Paper No. 85. Bulletin of the Indian Society of Earthquake Technology, Vol. VII, No. 1, pp. 47-53, March 1970

EARTHQUAKE RESPONSE OF FRICTION MOUNTED MASS

A. R. Chandrasekaran*

INTRODUCTION

The problem of dynamic behaviour of masses which are only friction mounted has received very little attention so far, compared to the problem in which masses are spring and dashpot mounted. Such studies are useful to understand the behaviour of (a) live loads which are usually friction mounted, (b) sliding bearings in bridges, (c) a portion of a structure which is cracked and resting through friction on another part.

In this paper, the earthquake response of objects, which rest on ground through friction, are investigated. If the object is not firmly tied to the ground, it will have a motion relative to the ground causing it to slide, rock or overturn. So far, the motion of such objects have been analysed on an equivalent static basis. Such analysis have also been the basis of estimation of probable ground acceleration contours. However, dynamic analysis indicates that generally such approximations give erroneous results.

For this study, the response of a mass connected to ground through suitable dashpot and having a single degree of freedom has been worked out. One type of dashpot is viscous, in which the damping force is a linear function of the velocity. The other is coulomb type in which the damping force is independent of the magnitude of velocity but dependent on its phase. These two dashpots represent two extreme cases.

Relative displacement will always take place irrespective of the value of friction. Displacement decreases with increase in friction in case of viscous damping whereas in case of coulomb damping the displacement pattern is irregular. For the four earthquakes analysed, the displacement is minimum corresponding to a coulumb friction factor of the order of 0.2 to 0.3.

EQUATION OF MOTION AND CHOICE OF PARAMETERS

Equation of motion of friction mounted mass, as shown in figure 1, is given by

$$m\dot{v} + \phi(v) = -m\ddot{y} \qquad (1)$$

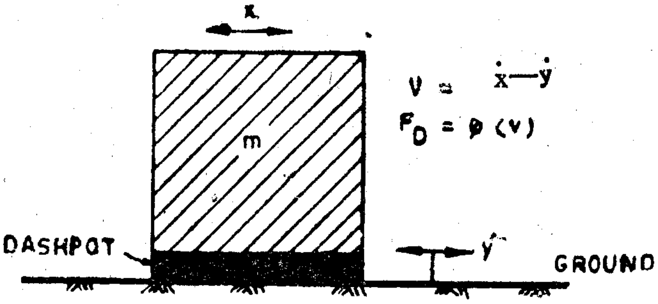
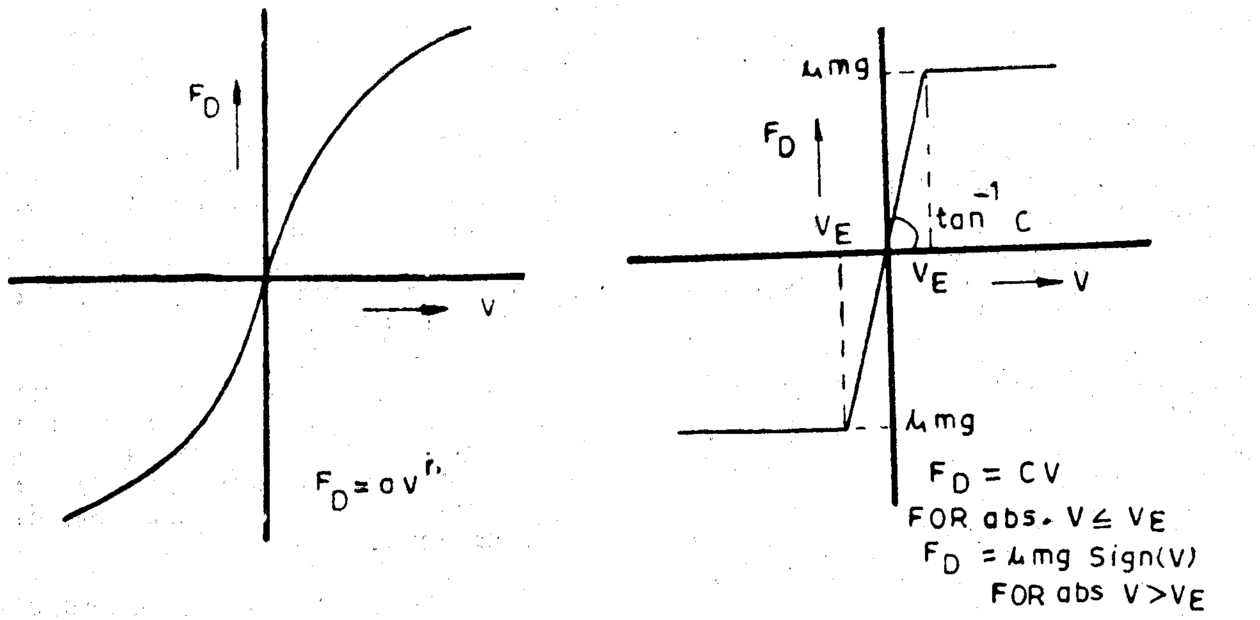


