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DYNAMIC RESPONSE OF MULTI STOREY FRAMES SUBJECTED TO EARTHQUAKE GROUND MOTION.

BY

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INTRODUCTION

The response of multi-degree systems to earthquake ground motion can be determined by modal analysis[1, 4, 8]. Computer programmes are prepared for generating the stiffness matrix for a frame and then reducing it to the lateral stiffness matrix. Finally an eigen value subroutine is called to obtain the normal modes of vibrations with the characteristic shapes.

Three R. C. C frames having three, eight and eleven storeys, shown in Figs. 1, 2 and 3 respectively are analysed by using actual spectrum inputs taken from the actual spectrum of Koyna earthquake average spectrum and assumed elasto-plastic spectrum. In the analysis, damping equal to 10% of critical damping is used.

ACTUAL SPECTRUM CURVE

The displacement, velocity and acceleration response spectrum curves for December 11, 1967, Koyna's earthquake motion are obtainad by Chandrasekaran and Saini [3]. The displacement spectrum curve for the transverse component (with respect to Koyna Dam) of ground motion is used for the analysis.

AVERAGE SPECTRUM CURVE

Housner[9] has averaged the normalised spectra for eight components of the four strongest ground motion in U. S. A. and obtained a smooth set of curves.

Housner has also suggested that the ordinates of these curves should be multiplied by a scale factor to bring them in agreement with the recorded ground motion. Housner has given an empirical relation to find out this scale factor. This relation depends on the value of spectrum intensity, which is the area under zero damping velocity spectrum curve between the limits 0·1 second and 2·5 second. The value of spectrum intensity for Koyna earthquake is taken from reference 4. The scale factor for Koyna's ground motion works out to 1·9[5].

ELASTO-PLASTIC SPECTRUM

Blume, Corning and Newmark[2] have suggested that a reasonable 'EQUIVALENT' design spectrum for an elasto-plastic system can be derived merely by considering the fact that the spectra for the elasto-plastic system have the same general-characteristics as spectra for elastic systems. Elasto-Plastic spectra are obtained by Veletsos and Newmark[10] for simple systems by considering the ductility factor μ. The ductility factor μ for a one degree system is defined as the ratio between the maximum displacement and yield displacement. Comparing the elasto-plastic spectra with the elastic spectra, Blume, Corning and Newmark have concluded that though their general characteristics are similar, the elasto-plastic spectrum curves appear to be displaced downward with respect to elastic spectrum curves at each frequency by an amount dependent on the ductility factor and is roughly equal to the ductility factor itself.

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A slightly different approach to the elasto-plastic system is also suggested by Velestos and Newmark[10]. It is based on the criteria that the energy absorbed in the elasto-plastic system is the same as in the elastic system. This energy criterion leads to a slightly different formulation that corresponds also to a shifting down of a spectrum by a ratio which, instead of μ as discussed above, is obtained by dividing the elastic spectrum by the factor $\sqrt{2\mu-1}$.

It has been also suggested that the effect of damping may be ignored in elasto-plastic spectrum. Hence one could obtain a design spectrum for the elasto-plastic system by dividing the ordinates of the undamped spectrum response curve for the elastic system at a such period by a factor μ or $\sqrt{2\mu-1}$ where μ is the ductility factor for which it is desired to design. For R. C. C. structures, the ductility factor may be taken as four[2], and the structures must be designed, detailed and constructed in such a manner that the ductility factor will be at least 4 upto the point of beginning of visible damage.

It may be emphasised here that response spectrum as obtained above for an elasto plastic system is essentially a *reduced elastic spectrum* which is approximately equivalent to the elasto-plastic spectrum.

ANALYSIS OF MULTI-STOREY FRAMES

Multi storey frames (in Figs. 1, 2 and 3) are analysed for the storey shears by using the approximate elasto-plastic spectrum. Three cases are considered as below :

CASE–I

The storey shears are computed by using the first mode of vibration only and by using the reduction factor equal to μ for converting the elastic spectrum into an equivalent elasto-plastic spectrum.

