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TORSION OF RECTANGULAR SHEAR CORES OF TALL BUILDINGS

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INTRODUCTION

Rectangular shear core is one of the commonly used lateral load bearing element of a tall building. This is because of its superior structural strength and the advantage it offers from the functional and constructional points of view. The behaviour of such cores (which are invariably perforated) in bending, has been now generally well established. This paper deals with the problem of torsional behaviour of such perforated shear cores. This is relevant to the current developments which are receiving increased attention.

Torsion of a building invariably occurs in practice. In the case of earthquake loads this would occur on account of the eccentricity between the centre of mass and centre of rigidity of the structure. Due to wind loads it would arise on account of the likely eccentricity of the resultant wind load whose direction can in general be arbitrary. Some of the building codes recognise this aspect and recommend provision for the same. The Indian Code I. S. 1893 (1970) on Earthquake Resistant Design of Structures recommends the design eccentricity to be 1.5 times the deviation between the calculated centre of mass and centre of rigidity. Rosman[1] reports that SEAOC Building Code recommends a minimum eccentricity equal to 5 percent of the largest building dimension be assumed for the wind load.

Considertion of the floor torsions is invariably necessary for correct determination of the loads between the various load bearing elements. There have been some publications in the recent past dealing with the three dimensional interaction of the lateral load bearing elements on account of floor translations and rotations[2–6]. The common approach of determining the load distribution is the displacement approach which is formulated in terms of the floor displacements and rotations. The shear walls of narrow rectangular sections and of those having negligible warping rigidities are assumed to carry only shears in their own plane, while as a core-type (closed thinwalled) element is assumed to carry in addition to the shears (along the principal axes through the shear centre) a torsional moment. The displacements of the elements at any floor level are expressed in terms of the displacements and torsional angle of the floor which is assumed to move as a rigid body on account of its large in-plane stiffness. The three equations of equilibrium either written at each floor level[5] or in a differential form[6] will lead to a system of simultaneous algebraic or differential equations respectively. Where core type shear elements are present it becomes necessary to account for the torsion carried by such elements since these possess considerable stiffness for this action. This requires the knowledge of the torsional characteristics of such cores, chiefly, the stiffness relations accounting for the restrained warping effect.

Another important aspect of the determination of the structural characteristics of a core wall lies in arriving at the relative participation of flexural and torsional modes of vibration. Some results can be found in Reference (1), in which approximate energy methods are used. Results of the exact mode shapes and frequencies and response to earthquake forces are yet not fully established. Work being undertaken is pending publication[14].

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The present paper deals with the basic problem of the torsional behaviour of a single core. Expressions for the torsional stresses and stiffness have been obtained. The commonly occurring case of the rectangular core wall with symmetric openings on an opposite pair of sides has been considered. The method followed is quite general and can be extended to the cases of unsymmetric coupling of walls. A few papers have been published in the recent past concerning this aspect. The paper by Jenkins and Harrison[7] is probably the first giving the analysis and test results for the torsion of a core wall. A Ritz type energy approach has been used. Michael[8] has presented a formulation which is suitable for symmetrical walls only. The method followed is not very general and cannot be extended to unsymmetric cores. The treatment given by Rosman[1] follows the theory of the torsion of thin walled sections using sectorial properties. The method is capable of being generalised. In the present paper the authors employ the familiar folded plate theory in which the core is assumed to be a redundant combination of plates. The formulation avoids the calculation of the sectorial properties and lends itself for generalisation to unsymmetric cores. Though the constitutive equations are different they are equivalent to Rosman's[1] formulation. However, on account of certain differences in assumptions they are not exactly equivalent to those of Michael's[8] formulation.

CONSTITUTIVE EQUATIONS

Consider the rectangular core with bands of openings on a pair of opposite sides, as shown in Fig. 1 (In this figure the floors are not shown). The core may be considered as a

