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DYNAMIC MODULI AND DAMPING FOR SANDS AT LOW STRAIN AMPLITUDES

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ABSTRACT

The available empirical relationships for maximum dynamic shear modulus (*Gmax*) and dynamic shear damping (Ds) at low strain amplitudes for sands are reviewed. Based on tests with dry Monterey No. 0 sand in resonant column device, new relations for *Gmax* and *Ds* are proposed. In addition, based on test data in longitudinal mode of vibration, relations for dynamic Young's modulus (*Emax*) and dynamic longitudinal damping (D_l) are developed for the first time. The developed relations are convenient to use as they are dimensionally correct and are applicable for any system of units. The observed experimental data and proposed relations are in good agreement. The proposed relations for Gmax and Ds are compared with reported relations and a difference of approximately 20% and 100% is observed in some cases for dynamic shear modulus and shear damping, respectively. More field and experimental results are warranted to put the developed relations for dynamic Young's modulus and longitudinal damping on firm foundation.

INTRODUCTION

The progress of advanced computational methods for dynamic soil structure interaction analysis has necessitated the accurate determination and estimation of dynamic soil properties. The design of engineering structures such as radar tower, power plant etc. requires the dynamic properties of soils at low strain ampitude and high frequencies. The Resonant Column Test, in such circumstances, is an indispensable tool and provides the values of dynamic moduli and damping ratio. The test though unique is not very commonly used. As such any empirical relations which can provide the modulus and damping values close to those obtained from resonant column tests are of great help to practicing engineers. This paper reviews such reported empirical

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relations for dynamic moduli and damping values for sands; discusses their limitations, and proposes new relations based on recent experimental data of resonant column tests.

THE RESONANT COLUMN TEST

The most common types of laboratory tests used to investigate the dynamic behaviour of soils cyclic triaxial, resonant column, simple shear and torsional shear tests, Each of these tests has advantages and disadvantages when compared with others (Woods, 1978). The resonant column test is the most recommended test to evaluate dynamic moduli and damping values of soils at strains ranging from 10^{-4} to 10^{-2}%.

The basic principle of the resonant column device is to excite a confined cylindrical solid or hollow specimen of soil with either one end rigidly fixed at the base or free in a fundamental mode of vibration, typically in torsional or axial vibration. Once the fundamental mode of vibration is established, measurments are made of the resonance frequency and amplitude of vibration from which wave propagation velocities and strain amplitudes are calculated using the theory of elasticity. From the measured velocities of waves, longitudinal and shear moduli can be computed. By considering single degree of freedom system with linear viscous damping (Kelvin-Voigt Model) and free vibrations, damping values are calculated. The mathematical expressions and computer programs for data reduction are overviewed by many authors, the recent among them being Drnevich (1985) and Avramidis and Saxena (1985). The definitions of moduli, damping and strain amplitude commonly adopted in analyzing resonant colomn test data and used in this paper, are shown in Fig. 1.

REVIEW OF EMPIRICAL RELATION

In this section the existing relationships for evaluating dynamic maximum shear modulus (G_{max}), dynamic maximum Young's moduls (E_{max}), dynamic shear damping (D_s) and dynamic longitudinal damping (D_l) are reviewed.

Relations for Dynamic Shear Modulus : A summary of reported equations for estimating G_{max} is provided in Table 1. It may be noted that in the Hardin and Drnevich (1972) equation, the value of k depends on plasticity index of soil (provided in Table 2). This relationship is

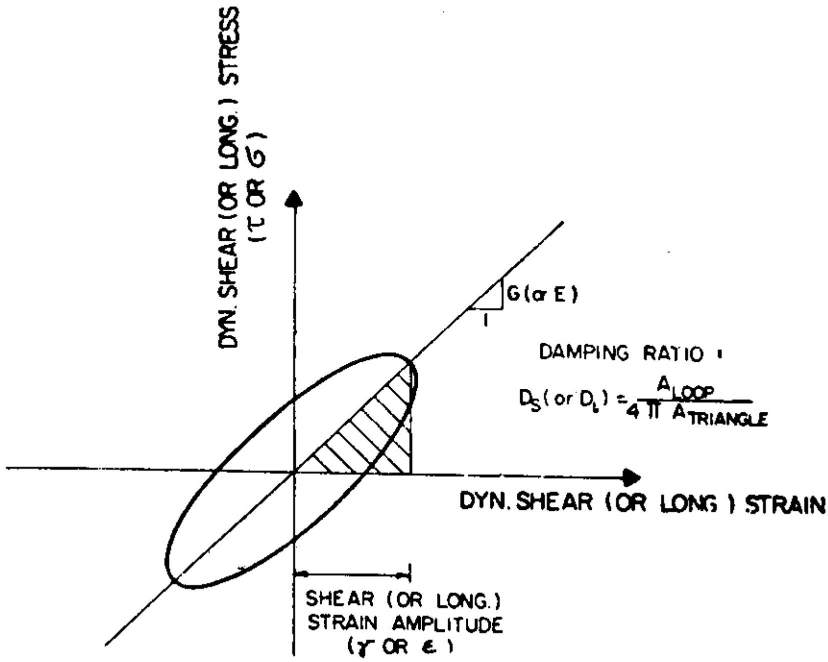

Fig. 1 Definitions of Strain amplitude, moduli and damping

found to be applicable for void ratio ranging from 0.4 to 1.2 only. For higher values of void ratio, the relation underestimates G_{max}, and as such a modified equation proposed by Hardin (1978), provides better results. Isenhower (1979) conducted resonant column tests with highly plastic silts and developed the following form of empirical relation for G_{max}.

$$\ln(G_{max}) = A + B\ln(\bar{\sigma}_0) + C\ln(\rho_0) + D\ln(\bar{\sigma}_0)\ln(\rho) + E[\ln(\bar{\sigma}_0)]^2 + F[\ln(\rho)]^2 \quad (1)$$

where ρ is the mass density. The units of G_{max}, $\bar{\sigma}_0$ and ρ and the values of constants A, B, C, D, E and F are given in Table 3. Ohsaki and Iwasaki (1973) suggested the following correlation for estimating G_{max} at strains less than $10^{-3}\%$, from standard penetration tests :

$$G_{max} = 1200N^{0.8} \quad (2)$$

where N = Number of blows in a standard penetration test and units for G_{max} are tonnes per square meter. The above equation, nevertheless, does not distinguish type of soil nor the effect of the depth of embedment. Seed and Idriss (1970) proposed the equation for estimating G_{max} for sands as follows :

$$G_{max} = 1000\ K_{2max}\ (\bar{\sigma}_0)^{0.5} \quad (3)$$

In this equation the units of G_{max} and $\bar{\sigma}_0$ are pounds per square feet and K_{2max} is an empirical factor which varies according to density. The values of K_{2max} were obtained by geophysical tests are presented in Table 4. Anderson et. al. (1978) based on field and laboratory tests data concluded that Equation 3 underestimates and that proposed by Ohsaki and Iwasaki (1973) overestimates the value of G_{max} for sandy soils.

Hardin and Drnevich (1972) proposed an approximate method of computing G at any strain level γ. Assuming hyperbolic stress-strain relations, the following expressions are obtained. :

$$\left(G/G_{max}\right) = \frac{1}{1+(\gamma/\gamma r)} \quad (4)$$

$$\gamma r = \tau_{max}/G_{max} \quad (5)$$

and

$$\tau_{max} = \left\{\left[\frac{(1+K_0)}{2}\ \bar{\sigma}\upsilon \sin\bar{\phi} + \bar{c}\cos\bar{\phi}\right]^2 - \left[\frac{(1-K_0)}{2}\bar{\sigma}\upsilon\right]^2\right\}^{0.5} \quad (6)$$

where K_0 = coefficient of lateral strain at rest

γr = reference strain

and $\bar{\sigma}\upsilon$ = effective vertical stress

Edil and Luh (1978) developed equations which can also be used to find G at given strain level γ. According to these investigators, for strains less than 0.25×10^{-4} radians the values of G are found to be nearly constant and hence it can be taken as G_{max}. However in the publication G at $\gamma = 0.25\times10^{-4}$ is termed as reference dynamic shear modulus G_0. The developed equations are :

$$\frac{G}{G_{max}} = 1.004 - 345.4\gamma. \quad (7)$$

$$G_{max} = 10^4\ [-5.899 + 0.305\ (\bar{\sigma}_0)^{0.7} \exp(D_r) + 4.02\ (\bar{\sigma}_0)^{0.25} \quad (8)$$

Where G, G_{max} and $\bar{\sigma}_0$ are in KPa; γ in radians and D_r (relative density) is a fraction of one. Expression in terms of void ratio was also provided.

