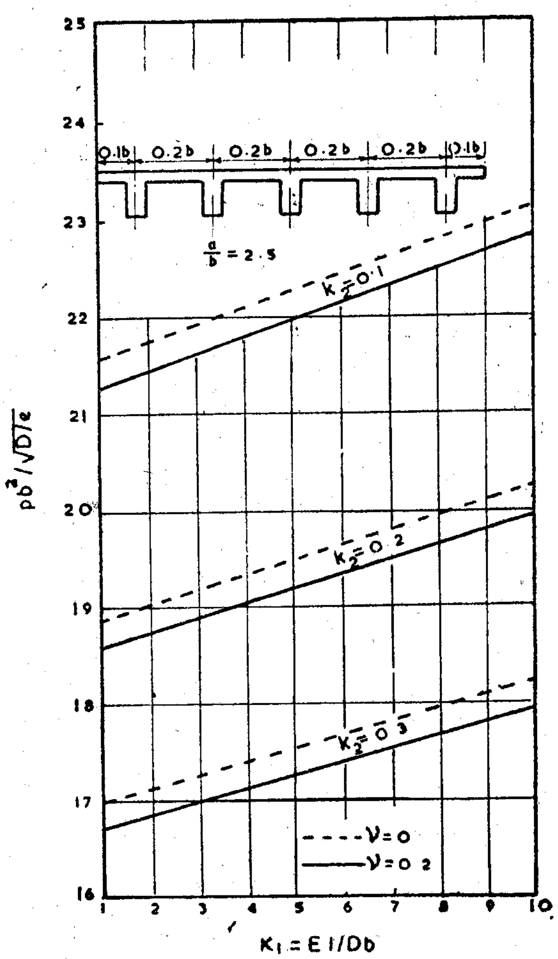

Fig. 8.

Values of $pb^2\sqrt{D/e}$ for a bridge with a/b=2.5 for the first symmetric mode.

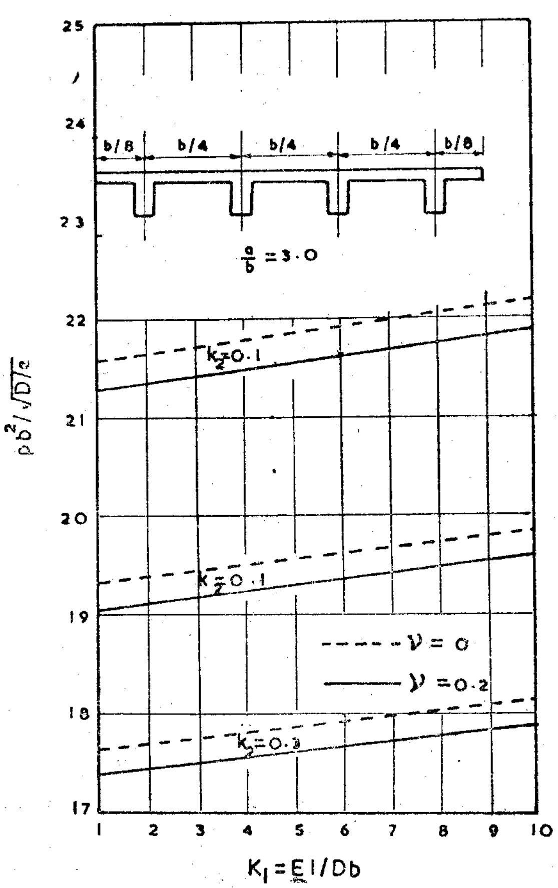

Fig. 9.