Fig. 1. Dashpot Mounted Mass

The functional relationship between frictional force and relative velocity would, in general, be non-linear as shown in figure 2a. However, it is the practice to assume the function to be one of two extreme types, namely either as linear (figure 2c), which is known as viscous type or as rigid-plastic (figure 2d), which is known as coulomb type. Both viscous and coulomb type can be obtained as special cases of the elasto-plastic type relationship shown in figure 2b.

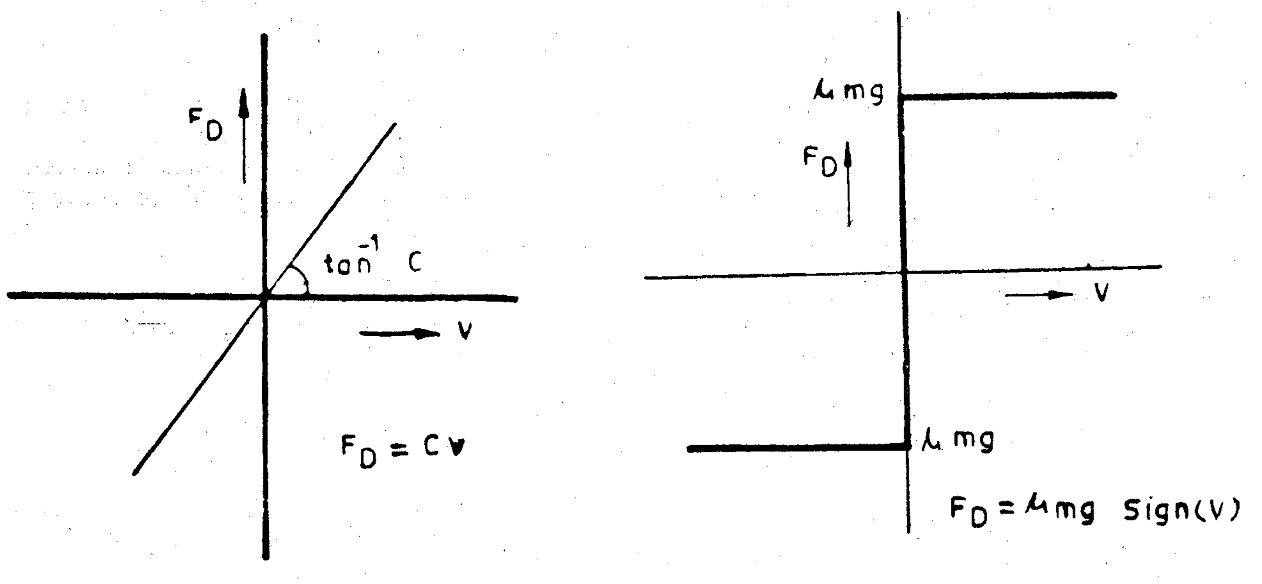
---

* *Member* Professor of Structural Dynamics, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee, U. P.



a_ NON-LINEAR DAMPING

b _ ELASTO PLASTIC TYPE DAMPING



c_VISCOUS DAMPING

d_ COULOMB DAMPING

Fig. 2. Damping Force as A Function of Relative Velocity

Equation 1 has been solved numerically using a step-by-step integration procedure(3). The ground motion has been assumed to be composed of a series of trapezoidal acceleration pulses and the duration of the pulse was kept equal to or lower than 0.005 second in order to improve the accuracy of the result.

