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DEFLECTION OF FLEXIBLE RETAINING WALLS ON RIGID FOUNDATIONS DURING EARTHQUAKES

BY

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INTRODUCTION

In the field of Soil Mechanics and Soil Dynamics, quite a lot of literature have been made available by the researchers on theories of earth pressures, methods of calculating earth pressures over rigid and flexible retaing walls under static and dynamic conditions and for calculating the point of action of earth pressures over retaining walls.

Mononobe-okabe (1929) has proposed an analytical solution for dynamic earth pressures on rigid walls based on a modification of Coulomb's equation for static conditions. The point of action of the total earth pressure (static+dynamic) has been assumed to be at one third the height from the base. Prakash and Basavanna (1969) determined the dynamic earth pressure and its distribution using Pseudo-static analysis and indicated that the dynamic increment will act at about 2/3 H for the particular case considered. Ishii et al (1960) concluded from experimental data that the point of action of the total earth pressure (static+dynamic) is at 0.35 H to 0.4 H above the base, where H is the height of the wall. Prakash and Nandakumaran (1969) have constructed a steel model wall and determined the dynamic pressure increment and its point of action. It has been indicated that the ratio of dynamic pressure increment to the static pressure bears a linear relationship with the acceleration to which the wall is subjected and the point of action lies around 0.48 to 0.54 H with an average value of 0.51 H above the base.

Recently engineers have been facing the problem of calculating the deflection of flexible retaining walls, such as counterfort retaining walls, in order to limit the deflection and movement of such walls for safety for which no solution is readily available.

DEFLECTION OF FLEXIBLE RETAINING WALL

The most important consideration in the seismic design of retaining walls is the realisation of the dynamic nature of the problem. None of the present methods, however, have taken into account this aspect while giving the total dynamic earth pressure or its distribution. Here an attempt is made to consider the dynamic increment as acting permanently on the wall for computation of elastic displacements. This may be considered to represent the case of cantilever retaining walls on rigid foundations which means that the foundations do not permit any displacement to occur even during an earthquake and the elastic displacements resulting out of a momentary increase in pressure is not regained because of the backfill.

STATIC CONDITION

A cantilever retaining wall of height L top width b and base width a with a fixed base acted upon by earth pressure (Fig. 1a) can be treated as a cantilever beam with variable moment of intertia subjected to a defined inclined load (Fig. 1b) which can be resolved into a horizontal component and a vertical component responsible for the deflection at the end of cantilever walls. The vertical component p of force p' is given by $p=p' \cos \psi$ where p is the earth pressure at base under static condition and ψ is the angle defining the variation of earth pressure with height.

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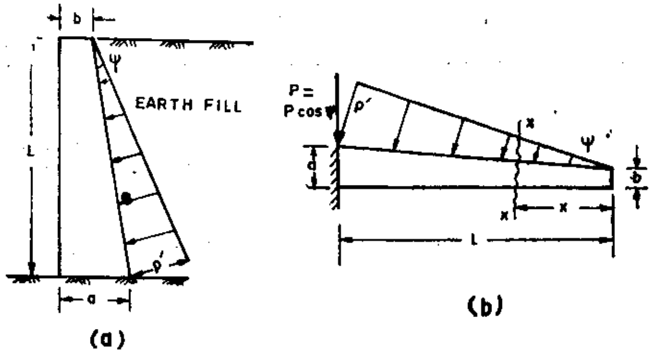

Fig. 1. Cantilever Retaining Wall-Cantilever Beam.

Considering any section at a distance x from the free end (Fig. 1b), moment of inertia I_x is given by

$$I_x = \frac{1}{12}(b+\theta.x)^3 \qquad (1)$$

where $\theta = \frac{a-b}{L}$

The vertical pressure intensity at distance $x = \frac{p_x}{L}$ (2)

The bending moment at section x, $M_x = \frac{p_x^3}{6L}$ (3)

The deflection at the free end is given by the differential equation

$$EI_x \frac{d^2y}{dx^2} = -M_x = \frac{p_x^3}{6L} \qquad (4)$$

in which y is the deflection at the free end.

Equation 4 can be rewritten as, $E\frac{d^2y}{dx^2} = \frac{2px^3}{Lb^3}(1+\lambda x)^{-3}$ (4a)

for the condition $a < 2b$.