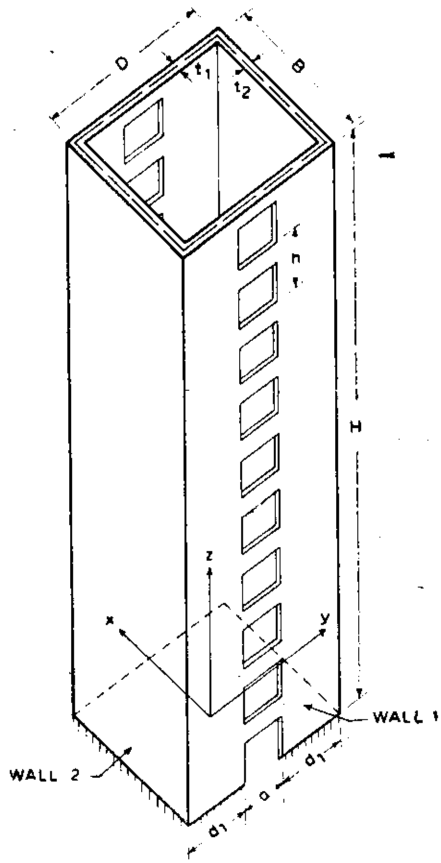

Fig. 1. Perforated core shear wall

combination of two coupled shear walls and two solid walls. On account of symmetry, a single variable. θ(Z) defines the torsion of the wall. By replacing the cross-beams as an equivalent continuous medium, θ becomes a continuous function of the height and a differential equation can be set up for its determination. The wall with openings is here considered as two walls, each designated as wall 1, coupled by the cross-medium (Fig. 2). The solid wall has been designated as wall 2.

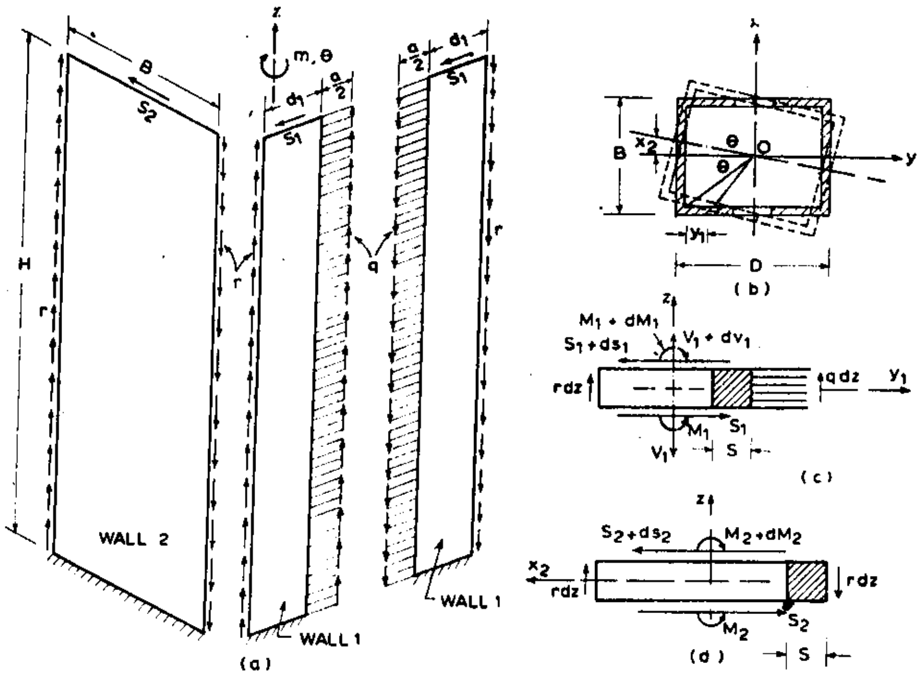

Fig. 2. Stress Resultants of wall elements

Figure 2(a) shows the equilibrium diagrams of the individual wall elements. The two redundants are, the edge shear r between wall 1 and 2, and the shear force q at the point of contraflexure of the cross-medium. Figures 25 (c) and (d) show the differential element with the internal forces. The redundant shears r and q are determined from the conditions of continuity as in the folded plate theory. The derivation of the differential equation is briefly given in Appendix B. The governing differential equation obtained in the following form:

$$EI_{\omega}\theta'''' - GJ^{*}\theta'' = m \tag{1}$$

This is of the form well known in the torsion of thin walled beams. The primes denote differentiations with respect to Z. The quantity I_{ω} and J^* are wall properties defined by eqs. (23–25) of Appendix B. The quantity I_{ω} denotes the warping constant. This quantity has been verified[9] to be the same as the warping constant determined from the sectiorial properties used in the general theory of torsion[10]. The section in this case consists of two channel cross sections of web of depth B and flange width d_1; the principal pole of the sectorial coordinates is 0, the centre of the shear-core. The contribution of the cross-beams is thus seen to be to the free torsion (St. Venant's) constant. This is given by J_c, eq. (25) of Appendix B.

The solution of the above equation is

$$\theta = A\cosh \alpha Z + B\sinh \alpha Z + CZ + D + \theta_p \tag{2}$$

where,

$$\alpha = \sqrt{\frac{J^*}{EI_\omega}} = \sqrt{\frac{J_o + J_c}{EI_\omega}} \tag{3}$$

Here θ_p is a suitable particular solution. The various quantities such as wall forces and moments are obtained in terms of θ and wall properties.