Sherif and Ishibashi (1976) conducted the correlation for shear modulus as:

$$G_{eq} = 2.8\bar{\phi}\gamma^{-0.6}\ (\bar{\sigma}_c)^{0.85} \quad (9)$$

where G_{eq} is the equivalent dynamic shear modulus cooresponding to the second cycle; $\bar{\phi}$ is the angle of internal friction, γ is the shear strain and $\bar{\sigma}_c$ is the effective confining pressure, the units of $\bar{\sigma}_c$ and G_{eq} are in KPa. In the above correlation the density (or void ratio) effects are reflected by an appropriate selection of $\bar{\phi}$.

Relations for Dynamic Young's Modulus : Currently the normal practice has been to calculate E_{max} from the estimated values of G_{max} using "appropriate Poisson's ratio" (Hardin, 1978). To theauthors knowledge, no direct relation for dynamic Young's modulus is reported in the literature as of today (1987). This paper proposes a relation for E_{max} for the first time and elaborates the reliability of Poisson's ratio values obtained based on the moduli from empirical relations.

Relations for Dynamic Shear Damping : Hardin and Drnevich (1972) proposed a relation for shear damping as given by :

$$D = D_{max} \left(1 - \frac{G}{G_{max}} \right) \tag{10}$$

The value of G_{max} can be obtained by choosing any relation in Table 1. However, determination of D_{max} is slightly more cumbersome (D_{max} is the value of damping ratio when shear modulus theoretically equals to to zero). Based on experimental data, empirical relations for D_{max} are proposed for various soils and are given in Table 5 It can be seen that for sands D_{max} depends on number of cycles of loading N, whereas, for silts and clays it depends on the number of cycles of loading N, frequency f and mean principal effective stress $\bar{\sigma}_0$. Therefore knowing G_{max}, D_{max} and G for any strain, the damping value D can be obtained.

Hardin (1968) proposed an empirical equation for damping ratio of clean dry sands for shear strains of 10^{-4} or less. and, confining pressures ($\bar{\sigma}_0$) varying from 24 to 163 KPa based on comprehensive resonant column tests, as follows :

$$D = 4.5\gamma^{0.2} (\bar{\sigma}_0)^{0.5} \tag{11}$$

where $\bar{\sigma}_0$ is in pounds per square feet. and γ as fraction of one. Tatsuoka et. al. (1978) found that the exponent (n) of $\bar{\sigma}_0$ in the above equation is not equal to a constant value of 0.5 but, is dependent on strain level as shown in Fig. 2. These researchers reported no effect of void ratio on damping and proposed a relation (Eqn. 12) to evaluate damping ratio (D_2) at any confining pressure ($\bar{\sigma}_{02}$), provided the value of damping ratio (D_1) at a specific confining pressure ($\bar{\sigma}_{01}$) is known. The proposed relation is as follows :

$$\frac{D_1}{D_2} = \left(\frac{\bar{\sigma}_{01}}{\bar{\sigma}_{02}}\right)^n \quad (12)$$

The value of n are obtained from Fig. 2. Sherif et. al. (1977) established

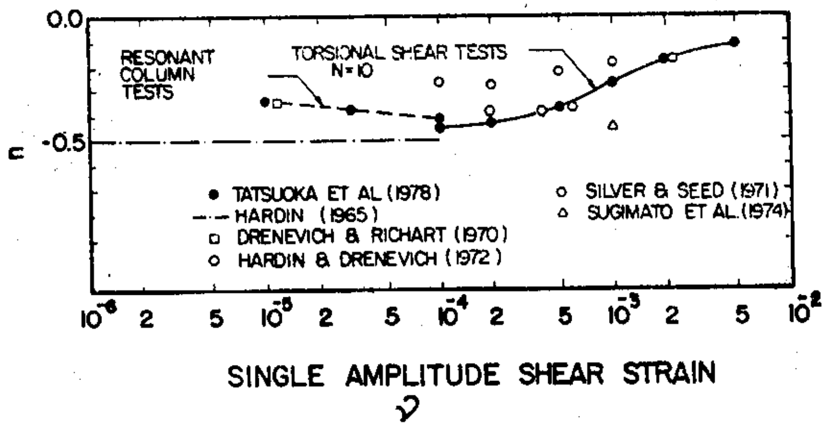

Fig. 2 n versus γ relationship

an empirical equation for damping at N = 2 (based on cyclic torsional shear tests on dry Ottawa sand) as :

$$D = (50 - 0.6\,\bar{\sigma}_0)\,\gamma^{0.3} \quad (13)$$

In the above equation D and γ are in percentages and $\bar{\sigma}_0$ in pounds per square inch. In an attempt to evolve a general relationship, soil gradation and sphericity factor F, and correlation factor for number of cycle N_c, were introduced and the subsequent expression for damping as :

$$D = \frac{(50 - 0.6\bar{\sigma}_0)}{38}(73F - 53)(1.01 - 0.046\log N_c)\gamma^{0.3} \quad (14)$$

Typically F varies from 1.0 to 2.0. In the above expression damping D and shear strain γ are in percentages and $\bar{\sigma}_0$ units are pounds per square inch.

given situation, only few of the listed factors may be important (e.g. Edil and Luh, 1978; Avramidis and Saxena, 1985). Therefore it is important to bear in mind that any reported relation can be valid for the parameters (e.g. soil type, vibration characteristics) that accounted in its development. For other parameters or condition the developed relation may or may not be applicable. As an example Hardin and Drnevich (1972) relations are mostly based on tests on clean Ottawa Sand, Lick Creek Silt and data obtained from other investigators with other soil at low confining pressures. Later many Japanese investigators (e.g. Iwasaki et. al. 1977, Tatsuoka et. al. 1978) in their series of investigations relating to moduli and damping values for different soils found that the grain size distribution, percent fines etc. affect the dynamic characteristics of sandy soils. Isenhower (1979) also observed that the relation developed by Hardin (1978) for normally consolidated soils results in an average error in the values of shear modulus upto 12.7% (Table 1), though one may also argue that the relation developed by Isenhower (1979), given as Eq. 1, is lengthy and involves too many constants and yet has an avrage error upto 8.8%. Chung et. al. (1984) found that the widely used equation of Hardin and Drnevich (1972) predicts moduli that are slightly lower than the average of the test results for Montrey No. 0 sand, though this tests results are limited to only one value of void ratio (e = 0.676).

Based on the reported studies, it can be said that the relations for shear modulus are fairly well established and relations for shear damping are still being developed. No relations for dynamic longitudinal modulus and longitudinal damping are available. In the following section, investigations conducted at Illinois Institute are reported along with newly proposed relations.

INVESTIGATIONS AT ILLINOIS INSTITUTE

In this paper the data of only resonant column tests on dry Monterey No. 0 sand specimens is reported.

Test Equipment : During this investigation all tests are conducted with the modified Drnevich Longitudinal and Torsional Resonant Column device. The device can accommodate cylindrical soil specimens with height and diameter of 135 mm and 71.12 mm respectively. The samples can be subjected to desired confining pressure. The samples are fixed at the base with excitation forces applied at the top. The

Edil and Luh (1978) observed a significant effect of the number of cycles on damping for Ottawa Sand and other such dry sands, and, found that the correlations expressed in Eqn. 15. D_{max} is the damping ratio defined at number of cycles, N equal to 1000.

$$\frac{D}{D_{max}} = 1.131 - 0.453 \log N \tag{15}$$

$$D_{max} = 0.70 + 7800 \gamma D_r^{0.5} - 0.36 \gamma^{0.33} (\bar{\sigma}_0/98)^2$$

where D and D_{max} are in percentage, D_r in decimal form, $\bar{\sigma}_0$ in KPa, γ in radians and N is dimensionless.

Relations for Dynamic Longitudinal Damping : In dynamic response of soils, the predominant energy input comes from shear waves; consequently, shear or torsional damping being the most important, many investigators tried to develop relations for shear damping. There may be situations where the longitudinal damping needs to be considered (Marcuson and Curro, 1981). As of today, no reliable empirical relation for dynamic longitudinal damping is reported in the literature. However, it is the persent practice to assume longitudinal damping about 3% for strains less than 10^{-4} and 12% for strains above 10^{-1}.

COMMENTS ON REPORTED EMPIRICAL RELATIONS

The development of simple equations to make preliminary estimates of soil moduli and damping at low strain amplitudes as initiated by Hardin and Drnevich (1972) is indeed necessary for engineers. Though the relationships reported in the literature are based on exhaustive test data, such relations should be periodically reviewed in view of new experimental data, while recognising that the development of unique expressions for moduli and damping which can account for all the factors would be difficult, if not impossible.

