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VIBRATION CHARACTERISTICS OF CABLE SUSPENDED SYSTEMS

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INTRODUCTION

Cable, as a structural element, has many advantages over other types of structural systems on account of efficient utilization of material, easy and speedy construction etc. Suspended systems, mostly made out of cables, are being used increasingly for the support of long span roofs to cover large column free spaces. These systems, being flexible in nature, can be moulded to any architectural shape desired for elegance and possess high degree of aesthetic potential as evidenced by many famous examples, the most recent being the cable roof systems at Munich Olympics 1972.

The characteristics of cable suspended structures have made an attractive proposition for covering large column free areas but, on the other hand, because of its flexible form, these systems have a disadvantage of inherent instability under dynamic forces of wind, blasts or earthquakes. An oscillatory motion is caused by dynamic force of wind giving rise to the flutter phenomenon. This in a way tends to damage the roofing material over the suspended systems. In flutter, forcing function is time dependent and requires the study of critical wind velocity and the dynamic response of structure at this critical velocity.

Very little is known about the dynamic response of cable suspended structures both because they are a relatively new form of construction and because in case of wind it is difficult to find the nature of the forcing function. If the cable suspended systems are located in seismic zones or other areas which have frequent blasts in neighbourhood they would be required to resist the effect of earthquake ground motion or blasts as the case may be. It is, therefore, important to be able to predict the dynamic response of such systems under various types of dynamic loading.

The example of aerodynamic collapse of Tacoma Narrows Bridge in 1940 was mainly due to the excessive deflection under flexural and torsional vibrations with only moderate velocity of wind. Similar failures are evidenced by many examples of suspension bridges in the earlier periods. Considerable amount of work has been done since the Tacoma Narrows bridge collapse, to study the aerodynamic problem of suspension bridges and a good deal is known on this aspect of the problem. Similarly the phenomenon of galloping of electric transmission cables has been investigated for a long time now. However, no such failure has been reported in suspended roof systems so far because these have been recently developed and time alone will show the behaviour of this type of structures. It is possible that new problems of aerodynamic instability, self excited oscillations and flutter may come up in suspended roof systems also. Therefore, there is yet much research work to be done in the study of vibration characteristics of such systems. The present paper is an attempt in this direction.

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DYNAMIC RESPONSE OF CABLE SYSTEMS

A pretensioned cable structure consists of a number of cable elements jointed at their intersections. These elements can be treated as perfectly flexible, straight and weightless for purposes of analysis. The number of elements in these structures is usually large, and the governing stiffness or flexibility matrix, formulated for their analysis is large too. With the help of digital computers, however, this does not present any special problem.

FORMULATION OF EQUATIONS

A typical cable system acted upon by different force elements is shown in Fig. 1. The weights of the cable system are assumed to be lumped at nodes. The vibration of the node

