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EARTHQUAKE RESPONSE OF HOMOGENEOUS EARTH DAMS USING FINITE ELEMENT METHOD

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Synopsis

This paper presents a dynamic analysis of a homogeneous earth dam to actually recorded strong ground motion. Finite element method has been used in the analysis. Two finite element idealizations of the dam have been considered in the analysis and the response compared. The natural frequencies and modes of vibration have been computed by inverse iteration. Mode superposition method has been used to evaluate the dynamic response. The response evaluted are dynamic displacements and dynamic stresses. Static stresses have also been computed as these form a major portion of the stress distribution in earth dams. The effect of the vertical component of ground motion on the dynamic response has also been investigated.

Introduction

Earth dams usually form an important element of multipurpose projects like hydroelectric, irrigation and flood control. The earthquake behaviour of earth dams is an extremely important problem since many important dams are being built in regions of high seismicity at the present time and others will be built in future. Thus, it is essential to obtain some understanding of the response of earth dams to earthquake excitation in an effort to explain their observed behaviour and to arrive at improved methods of design.

Very few studies have been reported on the dynamic analysis of earth dams. An earth dam is a three dimensional continuous system which is highly indeterminate. To attempt the problem, it is necessary to make some simplifying assumptions regarding their behaviour. In all studies reported so far, the true three dimensional nature of the geometry has been ignored. The problem has essentially been analysed by treating the earth dam as a shear structure based on a beam type solution. The beam model is converted into a lumped mass system and analysed[1]. The analysis, in a way, assumes that the stresses do not vary along the width of the cross section of the dam. Also, with this analysis, it is not possible to take into account the effect of variation of material and material properties as is generally encountered in the core of earth dams. Such studies do not furnish adequate information for design purposes. Due to lack of such technical data, the design is essentially based on the experience gained from the past behaviour of dams during earthquakes.

Earth dams are huge structures and their dimensions are such that length to height ratio is large. Further the width of an earth dam is quite significant as compared to height. In such cases, their behaviour will predominantly be two dimensional. Finite element technique[2,3] is more versatile for such analysis and has been used here. This paper describes the application of the finite element technique for earthquake response of homogeneous earth dams. The study has been illustrated with the help of an example of an earth dam 300 feet high. Two finite element idealizations of the earth dam section have been considered and the response compared. The response evaluated are dynamic displacements and dynamic stresses. Static stresses have also been computed as these form a major part of the stress distribution in earth dams.

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To determine the effect of the vertical component of ground motion, the response have been evaluated for horizontal ground motion alone as well as for the combined effect of hoirzontal and vertical components of ground motion. The response to the vertical component of ground motion alone has not been evaluated because such a ground motion will rarely occur. The normal practice in the design of earth dams is to take the seismic coefficient in the vertical direction equal to 50% of that in the horizontal direction. So, accordingly, in one case, the vertical component of ground motion has been taken as 50% of the horizontal component of ground motion and in the other case, the actually recorded vertical component of ground motion has been considered.

This study indicates that the use of a combination of rectangular and triangular elements in the structural idealization is better because for the same order of accuracy, the number of degree of freedom is much less when a combination of rectangular and triangular elements is employed than when triangular elements alone are used. The effect of the vertical component of ground motion on the horizontal dynamic displacements and dynamic stresses is small but the effect on the vertical dynamic displacements is large.

Finite Element Method

The finite element method of analysis is a powerful structural analysis technique. The method is well known for static analysis(3) but its application to vibration problems has been made only in a few cases(4,5,6). In this method, the continuous system is idealized by introducing finite elements thus converting it into a multiple degree freedom system. In this investigation, it has been assumed that the dam is uniformly loaded along the length so as to produce plane strain behaviour of the cross section. Further it has been assumed that the material of the dam behaves linearly elastic. On the basis of these assumptions, it is. possible to calculate the stiffness properties of the dam section which define nodal force deflection relationships and can be represnted as(2).

$$\{R\} = [K] \ \{Z\} \qquad (1)$$

Where $\{R\}$ is the vector of nodal point forces, $\{Z\}$ is the vector of nodal point displacements and $[K]$ is the stiffness matrix. Here, each nodal point has been assumed to possess two degrees of freedom. Support conditions may be applied by eliminating the rows and columns corresponding to nodal points which impose displacement constraints.

The stresses $\{\sigma\}$ at the nodal points can be computed as(3)

$$\{\sigma\} = [S] \ \{Z\} \qquad (2)$$

Where $[S]$ is the stress transformation matrix.

In addition to this, for dynamic analysis, the mass matrix for the idealized model of the dam is needed. This may be defined in various ways including the consistant mass matrix procedure(7). However, it is convenient if the total mass of an element is assumed to be concentrated equally at its various nodal points as this some what simplifies the analysis as it includes only diagonal terms in the mass matrix. Thus in this investigation, one-third of the mass of each triangular element and one-fourth of the mass of each rectangular element has been assumed to be lumped at nodal point.

Dynamic Analysis

Using Finite elements, the dam structure is reduced to a model with multiple degree of freedom. The equations of motion of such a system can be written using matrix notation as :

$$[M] \{\ddot{Z}\} + [C] \{\dot{Z}\} + [K] \{Z\} = \{R(t)\} \qquad (3)$$

Where [M] is the mass matrix, [C] the viscous damping matrix, [K] the stiffness matrix {Z} the vector of nodal point displacements relative to base {R(t)} the load vector caused by earthquake ground motion and dots define differentiation with respect to time.