wherein $\lambda = \theta/b$

Expanding the polynomial in equation 4a, using the Bionomial Theorem and integration on both sides yields,

$$E.y = \frac{2p}{Lb^3}\left(\frac{x^5}{20} - \frac{3\lambda x^6}{30} + \frac{\lambda^2 x^7}{7} - \frac{10\lambda^3 x^8}{56}\right) + c_1 x + c_2 \qquad (5)$$

in which c_1 and c_2 are the integration constants which can be solved for the boundary conditions :

when $x = L$, $y = O; \frac{dy}{dx} - O$ i.e. $c_1 = \frac{-2p}{b^3}\left(\frac{L^3}{4} - \frac{3\lambda L^4}{5} + \lambda^2 L^5 - \frac{10\lambda^3 L^6}{7}\right)$ (5a)

when $x = L$, $y = O$ because of fixity

$$i.e.\ c_2 = \frac{2p}{b^3}\left(\frac{L^4}{5} - \frac{\lambda L^5}{2} + \frac{6}{7}\lambda^2 L^6 - \frac{5}{4}\lambda^3 L^7\right) \qquad (5b)$$

Substitution of equations $5a$ and $5b$ in equation 5 gives

$$y = \frac{2p}{LEb^3}\left(\frac{x^5}{20} - \frac{\lambda x^6}{10} + \frac{\lambda^2 x^7}{7} - \frac{10\lambda^3 x^8}{56}\right) - \frac{2px}{ELb^3}\left(\frac{L^4}{4} - \frac{3\lambda L^5}{5} + \lambda^2 L^6 - \frac{10\lambda^3 L^7}{7}\right)$$
$$+ \frac{2p}{Eb^3}\left(\frac{L^4}{5} - \frac{\lambda L^5}{2} + \frac{6}{7}\lambda^2 L^6 - \frac{5}{4}\lambda^3 L^7\right) \qquad (5c)$$

y_0, the deflection at the free end due to load over full span on the cantilever beam, is obtained by taking $x=L$ and is given by

$$y_0=\frac{2p}{Eb^3}\left(\frac{L^4}{5}-\frac{\lambda L^5}{2}+\frac{6}{7}\ \lambda^2L^6-\frac{5}{4}\ \lambda^3L^7\right) \tag{6}$$

DYNAMIC CONDITION

The dynamic earth pressure increment can be considered as shown in Fig. 2 which agrees with the results of Prakash and Nanda Kumaran (1969). The pressure diagram can

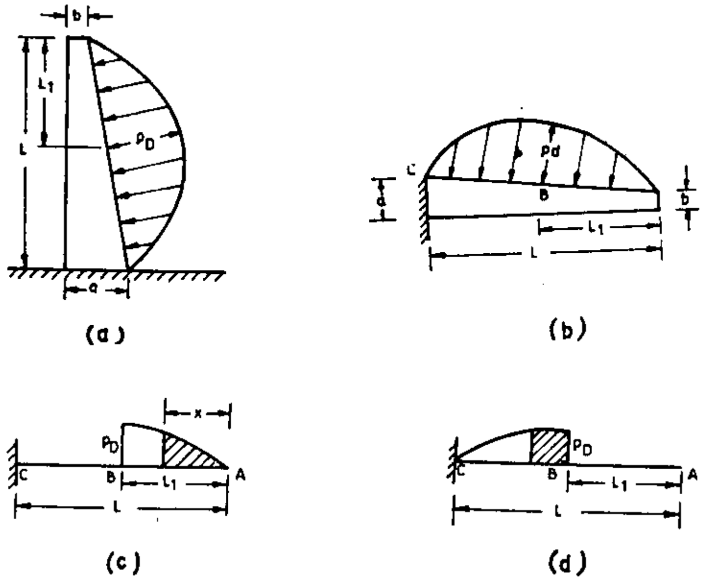

Fig. 2. Dynamic Earth Pressure Increment.

be considered as 2 parabolas of maximum ordinate p_D over spans $AB=L_1$ and $CB=(L-L_1)$. By taking A as origin the equation of parabola on AB can be expressed by

$$p_D=\alpha x^2+\beta_x+\gamma \tag{7}$$

where $\alpha=-\frac{p_D}{L_1^2}$ and $\beta=\frac{2p_D}{L_1^2}$ and $\gamma=O$ on span AB (7*a*)

Similarly, the equation of parabola on CB can be expressed, by taking A as origin, as

$$p_D=A^2y+B_x+C \tag{8}$$

where $A=-\frac{p_D}{(L-L_1)^2}$ and $\beta=\frac{2p_DL_1}{(L-L_1)^2}$ and $C=\frac{p_DL}{(L-L_1)^2}\ (L-_2L_1)$ (8*a*)