The factors affecting moduli and damping values can be listed as follows : 1. Type of soil, 2. Mode of vibration, 3. Strain Amplitude, 4. Effective mean normal stress, 5. Void ratio, 6. Number of cycles, 7 Prestrain, 8. Moisture content, 9. Stress History (OCR), 10. Frequency 11. Preloading, 12. Capillary action, 13. Strain rate, 14. Sampling and sample preparation, 15. Specimen geometry, 16. Saturation, 17. Grain size characteristics, 18. Time, 19. Temperature, 20. Testing technique and apparatus, 21. Data interpretation and others. For any apparatus is capable of applying both longitudinal and torsional excitations. Samples can be tested either in dry condition or completely saturated condition. (For the complete details of the apparatus, refer to Avramidis and Saxena, 1985).

Soil Type : Monterey No. 0 sand is used in all the tests. Monterey No. 0 sand is a uniform medium dense sand with a coefficient of about 1.5 and a mean particle diameter of about 0.4mm. The sand grains are predominantly quartz and feldspar with some mica; they have a specific gravity of 2.65 and are rounded to subrounded. The grain size analysis of the sand is given by several investigators (e.g. Mulilis et al., 1972; Saxena and Reddy, 1987). This soil has been chosen because of the availability of data from various geotechnical researchers and, thus, ease of comparison.

Sample Preparation : All the samples are prepared by method of undercompaction developed by Ladd (1978). The samples are prepared at relative densities of 25,43,60 and 80 percent.

Test Procedure : The details of test procedure and data reduction method can be found in Avramidis and Saxena (1985). Dry samples of different relative densities mentioned above are tested at effective confining pressure of 49, 98, 196, 392 and 588 KPa in such a way that the system response could be studied at different low strain amplitudes, All the tests are carried out in a single stage.

The variations of dynamic shear modulus (G) and dynamic longitudinal modulus (E) with strain at different densities under different confining pressures are shown in Fig. 3 and Fig. 4 respectively. Whereas, corresponding dynamic shear damping (D_s) and dynamic longitudinal damping (D_l) variations are shown in Fig. 5 and Fig. 6 respectively.

ANALYSIS OF TEST RESULTS

The typical test results as shown in Figs. 3-6, clearly show that dynamic shear modulus (G), and dynamic Young's (or longitudinal) modulus (E) decrease, and, dynamic shear damping (D_s) and dynamic longitudinal damping (D_l) increase with increase in strain amplitude. This trend was observed for all tests. The decrease in moduli is mainly due to the nonlinear behaviour of soils and the increase in damping ratios is caused by energy absorption due to particle rearrangement.

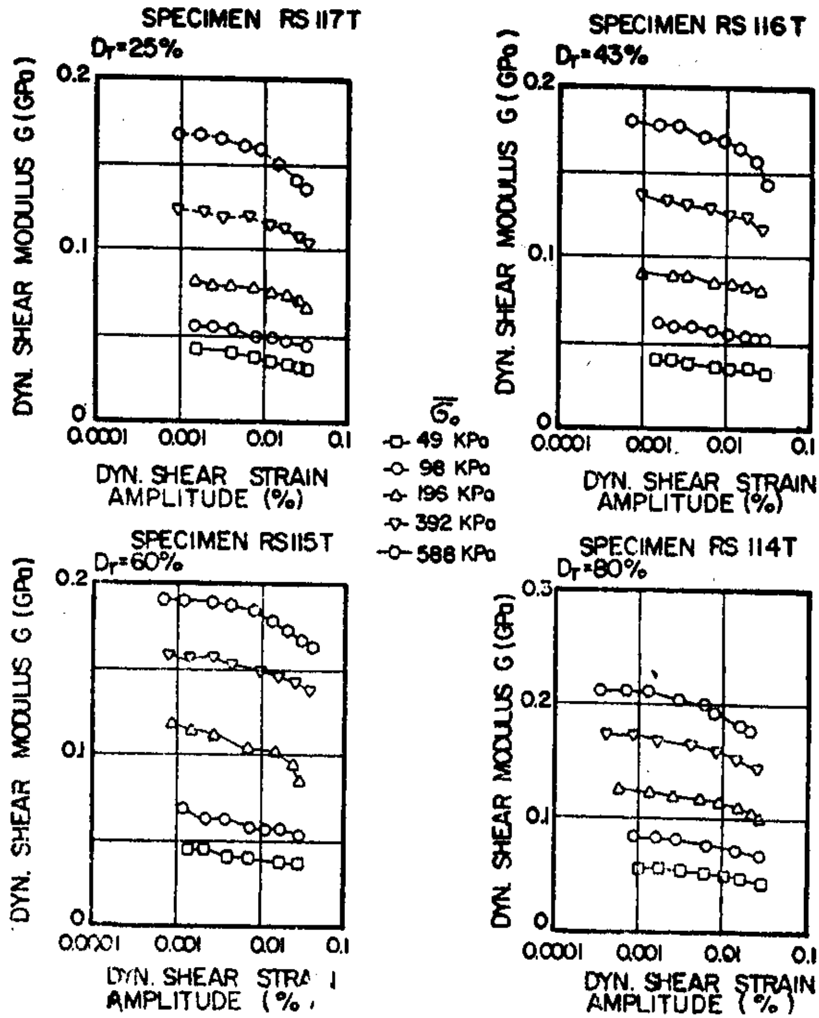

Fig. 3 Dynamic Shear Modulus versus shear strain amplitude for Monterey No. O sand with Dr=25, 43, 60 and 80%

Importantly it is obsereved that for strains less than 10^{-4}%, the G and E values remain constant, hence can be called maximum dynamic shear moduli(G_{max}) and maximum dynamic Young's Moduli (E_{max}) respectively. Any dynamic stress-strain relation should capture the nonlinear variation of G and E for strains greater than 10^{-4}%. The increase of D_s and D_l with strain is also nonlinear. Generally D_l are found to be smaller than D_s for similar conditions, The shape of D_l versus ϵ curves is more often erratic than Ds versus γ curves.

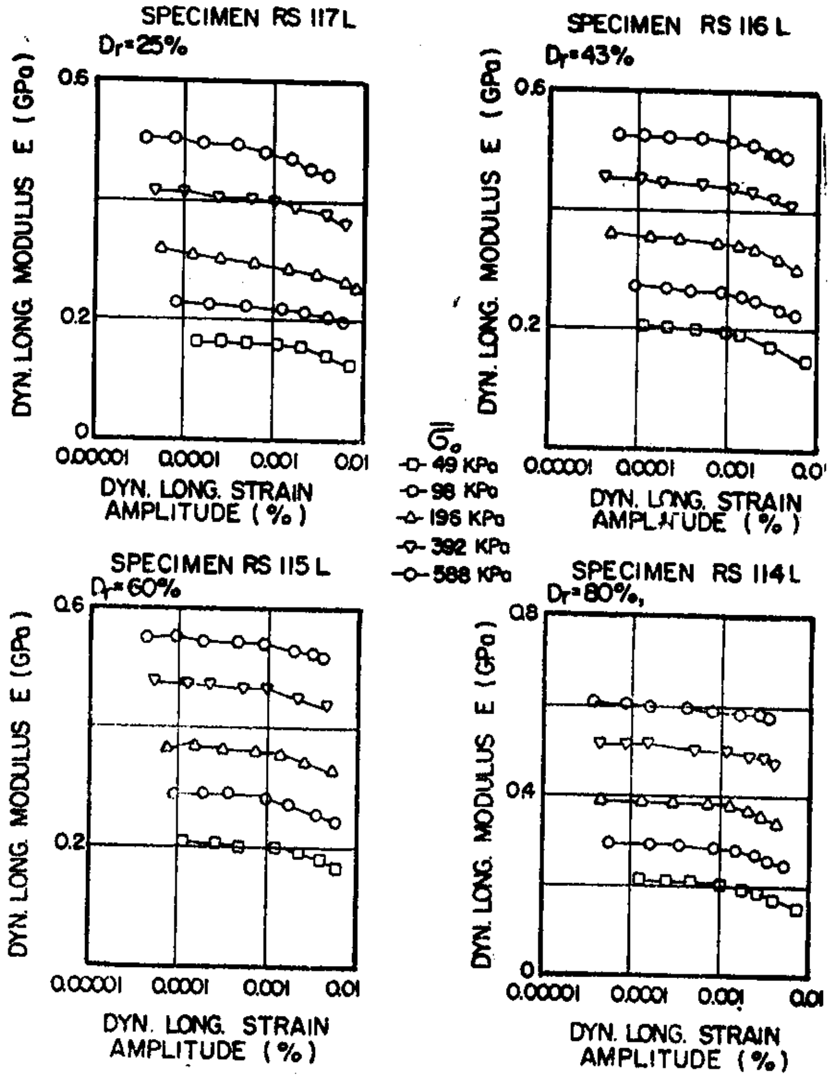

Fig. 4 Dynamic Young's Modulus versus Longitudinal Strain for Monterey No. O sand with Dr=25, 43, 60 and 80%

