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EFFECT OF DENSITY AND DYNAMIC LOADS ON SHEAR RESISTANCE OF SANDS

V.V.S. Rao*

Synopsis

At present no rational method is available to calculate the shear strength of sand at any void ratio when the same at a particular void ratio is known from laboratory tests. Similarly the reduction in shear strength due to dynamic loads can not be calculated. In the present paper results of laboratory investigations in this direction are reported. A modlfied Coulomb's equation is presented which relates the shear strength with static and dynamic normal stresses as well as with angle representing the void ratio.

For a design engineer the variation in the shear strength of sands due to variation in density and due to superposition of dynamic loads on normal static loads is yet an unsolved problem It is of course well known that decreasing denstty as well as increasing dynamic loads reduce the shearing resistance of sands. Lorenz (1) has proposed a rational method of estimating bearing capacity of shallow foundations on the basis of behaviour of soil under shear. But, the application of this method is very much limited as this behaviour directly depends upon void ratio and, as mentioned before, this behaviour is yet not very clear. Although many research engineers have investigated this problem, no useful method is available to predict the shear resistance at any density when the same under a particular density is known.

In a similar manner, we are still in dark about the effect of dynamic loads. It was Hertwing (2, 3, 4) who was the first person to throw some light on this problem. According to his famous resonance theory, the shear resistance under resonance. Thus, he stressed the influence of frequency on the shear resistance. On the other hand, Lorenz (5) is of the view that this reduction might be due to dynamic loads also.

The credit of conducting the pioneer series of dynamic shear tests on sands goes to Mogami and Kubo (6). They fixed a small shear box on a vibrating table and conducted the shear tests so that the vibrations are exerted in a plane perpendicular to the direction of shear force. On the basis of these and some other experiments, they suggested a " liquefaction theory " according to which the soil would behave like a liquid under vibrations. The cause for this effect they gave to be the accelerations of the vibrations. In many of the experiments reported by Mogami, the angle of internal friction of sand is very high, upto 67°. According to him this is due to the friction between the walls of the shear box and sand (7). As one can easily recognize, in the experiments, instead of applying dynamic loads, the sample has been subjected to high inertial forces by fixing it on the vibration table. As such it is the vibration of the table that has been measured aud not of the soil sample itself. Thus one has to be very cautious in applying these results to predict the behaviour of sands subjected to dynamic loads.

Similar experiments have also been conducted by Kutzner 8) who arrives at the same conclusion as Mogami.

On the other hand, Sawtshenko (9) constructed a new type of shear apparatus, wherein the direction of vibrations coincides with that of the shear force. On the basis of his results, Sawtschenko concludes that the reduction in shear resistance depends both upon amplitude as well as on the frequency of vibration. The results seems to be obvious as the vibrations induced coincide with the shearing direction, Hence, it is not possible to arrive at any useful conclusions for design engineers on the basis of the above available results.

*Assistant Professor, Indian Institute of Technology, Delhi.

As such in 1963 a research programme on "the shear strength of dynamically loaded soils" was undertaken by the author in the Institute for Foundation Engineering of the Technical University of Berlin. The main aim of this programme was to conduct intensive dynamic shear tests on sand and investigate the influence of various factors like density, dynamic and static loads on the shear strength of sand. A theoretical investigation assuming sand as a systematic packing of spheres led to the formula for shear resistance as

$$\tau = c + (\sigma_{st} - \sigma_{dyn}) \tan (\varphi + \alpha) \quad (1)$$

where

C = apparent cohesion
σ_{st} = Normal static stress
σ_{dyn} = Normal dynamic stress
φ = Frictional angle between the sand particles,
and α = the angle formed by the slip line at contact points with the direction of the shear stress. In an ideal packing of spheres, α depends upon the void ratio of the packing and varies between 0° and 19° 30′, corresponding to the variation of void ratio from 0.92 to 0.35 respectively. This variation is shown in fig. 1.

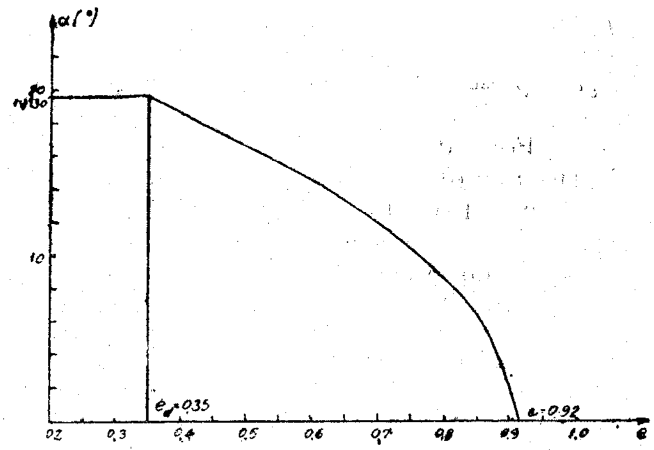

Fig. 1 Relation between the angle 'α' and void ratio 'e'