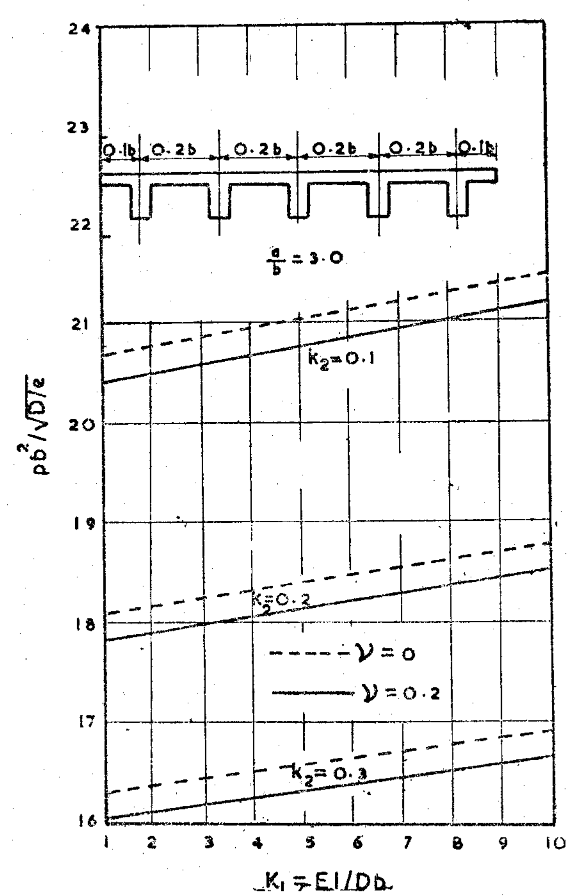

Fig. 10.

Values of $pb^2\sqrt{D/e}$ for a bridge with $a/b=3.0$ for the first symmetric mode.

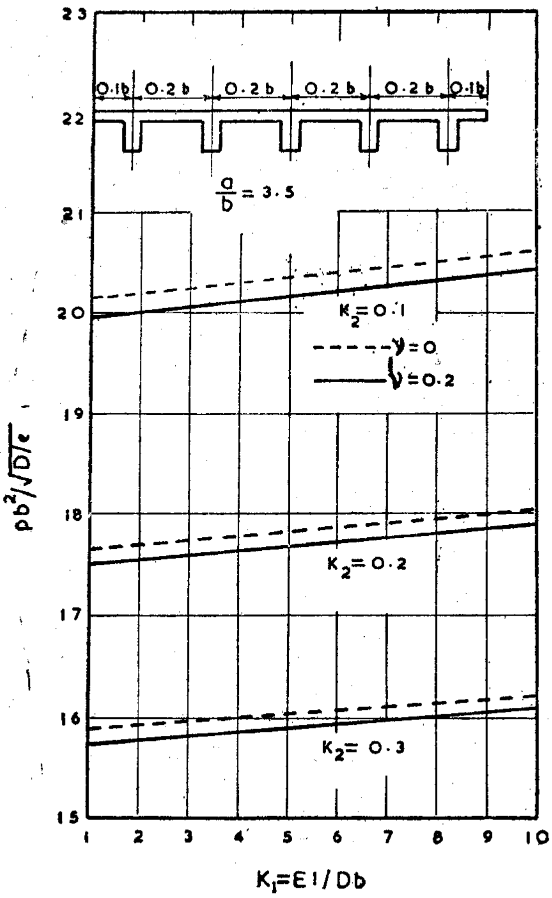

Fig. 11.

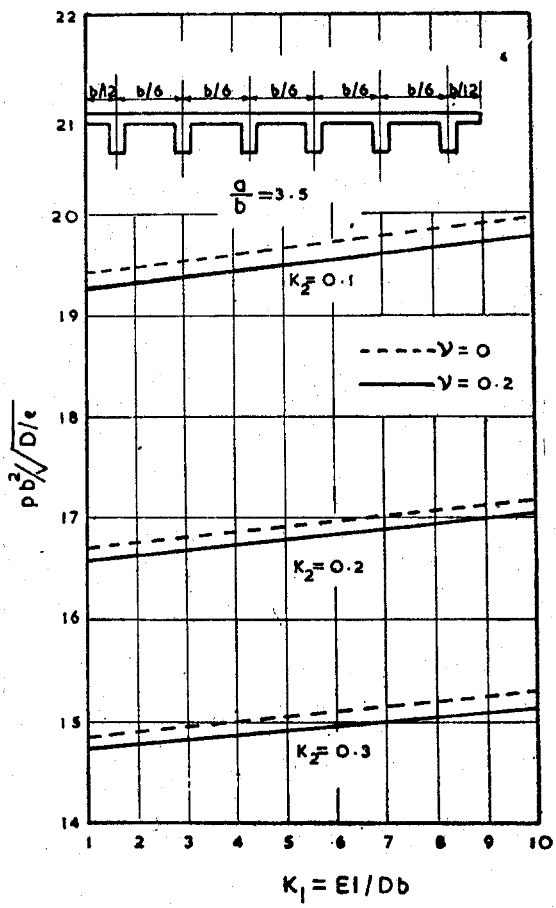

Fig. 12.