From Figs. 3/6, it can be concluded that with the increase of confining Pressure ($\bar{\sigma}_0$), the values of G and E also increase whereas D_s and D_l decrease with increase of confining pressure. The increase of moduli is mainly due to larger changes in void ratio causing the soil to become stiffer. The decrease of damping ratio is mainly due to more intergrain contacts as a result of higher confining pressures, thereby

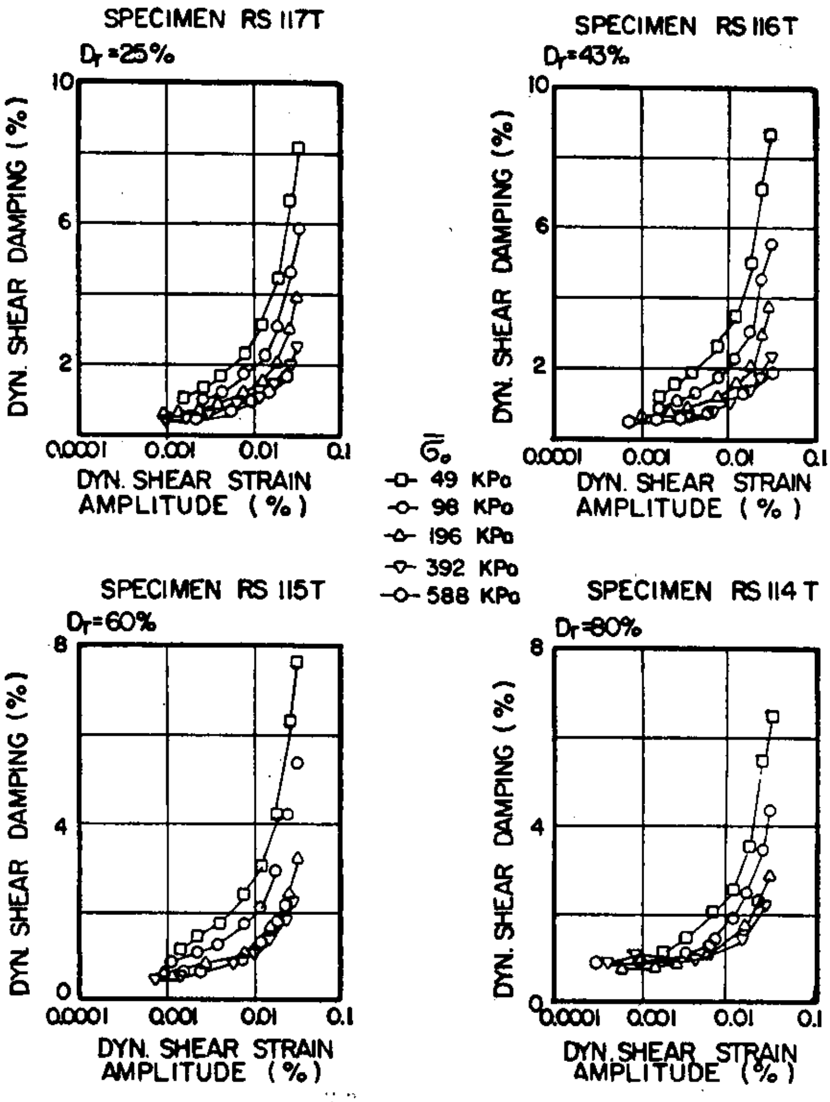

Fig. 5 Dynamic Shear Damping versus shear Strain Amplitude for Monterey No. O sand with Dr=25, 43, 60 and 80%

creating more wave travel paths and lesser dissipation of energy during wave propagation.

The effects of relative density, grain size and its distribution are indirectly reflected by void ratio (e). The increase of G and E with decrease of void ratio is clearly exhibited. The increase of G and

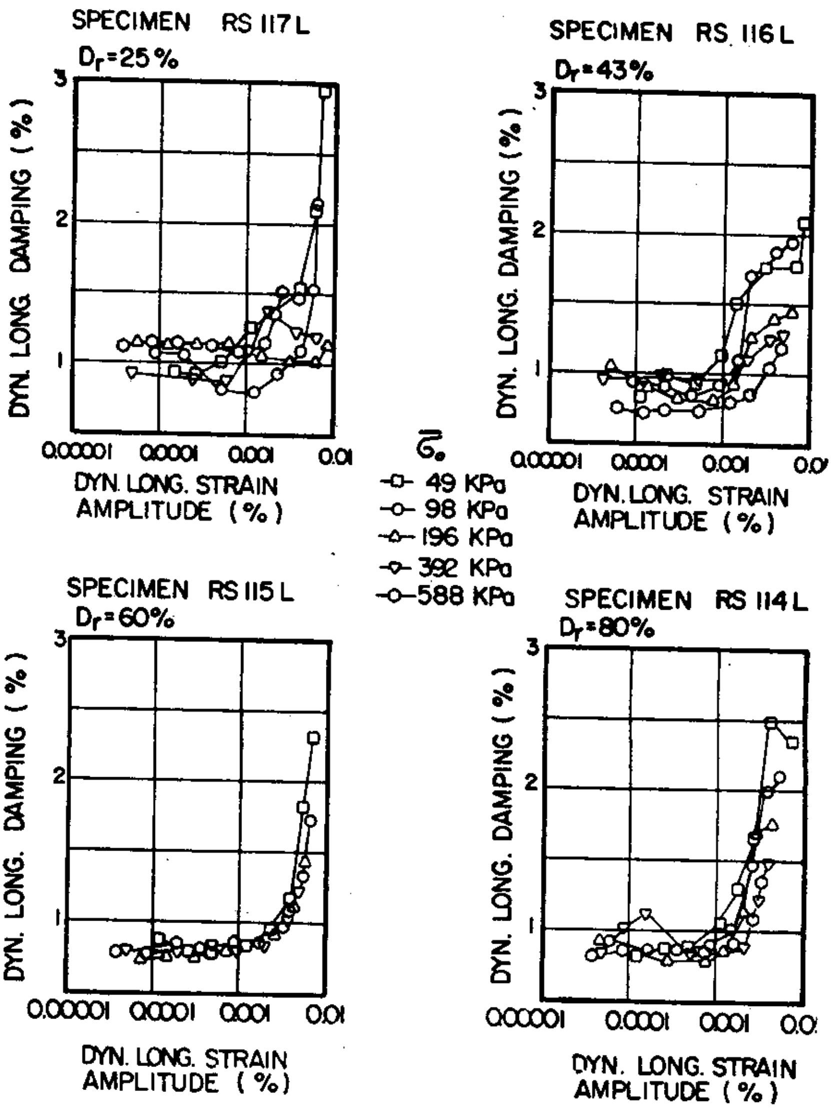

Fig. 6 Dynamic Longitudinal Damping versus Longitudinal Strain Amplitude for Monterey No. O sand with Dr=25, 43, 60 nd 80%

E with decrease of void ratio strongly depends on the increase in effective confining pressure. It may be observed from Fig. 7 that D_s and D_l are not affected stongly by void ratio.