CASE–II

The storey shears are computed by using the first mode of vibration only and by using the reduction factor equal to $\sqrt{2\mu-1}$.

CASE–III

The storey shears are again computed by using the modal super-position technique taking the first three modes of vibration and by using the reduction factor equal to μ.

For the three building frames shown in Figs. 1, 2 & 3 storey shears are also computed by using actual and average spectra. Each frame is analysed once assuming it as a shear structure and again as a flexible structure, which allows for joint rotations.

While using modal superposition technique, only first three modes are considered. The concept that the probable maximum response is equal to the square-root of the sums of the squares of the modal maximums[9], is used to obtain probable maximum by storey shears.

APPLICATION OF BUILDING CODE RECOMMENDATIONS

In design offices the storey shears for multi-storey frames are generally obtained using Code Recommendations.

Here, Uniform Building Code (UBC) Recommendations of U. S. A,[10] and I. S. I. Recommendations, (Revision of I. S. 1893—1966) are used to compute the storey shears.

UBC RECOMMENDATIONS

The UBC recommends that the maximum total lateral seismic force V, assumed to act non-concurrently in the direction of each of the main axes of the building, be determined by

$$V = ZCKW \qquad ...(1)$$

where Z=Zone factor which depends on expected severity of earthquake in various regions of the United States

K=Coefficient depending upon type of structure and varies from 0·67 to 3·0.

W=Total dead load.

The coefficient C is given by

$$C = \frac{0 \cdot 05}{(T)^{1/3}} \qquad ...(2)$$

where T is the fundamental period of vibration of the building in seconds in the direction considered. The lateral force V is to be distributed over the height of the structure in the following manner. A force F_t, at the uppermost level n is given by

$$F_t = 0 \cdot 04 \left(\frac{h_n}{D_s}\right)^2 \qquad (3)$$

A force F_x at each level x, including the uppermost level n is given by

$$F_x = (V - F_t) \frac{W_x h_x}{\sum_{i=1}^{n} W_i h_i} \qquad ...(4)$$

where h_n=height above base of uppermost level in main portion of structure, in feet.

D_s=Plan dimension of a vertical lateral force resisting system in feet.

h_x, h_i=Height of level x, i above the base in feet.

W_i, W_x=Portion of load W located at or assigned to level x, i. The force Ft need not exceed 0·15 V and may be taken zero if

$$\frac{h_n}{D_s} \leqslant 3.$$

The value of C as found out from eq. (2) need not exceed 0·10. UBC gives for different types of structures, the corresponding values of K. For the type of building frames under consideration, the value of K in eq. (1) may be taken equal to 1. While applying the UBC recommendations the value Z in eq. (1) is taken equal to 0·4. The storey shears obtained are multiplied by 1·5 to get the design storey shears.

I. S. I. CODE RECOMMENDATIONS

The I. S. I., Code states that the base shear V_B is given by the formula

$$V_B = C \alpha_h W \qquad ...(5)$$

where

$$C = \frac{0 \cdot 5}{(T)^{1/3}} \qquad ...(6)$$

α_h=Seismic coefficient which depends upon the regional location and type of the soil in which the structure is founded.

W=Total dead load+appropriate live load for stairs, balconies and garages.

T=Fundamental time period of the building in seconds.

Distribution of forces along the height of the building is given by the following relation :

$$Q_i = V_B \frac{W_i h_i^2}{\sum_{i=2}^{i=n} W_i h_i^2} \qquad ...(7)$$

where Q_i = lateral force at roof or floor i,

W_i = the weight considered to be acting at the level of the roof or floor i.

h_i = height of the roof or floor i above base of building

n = number of storeys including the basement floors.

The value of C as found out from eq. (6) should not exceed 1·00 for framed building. The minimum value of C will be limited to 0·33. The storey shears obtained are multiplied by 1·5 to get the design values.