The expressions for the shearing stresses have also been given in Appendix B. These are derived by the considerations of the equilibrium of differential segments, shown hatched in Figs. 2(c) and (d). The normal stresses in the wall elements are simply obtained by superposing the effects of M_1 and V_1 for wall 1 and from M_2 for wall 2.

Also of interest, from the design point of view, is the shear force Q in the connecting beam. This is obtained as

$$Q = \int_{Z-h/2}^{Z+h/2} q\,dZ \tag{4}$$

This can be obtained by direct integration either analytically or graphically.

Two particular limiting cases of the cross-beam properties are of interst;

Case I—Open Section-Cross-beam effect neglected.

Case II—Closed Section-beams infinitely stiff.

Case I denotes the limiting situation when the cross beams are assumed to be ineffective. The core wall section becomes in effect an open section consisting of two channel cross-sections rotating about the centre of the core 0, on account of floors acting as rigid diaphragms. For this case the parameter α can be taken as zero since, as will be seen later from an example, the contribution of the cross-beams towards the free torsion rigidity is quite large compared to that of the thin walled wall elements. The torque is wholly resisted by the inplane bending of the wall elements. With the second term of eq. (1) being zero the solution (2) degenerates to the polynomial solution

$$\theta = AZ^3 + BZ^2 + CZ + D + \theta_p \tag{5}$$

With q identically equal to zero the resulting expressions for moments and shear are easily derived. This assumption gives higher stresses in the wall elements and can be used for preliminary estimates.

The other limiting case can be considered to occur when the cross-beams are infinitely stiff, making the core to behave as a beam of closed-section. It is well known that the effect of restrained warping in such a case is of more local nature and is usual to neglect this effect and rely on the free torsion rigidity, which is very high. In the general expressions derived if the quantity J^* is replaced by the free torsion constant J_S of of closed section and limits taken for α tendihg to infiinity, the results of the free torsion of closed box are obtainable. The general expressions derived for the two loading cases considered in paper, have been tested for this limiting case by comparing them with the well known results of the free torsion of beams of closed section[11]. The shear flow obtained by this assumption may be used for the connecting beams. The bending effects in the wall would be completely absent in this case.

APPLICATIONS

Two cases of loading have been considered. The solutions are briefly described in the following paragraphs. A wall built into the foundation has been considered.

(a) Constant Torque

This is produced as a result of a torque T applied at the free top of the core. The boundary conditions are

$$\theta(o) = 0;\ \theta'(o) = 0;\ \theta''(H) = 0;\ T(H) = T \tag{6}$$

The particular integral is absent in this case. After evaluating the arbitrary constants, solution (2) becomes

$$\theta=\frac{T}{EI_\omega \alpha^3}\left[\alpha Z+\frac{\sinh \alpha (H-Z)}{\cosh \alpha H}-\tanh \alpha H\right] \tag{7}$$

All other quantities can be derived in terms of the derivatives of θ as per the equations given in the Appendix. B.

(b) The linearly varying torque

This case corresponds to the loading of the wall by an uniform torque of intensity m per unit length; the torque on the wall varies linearly from zero at top to mH at the base.

The particular integral in this case will be

$$\theta_p=\frac{m}{GJ^*}\frac{Z^2}{2}=\frac{m}{EI_\omega \alpha^2}\frac{Z^2}{2} \tag{8}$$

The first three of the boundary conditions (12) would remain the same the fourth one being T(H)=0. The solution for this case is obtained as follows:

$$\theta=\frac{m}{EI_\omega \alpha^4}\left[\frac{1+\alpha H \sinh \alpha H}{\cosh \alpha H}(\cosh \alpha Z-1)-\alpha H(\sinh \alpha Z-\alpha Z)-\frac{\alpha^2 z^2}{2}\right] \tag{9}$$

As in the previous case, all quantities are obtainable in terms of the derivations of θ. It may be noted here that for digital computation all the necessary calculation steps can be made in terms of matrix algebra.

The limiting cases discussed earlier, namely $\alpha=0$ and $\alpha=\infty$ for the two loading conditions are as follows: For $\alpha=0$, we obtain

$$\theta=\frac{T}{EI_\omega}\left(HZ-\frac{Z^2}{Z}\right) \tag{10}$$

for the constant torque loading, and

$$\theta=\frac{m}{EI_\omega}\left(\frac{Z^4}{24}-\frac{HZ^3}{6}+\frac{H^2Z^2}{4}\right) \tag{11}$$

for the linearly varying torque loading.