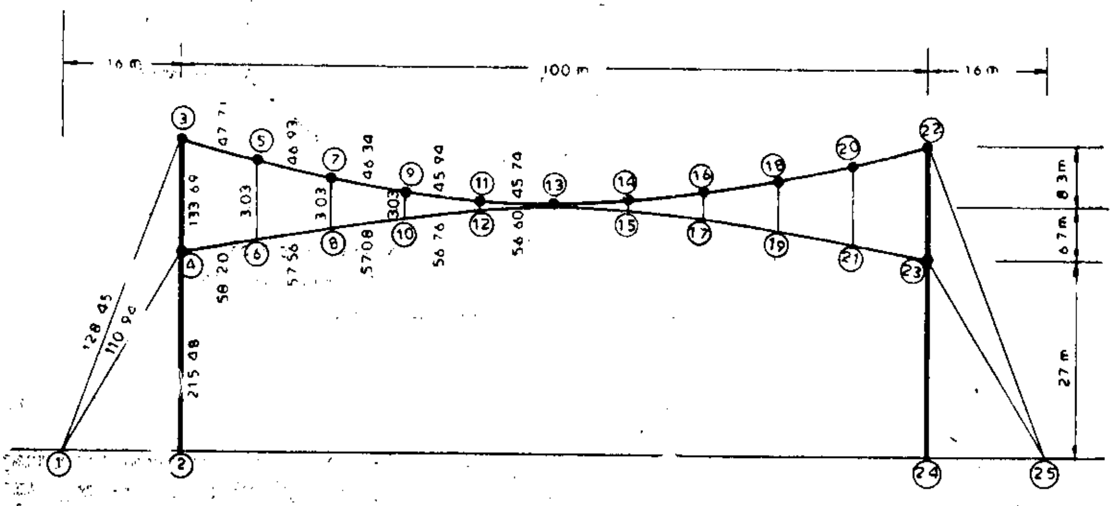


Fig. 1. Prestress cable truss (Node numbers are encircled. Member forces in tonnes are shown along them).

in any direction with respect to the static equilibrium position, is treated as a linear small deflection problem. During vibrations of the system the equations of equilibrium can be set up by considering the inertia force acting in a direction opposite to that of motion. These forces together with damping force and the restoring forces must equal the externally applied time dependent load. An equation in the standard form can be obtained as

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{G\}+\{F(t)\} \quad \ldots(1)$$

where

$[M]$=diagonal mass matrix
$[C]$=damping matrix
$[K]$=stiffness matrix
$\{u\}$=deformation vector
$\{G\}$=quadratic and cubic deformation vector
$\{F(t)\}$=time dependent external load.

Following a "Tension Coefficient" approach developed earlier for cable system by several authors[(1), (2), (3)] the stiffness matrix $[K]$ and the matrix $\{G\}$ containing the nonlinear terms obtained and is described as follows.

Consider a node q (x_q, y_q, z_q) shown in Fig. 2 connected to the other node $p(x_p, y_p, z_p)$ through members i, which have pretension force F_i and initial length l_i. The structure is in equilibrium under this set of forces. Therefore,

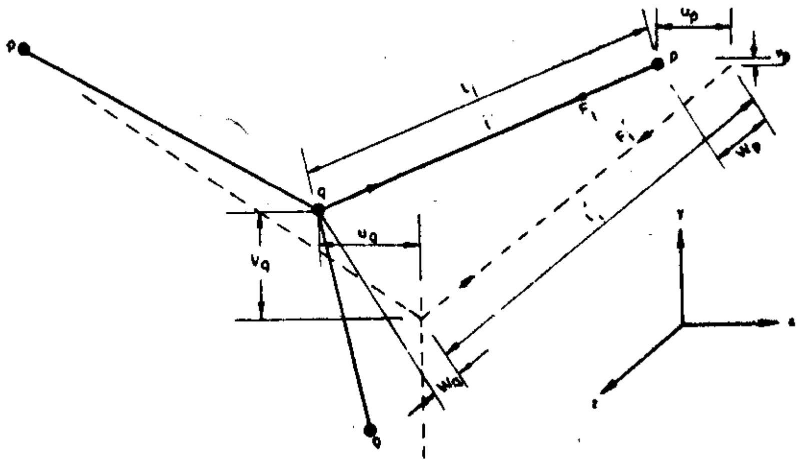


Fig. 2. Deformed state of node p and q.

$$\sum^{i} \frac{F_1 (x_p - x_q)}{l_1} = 0 \qquad \ldots(2)$$

Similar equations will hold good in y and z directions. Now the disturbing force may give rise to a vibration of the structure, thereby causing deflection (u_q, v_q, w_q) and (u_p, v_p, w_p) along x, y and z axis at nodes q and p respectively. The equations of equilibrium in x direction in the deformed state can then be written as

$$\sum^{i} \frac{F_1' \,[(x_p - x_q) + (u_p - u_q)]}{l_1'} = 0 \qquad \ldots(3)$$

where

$$l_1' = l_1 \,(1 + 2e_1 + e_2)^{1/2} \qquad \ldots(4a)$$

or

$$l_1 \left(1 + e_1 + \frac{1}{2} e_2 - \frac{1}{2} e_1^2 - \frac{1}{2} e_1 e_2 + \frac{1}{2} e_1^3 + \ldots\ldots \right) \qquad \ldots(4b)$$

and

$$\frac{1}{l_1'} = \frac{1}{l_1} \,(1 + 2e_1 + e_2)^{-1/2} \qquad \ldots(5a)$$

or

$$\frac{1}{l_1} \left(1 - e_1 - \frac{1}{2} e_2 + \frac{3}{2} e_1^2 + \frac{3}{2} e_1 e_2 - \frac{5}{2} e_1^3 + \ldots\ldots \right) \qquad \ldots(5b)$$