Dynamic Shear Modulus : A modified relationship among G_{max}, e and $\bar{\sigma}_0$ obtained by regression analysis of the observed data Fig.8 is :

$$G_{max}=\frac{428.2}{(0.3+0.7e^2)}(P_a)^{0.426}\left(\bar{\sigma}_0\right)^{0.574} \quad (16)$$

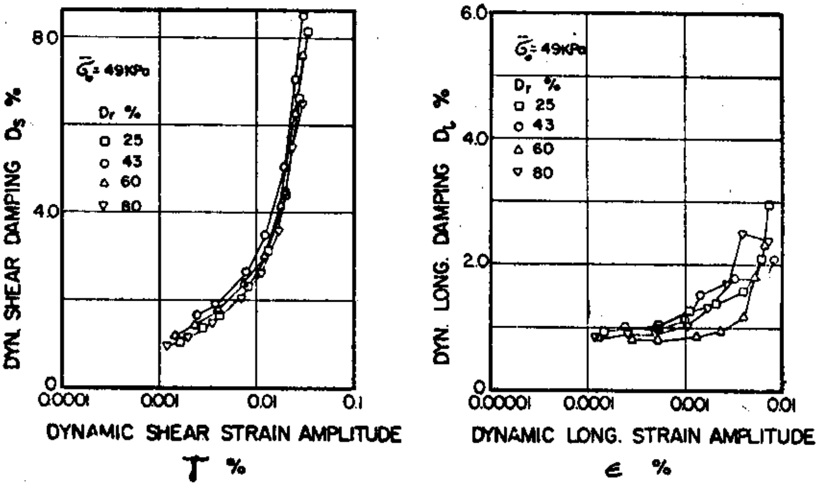

Fig. 7 Effect of Relative Density on Shear and Longitudinal Damping

where P_a is atmospheric pressure expressed in the same system of units as $\bar{\sigma}_0$. The coefficients in the above expression differ from those reported by earlier investigators. First of all, the above relation is based on tests on dry Monterey No. 0 sand for strains less than 10^{-3}%, void ratios 0.618 to 0 7775 (D_r=25 to 80%) and effective confining pressure from 49 to 588 KPa. The second noteworthv point is that this equation allows the shear modulus to approach zero only when void ratio approaches infinity. It may be pointed out that the void ratio function $1/(0.3+0.7\,e^2)$ is adopted as suggested by Hardin (1978) and also adopted by Chung et al. (1984). Thirdly this equation is applicable in all system of units because it is dimensionally correct.

Chung et. al. (1984) reported the data from resonant column testing using Monterey No. 0 sand with similar sample preparation technique for e = 0.676 (D_r = 60%) and $\bar{\sigma}_0$ ranging from 10 to 300 KPa. Defining maximum shear modulus at strain less than 10^{-3}% a relation was developed by them. However, their results are limited to one value of void ratio (e = 0.676). Drnevich (1978) conducted tests on Monterey No. 0 sand and obtained a relation for G_{max}. Since the soils used and testing technique are same, a comparison for G_{max} from these relation with Eqn. 16 for e = 0.676 (D_r = 60%) with effective confining pressure is shown in Fig. 9. It can be observed that Eqn. 16 predicts G_{max} values close to the results of Chung et. al. (1984). The Drnevich (1978) rela-

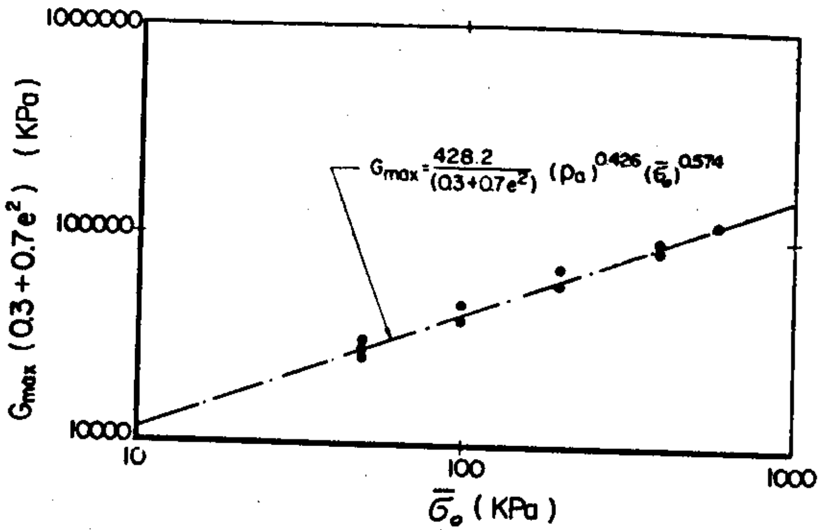

Fig. 8 Effective Confining Pressure versus Maximum Shear Modulus

tion overpredicts the G_{max} values, and the difference is significant at high confining pressures. Figure 10 presents such relationships for different soils along with the names of investigators. It may be noted that relations developed for one particular soils may not be applicable for another.

Dynamic Young's Modulus : The moduli values shown in Table 6 are compiled from the paper by Skoglund et. al. (1976) and are average values obtained independently by six investigators on uniform sand with an average void ratio of 0.635. The Poisson's ratio (ν) values are computed from theory of elasticity and listed in the table. It can be observed that the values of dynamic longitudinal moduli are about 2 to 3 times the dynamic shear modulus at the same confining pressure. However, such a conclusion may not be true all the time because of the basic variations in longitudinal vibration from those of torsional ones. The Poisson's ratio values obtained appear reasonable for sands. Importantly, the Poisson's ratio, as can be seen from the values in the table, decreases with an increase in the confining pressure.

The derivation of E_{max} from G_{max} requires an input of Poisson's ratio. The dependence of the values of Poisson's ratio on other parameters has been a matter of disagreement among researchers. For example, Ohsaki and Iwasaki (1973) observed that (*i*) dynamic Poisson's

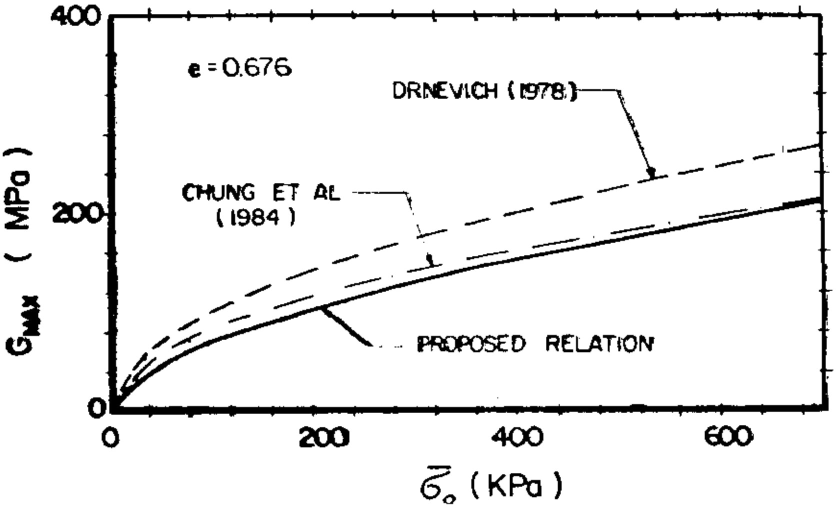

Fig. 9 Comparison of Proposed Relation and Available Relation for G_{max} for Monterey No. O sand

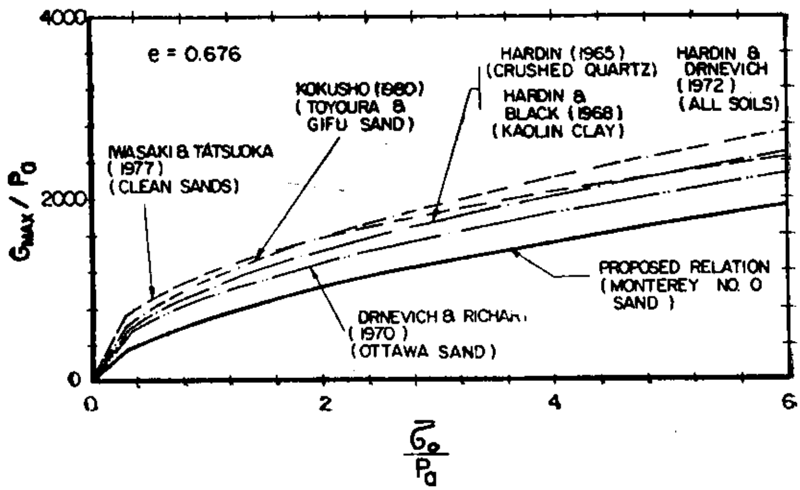

Fig. 10 Comparison of Proposed Relation for Monterey No. O sand with Reported Relations for Other Types of Soils

ratio increase with decrease of shear modulus and approaches 0.5 and (ii) different soil types cause no definite difference in values of Poisson's ratio. On the other hand, Hara (1973) disagrees with the above observations based on static and dynamic triaxial tests on clays and concludes that dynamic Poisson's ratios are not significantly influenced by the moduli, shear strain levels and frequencies or load application.

The relation developed by Ohasaki and Iwasaki for sandy soils, is presented as follows.

$$v=0.2+0.3\left[1-\frac{1}{16}(\log\ G_{max}-2)^2\right]^{1/2} \qquad (17)$$

The above relation is found valid for $500 < G_{max} < 100000$. The units of G_{max} are in tonnes per square meter.