If the damping is of viscous type, equation of motion is given by

$$\dot{v} + \frac{c}{m} v = -\ddot{y} \tag{2}$$

In the case of a system having spring mounting, c/m is equal to $4\pi\,\zeta/T$ where ζ is the percentage of critical damping and T is the period of the structure. A value of c/m equal to 4π would therefore correspond to the same frictional force per unit mass as of a single degree freedom system having ζ/T equal to 1.0. c/m was varied between π to 16π, that is, cor responding to to a system having ζ/T value between 0.25 to 0.4. This choice of parameter for viscous damping is such that it gives the same order of response as that for the coulomb damping parameters chosen.

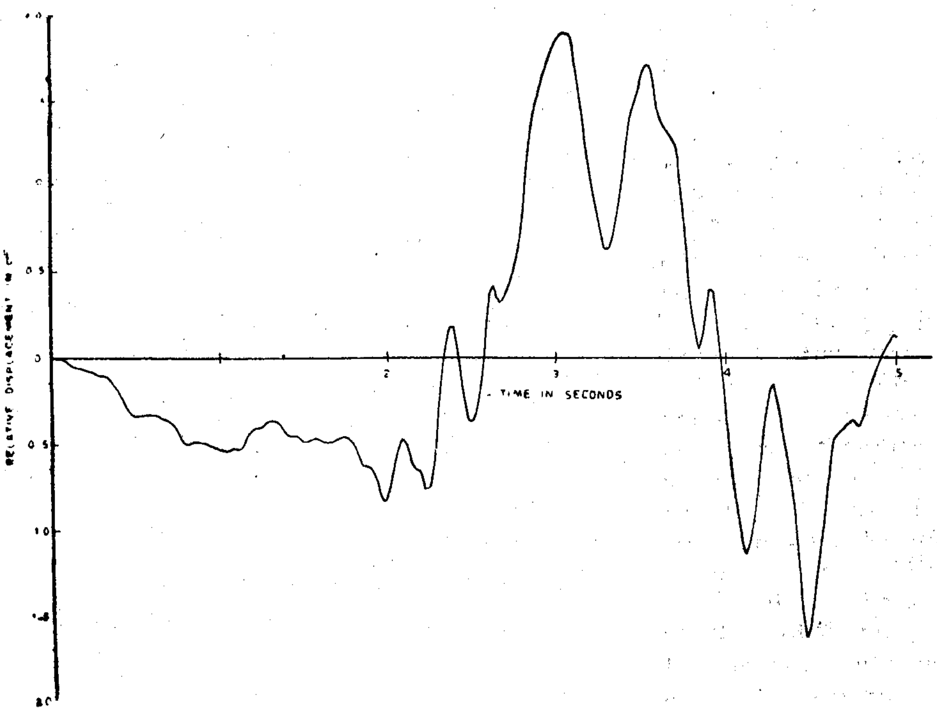


Fig. 3. Displacement Response of A Viscously Damped Mass. c/m=4 π, EQ=111, Koyna Long.

If the damping is of coulomb type, the equation of motion is given by

$$\dot{v} + \mu\, g\, \{\, \text{sign}\,(v)\,\} = -\,\ddot{y} \tag{3}$$

where μ is the coefficient of dry friction representing the ratio of frictional force to the weight of the structure and g is the acceleration due to gravity. μ had values varying between 0.1 to 1.0 which covers a wide range of values.

Four recorded strong motion accelerograms have been used in this study and their digitized versions are used as ground motion data. The four records are, namely, (a) El. Centro, Dec. 30, 1934, N. S. component (EQ--101). (b) El Centro, May 18, 1940, N.S. component (EQ—102), (c) Koyna, Dec. 11, 1967, Longitudinal component (EQ—111) and (d) Koyna, Dec. 11, 1967, Transverse component (EQ—112). The accelerogram for the above cases are given in references 1 and 2.