Here,

$$e_1 = \frac{1}{l_1^2} \,[(x_p - x_q)\,(u_p - u_q) + (y_p - y_q)\,(v_p - v_q) + (z_p - z_q)\,(w_p - w_q)]$$

$$e_2 = \frac{1}{l_1^2} \,[(u_p - u_q)^2 + (v_p - v_q)^2 + (w_p - w_q)^2]$$

Also $$F_1'=F_1+EA\left(\frac{l_1'}{l_1}-1\right) \qquad \ldots(6)$$

The terms on the right hand side of eqn. (4b) and (5b) are obtained by expanding the terms in eqn. (4a) and (5a) respectively by the binomial expansion and neglecting fourth and higher order terms.

Now substituting eqns. (4b), (5b) and (6) in eqn. (3) and rearranging the terms, after neglecting fourth and higher powers of deflections u, v, w, in such a manner that all terms containing single power of deflections are kept on the left hand side and all the nonlinear terms are grouped into g_x on the right hand side, then we get

$$\sum^{i}\left[\frac{F_1(u_p-u_q)}{l_1}+\frac{(EA-F_1)(x_p-x_q)e_1}{l_1}\right]=g_x \qquad \ldots(7)$$

Eqn. (3) and (7) and similar equations along y and z directions are the equations of equilibrium at each free node in the structure. Thus there will be simultaneous equations having 3N unknown (2N for a plane system) where N is the total number of interior nodes. Therefore, the coefficients of (u, v, w) will give rise to stiffness matrix K and non-linear terms of deflection (u, v, w) on the right hand side will give vector {G} of eqn. (1).

If the problem is treated as a linear one and term {G} are neglected, as well as the damping in the system is ignored, eqn. (1) reduced to,

$$[M]\{\ddot{U}\}+[K]\{U\}=\{F(t)\} \qquad \ldots(8)$$

For free vibrations of the system the equation of motion reduces to,

$$[M]\{\ddot{U}\}+[K]\{U\}=\{0\} \qquad \ldots(9)$$

Assuming harmonic solution with frequency ω, eqn. (9) is reduced to the standard eigen-value problem of the type,

$$[A]\{U\}=\lambda\{U\} \qquad \ldots(10)$$

Eqn. (10) can be solved by employing some of the powerful numerical methods for obtaining the eigenvalues and eigenvectors.

Methods of Solution :

There are generally two broad classifications of the methods for solving the eigenvalue problems.[4]

1. Transformation methods
   (a) Jacobi's method,
   (b) Givens' method,
   (c) Householder's method.
2. Iteration methods
   (a) Convergence of iteration,
   (b) Rayleigh's quotient,
   (c) Inverse iteration,
   (d) Matrix deflation,
   (e) Stodola's method,
   (f) Holzer's method.

The transformation methods are preferred when all the eigenvalues and eigen-vectors of a given system are required. These methods accomplish the solution by operating on the matrices of the system which necessitates the storage of the matrix, which may have a large size for cable systems. The methods use a sequence of matrix transformations to find a similar matrix [B] such that the evaluation of determinant | $B-\omega^2M$ | is simpler than the calculation of | $K-\omega^2M$ |. On the other hand, the iterative methods can avoid the necessity of storing the entire matrix. These methods yield a sequence of scalars and vectors that converge to some particular eigenvalue and its corresponding eigenvectors.