Based on the test results of present investigation the maximum dynamic longitudinal modulus of Monterey Sand can be closely approximated by regression analysis Fig 11 as:

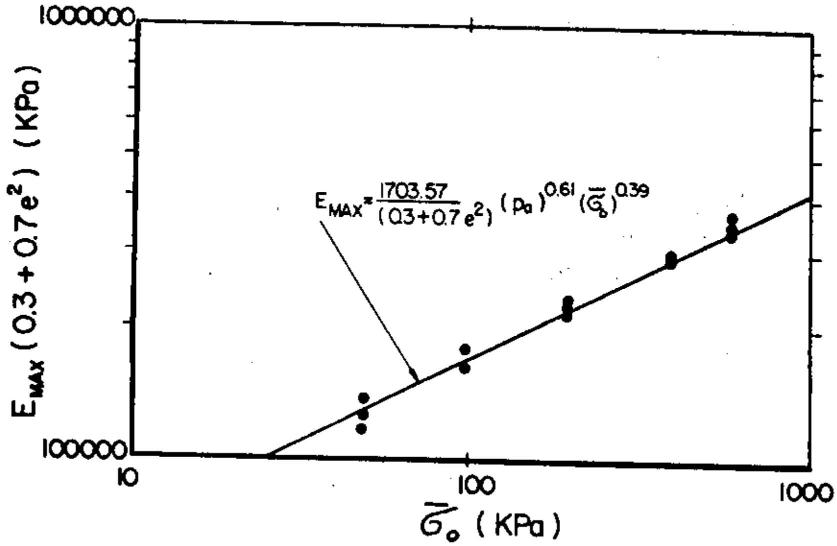

Fig. 11 Effective Confining Pressure versus Maximum Young's Modulus

$$Emax=\frac{1703.57}{(0.3+0.7e^2)}(Pa)^{0.61}(\bar{\sigma}_0)^{0.39} \qquad (18)$$

Since no such relation and no experimental data on Monterey No. 0 sand with samples prepared from method of undercompaction are available, no attempt is made herein to verify the validity of Eqn. 18. From the relations proposed for G_{max} and E_{max} (i.e. Eqn. 16 and 18), the ratio of maximum dynamic longitudinal modulus E_{max} to maximum dynamic shear modulus G_{max} is obtained and is as follows:

$$E_{max}=3.98\ G_{max}\ (Pa/\bar{\sigma}_0)^{0.184} \qquad (19)$$

From the theory of elasticity, Poisson's ratio can be expressed as:

$$v=0.5\ (E_{max}/G_{max})-1 \qquad (20)$$

Substituting from Eqn. 19, the empirical equation for dynamic Poisson's ratio becomes as given in Eqn. 21:

$$v=2(Pa/\bar{\sigma}_0)^{0.184}-1.0 \quad (21)$$

Eqn. 21 suggests that Poisson's ratio is dependent on effective confining pressure and as confining pressure increases the Poisson's ratio decrease. Similar observation was made by Skoglund et al. (1976) as shown in Table 6. However. the difficulty for finding reliable Poisson's ratio values from empirical relations such as Eqn. 21 has been recognized by many investigators for a long time. For instance, Hardin and Richart (1963) computed values of Poisson's ratio from the results for the shear and compressive wave velocities for dynamic shear and Young's moduli) of dry Ottawa sand. They found that the Poisson's ratio values computed based on the relationship between the velocities (or dynamic moduli) generates silly values except in higher ranges of confining pressure. The main reason for this is that a small error in the ratio of longitudinal and shear moduli can produce very large error in Poisson's ratio values. Hence, Poisson's ratios computed from empirical relations are not considered to be reliable.

Dynamic Shear Damping: The effect of density on dynamic shear damping is insignificant as observed from Fig. 7. This fact is supported by the observations of Hardin (1965), Tatsuoka et al. (1978), and Sherif et al. (1977). The test results can be best expressed by :

$$D_s=9.22\left(\frac{\bar{\sigma}0}{P_a}\right)^{-0.38}\gamma^{0.33} \quad (22)$$

The units $\bar{\sigma}0$ and P_a must be the same and γ must be expressed in percentage, hence the resulting value of damping would be in percent. Eqn. 22 has a distinct advantage over the reported relations; its being applicable to all systems of units, because it is non-dimensional.

Dynamic Longitudinal Damping : Fig. 7 also demonstrates the negligible effect of density on dynamic longitudinal damping values(D_l). Based on the statistical analysis of the data, the dynamic longitudinal damping can be expressed by :

$$D_l=\left(\frac{\bar{\sigma}_0}{Pa}\right)^{-0.13}\varepsilon^{0.33} \quad (23)$$

Eqn. 23 is also non-dimensional. The strain (ε) and damping are expressed in percentage. Comparing Eqn. 22 and 23, the dynamic shear dynamic shear damping (Ds) wirh $\gamma=\varepsilon$ and is expressed by Eqn. 24.

$$D_l = 1.08 D_s \left(\frac{\bar{\sigma}_0}{P\alpha}\right)^{0.25} \qquad (24)$$

It should be noted that D_l is the dynamic longitudinal damping in longitudinal mode of vibration and D_s is the dynamic shear damping in torsional mode of vibration. Depending upon the predominant mode of vibration, one of these values should be considered. In a real situation both dynamic longitudinal and shear damping exist during vibrations. The value of dynamic shear damping in longitudinal mode of vibration or dynamic longitudinal damping in torsional mode of vibration are difficult to assess, hence the proposed relations may help in estimating such values.

EVALUATION OF RELATIONSHIPS

The proposed relation for maximum dynamic shear modulus G_{max} (Eqn. 16) best fits the experimentally measured data as shown in Fig. 12. The experimental data are based on well controlled resonant column

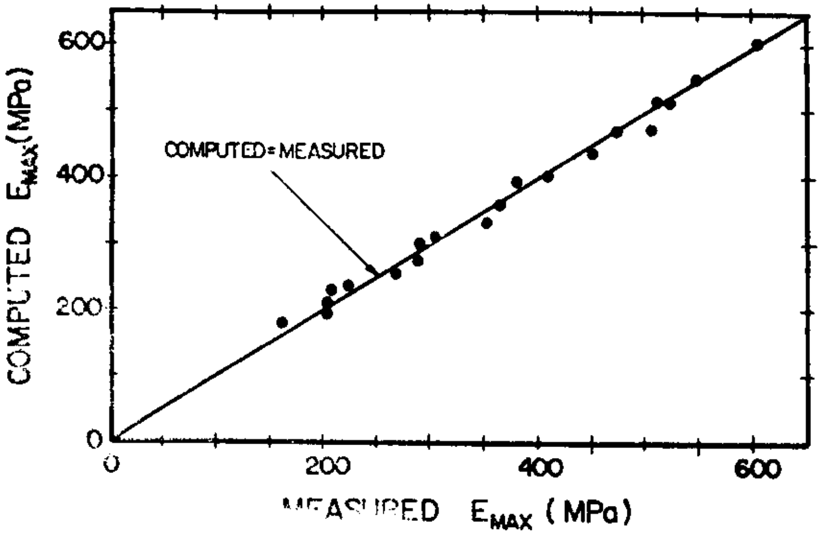

Fig. 12 Evaluation of Relationships for Maximum Shear Modulus

tests with samples prepared by method of undercompaction. As a matter of interest, the G_{max} values from different reported relations are compared with the measured values in Fig. 13. It can be inferred that the relation proposed by Seed and Idriss (1970) with density factor $K2_{max}$ equal to 40, also gave results similar to the measured values.

However, the K_{1max} value which depeds on density conditions is hard to select correctly. The relation proposed by Chung et al.