These can be derived by taking solution (11) instead of (2) or by taking the limits of the expressions derived earlier. For the other extreme case of α tending to infinity we obtain the well known relations

$$\theta=\frac{T}{GJ_S}Z \tag{12}$$

and

$$\theta=\frac{m}{GJ_S}\left(HZ-\frac{Z^2}{2}\right) \tag{13}$$

for the two loading cases respectively. These are the limits of the general expressions for α tending to infinity.* These can be directly derived by deleting the first terms of eq. (1) and replacing J* by J_S, the free torsion constant of the closed rectangular section. The shear flow in this case is constant all along the contour.

In this paper the expressions for other quantities such as wall shears moments etc. are not presented. These are available in References 11, and 13.

In practical situations, the wall properties and loading may vary along the height. The general method of solution is best obtained by using either the stiffness or transfer methods[14]. These matrices are available in the literature[10] and can be used conveniently.

* Note that J* is to be replaced by J_s the free torsion constant of closed section.

In fact for the problem of load distribution between the different load bearing elements, the core can be considered as a beam under bending and torsion with apppropriate stiffness used depending on whether the warping restraint is considered or not. Recently, Coull and Irwin[8] have considered the problem of the load-distribution for a building taking only the free torsion effect of shear core.

The findings of a limited study of the variation of actions due to the variation of the parameter $\bar{\alpha}$ are as shown in Fig. 3 The variations of core rotation θ and wall moment M_2 for different values of $\bar{\alpha}$ varying from zero to about 8 are shown. The practical values of $\bar{\alpha}$ are likely to be between 7 and 10. Evidently the contribution of the cross beams is quite substantial and has to be included.

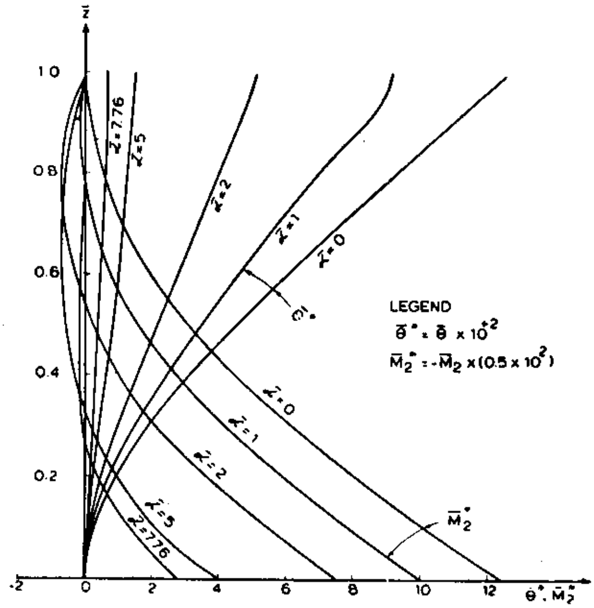

Fig. 3. Variations of $\bar{\theta}$ and ($\bar{M}_2$ or $\bar{M}_1$) with α
(Case of linearly varying torque)

EXAMPLE

A life shaft core, Fig. 4, square in plan of side 12 m, wall thickness of 0.6 m and height of 105 m has been considered. There are openings 2.5m × 3m on a pair of opposite sides giving 1 m deep beam in a storey height of 3.5m. The Poisson ratio has been taken as 0.1. Thus we have

$$B = D = 12;\ a = 3\text{m};\ d_1 = 4.5\text{m}$$

$$h = 3.5\text{m};\ t_1 = t_2 = 0.6\text{m};\ \mu = 0.1.$$

For this example we obtain $I_\omega = 33326\ \text{m}^6$; the free torsional rigidity of the walls, $J_0 = 2.94\ \text{m}^4$; the quantity denoting the contribution of the connecting beams to the total torsional rigidity, $J_c = 447.5\ \text{m}^4$. Thus it is seen that the cross-beams considerably add to the free torsion rigidity. Practically this only need be considered. The parameter α works out to be $0.0784\ \text{m}^{-1}$, giving the non-dimensional quantity, $\bar{\alpha} = \alpha H = 8.23$.