If it is assumed that the entire base section under the dam is subjected to the same ground motion at any instant of time, then the load vector {R(t)} associated with the ground acceleration caused by an earthquake can be written as

$$\{R(t)\} = -\{P_x\}\ \ddot{X}_g(t) - \{P_y\}\ \ddot{Y}_g(t) \tag{4}$$

where,

$$\{P_x\} = \begin{Bmatrix} M_1 \\ O \\ M_2 \\ O \\ M_3 \\ \vdots \\ \vdots \\ M_N \\ O \end{Bmatrix} \quad \text{and} \quad \{P_y\} = \begin{Bmatrix} O \\ M_1 \\ O \\ M_2 \\ O \\ \vdots \\ \vdots \\ O \\ M_N \end{Bmatrix} \tag{5}$$

$\ddot{X}_g(t)$ and $\ddot{Y}_g(t)$ represent the horizontal and vertical components of ground motion and N is the number of nodal points used in the structural idealization less support condition Thus the equation of motion (3) can be written as :

$$[M]\ \{\ddot{Z}\} + [C]\ \{\dot{Z}\} + [K]\ \{Z\} = -\{P_x\}\ \ddot{X}_g(t) - \{P_y\}\ \ddot{Y}_g(t) \tag{6}$$

Equation (6) represents a set of second order differential equations which can be solved numerically to compute dynamic respose. However, for the problem considered here, the degree of freedom is large. Also, the contribution of the first few modes of vibration is usually significant in the dynamic response and the contribution of higher modes is small. So, in this investigation the dynamic response was evaluated using mode superposition method. It was necessary first to determine the natural frequencies and mode shapes. These can be determined from the case of free undamped vibration because for small values of damping, damped natural frequency is approximalely equal to undamped natural frequency.

Determination of Natural Frequencies and Mode Shapes:

The equation of motion for free undamped vibrations is

$$[M]\ \{\ddot{X}\} + [K]\ \{X\} = O \tag{7}$$

where {X} is the vector of absolute displacements.

The solution of Eq. 7 takes the form

$$\{X\} = a.\ \ e^{ipt}\ \{\phi\} \tag{8}$$

where
a = scalar of dimension L
p = scalar of nimension T^{-1}
t = time

$$\{\phi\} = \text{a nondimensional vector} \quad \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \vdots \\ \phi_n \end{Bmatrix}$$

and $\quad i = \sqrt{-1}$

Substituting Eq. 8 into Eq. 7

$$-p^2 [M] \{\phi\} + [K] \{\phi\} = 0$$

or $\quad [K] \{\phi\} = p^2 [M] \{\phi\} \quad (9)$

which is a characteristic value problem, the solution of which will yield n distinct values of p^2 where $n = 2N$. For each value of p_r, there will be an associated vector $\{\phi_r\}$.

To obtain the solution of the characteristic value problem represented by Eq. 9, it should first be transformed to the form of standard eigevalue problem represented by $[A] \{Y\} = \lambda \{Y\}$. It may be recalled here that [M] is a diagonal matrix.

Let $[M] = [H]^T [H] \quad (10)$

where [H] is a diagonal matrix and the superscript 'T' indicates the transpose of matrix. Since [H] is diagonal $[H]^T = [H]$

Therefore $[M] = [H] [H] \quad (11)$

and $[H] = [M]^{1/2}$

Substituting Eq. 11 into Eq. 9

$$[K] \{\phi\} = p^2 [H] [H] \{\phi\} \quad (12)$$

Premultiply both sides by $[H]^{-1}$

$$[H]^{-1} [K] \{\phi\} = p^2 [H] \{\phi\} \quad (13)$$

or $\quad [H]^{-1} [K] [H]^{-1} [H] \{\phi\} = p^2 [H] \{\phi\} \quad (14)$

Let $\quad \{Y\} = [H] \{\phi\}$

and $\quad [A] = [H]^{-1} [K] [H]^{-1} \quad (15)$

Then Eq. (14) becomes

$$[A] \{Y\} = p^2 \{Y\} \quad (16)$$

which is of the standard form, $[A] \{Y\} = \lambda \{Y\}$.

Thus the eigenvalues obtained from Eq. (16) will be the true eigenvalues and the true eigenvectors can be obtained as

$$\{\phi\} = [H]^{-1} \{Y\} \quad (17)$$

Method of Inverse Iteration :

This is an iterative process and is based on the fact that if {Y} is an eigenvector corresponding to an eigenvalue λ for the matrix [A], then {Y} and $\mu = \frac{1}{\lambda - b}$ are the corresponding eigenvector and eignvalue for $[A - b\, I]^{-1}$.

This follows by writing the defining equation

$$[A] \{Y\} = \lambda \{Y\} \text{ as}$$

$$[(A) - b\,(I)] \{Y\} = (\lambda - b) \{Y\} \quad (18)$$

Then,

$$[(A) - b\,(I)]^{-1}\,\{Y\} = \frac{1}{\lambda - b}\,\{Y\} \tag{19}$$

Thus if the eigenvalues of [A] are $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ and λ_r together with $\{Y_r\}$ are desired, then any value of b sufficiently close to λ_r will make $| \lambda_r - b | < | \lambda_s - b |$ if $s \neq r$ and λ_r is not a multiple root. In the present case, [A] is a symmetric matrix and there are no multiple roots.