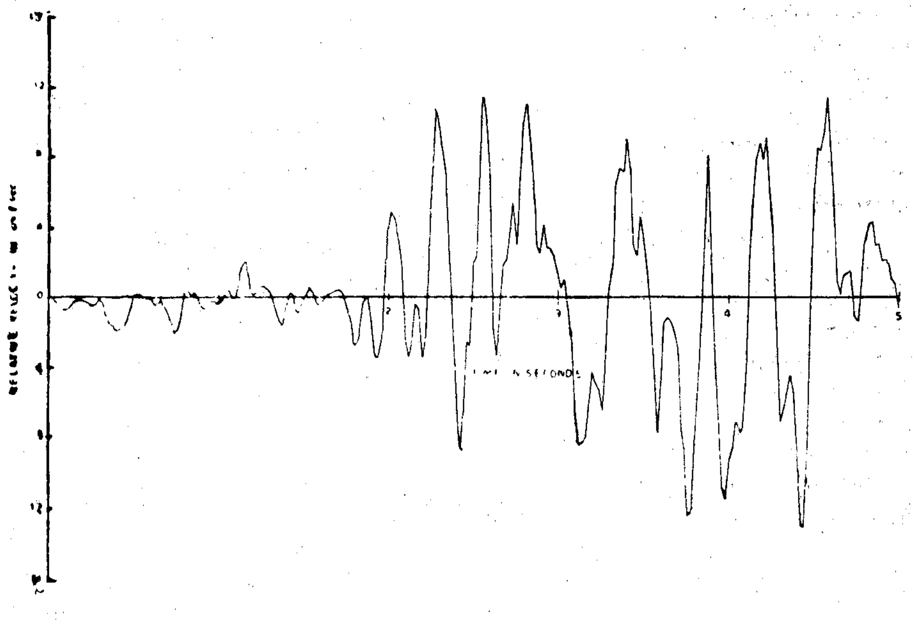


Fig 4 Velocity Response of A Viscously Damped Mass. c/m=4 π, EQ—111: Koyna Long.

DISCUSSION OF RESULTS

Figures 3 and 4 show respectively the displacement and velocity response of a viscous damped system as a function of time and figures 8 and 9 of a coulomb damped system. It is seen that higher frequency components are exhibited in the velocity response of coulomb damped system compared to viscous damped systems. It is, therefore, likely that coulomb damped systems are less prone to overtoppling as the velocity changes sign rapidly.

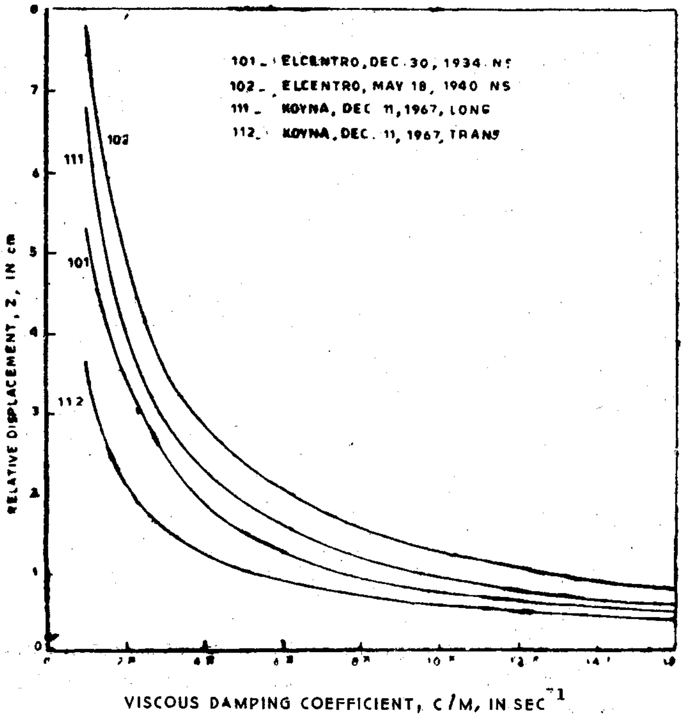


Fig. 5. Relative Displacement vs Viscous Damping Coefficient

Figures 5,6 and 7 show respectively maximum relative displacement, maximum relative velocity and maximum absolute acceleration as a function of viscous damping parameter c/m. It is seen that displacement and velocity decrease and acceleration increase with an increase in damping. Displacement decreases more rapidly compared to velocity. All the curves tend to be flat at higher values of damping.