SYSTEM CHOSEN FOR STUDY

A plane cable truss as shown in fig. 3 is chosen to investigate the dynamic characteristics under different loading conditions. This cable truss system is assumed to consist of

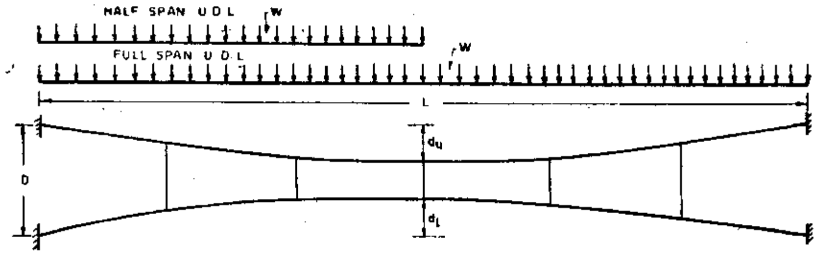


Fig. 3. System chosen for stydy (u. d. l. has been discretised at each node as concentrated load).

discrete, straight, weightless elements meeting at node points. The weights of these elements, are concentrated at the nodes. Masses are lumped at node points.

The natural frequencies and the mode shapes of cable truss systems have been obtained using Jacobi's method for solving eigenvalue problem. Variable structural parameters chosen are extensional rigidity of top cable, dip-span ratio of the top cable and different loading variations. This study helps the designer to arrive at the design range for these parameters for a particular type of loading so as to avoid the condition of resonance with the expected dynamic load.

The basic system chosen for study has the following data,

N = number of hangers = 5

H = horizontal component of initial tension at the end of the top cable = 50.0 tonnes,

L = span of truss = 100 metres,

AET = extensional rigidity of the top cable = 600 H

AEL = extensional rigidity of the bottom cable = 600 H

AEH = extensional rigidity of the hangers = 60 H

FH = initial pretension in each hanger = 0.08 H

d_u = sag of the top cable = 0.0625 L

d_l = sag of the bottom cable = 0.0625 L

D = difference in the elevation of ends of top and bottom cable = 0.15 L

Values of frequencies and mode shapes are obtained for W/H from 0.6 to 1.8 over full span and W/H from 0.3 to 0.9 over half span where W is the total load over the truss system. Results are obtained for dip-span ratio of upper cable d_u/L from 0.05. to 0.075 and for extensional rigidity of the top cable AET/H from 300 to 900.

PRESENTATION OF RESULTS

Dynamic characteristics of 100M span truss type cable system were studied. The frequencies and mode shapes were obtained for a number of cases in which the total load over the structure (W), extensional rigidity of top cable (AET) and sag span ratio (d_u) were varied.

Figures 4 and 5 show the effect of variation of AET and d_u respectively on the first three frequencies of the cable system for two conditions of loading—one when full span is

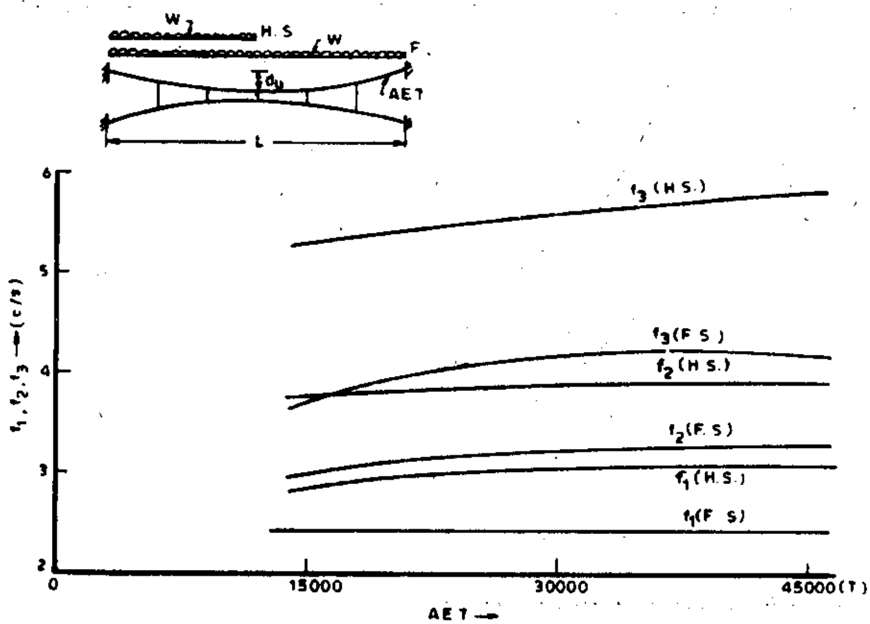


Fig. 4. Effect of variation in extensional rigidity of top cable on first three frequencies of the truss system.