Since the frames which are considered here, are located in Bombay, and assuming hard soil for the foundation, the value of α_h, in eq. (5) is taken equal to 0·04 based on the Code Recommendations. The value of Z in eq. (1) is taken equal to 0·4 to make proper comparison between UBC and ISI values.

For the frames, the storey shears computed by using actual, average and elasto-plastic spectra and UBC and ISI values are shown in Tables I, II and III.

DISCUSSIONS AND CONCLUSIONS

Comparison of values of storey shears obtained with different spectrum inputs for the frames, assumed as shear structures, suggests that the values for actual spectrum inputs are the largest while those obtained by using UBC Recommendations are the lowest. However, the variation of the storey shears from top to the base is approximately similar for different spectrum inputs. Approximate ratios of storey shears in terms of UBC values are given in

TABLE IV

APPROXIMATE RATIO OF STOREY SHEARS

$$\left\{\frac{\text{Spectrum input values}}{\text{UBC Values.}}\right\}$$

Type of Frames	Actual Spectrum	Average Spectrum	Elasto-Plastic Spectrum (Case III)	ISI Recommendations	UBC Recommendations
11 Storey	7·00	4·50	4·00	1·20	1·00
8 Storey	6·50	3·50	2·75	1·20	1·00
3 Storey	7·50	4·50	5·00	1·10	1·00

From Tables I to III it is observed that UBC and ISI recommendations give approximately same values for storey shears for the three frames. However, the values obtained by using UBC are slightly smaller. This may be due to the difference in the nature of force distribution as indicated by eq. (4) and eq. (7).

Results tabulated in Table IV above indicate that the indirect spectrum inputs such as the "average spectrum" and the "Elasto-plastic Spectrum" as also the UBC and ISI Recommendations are reasonable representations of dynamic response. The storey shears for Actual and Average spectra are very high as compared to other values since the analysis using these spectra are essentially elastic. These response spectra represent a rather strong earthquake and a structure should be expected to undergo considerable plastic distortion under such seismic loading conditions. The values for actual spectrum are consistently larger than those due to average spectrum. This may be due to the fact that the values from average spectrum depend upon the scale factor which itself depends upon the spectrum intensity. It has been pointed out[3] that though the maximum ground acceleration recorded at the time of Koyna earthquake is of the order of 0·63 *g* (highest ever) the spectrum intensity computed for the ground motion is comparatively less. The larger values of storey shears obtained from the actual spectrum may be also due to the local peaks in the spectrum curve.

It may be pointed out here that the draft Indian Standard criteria for earthquake resistant design of structure (Revision of I. S. 1893—1966) recommends the use of average spectrum as given by Housner with a scale factor which is designated as F. This factor is shown to vary, depending upon the type of zone in which a structure is built. The zones are classified on the basis of magnitudes, direction and form of the expected earthquake, soil conditions etc. The factor F for the Bombay City is given as 1/2, while the value found out by using spectrum intensity as suggested by Housner is 1·9. It has been pointed out in the Draft that the structures will be subjected to a bigger force during the expected maximum intensity of earthquake and the capacity of the structure in plastic range will be available for resisting such forces. If use of I. S. draft specifications is made, ordinates in the average spectrum curve will be multiplied by 0·5 instead of 1·9. The ratio of the scale factor obtained as suggested by Housner to the Factor F, as given in the Draft Specifications is roughly equal to 4. It may be noted that Housner's scale factor is for a purely elastic response, while the Draft takes into account energy absorption capacity in the inelastic range. This above ratio of the scale factor to factor F is similar to the ductility factor μ for the elasto-plastic system. It may be noted that the ductility factor μ is taken equal to 4 by the authors for finding out the elasto-plastic response.

Though the elasto-plastic analysis made here is not intended to represent truly inelastic response, the smaller values of storey shears obtained indicates that the inelastic analysis of the frames is not only realistic but also provides rational values for design storey shears.