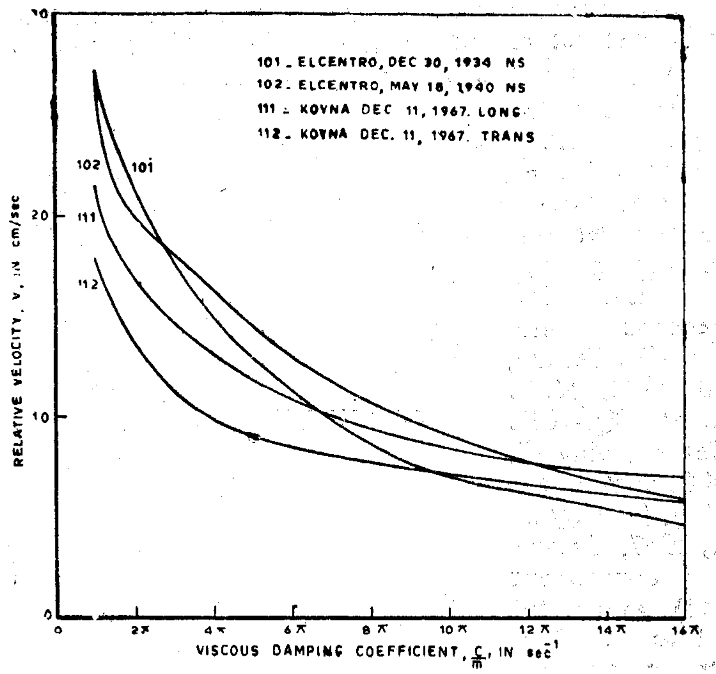


Fig. 6. Relative Velocity vs Viscous Damping Coefficient

Figure 5 reveals that El. Centro shock (EQ–102) is more intense than that of Koyna (EQ–111) in the sense that displacements are larger. Such a conclusion was also obtained based on spectrum intensity criterion[2]. Since El Centro shock had a peak acceleration less than Koyna shock, it can be concluded that response is not a function only of peak ground acceleration.

Figures 10 and 11 show respectively maximum displacement and velocity response of a coulomb damped system. There is no regular pattern of variation of response as a function of friction factor μ. The response is also not a well defined function of the peak ground acceleration as can be seen from the graphs for various ground motion. The least value of maximum relative motion is obtained for μ of the order 0.2 to 0.3. In the equivalent static approach of estimation of response, it is assumed that motion occurs when the ground acceleration exceeds μg. This analysis indicates that