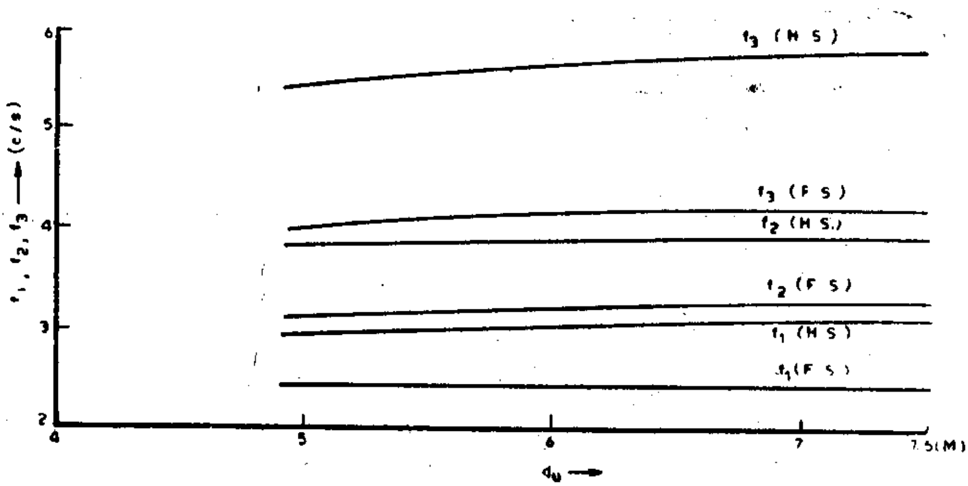


Fig. 5. Effect of variation in sag span ratio of upper cable on first three frequencies of the truss system.

loaded and the other when half span is loaded. The former will be referred to as F. S. condition and the latter as H. S. condition. It is seen that frequencies of the system are not significantly altered by variations in the values of AET of d_u. This would suggest that changing the cross sectional area or the sag of top cable of the truss will not materially alter the dynamic characteristics of the cable system.

Figure 6 shows the variations in the frequencies (first three) when the total load over

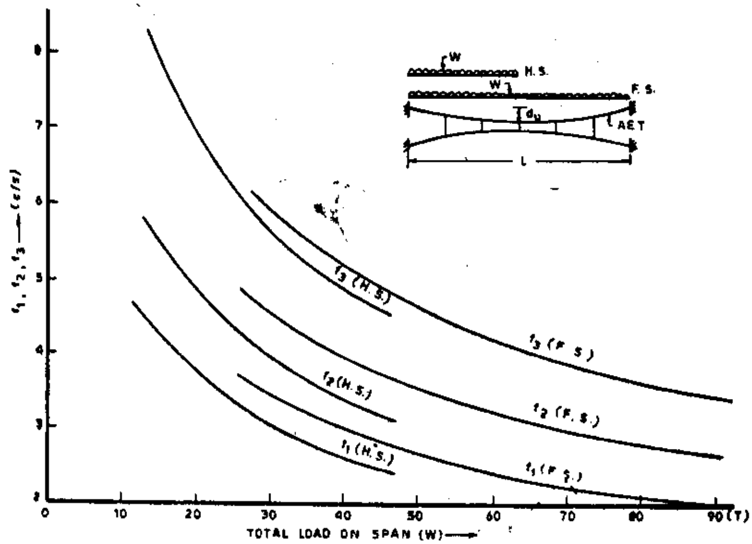


Fig. 6. Effect of variation in intensity of loading on first three frequencies of the truss system.

the structure is varied. As should be expected, the frequencies drop nonlinearly with the increase in load intensity both for F. S. and H. S. conditions.