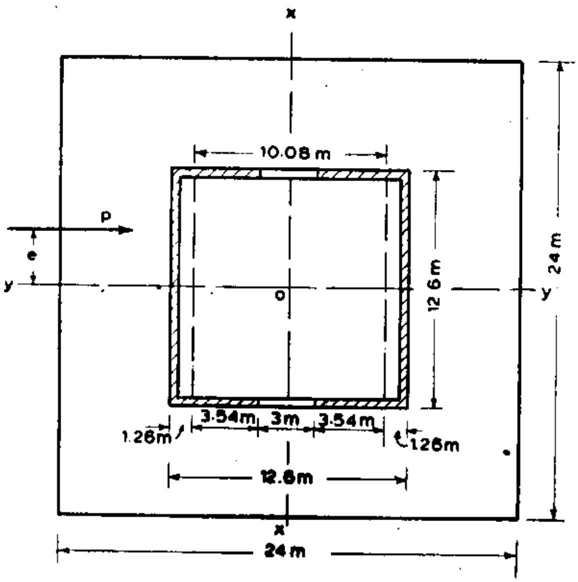

Fig. 4. Perforated core shear wall located inside a building

Figure 5 shows the variations of the wall rotation θ, and the shear force q at the points of contraflexure of the cross-beams, under uniform torsional loading of intensity m per

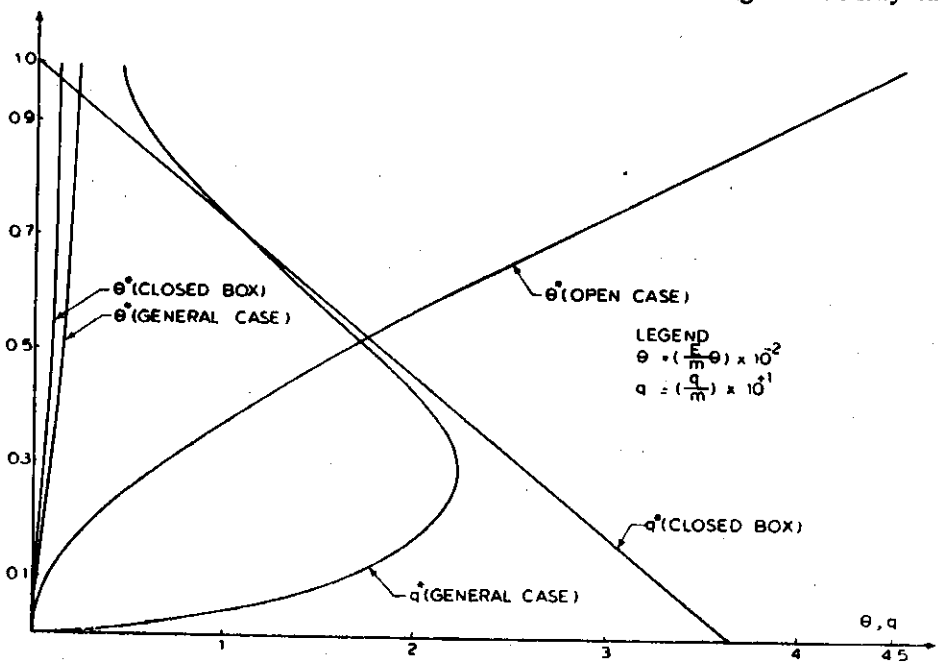

Fig. 5. Variations of θ and q (Case of uniformly distributed torque m per unit height)

unit height. The short term 'open case' denotes the case of the cross-beams being absent. Since J_0 is quite small compared to J_c the parmeter α has been taken equal to zero for this case. It is readily observed that the cross-beams provide considerable stiffness against rotation. In fact the stiffness of the core is more near to that of the closed box section. The variation of q_i is very similar to that occurring in the bending of coupled walls. This is naturally so because of the physical fact that each of the four walls are bending in their own plane due to restrained torsion. Variation of moments shown in Fig. 6 also conform to this pattern. Several figures showing the variations of other quantities such as the wall shears, the shear forces at the junctions etc. for this and the other case of the constant torque are available [11].