The storey shears for the frames assumed as flexible structures are less compared to the values for the frames assumed as shear structures. These results are quite reasonable since the effect of joint rotations is to make the structure flexible and thus increase the natural period of vibration. It should be noted that the frame will undergo longer lateral displacement but will be subjected to small lateral forces.

The response spectrum curves used for the analysis are based on the accelerogram traces recorded at the Koyna Dam. The frames analysed are parts of buildings designed to be constructed in Bombay. Since the epicentre (near Koyna Dam) of the earthquake is at a large distance from the structures analysed, the attenuation of the ground acceleration should be taken into account. However, since no accelerogram traces for Bombay are available, no spectrum curves can be obtained. It is suggested that rational information regarding the effects of earthquakes on structures away from the epicentre should be collected by installing a number of recording devices at various stations especially in large cities. The analysis based on such data when available will be realistic.

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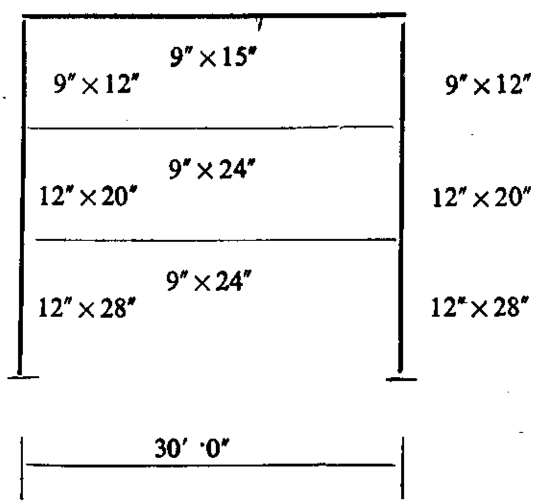


Fig. 1. Three Storey Frame
Height of bottom floor 15 feet. Other floors 12 feet.