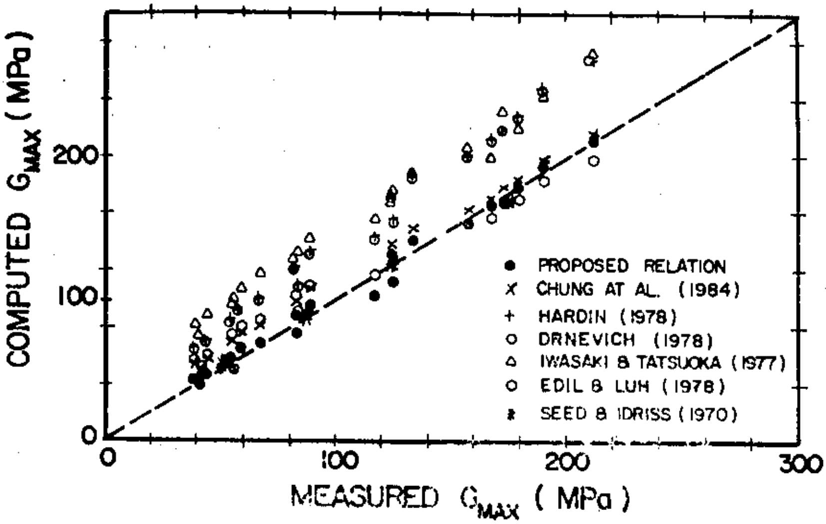

Fig. 13 Evaluation of Relationships for Maximum Shear Modulus as compared to Experimental Results

(1984) predicts the measured Gmax at higher confining pressures. However at low confining pressures, the difference between their prediction and our measured values is significant, perhaps because the relation is based on very limited test data. The relation proposed by Edil and Luh (1978) underestimates G_{max} at lower confining pressures. The relations proposed by Hardin (1978) and Drnevich (1978) are found to overpredict G_{max} values in case of Monterey No. 0 sand.

The proposed equation for dynamic longitudinal modulus, E_{max} (Eqn. 3) fits the experimentally determined values as shown in Fig. 14. The relation may be helpful in a situation where the longitudinal mode of vibration is dominant. More field and laboratory test data is, however, needed to put this relation on a firm foundation.

Fig. 15 shows the variation of dynamic shear damping with dynamic shear strain for a particular case of effective confining pressure equal to 98 KPa and relative density equals to 43% (e=0.7253). At low strains, there is no appreciable difference among measured values, proposed and reported relations. However at large strains, the difference in value of damping is significant as shown in Fig. 15. Previously Edil and Luh (1978) presented shear damping result as shown in

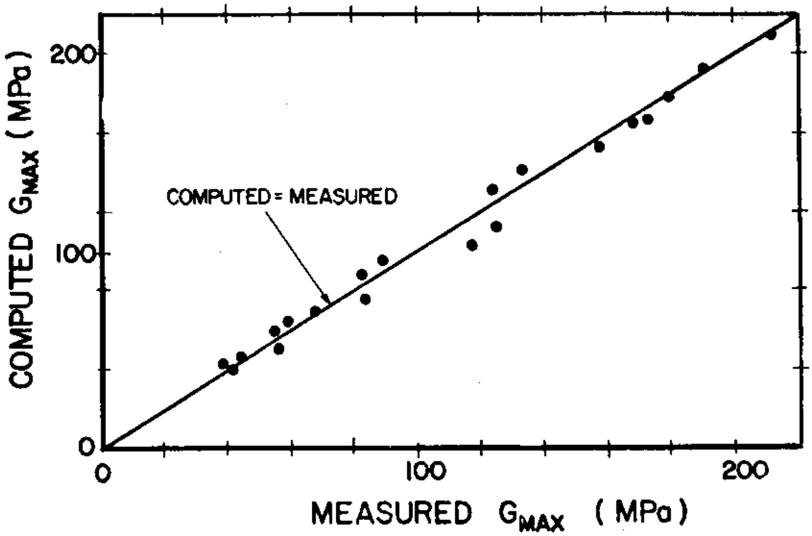

Fig. 14 Evaluation of Relationship for Maximum Young's Modulus

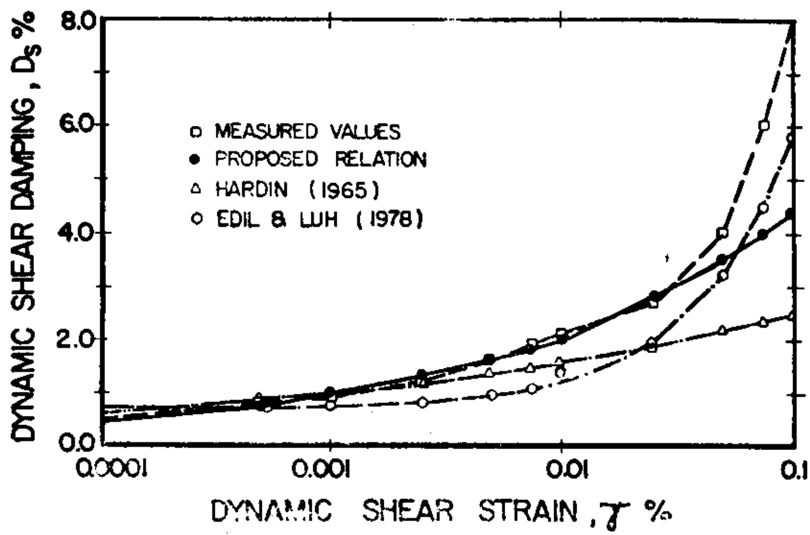

Fig. 15 Comprison of Shear Damping values for $\bar{\sigma}_0=98$ Kpa and e=0. 7253

Fig. 16(a) and the reason attributed was the experimental error while determining damping values. However, in this study a less scatter is observed with the proposed relation for shear damping as shown in Fig. 16(b).

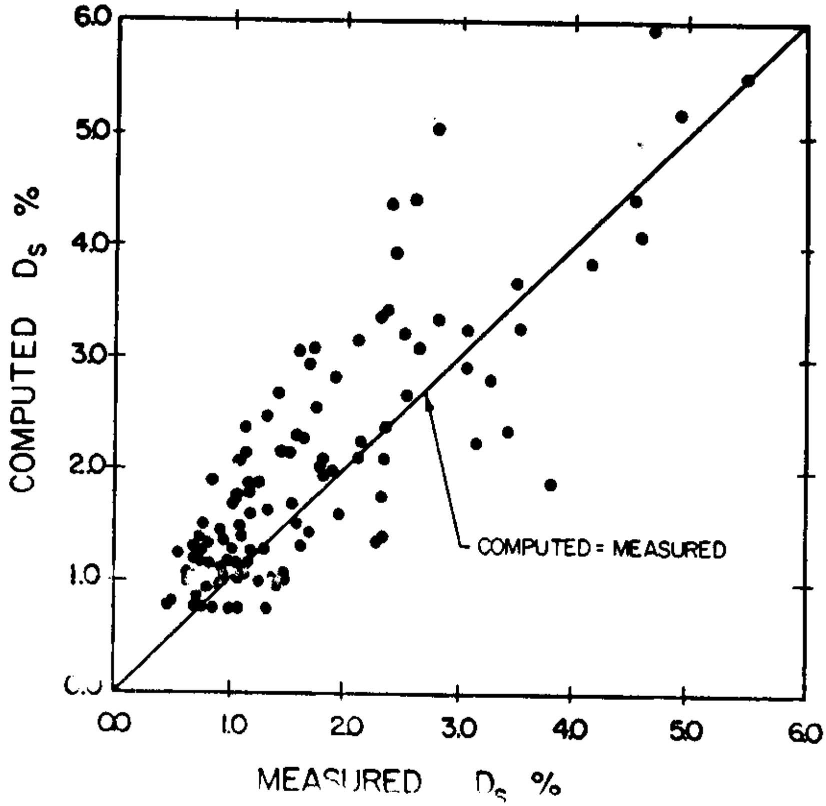

Fig. 16(a) Evaluation of Releationship for Shear Damping (Edil and Luh, 1978)

The measured dynamic longitudinal dynamic damping is compared with computed values from the proposed relationship in Fig. 17. The large difference between computed and measured values can not be attributed solely to experimental error since the shear damping values obtained from the same experiments appear to be correct. The difference therefore may be partly caused by the computational technique associated with the determination of longitudinal damping. Secondly, small experimental error may effect the values of longitudinal damping more than the shear damping values. The authors are currently undertaking an analytical study to explore this aspect in depth. However, the relation developed for longitudinal damping is proposed for the first time and its validation by more laboratory and field data is a task for the future. For the present, assuming longitudinal damping about 3% for strains less than 10^{-4} and