Therefore $\mu = \frac{1}{\lambda_r - b}$ is the dominant eigenvalue of $[(A)-b(I)]^{-1}\{Y\} = \mu\{Y\}$. Matrix iteration of Eq. (19) will yield $\{Y_r\}$ and μ. Knowing μ and b, λ_r and hence p_r can be obtained. To start the procedure, a close approximation to the fundamental frequency can be estimated by Rayleigh's method. In actual practice, it is not necessary to carry out the actual inversion process $[(A)-b\,(I)]^{-1}$ and in this study, the iterative scheme has been carried out using a special technique. The frequencies thus obtained are the true frequencies and the actual mode shapes can be obtained as

$$\{\phi_r\} = [H]^{-1}\ \{Y_r\} \tag{20}$$

These mode shapes satisfy the following orthogonality relationship.

$$\left.\begin{aligned}\{\phi_r\}^T\,[M]\ \{\phi_s\} &= 0\\ \{\phi_r\}^T\,[K]\ \{\phi_s\} &= 0\end{aligned}\right\}\quad r \neq s \tag{21}$$

where r and s are two different modes of vibration,

Normal Coordinates

The coupled equations of motion (6) can be reduced to a set of uncoupled normal equations using the orthogonality relationship given by Eq. 21. In order to transform the nodal coordinates Z to the normal coordinates ξ, let

$$\{Z\} = [\phi]\ \{\xi\} \tag{22}$$

where $[\phi]$ is a square matrix composed of modal vectors as columns and is given by

$$[\phi] = \begin{bmatrix} \phi_1^{(1)} & \phi_1^{(2)} & \cdots & \phi_1^{(n)} \\ \phi_2^{(1)} & \phi_2^{(2)} & \cdots & \phi_2^{(n)} \\ \vdots & \vdots & & \vdots \\ \phi_n^{(1)} & \phi_n^{(2)} & & \phi_n^{(n)} \end{bmatrix} \text{ and } \{\xi\} = \begin{Bmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_n(t) \end{Bmatrix} \tag{23}$$

Substituting Eq. 22 in Eq. 6 and premultiplying throughout by $[\phi]^T$.

$$\begin{aligned}[\phi]^T\,[M]\,[\phi]\,\{\ddot{\xi}\} + [\phi]^T\,[C]\,[\phi]\,\{\dot{\xi}\} + [\phi]^T\,[K]\,[\phi]\,\{\xi\}\\ = -\,[\phi]^T\,\{P_x\}\,\ddot{X}_g\,(t) - [\phi]^T\,\{P_y\}\,\ddot{Y}_g\,(t)\end{aligned} \tag{24}$$

Making use of orthogonality relationships, Eq. 21, it may be noted that $[\phi]^T\,[M]\,[\phi]$, and $[\phi]^T\,[K]\,[\phi]$ are diagonal matrices. If the damping is such that the same transformation which diagonalises the mass and stiffness matrices, also diagonalises the damping matrix, that is $[\phi]^T\,[C]\,[\phi]$ is a diagonal matrix, then Eq. 24 could be simplified and solved.

Using the notation

$$\{\phi_r\}^T [M] \{\phi_r\} = M_r^*$$
$$\{\phi_r\}^T [K] \{\phi_r\} = p_r^2 M_r^*$$
$$\{\phi_r\}^T [C] \{\phi_r\} = 2\, p_r\, \zeta_r\, M_r^* \qquad (25)$$
$$\frac{\{\phi_r\}^T \{P_x\}}{M_r^*} = Q_x^{(r)}$$
$$\frac{\{\phi_r\}^T \{P_y\}}{M_r^*} = Q_y^{(r)}$$

where ζ_r = Fraction of critical damping in rth mode of vibration.

Equation 24 can be written as a set of nnormal equations

$$\ddot{\xi}_r + 2\, p_r\, \zeta_r\, \dot{\xi}_r + p_r^2\, \xi_r = -\, Q_x^{(r)}\, \ddot{X}_g(t) - Q_y^{(r)}\, \ddot{Y}(t)$$
$$r = 1, 2 \ldots\ldots n \qquad (26)$$

Thus an n degree of freedom system represented by Eq. 6 has been reduced to n single degree freedom systems represnted by Eq. 26 using a transformation given by Eq. 22. It may be noted that either of the ground acceleration components can be considered separately or they may be combined to give the total effective force as

$$R_r^*(t) = -\, Q_x^{(r)}\, \ddot{X}_g(t) - Q_y^{(r)}\, \ddot{Y}_g(t) \qquad (27)$$

The solution of Eq. 26 can be written as

$$\xi_r = \frac{1}{p_{dr}} \int_0^t R_r^*(\tau)\, e^{-p_r\, \zeta_r (t-\tau)} \,\mathrm{Sin}\, p_{dr}\, (t-\tau)\, d\tau \qquad (28)$$

where $p_{dr} = p_r \sqrt{1-\zeta_r^2}$ = damped natural frequency.

The displacemet relative to base of ith mass in the rth mode of vibration can be written as

$$Z_i^{(r)} = \phi_i^{(r)} \frac{1}{p_{dr}} \left[\int_0^t R_r^*(\tau)\, e^{-p_r\, \zeta_r (t-\tau)} \,\mathrm{Sin}\, p_{dr}\, (t-\tau)\, d\tau \right] \qquad (29)$$

or

$$Z_i^{(r)} = \phi_i^{(r)}\, (W)_r^* \qquad (30)$$

where

$$(W)_r^* = \frac{1}{p_{dr}} \left[\int_0^t R_r^*(\tau)\, e^{-p_r\, \zeta_r (t-\tau)} \,\mathrm{Sin}\, p_{dr}\, (t-\tau)\, d\tau \right] \qquad (31)$$

Hence, the displacements relative to base in the rth mode of vibration can be expressed as

$$\{Z\}^{(r)} = \{\phi_r\}\, (W)_r^* \qquad (32)$$

The stresses at the nodal points in the rth mode of vibration can be computed using Eq. (2) as

$$\{\sigma\}^{(r)} = [S]\, \{Z\}^{(r)}$$
$$= [S]\, \{\phi_r\}\, (W)_r^*$$
$$\{\sigma\}^{(r)} = \{D_r\}\, (W)_r^* \qquad (33)$$

where $\{D_r\} = [S]\, \{\phi_r\}$

The total displacement and stresses can be computed by making use of the principle of superposition as :

$$\{Z\} = \sum_{r=1}^{n} \{\phi_r\} (W)_r^* \tag{34}$$

$$\{\sigma\} = \sum_{r=1}^{n} \{D_r\} (W)_r^*$$

Knowing normal and shear stresses, principal stresses can be computed using well known relationships from Theory of Elasticity.

