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ON INVERSION OF SLOWNESS AND TRAVEL TIME DATA FROM ARRAYS[1]

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Introduction

The travel times of seismic body waves, which travel through the interior of the earth, are the most directly measurable quantities used in deriving the internal structure of the earth. These are directly related to the velocity distribution in the interior by the Wiechert-Herglotz equation through their distance derivative, $dT/d\Delta$. The process of arriving at the velocity distribution using the measured values of travel time and slowness p, which is defined as $dT/d\Delta$, falls under the purview of inversion. Because such derivations are normally based on a set of data which are incomplete and inaccurate, the derived models are not unique (Backus and Gilbert, 1967, 68, 70; Dzeiwonski, 1970, Johnson and Gilbert, 1972). The procedure, which is used to determine the velocity structure in the interior of the earth, requires a $p(dT/d\Delta) - \Delta$ curve over the entire range of Δ and involves solving the Wiechert-Herglotz equation (Bullen, 1963)

$$ln\,(r_0/r_1) = \frac{1}{\pi}\int_0^{\Delta_1} \mathrm{Cosh}^{-1}\,(p/p_1).\,d\,\Delta$$

where r_0 is the mean radius of the earth and p_1 is the ray parameter for the ray which penetrates a depth r_0-r_1 and reaches the surface at a distance Δ_1 from the source. A more efficient approach is that of Gerver and Markushevich (1966) where one solves the equation

$$d(p) = \int_p^{p_s} \frac{\Delta(q)\,dq}{(q^2-p^2)^{1/2}} + \frac{2}{\pi}\int_{y_k}^{y_k'} \tan^{-1}\left[\frac{u^{-2}(y)-p_k^2}{p_k^2-p^2}\right]^{1/2} dy$$

for the depth of penetration d(p). Here y_k and y_k' are depths to the top and bottom of the k^{th} low velocity zone (LVZ) and p_k is the value of the ray parameter at which the k^{th} LVZ produces the shadow zone. u(y) is the velocity at a depth y. This method allows the determination of the velocity structure in the presence of one or more LVZs. It has been proved by Gerver and Markushevich that if sources are located between the LVZs the velocity structures can be determined accurately everywhere except within the LVZs. However, in actual array data, there are inaccuracies in the observations which are introduced by measurement errors, azimuthal variations, lateral inhomogeneity etc. Also, the ray parameter p is not known for all values of Δ leaving the data incomplete. There may be present LVZs at certain depths and the corresponding shadow zones may not be well determined. Such uncertainties in the data reflect on the derived velocity distribution. McMechan and Wiggins (1972) have developed an extremal inversion technique where an envelope of the $p-\Delta$ values, instead of the $p-\Delta$ curve is inverted

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to give a velocity-depth (V–D) envelope, so that (with a given set of data) the range of allowed velocities at a given depth (or allowed depth range for a given velocity) is computed. In this paper the extremal inversion technique has been exploited to provide the estimates of uncertainties involved while inverting array data.

2. Constructing the p–Δ envelope

Figure 1 shows the results of slowness measurements which were obtained on the basis of an optimum signal-to-noise ratio array beam (Birtill and Whiteway, 1965) for over one hundred and fifty events from the Banda Sea, Mindano and Philippine regions recorded at the Warramunga Seismic Array (WRA). In this figure, slownesses for which maxima were observed in the array beam computed every half second interval for 30 seconds from the beginning of the signal onset are plotted. Because it is not convenient to superimpose the amplitudes of the measured phases on this plot, the data are presented using four different sets. In Figure 1 (a) all slownesses, for which the maxima were observed in the array beam, are plotted. In Figure 1 (b) a minimum threshold has been used so that maxima for which the Nth root correlator output (which is obtained by adding array outputs into two groups, multiplying the two partial sums and averaging the product over an interval of time) is less than 5% of the largest value within the observation interval are excluded. In Figure 1 (c) and 1 (d) this threshold figure has been raised to 10% and 15% respectively. Becuase of