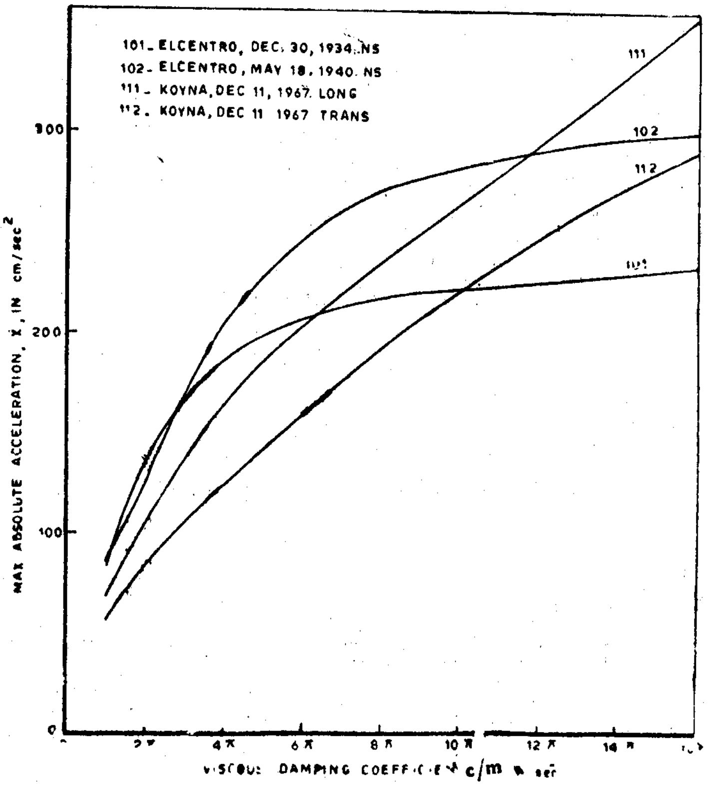


Fig. 7. Absolute Acceleration vs Viscous Damping Coefficient

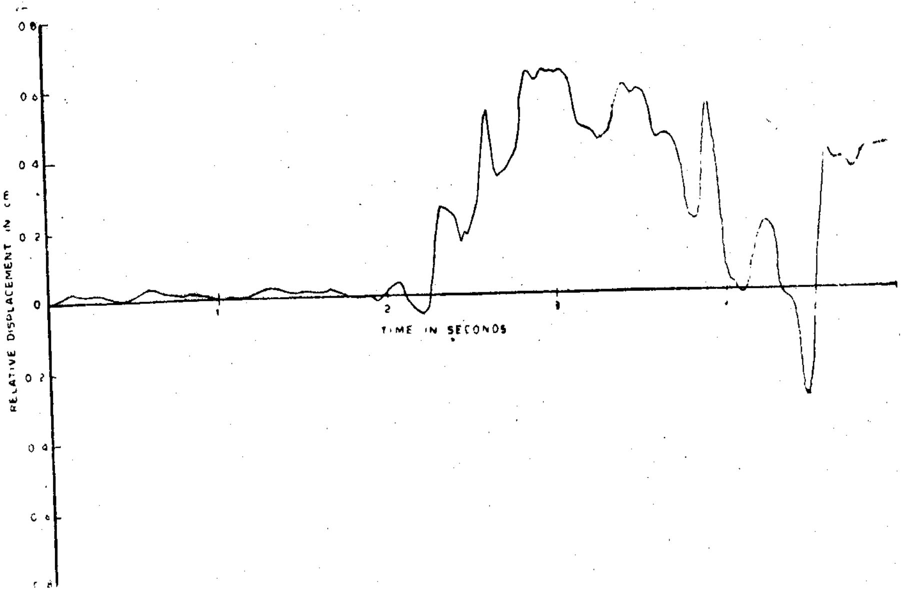


Fig. 8. Displacement Response of A Coulomb Friction Mounted Mass. μ=0.2, EQ–111, Koyna Long.

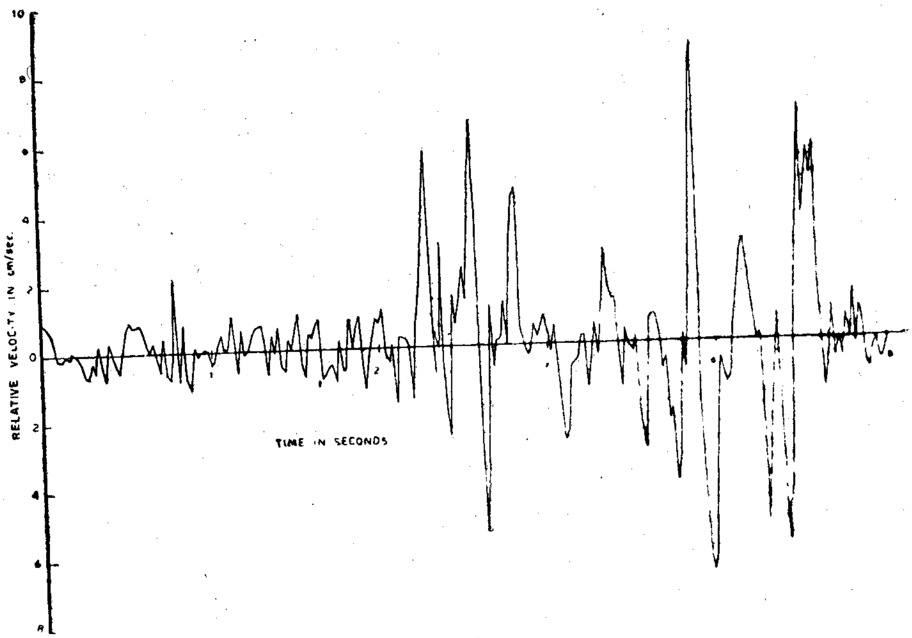


Fig. 9. Velocity Response of A Coulomb Friction Mounted Mass $\mu=0.2$, EQ-111, Koyna Long.