Specifications of the Example

The study has been illustrated with the help of an example. A homogeneous dam of height 300 feet and symmetric triangular crossection with side slopes of 1.5 has been taken. Following data has been taken for the material of the dam section :

Moduls of Elasticity of dam material	$E = 81{,}300$ p.s.i.
Weight density of dam meterial	$\rho = 130$ lbs/cft.
Poisson's ratio of dam material	$\gamma = 0.45$

These values are associated with a shear wave propagation velocity of $V_s = 1000$ ft/sec. and a longitudianal wave propagation velocity of $V_l = 1700$ ft/sec. The section of the dam taken for the example is the same as used in some of the previous investigations[4,5,6]. In the present study, only homogeneous dam has been analysed though the analysis described in this paper is general and can very conveniently take into account the nonhomogeneity of the dam material. In this investigation, two finite element idealizations of the dam section have been considered (Fig. 1) and the response compared. In one case (Case–A), only triangular elements have been used in the structural idealization and in the other case (Case–B), a combination of rectangular and triangular elements has been used. Case A consists of a 110 degree freedom system and Case B consists of a 50 degree freedom system only.

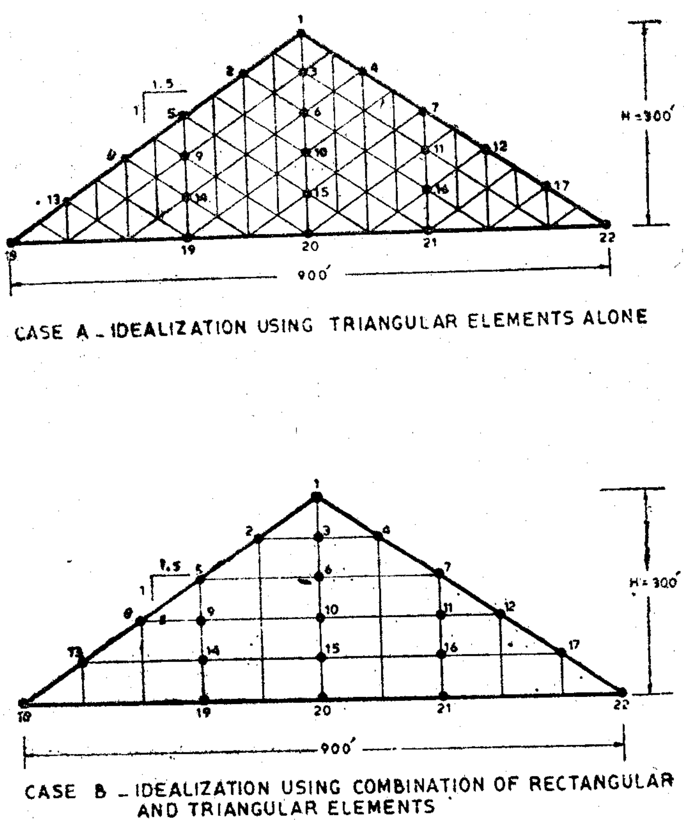


Fig. 1. Finite Element Idealization of the Earth Dam

Discussion of Results

Natural Frequencies

For the two finite element idealizations of the dam section considered, natural frequencies of vibration and mode shapes have been determined by inverse iteration for the first fifteen modes of vibration. The natural frequency of vibration is given by :