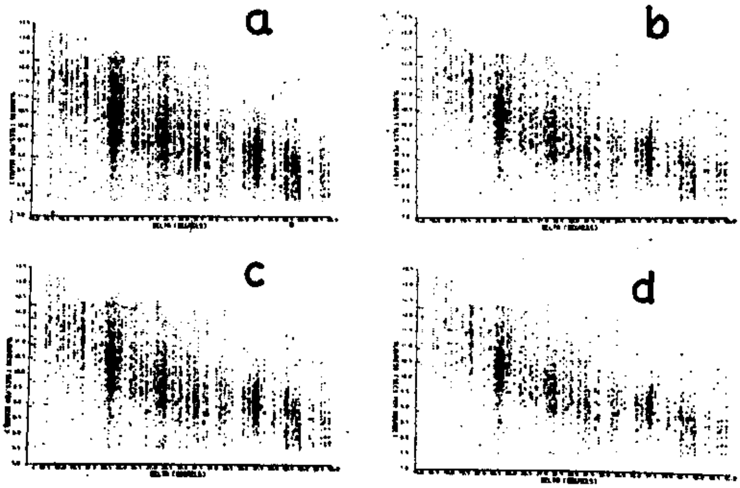

Figure 1 Measured slowness for events recorded at WRA. In Figure a-d, slownesses for which the N^{th} root TAP exceeded 0 5,10 and 15%, respectively, of the maximum TAP output along the trace are plotted as a function of the epicentral distance (DELTA)

the nonlinenr nature of the processing technique and the difference in the magnitudes of different events it is not possible to translate these ratios into the actual amplitudes. Introduction of the threshold improves the p–Δ plot in the sense that it becomes less complex, but there is no way to remove from the plot only those points which are produced either by interfering effects between various branches of the travel time curve or varied noise conditions. Considering the nature of these plots, it is also clear that the possibility of constructing a reliable p—Δ curve, or an envelope, is very much remote. To reduce the effects of data which are inaccurate and uncertain, and to avoid introducing large gaps (which would be created if only those parts of the data where accurate measurements are available, are used) an alternative approach is adopted. Here, observable jumps in the slowness of the first arrival are first determined on the basis of the first arrival data alone. If the jumps so determined are real, they should produce later arrival branches in certain epicentral distance ranges. Therefore in order to establish the authenticity of these jumps, the data are searched for later arrivals corresponding to the banches which are expected on the basis of the first arrival data. These branches, if observed, are then traced in the T—Δ plane and an empirical travel time curve is constructed to satisfy the observed data. Assuming the latest available model of the region for the upper layers (Simpson 1973) the velocities are adjusted to provide an approximate fit to the postulated branches. The travel time model and the corresponding velocity model, which were derived are illustrated in Figure 2 (Ram Datt, 1977). It should be pointed out here, however, that arrival on the E and F branches have been observed on most events up to epicentral distances as far as 50°. If these arrivals are associated with the triplication branches, then negative velocity gradients

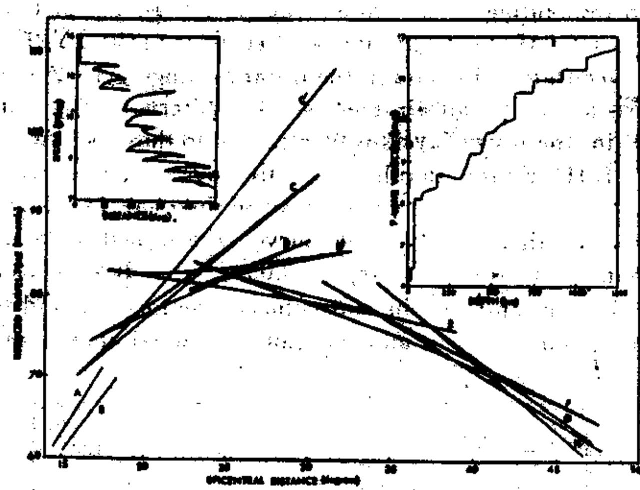

Figure 2. Velocity model derived on the basis of WRA data in the distance range 14-46° and the corresponding slowness-epicentral distance (p-Δ) and travel time-epicentral distance (T-Δ) curves