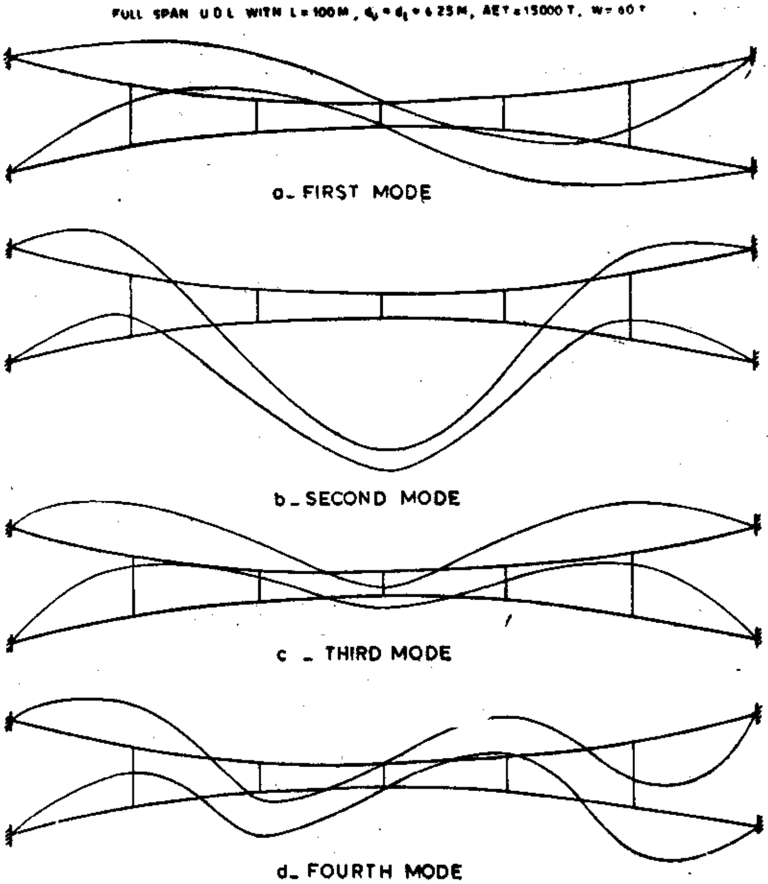
Mode shapes for a class of truss system are plotted in Fig. 7 and 8 for F. S. and H. S. conditions. It is seen that the fundamental mode shape is antisymmetric for both the conditions. Higher modes are alternately symmetric and antisymmetric.

The effect of variations in AET, d_u and W, on the fundamental mode shape of the system has been examined. The mode shapes for different cases have been plotted in Fig. 9 to 11. It is seen that the general nature of mode shapes remains the same in all cases although the relative mode shape coefficients at various points do change with such variations.