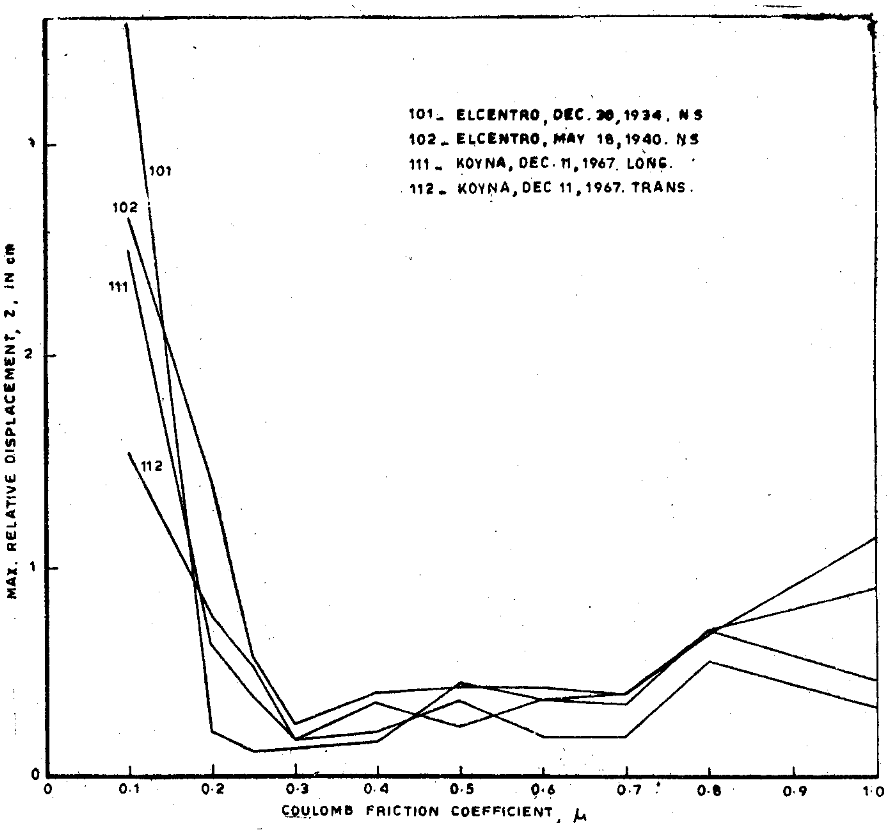


Fig. 10. Relative Displacement vs Coulomb Friction Coefficient

motion would occur irrespective of the value of friction and the motion cannot be estimated by the simplified static approach. Same results as are obtained for the various problems involving coulomb damping (figure 2 b) can also be had by solving the elasto-plastic type damping case (figure 2 b) in which it is assumed that c/m is greater than 1×10^{10}.