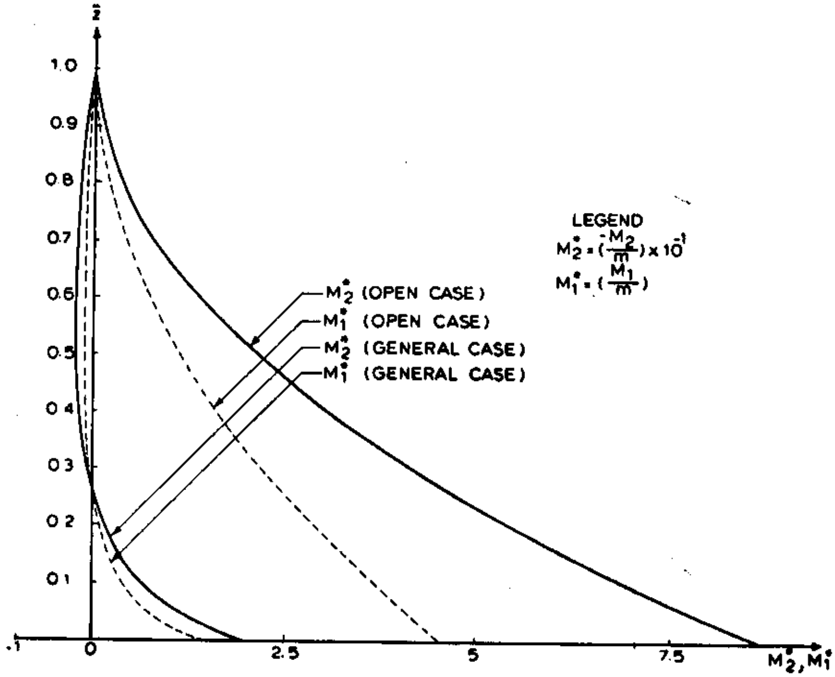

Fig. 6. Variations of wall moments M_1, M_2 (Case of uniformly distributed torque m per unit height)

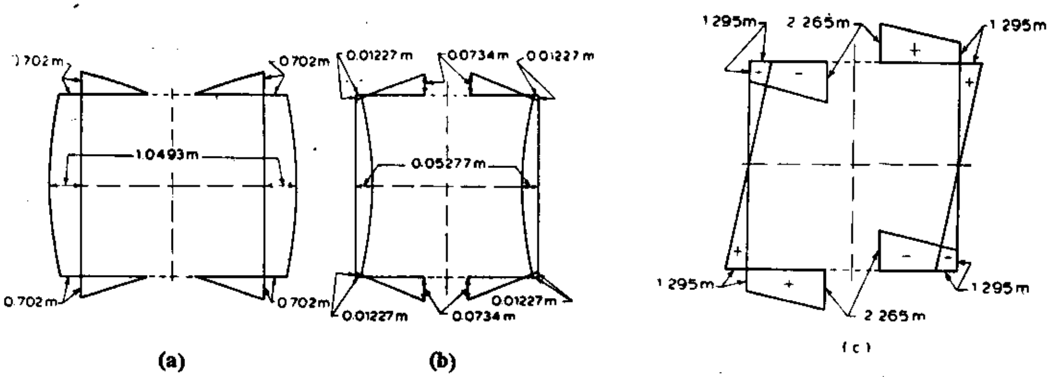

Fig. 7. Shearing and normal stress distributions for general case (Case of uniformly distributed torque m per unit height)

Figures 7 (a) and (b) show the shearing stresses, in terms of the applied loading m for the general case obtained by using eqs. (3.) and (32) of Appendix B. The normal stresses in wall 1 are due to M_1 and V_1 and in wall 2 due to M_2 alone. These are as shown in Fig. 7 (c).

COMBINED BENDING AND TORSION

The stresses due to torsion on account of the possible eccentricity of the lateral load were computed [11] for the example discussed above. A building plan of 24 m × 24 m was assumed as shown in Fig. 4. Taking possible eccentricity e of the lateral load at 5 percent of the lateral dimension, namely 1.2 m, the stresses were computed. The lateral load p was assumed uniform along the height. The stresses in the perforated wall due to the bending action were computed using the method given by Coull and Choudhary[12]. Figure 8 shows the comparison of the normal stresses due to bending as well as due to the torsion. It is observed that the effect of torsion is of the order of five percent that of bending. In case higher eccentricities are encountered these may become comparable. The shearing forces in the connecting beams were compared and it was observed that due to torsion and bending the values were 22.75p and 298p at Z/H=0.266, and 21.30p and 333p at Z/H= 0.17. The shear forces in the connecting beams are of the order of ten percent of those due to bending.

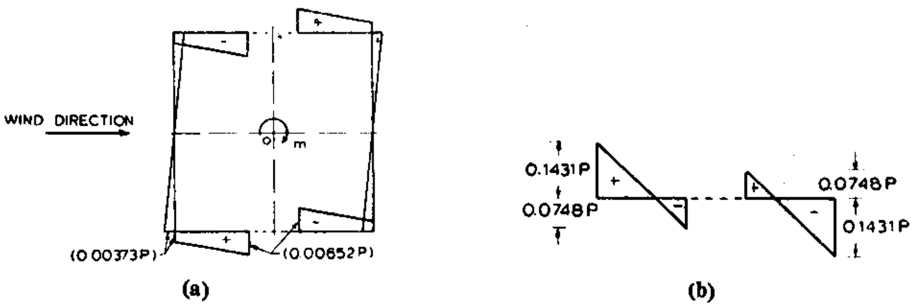

Fig. 8. Comparison of normal stresses at base section due to torsional and bending effects of wind load

However, when a core is located unsymmetrically in the plan of the building the effect of torsion will be significant.