are required in the layers which lie in the depth intervals 650—750 km and 750—920 km. Very small negative velocity gradients (of the order of 0.1 km/sec. per 100 km), which do not produce shadow zones (dv/dr < v/r), will make the arrivals on these branches to appear upto very large distances, though with very small amplitudes. An alternative, which has been considered to account for the observed extensions of the E and F branches, is in terms of the underside reflections produced by the interfaces (Ram Datt and Muirhead, 1977). As no evidence has accumulated on the basis of the analysed data to discriminate between these alternatives, flat velocity gradients have been chosen between the depths of the discontinuities, and hence are the shown travel time branches. Both the alternatives, however, demand sharp velocity gradients at the interfaces, so that the arbitrariness in the velocity model is not very large. Pending a choice among the alternatives, their implications in the determination of the velocity structure have been considered.

3. Non-uniqueness of the Model

It has been mentioned above that the travel time curve is only an approximation because it is possible to derive a set of velocity models, which satisfy this data. Hence no such model may be considered as unique. If accurate observations in the entire range of p are made available, and no LVZ is present, the velocity structure determined using these observations may be uniquely determined. If observations in a given distance range $0-\Delta_1$ are available only for a finite number of points, there exists a multitude of velocity models in the depth range $0-d_1$ that produce travel time curves conforming to the observations, d_1 being the bottoming depth for the ray which emerges at a distance Δ_1. For each of these models, given an accurate set of observations in the distance range $\Delta_1-\Delta_2$ ($\Delta_1 < \Delta_2$), there exists a velocity model which satisfies the observations in the distance range $\Delta_1-\Delta_2$. Therefore, the V–D (Velocity-depth) relationship in the depth range d_1-d_2 is not unique, though the data in corresponding epicentral distance range is complete and accurate. This type of non-uniqueness in the velocity model (let us call it TYPE I) always remains, irrespective of how accurately the data in the distance range $\Delta_1-\Delta_2$ are known, unless an accurate data set exists for the distance range $0-\Delta_1$. Alternatively, if the non-uniqueness of the V–D curve in the upper layers is transmitted to those below. The second type of non-uniqueness (TYPE II) in the depth range d_1-d_2 is caused by the errors in, and incompleteness of, the data for which the rays traverse these layers. Figure 3 shows a p–Δ curve for an earth model with no LVZ. The curve is assumed to be accurately known for p values greater than 12.5 s/deg (at steps of 0.05 s/deg). Beyond 12.5 s/deg the p values are assumed to be known only at the end-points of the forward and retrograde branches. In the latter data, measurement errors and small observational uncertainties (± 1 s and ± 1°

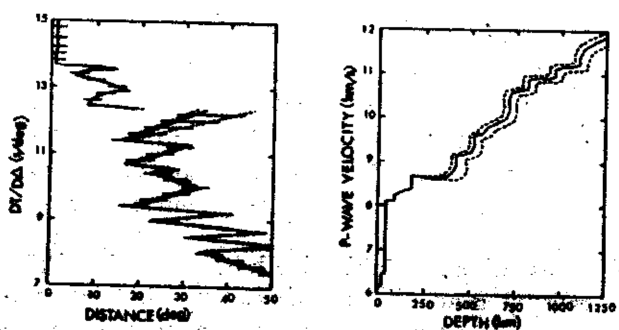

Figure 3. (p-Δ) and (V-D) envelope for an earth containing regions of high velocity gradients availability of data points with moderate accuracy (± 1° in Δ and ± 1 s in T) only at the end points of the triplication branches has been assumed

in travel time and epicentral distance respectively) are also allowed. The p—Δ ridge so determined is shown by broken lines in the diagram. On the right hand side of this figure is shown the V—D envelope obtained by using the extremal inversion method. The solid curve in this figure represents an average model (McMechan and Wiggins, 1972) It is apparent from this figure that the finite width of the model has been produced by the inaccurate and incomplete data for rays which traversed deeper than 175 km. Now consider a larger uncertainty at the point P, where the forward branch bottoming at depths between 175 and 300 km terminated (and the corresponding retrograde branch begins). Keeping the rest of the data identical to that of the previous figure the p—Δ and V-D envelopes are shown in Figure 4. The increase in the width of the V—D envelope at depths down to 1250 km has apparently been caused by additional uncertainty at the point P. It is therefore necessary to assess these two types of non-uniqueness when considering the validity of any model which is derived through array measurements. A third type of non-uniqueness is introduced by an LVZ and is more of a localized nature in the sense that