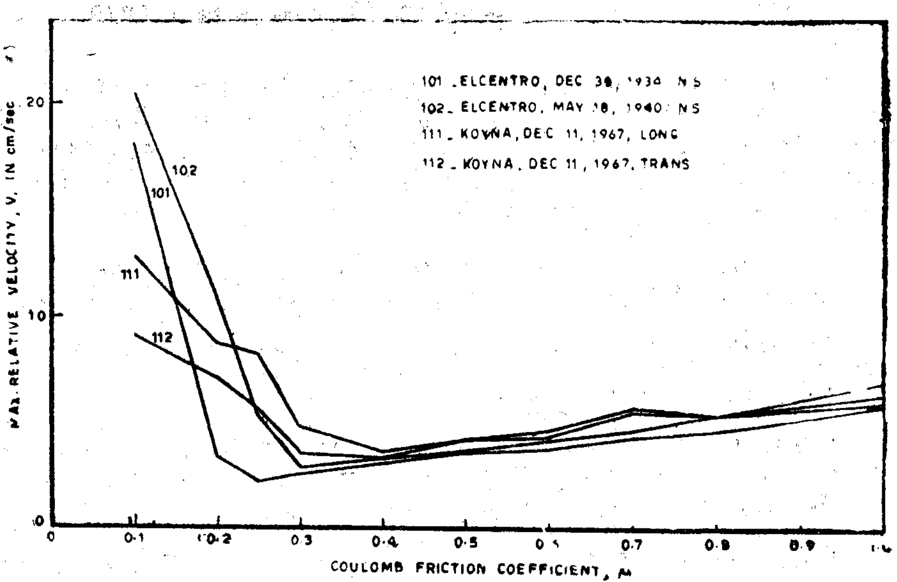


Fig. 11. Relative Velocity vs Coulomb Friction Coefficient

CONCLUSIONS

1. Estimation of peak ground accelerations by the equivalent static approach would generally give erroneous results.
2. The relative motion of the mass is not only a function of the maximum acceleration but also of the wave form of ground motion.
3. The relative velocity of the mass has higher frequency components in case of coulomb damping compared to viscous damping.
4. The relationship between maximum relative motion and the coulomb damping factor μ does not reveal a regular pattern. For the parameters considered, the least value of maximum relative motion is obtained for μ of the order of 0.2 to 0.3.
5. In the case of viscous damping, relative displacement and velocity decreases with increase in damping. However, the rate of decrease is larger in the case of displacement compared to velocity. In the case of absolute acceleration of the mass, it increases with increase in damping.
6. Viscous damping response indicates the El Centro shock is more intense than that of Koyna shock. Such conclusions had also been obtained by a comparision of spectral intensities for the two records.

ACKNOWLEDGEMENTS

The digital computer work was done on the I. B. M. 1620 computer at S. E. R. C., Roorkee.

The paper is being published with the kind permission of Director, S. R. T. E. E., University of Roorkee, Roorkee.

REFERENCES

1. Alford, J. L., Housner G. W. and Martel, R. R., "Spectrum Analyses of Strong-Motion Earthquakes" Engineering Research Laboratory, California Institute of Technology, Pasadena, U. S. A., August 1951.
2, Jai Krishna, Chandrasekearan, A. R. and Saini, S. S., "Analysis of Koyna Accelerogram of December 11, 1967", Bull. S. S. A., Vol. 59, No. 4, August 1969.
3. Norris, C. H., et. al, "Structural Design for Dynamic Loads", Chapter 8, Mc Graw Hill Book No., Inc., New York, 1959.

SPECIAL COURSE IN SOIL DYNAMICS
UNIVERSITY OF ROORKEE, ROORKEE
May 18, 1970 to June 6, 1970

A specialist Course on Soil Dynamics, the science of behaviour of soils and foundations under dynamic loading, is being arranged at the School of Research and Training in Earthquake Engineering, University of Roorkee from May 18 to June 6, 1970. The Earthquake School has well developed laboratory facilities for post graduate training. The curriculam for this Specialist course is as follows :

Historical development of Soll Dynamics : Damages to Foundations and Earthen Structures due to Earthquakes and other Vibration Problems.

Strength Characteristics of Soils Under Dynamics Conditions both Transient, Repeated

Dynamic Earth Pressures.

Earth Dams : Dynamic Stability, Vibration Characteristics, Deformation Analysis, Newmark's Method, Total and Effective Stress Analysis, Model Studies, Current Practise, I.S. Code of Practice, Recent Research.

Liquefaction of Sands : Principles, Factors Affecting, Recent Research, Remedial Measures.

Dynamic Bearing Capacity of Soils and Design of Footing in Earthquake Zones.

Lateral Loads on Piles, Buckling of Piles, Dynamic Characteristics of Pile Foundations, Design of Piles in Earthquake Zones.

Dynamic Compaction of Soils, Cratering by Nuclear Blast, Seismic Methods of Subsoil Explorations.

Theory of Vibrations, Single and Two Degree Freedom Systems, Free and Forced Vibrations, Damping Measuring Instruments, Vibration Absorbers.

Machine Foundations, Dynamic Soil Constants, Block Foundation, Hammer Foundations.

Response of Soil Mass to Earthquake Type Excitations.

The tuition fee for this Course will be Rs. 150/- per trainee and the Registration fee is Rs. 100/- per trainee. The number of seats is limited to 15. All interested in attending the above course are requested to apply to the Director, School of Research & Training in Earthquake Engineering, University of Roorkee, Roorkee, indicating their name, designation educational qualification, and papers published if any, along with the Tuition and Registration fees by April, 30, 1970.

Specialist Course on Earthquake Resistant Design of Buildings

Under the auspices of ECAFF, School of Research & Training in Earthquake Engineering, University of Roorkee, in collaboration with the National Building Organisation, New Delhi, is to conduct a specialist course in "Earthquake Resistant Design of Building" for the trainee officers from Southeast Asian Countries. The course shall be of four week's duration and is proposed to be held in the middle of October 1970.