$$p = c_p \frac{V_l}{H} \tag{35}$$

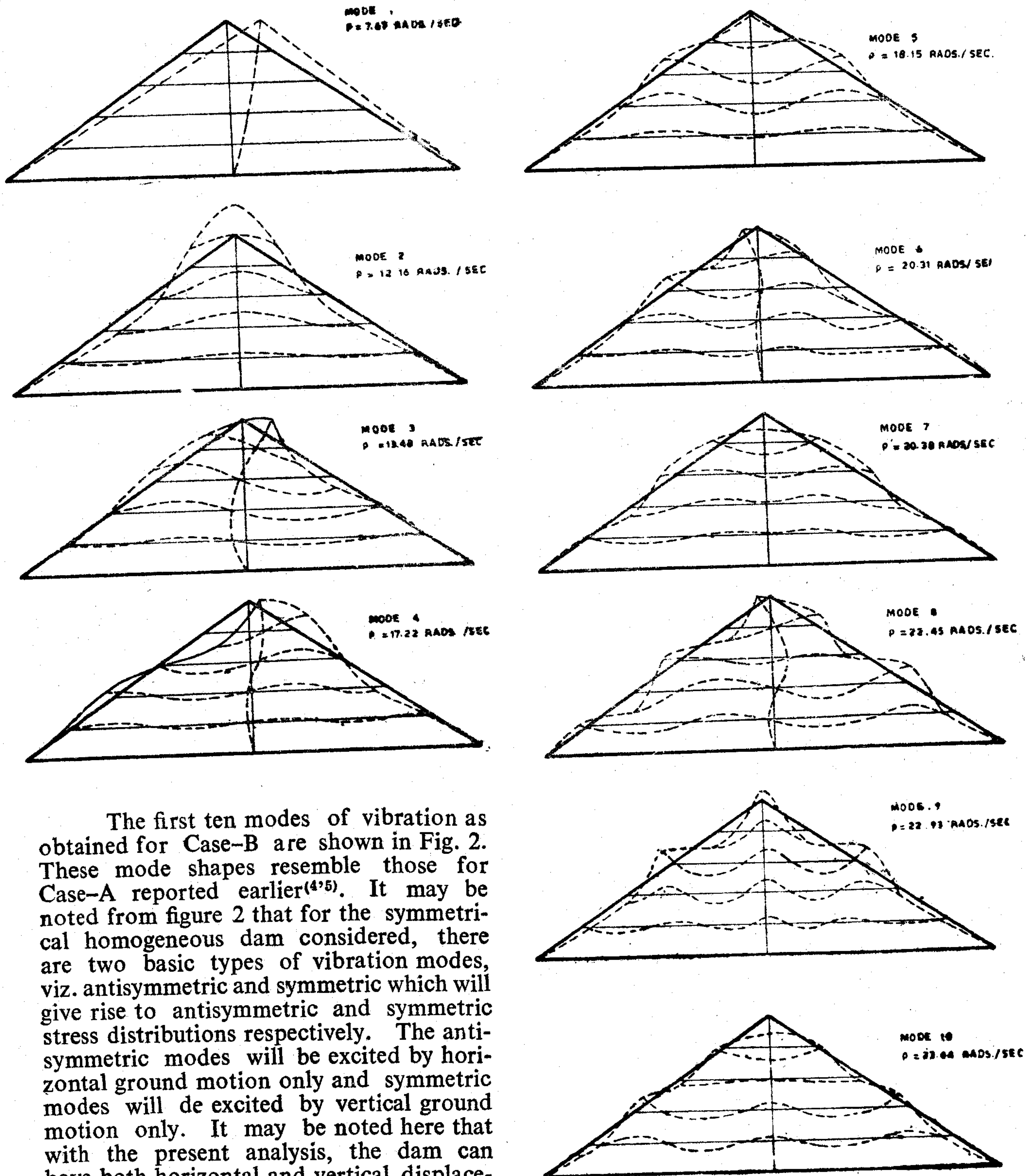
where p = undamped natural frequency in rad./sec.
c_p = frequency coefficient
V_l = longitudinal wave propagation velocity in ft/sec.
H = height of dam in feet

The value of the frequency coefficients for the two cases have been presented in Table 1. These coefficients can be used for homogeneous dams of any height H having side slopes of 1.5. Considering the case of the dam 300 feet high with the material properties given, $\frac{V_l}{H}$ works out to 5.67. Using this, the values of natural frequencies can be calculated using Eq. 35 and these values are also presented in Table 1. It may be noted from Table 1 that for Case–B, the frequencies in the various modes are somewhat lower than those for Case–A. The amount of difference is quite small in the fundamental mode but gradually increases in higher modes of vibration.

Table 1

Comparison of Frequencies

Freq. No.	Values of Cp		Values of p	
	Case–A	Case–B	Case–A	Case–B
1	1.36	1.35	7.71	7.67
2	2.21	2.14	12.51	12.16
3	2.57	2.38	14.58	13.48
4	3.40	3.04	19.29	17.22
5	3.54	3.20	20.09	18.15
6	4.07	3.58	23.07	20.31
7	4.18	3.59	23.72	20.38
8	4.57	3.96	25.92	22.45
9	4.71	4.04	26.73	22.93
10	5.07	4.13	28.73	23.44
11	5.47	4.29	31.04	24.34
12	5.53	4.50	31.33	25.54
13	5.70	4.63	32.34	26.23
14	5.93	4.68	33.60	26.51
15	6.02	4.84	34.14	27.43



The first ten modes of vibration as obtained for Case–B are shown in Fig. 2. These mode shapes resemble those for Case–A reported earlier[4,5]. It may be noted from figure 2 that for the symmetrical homogeneous dam considered, there are two basic types of vibration modes, viz. antisymmetric and symmetric which will give rise to antisymmetric and symmetric stress distributions respectively. The antisymmetric modes will be excited by horizontal ground motion only and symmetric modes will de excited by vertical ground motion only. It may be noted here that with the present analysis, the dam can have both horizontal and vertical displacements even under horizontal excitation alone unlike for the case of beam analysis where horizontal excitation will cause horizontal displacements only.

FIG 2(CONTD.) - NATURAL FREQUENCIES AND MODES OF VIBRATION OF EARTH DAM (CASE-B)

Table 2

Comparison of Mode Participation Factors

Nodal Point No.	Along the Central Axis of Dam				Nodal Point No.	Along the Slope of the Dam			
	First Mode		Second Mode			First Mode		Second Mode	
	Case–A	Case–B	Case–A	Case–B		Case–A	Case–B	Case–A	Case–B
1	1.651	1.580	0.877	0.869	1	1.651	1.580	0.877	0.869
3	1.513	1.481	0.895	0.857	2	1.477	1.466	0.595	0.564
6	1.254	1.258	0.732	0.723	5	1.142	1.151	0.078	0.006
10	0.880	0.882	0.448	0.450	8	0.718	0.725	−0.066	−0.078
15	0.440	0.441	0.158	0.159	13	0.295	0.292	−0.014	0.012