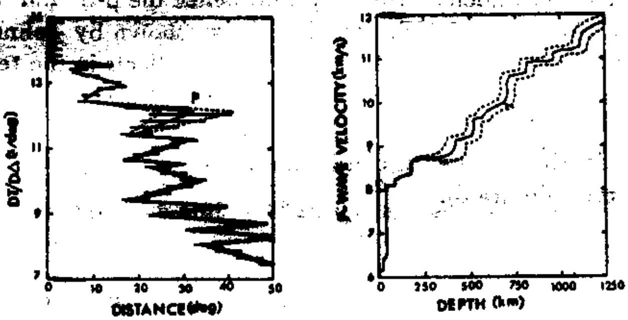

Figure 4. Same as figure 3, but with larger uncertainty in one of the travel time branches (Point P)

the velocity in the LVZ cannot be determined uniquely (except in an ideal case when the LVZ is effectively a discontinuity, so that velocity again increases in the LVZ after a rapid decrease and sources are located in the LVZ). This type of non-uniqueness is limited to the LVZ and the region immediately below. A more important aspect of the LVZ is, however, due to the fact that there are inherent ambiguities in the estimates of the epicentral distance at which the shadow zone begins, and that where the retrograde branch begins. Also the accuracies, with which the relevant parameters (travel time, slowness and hypocentre) are known, do not allow the estimate of the jump in the travel time intercept (Kennett, 1976)

$$\tau = T - \int p \, d\Delta$$

A suitable method of examining the implications of the observations in different distance ranges and estimating the uniqueness may be based on the extremal inversion method. In the following sections we examine the uniqueness of the velocity model, which was derived on the basis of Warramunga array data by considering the factors, which are responsible for uniqueness, separately.

3.1 Measurement Errors in the estimates of p-Δ and T-Δ curves

Suppose the p values which may be obtained by differentiating the travel time curve are available in the entire distance range at intervals Δp where Δp is as small as desired. If there are no inaccuracies in the p-Δ and T-Δ data, the velocity structure can be determined uniquely everywhere except in the LVZ, and a limited region below, provided the jump in the travel time intercept "τ" introduced by the LVZ is known accurately. Figure 5 shows an inversion model of the travel time curve of Figure 2. This model was constructed by differentiating the travel time curve so as to provide Δ values for p at intervals of 0.05 s/deg. The model has been averaged over 0.2 s/deg intervals of p. (This is because when the path of integration jumps from the right to the left the corresponding V-D curve is triple valued; for details see Wiggins et al 1973, pp. 93). Now consider the first cause of non-uniqueness, viz. uncertainty in the source location. To illustrate the effect an uncertainty of ± 1° is introduced for all p-Δ pairs for which p is less than 12.5 s/deg. A measurement error in travel times of ± 1 s has also been allowed to account for the uncertainty in travel times. In doing so, it has been the velocity structure, and hence the p-Δ and T-Δ data are known accurately for layers above the LVZ. (It has been shown by Johnson and Gilbert that uncertainties in the p values produce only second order effects in the results). The inaccuracies make the p-Δ envelope relatively wider near those values o p which correspond to the critical angles at the boundaries of the discontinuities and produce relatively larger uncertainties in the depths of the discontinuities. All other parts of the V-D curve are affected almost uniformly (Figure 6).