TEST RESULTS

Two perspex models, one with and the other without the cross beams were tested[11] for the loading case of the torque applied at the free end of the core. Figure 9 shows the comparison of the test results with the analytical calculations. The agreement has been very satisfactory. The warping strength of the floors has not been accounted for in the proposed analysis, so also the effect of shearing deformations of the wall elements of the perforated wall. These effects are likely to be small from the practical point of view.

CONCLUSION

The folded plate approach used in deriving the differential equation is simpler and permits generalisation to cover unsymmetrically coupled walls. It has been shown that the cross beams provide considerable stiffness. The stiffness is nearer to that of a closed section. With differential equation in terms of the wall rotation available calculation of the stiffness or transfer matrices is straightforward. The stiffness of perforated core wall including the restrained warping effect can be used in load distribution analysis.

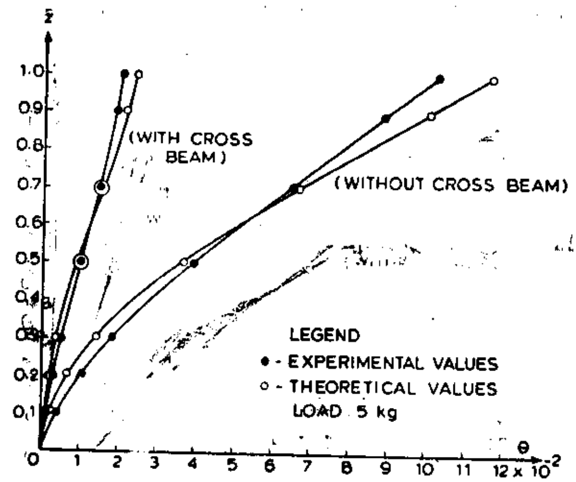

Fig. 9. Test Results

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APPENDIX-A

NOTATIONS

H	—Height of the core wall,
B, D	—Sides of the rectangular cross-section,
t_1, t_2	—Wall thickness,
h	—Spacing of wall openings,
a, d_1	—Distances for openings (Fig. 3),
M_1, M_2, S_1, S_2	—Wall moments and shears,
V_1	—Axial torsion in wall 1,
r, q	—Shear flows,
A_1, I_1, A_2, I_2	—Areas and moments of inertia of wall 1 and 2 respectively,
A_b, I_b	—Area and moment of inertia of connecting beam,
γ, β^2	—non-dimensional parameters for the cross-medium,
J_o, J_c	—Free torsion rigidities,
I_ω	—Warping inertia,
α	—non-dimensional stiffness parameter,
m	—torsional moment intensity.

APPENDEX-B

DERIVATIONS OF BASIC EQUATIONS

A. The differential Equation (Fig. 1 (a), (b), (c) and (d))

(a) Wall displacements:

$$y_1 = -\frac{B}{2}\,\theta;\; x_2 = \frac{D}{2}\,\theta \qquad (1, 2)$$

(b) Equilibrium of wall 1:

$$V'_1 = -(r+q);\; S_1 = M_1' + r\frac{d_1}{2} - q\left(\frac{d_1+a}{2}\right) \qquad (3, 4)$$

$$M_1 = EI_1\, y_1'' = -EI_1\,\frac{B}{2}\,\theta'' \qquad (5)$$

where,

$$I_1 = t_1\,\frac{d_1^3}{12} \qquad (6)$$

(c) Equilibrium of Wall 2:

$$S_2 = M_2' + Br;\; M_2' = -EI_2\, x_2'' = -EI_2\frac{D}{2}\,\theta'' \qquad (7, 8)$$

where

$$I_2 = t_2\,\frac{B^3}{12} \qquad (9)$$

(d) Strain Compatibility between wall 1 and 2

$$y_1''\,\frac{d_1}{2} + \frac{V_1}{A_1 E} = x_2''\,\frac{B}{2} \qquad (10)$$

Use of eqs. (1, 2) leads to: $V_1 = EC_1\,\theta''$ (11)

where, $$C_1=\frac{A_1 B}{4}(D+d_1) \tag{12}$$

(e) Continuity at the point of contraflexure of the cross medium.