Mode Participation Factors

Mode participation factors for displacement are presented in Table 2 for the first two modes of vibration for the two cases under consideration. The comparison has been made along the central axis of the dam as well as along the slope of the dam. The displacement in any mode of vibration will be given by mode participation factor multiplied by spectral displacement in that mode of vibration. From Fig. 2, it is noted that the first mode of vibration is an antisymmetric mode and will be excited by horizontal ground motion only and the second mode is a symmetric mode and will be excited by vertical ground motion only. In Table 2, are presented, the participation factors in the first mode, for horizontal displacements due to horizontal vibrations and the participation factors in the second mode, for vertical displacements due to vertical vibrations. In the first mode, the participation factors for vertical displacements due horizontal vibrations and in the second mode, participation factors fcr horizontal displacements due to vertical vibrations are not presented as these are small for purposes of comparison. From Table 2, it is noted that the participation factors for the two cases A and B compare quite well though the number of degree of freedom in case B is much less than that in case A.

Static Stresses

Static stresses form a major portion of the stress distribution in earth dams and these have been obtained for the two cases under consideration. Principal stresses have also been computed and are give by :

$$\sigma_{s1} = C_{s_1} \cdot \rho H$$
$$\sigma_{s_2} = C_{s_2} \cdot \rho H \qquad (36)$$
$$\sigma_{s12} = C_{s12} \cdot \rho H$$

where σ_{s1} = major static principal stress
σ_{s2} = minor static principal stress
σ_{s12} = maximum static shear stress
c_{s1} = coefficient for major static principal stress
c_{12} = coefficient for minor static principal stress
c_{s12} = coefficient for maximum static shear stress.

For the example considered, $\rho H = 270.83$ lbs/sq. in. Using this value, the principal stresses can be evaluated and these are presented in Table 3*, for the two cases. From the Table, it is noted that the stresses in the two cases are close to each other though the number of degree of freedom in Case–B is much less than that in Case–A. Thus the use of a combination of rectangular and triangular elements may be preferred in comparison to the use of trıngular elements only as this results in a considerable saving of memory space and computation time. Further, it is noted form Table 3 that the static stresses for the two cases are wholly compressive. The stresses are maximum at the base and decrease towards the top of the dam. As regards the variation of stresses along the width of the dam, the stresses are maximum along the central axis and decrease towards the slopes and are minimum at the slopes.

Dynamic Response

Dynamic response has been evaluated for the two cases A and B. The contribution of the first fifteen modes of vibration has been considered. Damping has been considered to be equal to 20% of critical damping in all the modes of vibration. The digitalized ground motion data of El Centro earthquake of May 18, 1940 has been used. For the horizontal ground motion, NS component has been taken. To study the combined effect of the horizontal and vertical components of ground motion, in one case, the vertical componet has been considerd to be 50% of the horizontal component and in the other case, the actually recorded data of the vertical component has been used. Results have been obtained for the horizontal ground motion alone as well as for the combined effect of horizontal and vertical components of ground motion. The response obtained are dynamic displacements and dynamic stresses.

Dynamic Displacements

Dynamic displacements have been obtained for the two Cases A and B due to horizontal ground motion alone and due to the combined effect of horizontal and vertical components of ground motion and these are presented in Table 4. It may be noted from the table that due to the horizontal ground motion alone, the maximum horizontal displacement occurs at the top and is of the order of 3.0 inches and decreases towards the base. These are maximum near the central axis of the dam and decrease towards the slopes. The maximum vertical displacement is of the order of 0.15 inches and occurs near the top of the dam.

Further it is noted from Table 4 that the combined effect of the horizontal and vertical components of ground motion has little influence on the horizontal displacements in comparison to those obtained for the horizontal ground motion alone but the vertical displacements are increased. The maximum vertical displacement is of the order of 0.60 inches when the vertical component is taken to be 50 per cent of the horizontal component and is of the order of 0.25 inches when the actually reeorded data of the

* Comparison of static and dynamic response has been made at twenty two selected points as marked in Fig. 1. (Case A and B). In this study, compressive stresses have been marked as positive and tensile stresses as negative.

TABLE 3*

Comparison of Static Stresses

Nodal Point No.	Case–A			Case–B		
	σ_{s1}	σ_{s2}	σ_{s12}	σ_{s1}	σ_{s2}	σ_{s12}
1	13.19	14.25	.51	.30	18.06	8.91
2	3.66	33.58	14.98	6.17	29.87	11.86
3	6.50	32.74	13.14	12.76	40.08	13.65
4	3.66	33.58	14.98	6.17	29.87	11.86
5	5 17	32.39	13.62	9.10	30.90	10.91
6	14.92	81.68	33.39	10.37	84.85	37.24
7	5.17	23.39	13.62	9.10	30.90	10.91
8	11.67	42.11	15.22	14.92	40.19	12.65
9	27.76	65.84	19.04	25.92	63.02	18.55
10	46.45	129.19	41.38	41.06	131.84	45.39
11	27.76	65.84	13.04	25.92	63.02	18.55
12	11.67	42.11	15.22	14.92	40.19	12.65
13	13.03	63.16	25.05	13.97	63.56	24.78
14	66.87	125.37	29.25	65.03	125.07	30.01
15	101.78	174.52	36.35	94.01	175.88	40.92
16	66.87	125.37	29.25	65.03	125.07	30.01
17	13.03	63.16	25.05	13.97	63.56	24.78
18	8.77	52.08	21.64	0.00	56.20	28.09
19	96.77	163.61	33.42	93.44	158.38	32.47
20	152.50	201.77	24.65	157.03	195.67	19.31
21	96.77	163.61	33.42	93.44	158.38	32.47
22	8.77	52.08	21.64	0.00	56.20	28.09

* Static Stresses presented are in p.s.i.