The deflection, δ, of the cantilever beam (Pig. 2b) at the free end can be considered as

$$\delta=\delta_1+\delta_2+\delta_2 \tag{9}$$

where δ_1 is the deflection at the free and due to load over AB (its moment over AB) and is given by

$$\delta_1={}_A\int^B \frac{M_x\, xdx}{EI_x} \tag{9a}$$

δ_2 is the deflection at the free and due to load over AB (its monent over BC) and is given by

$$\delta_2={}_B\int^C \frac{M_x\times d_x}{EI_x} \tag{9b}$$

δ_3 is the deflection at the free end due to load over BC and is given by

$$\delta_3 = \int_B^C \frac{M_x \times d_x}{EI_x} \tag{9c}$$

in which M_x is the bending moment at any section X measuring from the free end. For the load over AB and its moment over AB (Fig. 2c)

$$M_x = \left(\alpha \frac{x^3}{3} + \frac{\beta x^2}{2}\right)\left[x - \frac{\left(\frac{\alpha x^2}{4} + \frac{\beta x}{3}\right)}{\left(\frac{\alpha x}{3} + \frac{\beta}{2}\right)}\right] \tag{10a}$$

and $I_x = \frac{1}{12}(b + \theta x)^3$ as given in equation 1.

$$\text{hence } \delta_1 = \int_0^{L_1} \frac{M_x \times dx}{EI_x} = \frac{12}{Eb^3} \int^{L_1} x^4 \left(\frac{\alpha x}{12} + \frac{\beta}{6}\right)(1+\alpha_1 x)^{-3}\, dx \tag{10b}$$

which after expanding by Bionominal theorem and integration yields

$$\delta_1 = \frac{12}{Eb^3}\left[\frac{\beta L_1^5}{30} + \frac{\beta_1 L_1^6}{6} + \frac{\beta_2 L_1^7}{7} + \frac{\beta_3 L_1^8}{8} + \frac{\beta_4 L_1^9}{9}\right] \tag{10c}$$

$$\left.\begin{aligned} \text{wherein } \alpha_1 &= \frac{a-b}{Lb} \\ \beta_1 &= \frac{\alpha - 6\alpha_1\beta}{12} \\ \beta_2 &= \frac{4\alpha_1^2\beta - \alpha\alpha_1}{4} \\ \beta_3 &= \frac{3\alpha\alpha_1^2 - 10\beta\alpha}{6} \\ \text{and } \beta_4 &= \frac{-5\alpha\alpha_1^3}{6} \end{aligned}\right\}$$

For the load over AB and its moment over BC (Fig. 2C)

$$M_x = \left(\frac{\alpha L_1^3}{3} + \frac{\beta L_1^2}{2}\right)(x - X) \tag{11a}$$

in which X is the distance of centre of the load over AB from A and is given by

$$X = \left(\frac{3\alpha L_1^2 + 4\beta L_1}{4\alpha L_1 + 6\beta}\right) \tag{11d}$$

$$\begin{aligned} \text{hence } \delta_1 = \int_{L_1}^{L} \frac{M_x \times d\partial}{EI_x} = \frac{2}{Eb^3}\left(2\alpha L_1^3 + 3\beta L_1^2\right)\Big[&-\frac{x}{2}\left(L^2 - L_1^2\right) + \frac{\Phi_1}{3}\left(L^3 - L_1^3\right) \\ &+ \frac{\Phi_2}{4}\left(L^4 - L_1^4\right) + \frac{\Phi_3}{5}\left(L^5 - L^5\right) + \frac{\Phi_4}{6}\left(L^6 - L_1^6\right)\Big] \end{aligned} \tag{11c}$$

$$\left.\begin{aligned} \text{wherein, } \Phi_1 &= 1 + 3\alpha_1 X \\ \Phi_2 &= -3\alpha_1 - 6\alpha_1^2 X \\ \Phi_3 &= 6\alpha_1^2 x + 10\alpha_1^3 X \\ \text{and } \Phi_4 &= \frac{-10\alpha_1^3}{6} \end{aligned}\right\}$$

For the load over BC and its moment over BC (Fig. 2*d*) :

$$M_x = \left[\left(\frac{Ax^3}{3} + \frac{Bx^2}{2} + cx\right) - \left(\frac{AL_1^3}{3} + \frac{BL_1^2}{2} + CL_1\right)\right]\left(X - X_1\right) \tag{12a}$$

in which X_1 is the distance of the centre of load over BC from A and is given by

$$X_1=2\,\frac{(3A_x^4+4B_x^3+6C_x^2-3AL_1^4-4BL_1^3\times 6CL_1^2)}{(2A_x^3+3B_x^2+6C_x-2AL_1^3-3BL_1^2+6CL_1)} \qquad (12b)$$

hence

$$\delta_3=\int_{L_1}^{L}\frac{M_x dx}{EI_x}=\frac{12}{Eb^3}\Big[-\left(3AL_1^4+4BL_1^3+6CL_1^2\right)\left(\frac{L^2-L_1^2}{24}\right)+\frac{a_1}{3}\left(L^3_1-L^3_1\right)+\frac{a_2}{4}\left(L^4-L_1^4\right)+\frac{a_3}{5}\left(L^5-L_1^5\right)+\frac{a_4}{6}\left(L^6-L_1^6\right)+\frac{a_6}{8}\left(L^8-L_1^8\right)+\frac{a_7}{9}\left(L^9-L_1^9\right) \qquad (12c)$$