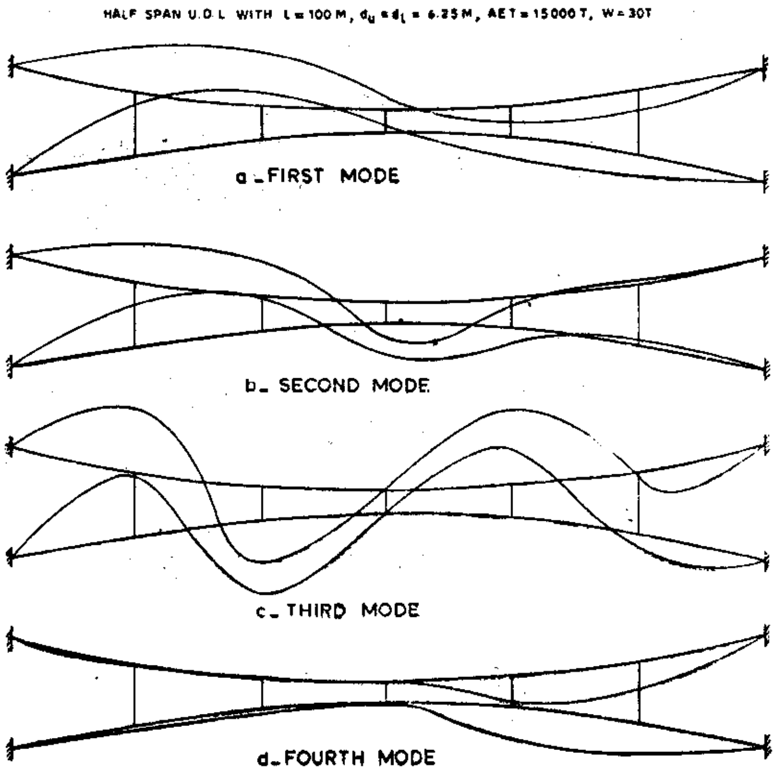
Dynamic characteristics of trusses with concentrated loads at isolated single points (nodes) have also been examined. For this purpose a load of 5 tonnes has been considered at nodes 1, 2 and 3 (one node only at a time). The fundamental mode shape in each of the above three cases is plotted in Fig. 12. The associated frequencies are also indicated for each case in the same figure. As should be expected, the mode shapes of the truss when load acts at node 1 and 2 are asymmetric while for load acting at node 3 (mid point), the mode shape is symmetrical. It is seen that the fundamental frequency increases with the movement of load point towards the centre of span. This conforms to the normal thinking that cables with central loads are stiffer than those with loads acting elsewhere.

CONCLUSIONS

Truss type cable systems are generally low frequency systems with antisymmetrical fundamental mode. Some of the most important structural parameters like the extensional rigidity and dip of the top cable have little effect on the fundamental and higher frequencies as also on the nature of the mode shapes. Systems with concentrated loads at the centre are found to be stiffer than those with loads elsewhere.



→ **Fig. 7. First four mode shapes for full span U. D. L.**



→ **Fig. 8. First four mode shapes for half span U. D. L.**

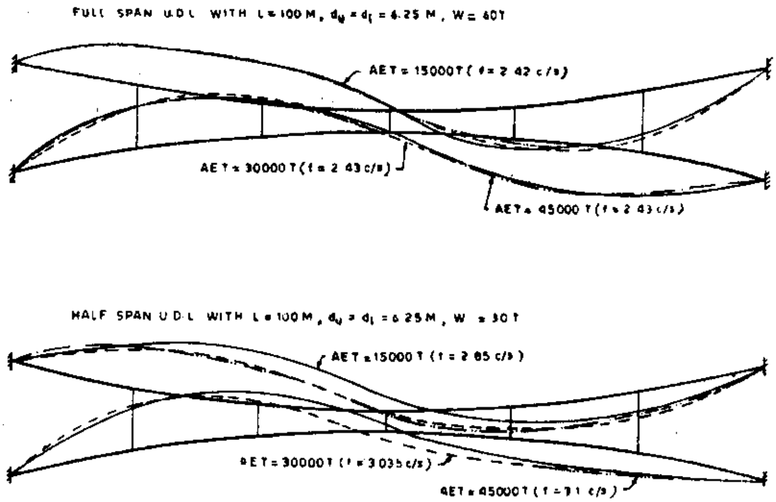


Fig. 9. Mode shapes with variation in AET.