TABLE 4*

Compariso of Dynamic Displacements Due to

Nodal Point No.	Horz. Ground Motion				Horz. and Vert. Motion Vert. Being 50% of Horz.				Horz. and Vert. Motion Actually Recorded Vert.			
	Case–A		Case–B		Case–A		Case–B		Case–A		Case–B	
	Z_x	Z_y	Z_x	Z_y	Z_x	Z_y	Z_x	Z_y	Z_x	Z_y	Z_x	Z_y
1	2.96	0.00	2.74	0.00	2.96	0.59	2.74	0.60	2.96	0.19	2.74	0.23
2	2.58	0.15	2.54	0.15	2.57	0.40	2.51	0.42	2.59	0.21	2.55	0.24
3	2.67	0.00	2.57	0.00	2.67	0.59	2.57	0.57	2.67	0.19	2.57	0.22
4	2.58	0.15	2.54	0.15	2.59	0.46	2.57	0.48	2.57	0.21	2.53	0.25
5	1.87	0.13	1.89	0.15	1.98	0.22	2.01	0.27	1.94	0.13	1.95	0.14
6	2.10	0.00	2.13	0.00	2.10	0.47	2.13	0.46	2.10	0.17	2.13	0.18
7	1.87	0.13	1.89	0.15	1.77	0.19	1.83	0.18	1.79	0.16	1.83	0.17
8	1.17	0.05	1.23	0.08	1.32	0.12	1.38	0.11	1.26	0.07	1.28	0.09
9	1.34	0.09	1.36	0.11	1.49	0.19	1.53	0.21	1.44	0.13	1.45	0.14
10	1.46	0.00	1.50	0.00	1.46	0.29	1.50	0.28	1.46	0.10	1.50	0.11
11	1.34	0.09	1.36	0.11	1.29	0.19	1.34	0.20	1.30	0.12	1.36	0.14
12	1.17	0.05	1.23	0.08	1.16	0.11	1.20	0.10	1.16	0.12	1.23	0.07
13	0.50	0.02	0.56	0.02	0.56	0.03	0.57	0.03	0.51	0.03	0.55	0.03
14	0.71	0.06	0.81	0.06	0.81	0.09	0.85	0.09	0.77	0.08	0.81	0.08
15	0.79	0.00	0.85	0.00	0.79	0.12	0.85	0.10	0.79	0.07	0.85	0.04
16	0.71	0.06	0.81	0.06	0.74	0.12	0.83	0.12	0.72	0.07	0.82	0 07
17	0.50	0.02	0.56	0 02	0.47	0.02	0.56	0.03	0.50	0.02	0.57	0.02

* z_x — Horizontal Displacement in inches.
z_y — Vertical Displacement in inches.

vertical component is used. This indicates that the vertical component when taken to be 50 per cent of the horizontal component is more intense than the actually recorded vertical component of ground motion. It may be of interest to mention that based on specrum intensities, for damping equal to 20 per cent of critical, the actually recorded vertical componet is only about 25 per cent as intense as the horizontal component.

For the two cases A and B, the displacements are close to each other.

Dynamic Stresses

Dynamic stresses are the changes in stresses over the static stress condition. These have been evaluated for the two cases A and B. Principal stresses have also been computed and are given by :

$$\sigma_{d1} = c_{d1} \cdot \rho \cdot H$$
$$\sigma_{d2} = c_{d2} \cdot \rho \cdot H$$
$$\sigma_{d12} = c_{d12} \cdot \rho \cdot H \qquad (37)$$

where σ_{d1} = Major dynamic principal stress
σ_{d2} = Minor dynamic principal stress
σ_{d12} = Maximum dynamic shear stress
c_{d1} = Coefficient for major dynamic principle stress
c_{d2} = Coefficient for minor dynamic principal stress
c_{d12} = Coefficient for maximum dynamic shear stress

For the example considered, ρ H = 270.83 lbs/sq. in. Using this the principal stresses can be evaluated using Eq 37. Maximum values of the principal stresses during the history of ground motion have been obtained and these are presented in Tables 5, 6 and 7.

It may be noted from Table 5 that the maximum principal dynamic stresses do not occur at the top but occur at about 2/5 height from the top of the dam. The maximum principal stress is of the order of 54 p.s.i. These are maximum near the slopes and decrease towards the central axis of the dam. The maximum shear stress is of the order of 31 p.s.i. and occurs at the base of the dam. These are generally maximum near the central axis of the dam and decrease towards the slopes. Due to the combined effect of horizontal and vertical components of ground motion, the maximum principal stress is of the order of 58 p.s.i. and the maximum shear stress is of the order of 32 p.s.i. The general pattern of stress distribution is similar as obtained for horizontal ground motion alone. A comparison of tables 5, 6 and 7 indicates that due to the combined effect of horizontal and vertical components of ground motion, the dynamic stresses are not appreciably altered than those obtained due to horizontal ground motion alone. Further for the two cases A and B, the stresses are close to each other.

Conclusions

On the basis of this study, following conclusions can be drawn :

1. The use of a combination of rectangular elements in the structural idealization is better in comparison to the use of triangular elements alone as for the same order of accuracy, the number of degree of freedom in the former case is much less than in the latter thus resulting in considerable saving of memory space and computation time of the digital computer.