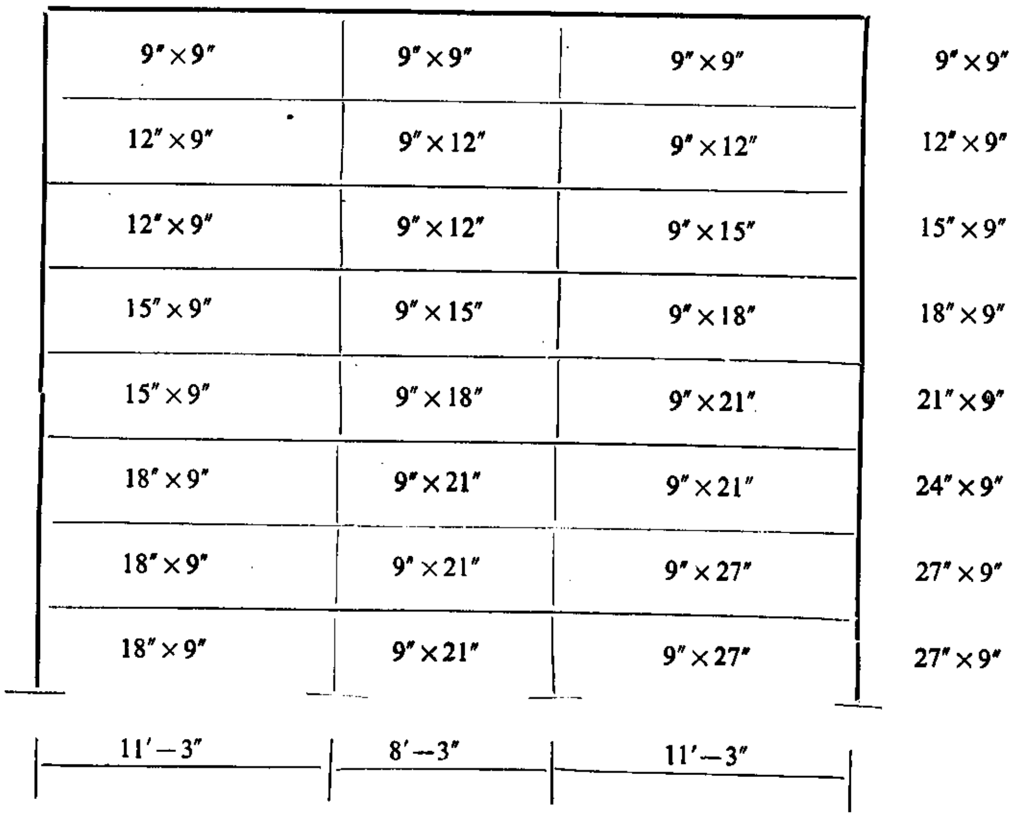
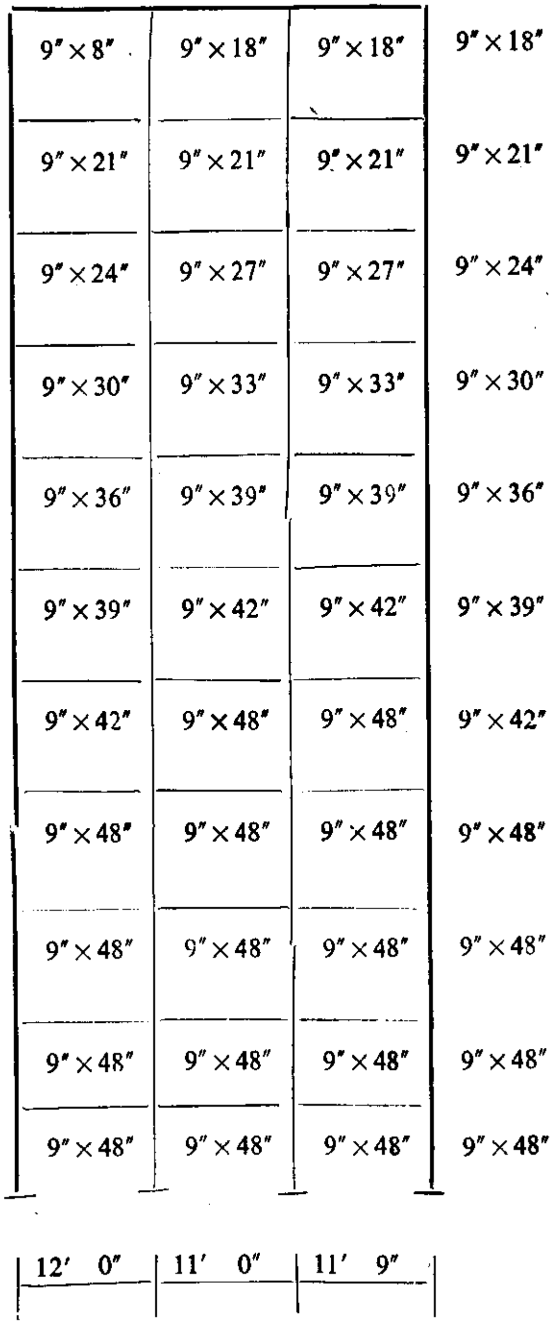


Fig. 2. EIGHT STOREY FRAME

(1) All Floors are of 10' height ; Beams 9"×21" size.

(2) For all beams and cols., value of Modulus of Elasticity E is taken equal to 2000 KIPS²/IN².