This analysis is similar to the one followed in the bending of the coupled walls. The relative movement between the two ends at the cut due to bending, and stretching of the wall and due to the redundant q is set equal to zero. Therefore,

$$(d_1+a_1)\, y_1'-2\int_0^z \frac{V_1\, dz}{A_1 E}-q\,\frac{a_1^3}{12E\left(\frac{I_b}{h}\right)}\,\beta^2=0 \tag{13}$$

where β^2 is the factor to include the effect of the shear flexibility of the cross-medium:

$$\beta^2=1+12\,\frac{E}{G}\,\frac{I_b}{A_b^* h^2} \tag{14}$$

where A_b^* is the effective area for deflection due to shear. (The factor β^2 is the same as introduced by Beck.)

(f) Derivation of the differential equation.

Use eqs. (1) and (11) in eq. (13); after integrating, we obtain,

$$q=-E\gamma BD\theta' \tag{15}$$

where $$\gamma=\frac{12\left(\frac{I_b}{h}\right)}{a^3\beta^2} \tag{16}$$

Eq. (3) gives r in terms of V_1' and q:

$$r=-V_1'-q=-EC_1\,\theta''-q \tag{17}$$

The shear forces S_1 and S_2 can be obtained from eqs. (3) and (7), in which r is replaced in terms of q from eq. (3). Thus

$$S_1=-\frac{E}{2}(I_1 B+C_1 d_1)\,\theta''-\frac{D}{2}q \tag{18}$$

$$S_2=-E\left(I_2\frac{D}{2}+C_1 B\right)\theta''-Bq \tag{19}$$

The torque due to these shears is $2S_1 B+S_2 D$. The torque due to the free (St. Venant's) torsion is $GJ_0\,\theta'$ where J_0 is the free torsional rigidity of the wall elements, namely

$$J_0=2\left(\frac{Bt_2^3}{3}+2\,\frac{d_1 t_1^3}{3}\right) \tag{20}$$

Thus the torque at any section is given by

$$T=2S_1B+S_2 D+GJ_0\,\theta' \tag{21}$$

This in terms of θ, given by eqs. (18, 19, and 15) becomes

$$T=-EI_\omega\,\theta'''+GJ^*\,\theta \tag{22}$$

where,

$$I_\omega=(I_1 B+C_1 d_1)\,B+\left(I_2\frac{D}{2}+C_1B\right)D \tag{23}$$

$$J^*=J_0+J_c \tag{24}$$

$$J_c=2\frac{E}{G}\gamma B^2D^2=2\frac{E}{G}\frac{12I_b}{ha^3\beta^2}B^2D^2 \tag{25}$$

The equilibrium of the wall under a torsional loading of intensity, m is

$$T'=-m \tag{26}$$

Thus

$$EI_\omega\theta''''-GJ^*\theta''=m \tag{27}$$

B. Shearing Stresses

Wall 1. Consider the equilibrium of the hatched portion shown in Fig. 2 (c). If $q(y_1)$ is the shear flow (stress multiplied by thickness), then equilibrium of the shaded portion requires,

$$(q(y_1)+q)\,dz=\int_{y_1}^{d_{1/2}} d\sigma\, t_1\, d\eta \tag{28}$$

where y_1 is the distance of any point from the centre of the wall. The stress σ is given by

$$\sigma=-\frac{M_1}{I_1}y_1+\frac{V_1}{A_1} \tag{29}$$

Dividing eq. (28) through by dZ and using the above equation we get

$$q(y_1)=-q+\frac{M_1'}{I_1}K_1^*-\frac{V_1'}{A_1}A^* \tag{30}$$

where $K_1^*=\int_{y_1}^{d/2}\eta t_1\,d\eta$, is the moment of the shaded area about the centre of wall. A^* is the shaded area.

Using eqs. (3) and (4) for V_1' and M_1' we get

$$q(y_1)=-q+\frac{S_1-\gamma\frac{d_1}{2}+q\frac{d_1+a}{2}}{I_1}K_1^*+\frac{r+q}{A}A^* \tag{31}$$

Wall 2. Similarly for wall 2 we obtain for a point distant x_2 from the centre of the wall, the shear flow

$$q(x_2)=\gamma+\frac{S_2-Br}{I_2}K_2^* \tag{32}$$

It may be verified that the sum of shear flows of wall 1 works out to $S_1+q\frac{a}{2}$ and for wall 2 to S_2. The cross medium applies a variable moment $q\frac{a}{2}$ which causes the additional shear flow of $q\frac{a}{2}$.