TABLE 5**

Comparison of Dynamic Stresses
Due to Horizontal Ground Motion

Nodal Point No.	Case–A			Case–B		
	σ_{d1}	σ_{d2}	σ_{d12}	σ_{d1}	σ_{d2}	σ_{d12}
1	—14.08	14.08	14.08	—11.05	11.05	11.05
2	—38.35	28.95	23.72	—31.82	24.16	19.80
3	—19.15	19.15	19.15	—17.22	17.22	17.22
4	—28.95	38.35	23.72	—24.16	31.82	19.80
5	—53.90	45.50	28.71	—51.05	41.27	27.79
6	—27.81	27.81	27.81	—27.68	26.68	27.68
7	—45.50	53.90	28.71	—41.27	51.02	27.79
8	—47.23	45.91	23.56	—50.18	46.20	25.32
9	—50.54	46.37	26.92	—49.83	43.31	28.36
10	—29.47	29.47	29.47	—30.82	30.82	30.82
11	—46.37	50.54	26.92	—43.31	49.83	28.36
12	—45.91	47.23	23.56	—46.20	50.18	25.32
13	—35.10	38.57	20.39	—35.86	39.92	22.18
14	—42.57	42.74	26.22	—38.65	40.33	26.79
15	—28.76	28.76	28.76	—28.90	28.90	28.90
16	—42.74	42.57	26.22	—40.33	38.65	26.79
17	—38.57	35.10	20.39	—39.92	35.86	22.18
18	—22.91	26.92	15.79	—24.21	30.90	21.96
19	—37.65	40.68	28.00	—35.37	39.70	31.04
20	—31.12	31.12	31.12	—32.80	32.80	32.80
21	—40.68	37.65	28.00	—39.70	35.37	31.04
22	—26.92	22.91	15.79	—30.90	24.21	21.96

**Dynamic Stresses presented are in p.s.i.

Table 6**

Comparison of Dynamic Stresses Due to Horizontal and Vertical Ground Motion Vertical Component Being 50% of Horizontal Component

Nodal Point No.	Case-A			Case-B		
	σd_1	σd_2	σd_{12}	σd_1	σd_2	σd_{12}
1	−14.41	15.00	14.16	−12.67	10.86	11.21
2	−36.10	26.08	24.24	−32.72	24.65	21.69
3	−18.17	21.45	19.20	−18.28	18.50	17.28
4	−34.64	41.22	23.32	−24.86	32.47	18.52
5	−49.43	41.71	28.36	−48.40	39.43	27.57
6	−34.02	35.88	27.81	−25.89	31.39	27.68
7	−50.67	58.36	29.11	−44.09	53.95	28.25
8	−47.26	46.85	25.33	−46.53	46.12	24.70
9	−46.56	44.80	28.33	−43.22	41.79	28.46
10	−43.52	42.52	29.66	−36.29	37.97	31.01
11	−52.05	56.17	26.24	−51.62	57.33	28.82
12	−46.56	48.91	22.21	−49.83	54.95	27.33
13	−37.62	43.87	23.26	−40.49	44.63	24.08
14	−40.46	47.86	29.60	−38.38	47.31	30.28
15	−43.79	48.61	29.01	−38.30	43.03	29.47
16	−51.51	53.62	25.62	−45.80	47.53	26.79
17	−37.70	32.80	19.72	−38.16	33.53	21.12
18	−24.70	30.01	17.52	−30.14	31.31	22.40
19	−40.87	50.40	31.61	−38.81	48.97	32.66
20	−41.98	53.14	31.23	−38.57	47.99	32.85
21	−49.29	51.16	29.11	−41.44	44.01	31.50
22	−25.35	21.42	14.87	−32.64	22.72	21.69

**Dynamic Stresses presented are in p.s.i.

Table 7**

Comparison of Dynamic Stresses Due to Horizontal and Vertical Ground Motion

Actually Recorded Vertical Component

Nodal Point No.	Case–A			Case–B		
	σd_1	σd_2	σd_{12}	σd_1	σd_2	σd_{12}
1	–14.62	13.57	14.08	–11.59	10.51	11.05
2	–37.97	30.09	23.78	–32.39	25.43	20.45
3	–19.28	19.20	19.15	–18.23	16.25	17.22
4	–28.17	38.97	23.70	–22.89	31.25	19.15
5	–53.35	44.98	28.79	–50.86	41.08	28.03
6	–27.71	29.01	27.87	–27.52	28.09	27.81
7	–46.53	54.44	28.65	–41.49	51.21	27.57
8	–47.86	46.91	25.02	–50.51	46.18	25.76
9	–51.16	46.99	27.44	–49.51	42.90	28.87
10	–35.32	30.60	29.55	–31.23	31.63	30.87
11	–45.80	50.54	26.41	–44.17	50.73	27.84
12	–44.90	46.64	22.13	–46.20	49.91	25.02
13	–34.40	41.11	21.53	–35.26	41.36	22.18
14	–44.77	43.68	28.57	–41.03	42.33	28.33
15	–44.80	36.89	28.76	–38.08	34.42	29.03
16	–42.22	41.87	25.57	–40.00	38.92	26.89
17	–38.02	35.86	20.39	–39.84	36.43	22.18
18	–22.51	27.52	16.09	–25.05	30.44	21.75
19	–42.36	44.36	30.06	–39.54	42.71	30.79
20	–48.10	40.87	31.12	–38.13	38.05	32.80
21	–42.25	40.35	28.27	–40.79	37.73	31 28
22	–26.92	23.43	15.79	–31.63	23.67	22.18

** Dynamic stresses presented are in p.s.i.

2. Due to the combined effect of horizontal and vertical components of ground motion, the dynamic stresses are not appreciably altered than those obtained due to the horizontal ground motion alone. Also the effect on the horizontal displacments is small though the effect on the vertical displacements is significant and is more pronouced near the central axis of the dam.

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