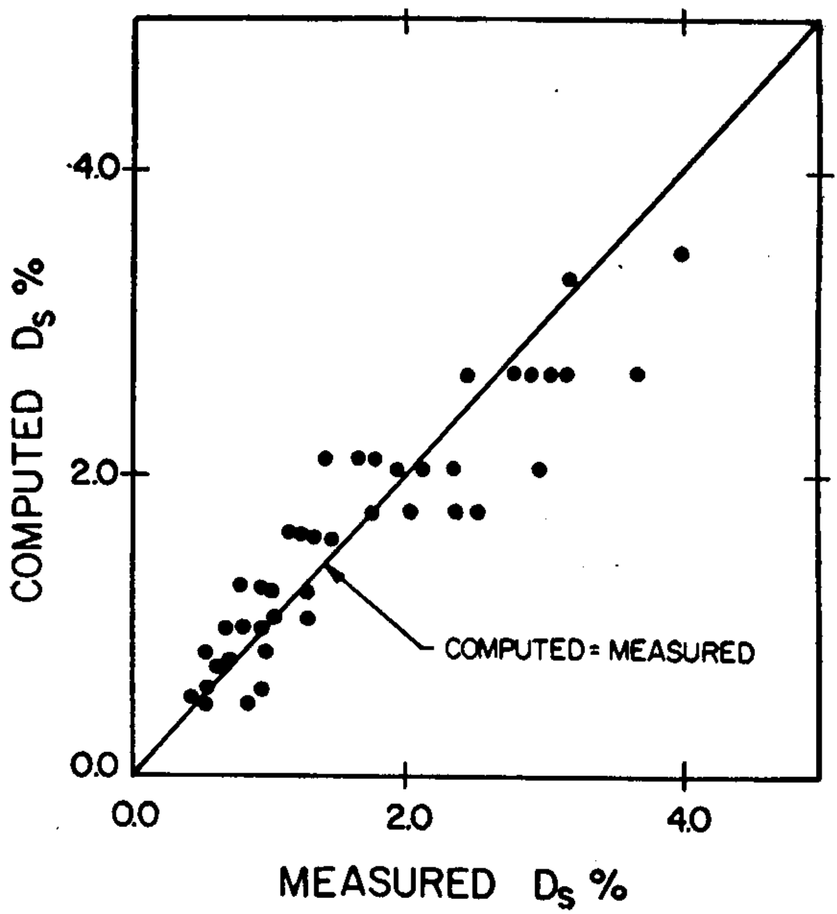

Fig. 16(b) Evaluation of Relationship for Shear Damping from Present Study

12% for strains above 10^{-6} appears to be good for solutions of practical problems.

CONCLUSIONS

The following conclusions are drawn from this study :

1. The reported relations for dynamic shear modulus (G_{max}), dynamic Young's modulus (E_{max}), dynamic shear damping (D_s) and dynamic longitudinal damping (D_l) for sands at low strain levels are listed and discussed.

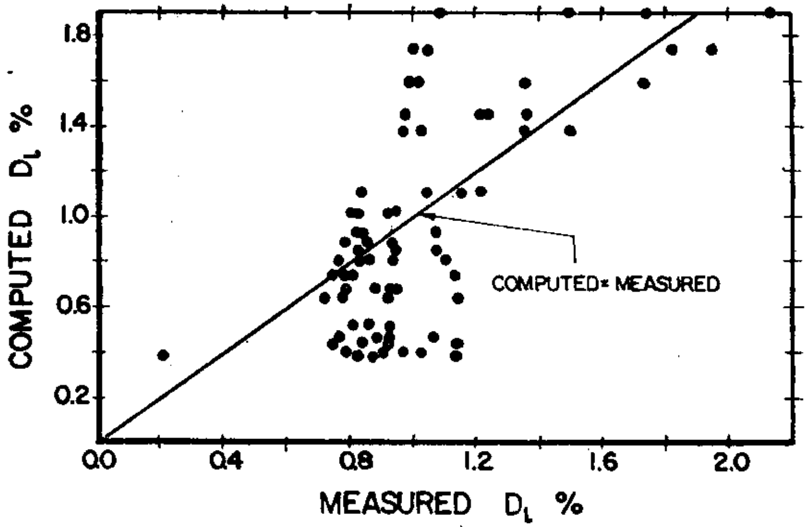

Fig. 17 Evaluation of Relationship for Longitudinal Damping

2. Based on experimental results with resonant column device on Monterey No. 0 sand, new and modified relations for G_{max}, E_{max}, D_s and Deare proposed. These relations being non-dimensional can be used for any system of Units.

3. An expression for computation of dynamic Poisson's ratio is given and its validity discussed.

4. The developed relations for G_{max} and D_s are compared with experimental results and other reported relations. It is found that hitherto previously reported relations overpredict moduli and underpredict damping in case of Monterey No. 0 sand.

5. The relation of E_{max} and D_l are proposed afresh. More field and experimental results are required to establish these relations accurately.

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Table . 1 Reported Relationships for G_{max} estimation

Soil Type	Empirical Relation	Strain γ	Valid Units G_{max}	Valid Units $\bar{\sigma}_0$	Ref.
Ottawa Sand	For $\bar{\sigma}_0 > 2000$ Psf $G_{max} = [(32.17 - 14.8e)^2/(1+e)]\bar{\sigma}_0{}^{0.5}$ For $\bar{\sigma}_0 < 2000$ Psf $G_{max} = [(22.52 - 10.6e)^2/(1+e)]\bar{\sigma}_0{}^{0.5}$	10^{-5}	Psi	Psf	Hardin (1965)
Kaolin Clay	$G_{max} = 1230[(2.973 - e)^2/(1+e)]\bar{\sigma}_0^{0.5}$	10^{-4}	Psi	Psi	Hardin & Black (1968)
Clays and Sands	$G_{max} = 1230[(2.97 - e)^2/(1+e)]\bar{\sigma}_0^{0.5}$	10^{-5}	Psi	Psi	Hardin and Drenevich (1972)
Clean Sands	$G_{max} = 900[(2.17 - e)^2/(1+e)]\bar{\sigma}_0^{0.4}$	10^{-6}	kg/cm^2	kg/cm^2	Iwasaki & Tatsuoka (1977)
Clays and Sands	$G_{max} = [625(OCR)^k/(0.3 + 0.7e^2)](P_a\bar{\sigma}_0)^{0.5}$	10^{-5}	any units	same as G_{max}	Hardin (1978)
Mont. No.O Sand	$G_{max} = 1230[(2.973 - e)^2/(1+e)]\bar{\sigma}_0^{0.5}$	10^{-5}	Psi	Psi	Drenevich (1978)
Sand	$G_{max} = 840[(2.17 - e)^2/(1+e)]\bar{\sigma}_0^{0.5}$	10^{-6}	kg/cm^2	kg/cm^2	Kokusho (1980)
Mont. No.O Sand	$G_{max} = [523/(0.3 + 0.7e^2)]P_a^{0.52}\bar{\sigma}_0^{0.48}$	10^{-5}	any. units	same as G_{max}	Chung & Others (1984)

Table 2 Values of k (Hardin and Drnevich, 1972)

PLASTICITY INDEX PI	CONSTANT k
0	0
20	0.18
40	0.30
60	0.41
80	0.48
> 100	0.50

Table 3 Values of Constants for Equn. 1 (Isenhower, 1979)

CONSTANT	VALUES BASED ON		
	SOILD SPECIMEN	HOLLOW SPECIMEN	BOTH
A	-505.7	535.8	445.6
B	-62.3	17.9	65.2
C	1082.3	-1009.9	-963.2
D	61.1	-15.4	-69.0
E	-1.9	-0.1	2.0
F	-567.0	484.4	529.7

Note: G is in Psf, $\bar{\sigma}_o$ is in Psi, and ρ is in Slugs/cu ft.

Table 4 Values of K_{2max} for Eqn 3
(Seed and Idriss, 1970)

SOIL	LOCATION	DEPTH (ft)	K_{2max}
Loose moist sand	Minnesota	10	34
Dense dry sand	Washington	10	44
Dense saturated sand	S. California	50	58
Dense saturated sand	Georgia	200	60
Dense saturated silty sand	Georgia	60	65
Dense saturated sand	S. California	300	72
Extremely dense silty sand	S.California	125	86
Dense dry sand (slightly cemented)	Washington	65	166
Moist clayey sand	Georgia	30	119

Table 5 Values of D_{max} for Eqn 10 (Hardin and Drnevich, 1972)

SOIL TYPE	VALUE OF D_{max} %
Clean dry Sands	$33 - 1.5 \ \text{Log} \ N$
Clean saturated sands	$28 - 1.5 \ \text{Log} \ N$
Saturated Lick Creek silt	$26 - 4(\bar{\sigma}_o)^{0.5} + 0.7 f - 1.5 \ \text{Log} \ N$
Various saturated cohesive soils including Rhodes Creek Clay	$31 - (3 + 0.03 f)(\bar{\sigma}_o)^{0.5} + 1.5 f - 1.5 \ \text{Log} \ N$

Note: f is in cycles per second and $\bar{\sigma}_o$ is in Kg/sq cm