3.2 Incomplete Data

It is rarely possible to make accurate measurements in all distance ranges, either because data are not available or they are of poor quality in some places. First suppose that the uncertainties in the p—Δ and T—Δ data are of the same order as above but that the data points are available only at the end points of various branches. This assumption is based on the ease with which accuracy in measurements could be improved in those parts of the T—Δ plane where the travel time branches are well separated. Figure 7 shows the p—Δ and the corresponding V-D envelope for such data. Though the width of the V-D envelope

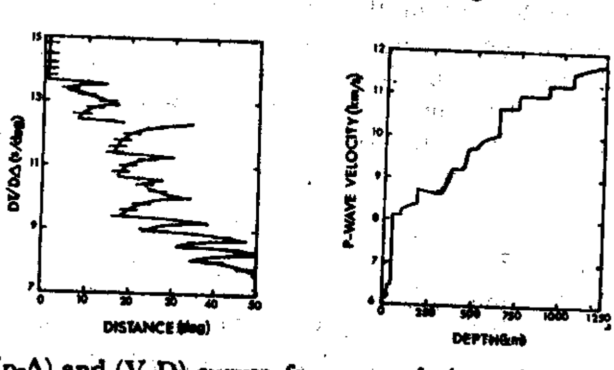

Figure 5. (p-Δ) and (V-D) curves for an earth (containing regions of high velocity gradients and a low velocity zone) for which accurate T-Δ values are available for all slownesses in steps of 0.05 sec/deg

in Figure 7 is not much different from that of Figure 6, significant difference exists between the V-D determinations with respect to the depth of the discontinuities. This effect results from the nature of the interpolation used to calculate p values at intermediate distances in order to fill the gaps. Linear interpolation reduces the curvature in the p—Δ curve and provides comparatively smaller estimates of travel times at all distances. This results in increased depth estimates for all velocities. In Figure 7, for example, the depth of the 650 km discontinuity has increased to 680 km, compared to 640 km in Figure 6. Finer details of the p—Δ curve, therefore, become more important in determining the depths of the discontinuities accurately.

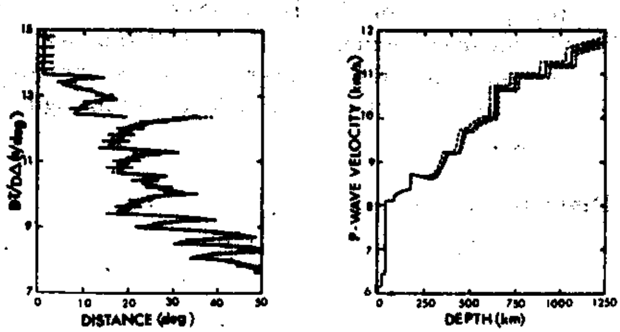

Figure 6. Same as Figure 5, except that observational errors of ± 1 s in travel time and ± 1° in epicentral distance are introduced

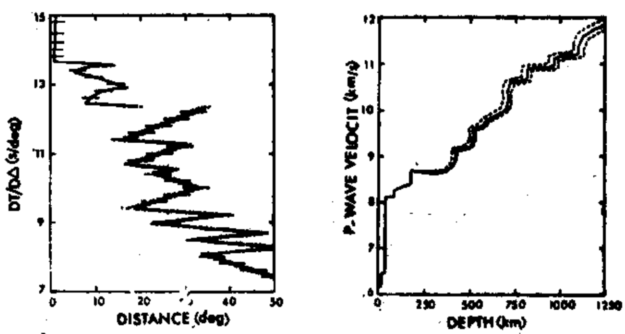

Figure 7. Same as Figure 6, when, the data for only the end-points of the travel time curve are available

3.3 Observational Uncertainties

There are always uncertainties in velocity models and hence the travel time curves, when they are constructed on the basis of observed data. For instance, in the travel time curve referred above, a low-velocity zone between 210 and 330 km depths has been postulated, though it has not been possible to determine exactly either the epicentral distance at which the shadow zone begins or that where the corresponding retrograde branch produced by the LVZ begins. Though this branch was identified to distances as far as 32°, its extension to larger distances is not ruled out (see the slowness data of Figure 1).