Values of $pb^2\sqrt{D/e}$ for a bridge with $a/b = 3.5$ for the first symmetric mode.

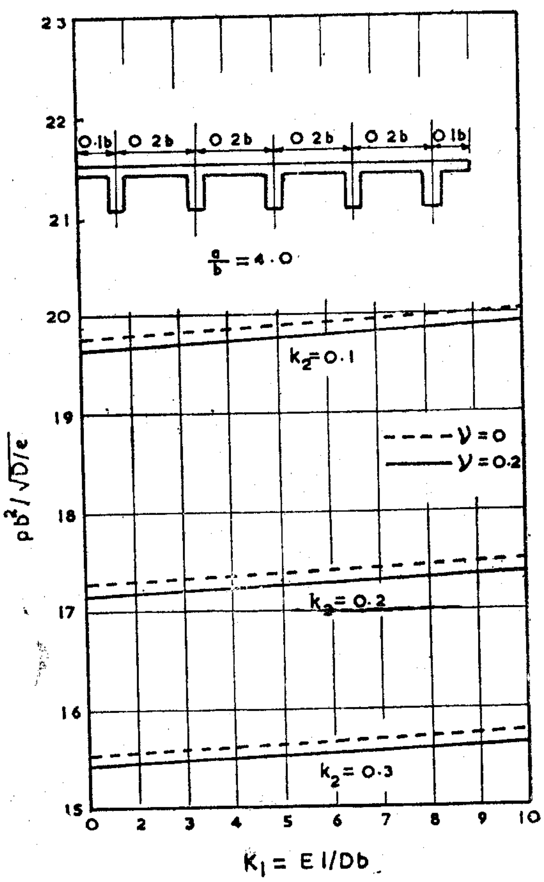

Fig. 13.

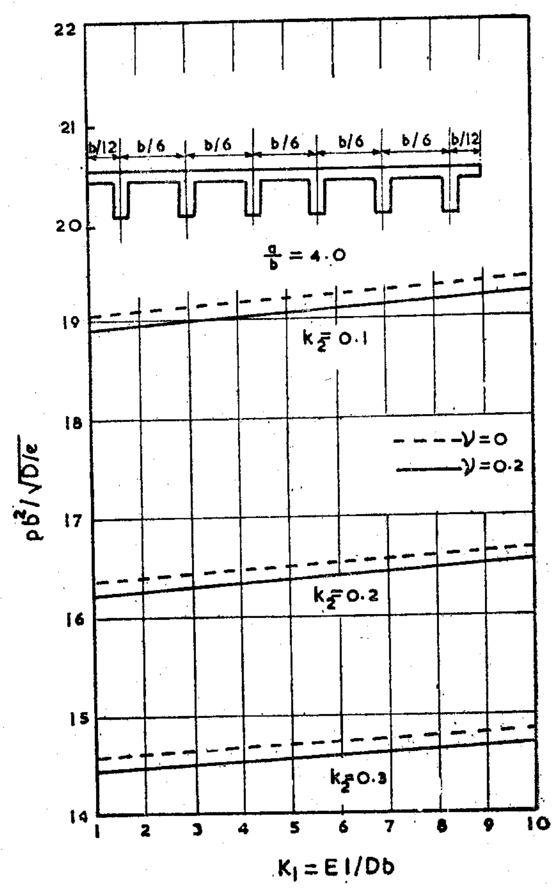

Fig. 14.

Values of $pb^2\sqrt{D/e}$ for a bridge with $a/b=4.0$ for the first symmetric mode.