wherein

$$\left.\begin{aligned} a_1&=3\alpha_1 z_2=z_1\\ a_2&=\frac{c}{2}-3\alpha_1 z_1-6\alpha_1^2 z_2\\ a_3&=\frac{B}{6}-\frac{3\alpha_1 c}{2}+6\alpha_1^2 z_1+10\alpha_1^3 z_2\end{aligned}\right\}$$

$$\left.\begin{aligned} a_4&=\frac{A}{12}-\frac{\alpha_1 B}{2}+3\alpha_1^2 C-10\alpha_1^3 z_1\\ a_5&=B\alpha_1^2-\frac{A\alpha_1}{4}-5\alpha_1^2 C\\ a_6&=\frac{A\alpha_1^2}{2}-\frac{5B\alpha_1^3}{3}\\ \text{and}\quad a_7&=-\frac{5}{6}A\alpha_1^3\end{aligned}\right\} \qquad (12d)$$

The maximum deflection (δ_{max}) under dynamic gonditions is the sum of that under static condition and that due to the dynamic earth pressure increment

Thus

$$\delta_{max}=y_0+(\delta_1+\delta_2+\delta_3) \qquad (13)$$

From the experimental study on a 1M high steel model wall, Prakash and Nandakumaran (1969) indicated that the ratio of dynamic increment to the static pressure is 0 185 times the acceleration under shock loading to which the wall is subjected to. The steel flexible wall is of 1 cm thick whose $E=2.11\times10^6$ kgms/cm². The deflection of the steel (flexible) retaining model wall under different acceleration to which the wall has been subjected to (shock loading), have been measured and given in Table 1. These measured values have been compared with those calculated by using the equations developed herein this paper in Table 1.

Table 1 Deflections—Calculated and Measured

Details	Test 1	Test 2	Test 3
Acceleration Static earth pressure calculated $P=K_A.\gamma H$ (kg/cm²)	3·34 g 0·056 with $\Phi=40^0$ for	4·22 g 0·056 the sand used	4·55 g 0·056 as backfill
Measured dynamic earth pressure increment (g/cm)	1680	1961	2177
Point of action above base $L-L_1$ (cm)	50	55	48·5
Static deflection from eqn. 6 (cm) observed (cm)	1·06 0·84	1·06 0·85	1·06 0·85
Dynamic deflection from eqn. (13) (cm) observed (cms)	2·24 →about	2·36 2 00	2·34 →

Sixteen shocks have been given to the shaking table on which the wall is mounted. Each shock given to the shaking table is said to have caused larger acceleration response then the koyna shock of 1967. The displacement of the wall at top at the end of 16 shocks was more or less equal to the computed displacement by the proposed theory. Now, if the 16 shocks represent one earthquake, the theory may be considered sufficient at least for the present. If the damage potential of the shock is considered three table shocks may be considered to have the same damage potential as a modified Koyna shock. Thus it will be seen that the displacement as computed is larger than the actual displacement that will be caused by an earthquake as big as the Koyna shock and probably may be equal to the displacement resulting from 5 to 6 earth quakes. Thus a factory of safety of 5 to 6 regarding displacements is available.

CONCLUSIONS

Based on theory of deflection of simple beams, equation for deflection of a flexible retaining wall with a rigid base have been obtained under static conditions. Similar equations have been obtained for under dynamic conditions with the consideration of proper dynamic earth pressure increment and its point of action.

Comparison of calculated and observed deflections, of a lm high steel model wall, under static and dynamic conditious showed an excellent agreement, with the calculated values slightly on the safe side.

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APPENDIX—NOTATION

The following symbols are used in this paper :

A Algebraic constant
a Base width of the retaining wall
a_1, a_2, a_3. a_4, a_5, a_6, a_7, Functional parameters
B Algebraic constant
b top width of the retaining wall
C Algebraic constant
C_1, C_2 Constants of integration.
E Young's modulus of the wall material

H height of the retaing wall
I_x Moment of inertia at section xx
L height of retaining wall
L_1 Distance at which the nature of loading changes under dynamic conditions
M_x Bending moment at section xx
P Vertical component of the load (earth pressure) at the fixed end.
p′ Loading (earth pressure) at the fixed end.
p_D Maximum dynamic earth pressure increment
X, X_1 Distances of centre of load from the fig. and
x Distance of section xx from the free end
y Deflection
y_0 deflection at free and under static conditions
α, β, γ, algebraic constants
$\alpha, \beta_1, \gamma_2, \beta_3, \beta_4$ Functional parameters
$\delta_1, \delta_1, \delta_2, \delta_3$, Deflections
θ Dimensionless parameter
$\Phi_1, \Phi_2, \Phi_3, \Phi_4$ Functional parameters
ψ Angle defining the variation of earth pressure under static conditions
λ Constant.