Assuming that the rest of the observations are similar to those of Figure 7, the effect of this uncertainty is illustrated in Figure 8. The widths of V-D envelopes in Figures 8 and 4 (a model with no LVZ) are very similar, as are the depths of the discontinuities. These figures demonstrate that the non-uniqueness at greater depths is similar, irrespective of the presence or absence of a LVZ. The main factor that dominates in the non-uniqueness, of the model is the uncertainty in the extensions of the triplication branches. It should be however, pointed out that if the LVZ is ignored where it exists (or conversely if an LVZ is assumed at a certain depth when none is present) calculations of the velocity structure at, and the depths of, the discontinuities will be inaccurate, though the error will decrease with increasing depth. Assuming that the presence of the LVZ has been interpreted correctly, the next observational uncertainty is due to the extensions of the 650 and 750 km branches. Figure 9 shows the results of inversion when these uncertainties are included for the 650 km and 750 km branches. The extra spread in the V-D envelope in this figure illustrates the effects of these increased uncertainties.

3.4 Depth of the Discontinuities

On the basis of the p$-\Delta$ data and the corresponding velocity model of Figure 6 and process of extremal inversion the depths of various discontinuities are 330 $\pm$ 15 km, 469 $\pm$ 22

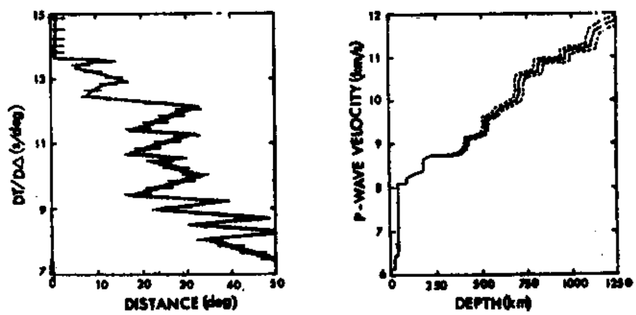

Figure 8. A large uncertainty exists in the retrograde branch produced by the low-velocity zone

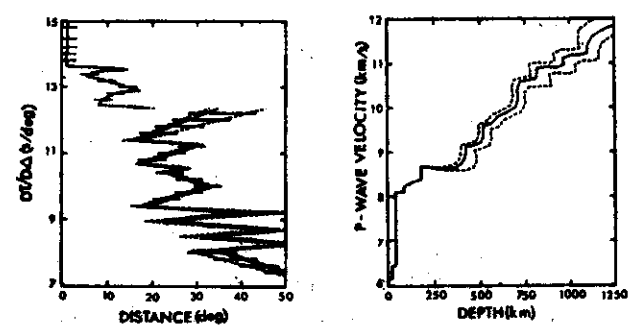

Figure 9. (p-Δ) and (V-D) envelopes for an earth with a low-velocity zone and regions of high velocity gradient, taking into account most observation errors and uncertainties

km, 520 ± 26 km, 640 ± 22 km, 750 ± 24 km, 920 ± 29 km and 1065 ± 30 km. These are almost the best estimates that can be obtained if accurate measurements corresponding to the end points of the travel time branches are assumed. However, a larger uncertainty exists in the data regarding the location of end points. For example, the data provide no estimate of the termination of various branches on either side of the triplications. The observed energy, which arrives near the arrival times of these branches may be associated with underside reflections. In the light of these uncertainties the estimates of the depths of these discontinuities may be revised (on the basis of Figure 9) to 330 ± 40 km, 460 ± 35 km, 520 ± 45 km, 640 ± 28 km, 750 ± 36 km, 920 ± 37 km, and 1065 ± 44 km. The ambiguities in these observations therefore increases the uncertainties in the depths of these discontinuities by an amount which is of the order of 15 km.

Conclusions

The extensions of the triplication branches play an important role in the determination of the velocity structure in the interior of the earth. After the main features of the data are determined and identified (particularly with respect to the travel time and slowness data), it is more convenient to adopt an empirical approach to determine a velocity model and the travel time curve. Using the empirically determined travel time curve, the extremal inversion technique can be exploited to estimate the uncertainties in the estimates of the depths of various discontinuities.

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