FLOOR	LUMPED MASS M.
1	0·2207 KIPS. SEC²/IN.
2	0·2207 ,, ,,
3	0·2195 ,, ,,
4	0·2164 ,, ,,
5	0·2143 ,, ,,
6	0·2115 ,, ,,
7	0·2101 ,, ,,
8	0·1796 ,, ,,



FLOOR	LUMPED MASSM.
1	0·222 KIPS. Sec²/In.
2	0·222 ,,
3	0·222 ,
4	0·222 ,,
5	0·219 ,,
6	0·215 ,,
7	0·2125 ,,
8	0·207 ,,
9	0·201 ,,
10	0·197 ,,
11	0 1485 ,,

COLUMNS	VALUE OF E KIPS/IN.²
GROUND FLOOR	2700
FIRST FLOOR	2300
SECOND FLOOR	2300
REMAINING FLOORS	2000

Fig. 3. ELEVEN STOREY FRAME

(1) All Floors are of 10′ Height ; Beams 9″×24″ size.

(2) For all Beams Value of Modulus of Elasticity E is taken equal to 2000 KIPS/IN.².

TABLE I

STOREY SHEARS FOR THREE STOREY FRAME

(All the values are in Kips)

STOREY	SHEAR STRUCTURE							FLEXIBLE STRUCTURE						
	SPECTRUM INPUT							SPECTRUM INPUT						
	Actual	Average	Elasto–Plastic			UBC	ISI	Actual	Average	Elasto–Plastic			UBC	ISI
			Case I	Case II	Case III					Case I	Case II	Case III		
1	57	34	23	35	40	7·6	7·5	36	22	12	19	16	6·4	6·7
2	43	27	20	30	28	6·0	6·9	29	16	10	15	13	5·2	6·0
3	29	18	15	23	17	3·2	4.2	15	9	4·7	7·1	6·2	2·6	3·6

TABLE II

STOREY SHEARS FOR EIGHT STOREY FRAME

(All the values are in Kips)

STOREY	SHEAR STRUCTURE							FLEXIBLE STRUCTURE						
	SPECTRUM INPUT							SPECTRUM INPUT						
	Actual	Average	Elasto-Plastic			UBC	ISI	Actual	Average	Elasto-Plastic			UBC	ISI
			Case I	Case II	Case III					Case I	Case II	Case III		
1	115	65	40	60	50	18	18	94	50	38	57	50	17	16
2	110	62	39	59	48	18	18	89	47	36	55	47	16	16
3	103	57	38	58	42	16	18	82	44	35	53	44	15	16
4	94	51	35	53	37	15	17	71	38	31	48	37	14	15
5	83	45	32	49	34	13	15	60	33	27	40	29	12	14
6	71	39	27	41	30	10	13	49	27	20	30	21	9·6	12
7	54	31	19	29	23	7·2	9.5	43	20	13	19	13	6·8	7·9
8	35	20	10	15	18	3·6	5·0	26	13	3·8	5·7	8·2	3·2	4·6

TABLE III

STOREY SHEARS FOR ELEVEN STOREY FRAME

(All the values are in Kips)

STOREY	SHEAR STRUCTURE								FLEXIBLE STRUCTURE							
	SPECTRUM INPUT								SPECTRUM INPUT							
	Actual	Average	Elasto–Plastic			UBC	ISI		Actual	Average	Elasto–Plastic			UBC	ISI	
			Case I	Case II	Case III						Case I	Case II	Case III			
1	259	156	115	174	154	36	36		131	68	45	68	57	26	26	
2	254	154	114	172	150	36	36		152	79	54	81	64	26	26	
3	242	149	111	168	140	34	36		151	79	54	82	61	25	26	
4	226	141	108	163	128	32	35		136	73	47	71	56	24	25	
5	206	132	103	156	115	30	34		133	71	46	69	55	22	25	
6	186	121	99	150	106	27	33		114	61	41	62	44	20	23	
7	167	109	88	133	95	24	29		118	63	41	62	47	17	21	
8	150	96	77	117	87	19	25		104	58	36	55	47	13	18	
9	129	82	64	97	76	15	20		89	48	29	44	39	11	15	
10	103	62	46	70	58	10	15		61	33	18	27	31	7·2	10	
11	66	34	22	33	46	4·4	6·7		30	16	7·8	12	18	3·6	4·9	