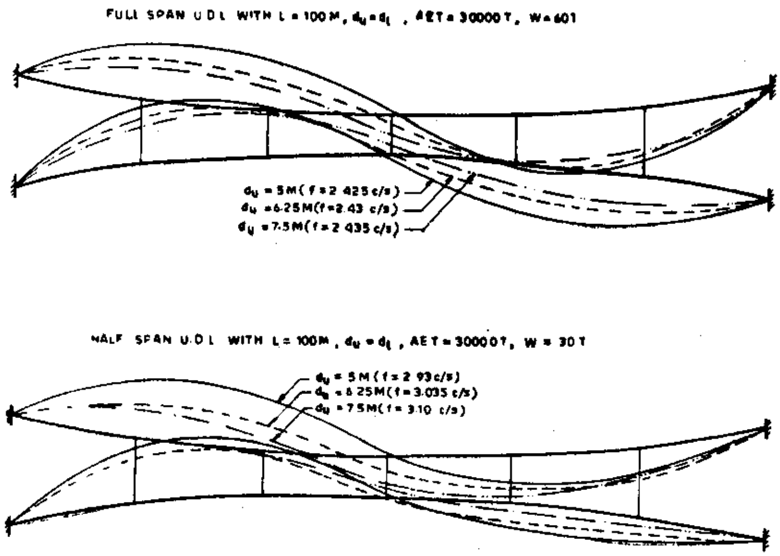


Fig. 10. Mode shapes with variation in d_u.

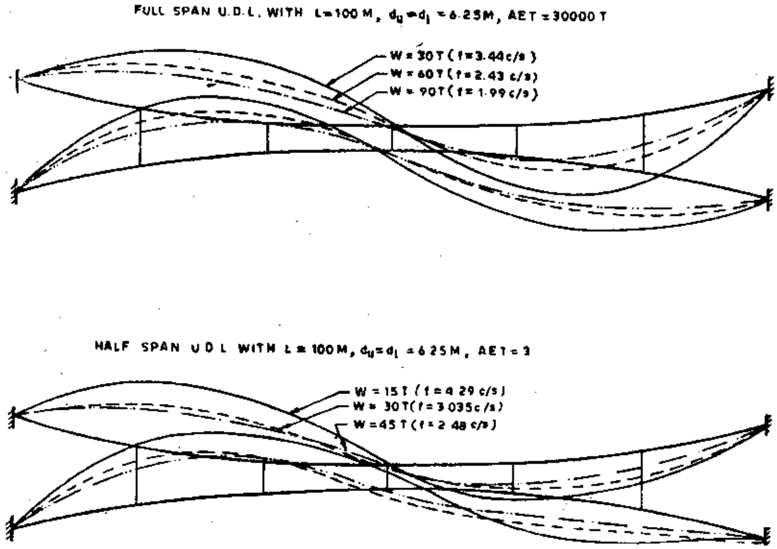


Fig. 11. Mode shapes with variation in W.

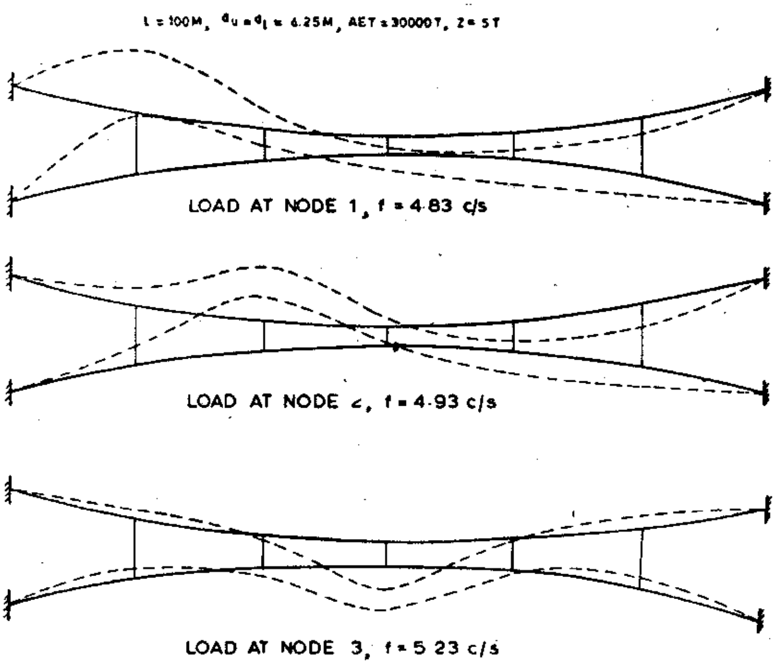


Fig. 12. Mode shapes with 5T concentrated load applied at node 1, 2 and 3.

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