In deriving this formula, the shear strength of sand is taken to comprise of two resistances; frictional resistance due to interparticle friction, and dilational resistance due to particles "riding" over each other. During the analysis it was found that as the shear takes place, the soil gets compacted in the beginning and when the frictional resistance is fully mobilized, the soil starts expanding, beginning to mobilize dilational resistance. If the vertical deformations during a shear test is measured, a turning point will be observed, when the sample which had all along been consolidating upto that point. starts to expand. This point is called as Point of Reversal (R–Point) (Fig. 2). The shear stress corresponding to this point is the total frictional resistance offered by the sample and the τ in Equation 1. represents this stress. The excess shear stress measured after this point corresponds to the dilational resistance. But, as the dilational resistance depends upon various factors like particle shape and size and direction of shear with respect to particle orientation, it can not be taken into account in designs.

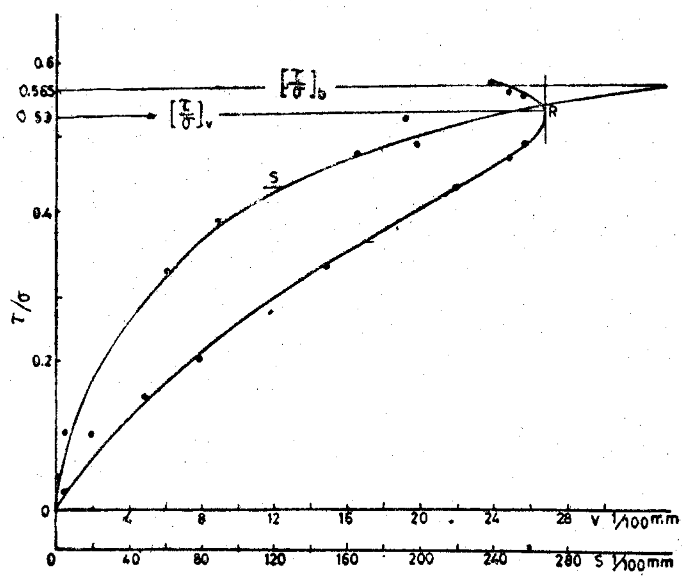

Fig. 2 Typical S and V Diagramme from a static shear test.

In case of purely static loads, Equation 1 reduces to

$$\tau_{st} = c + \sigma_{st} \tan (\varphi + \alpha) \quad (2)$$

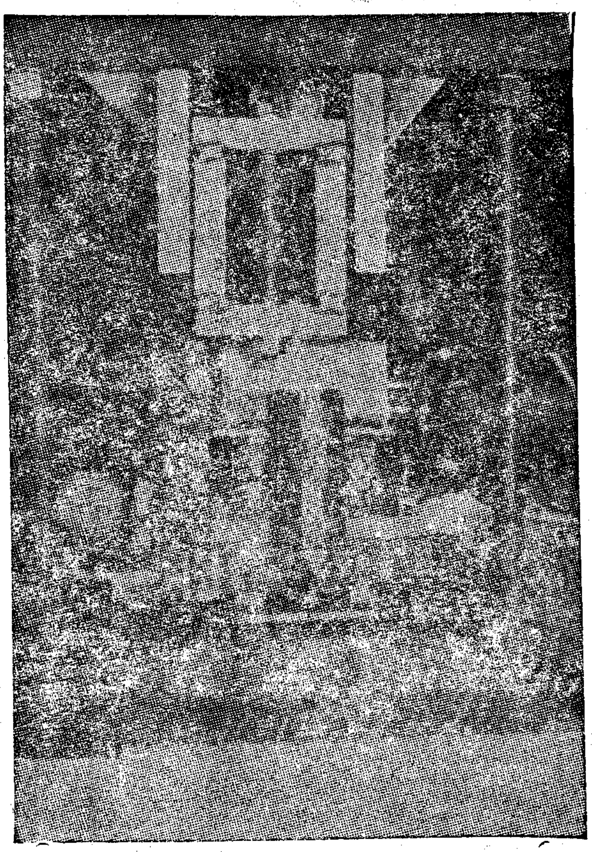

Fig. 3

Fig. 4

A modified constant stress type of Casagrande shear box apparatus was used for conducting the shear tests. In this modified setup, the shear box is fitted with wheels, and rests on a level steel plate fixed over a masonary pillar. The top half of the box is connected to a lever arm on which a wagon loaded with dead weight can be made to move at any required speed with the help of a variable speed motor and chain system. The bottom half is linked to a pendulum with a heavy weight so that the shear load applied at any instant, which is proportional to the movement of the pendulum can be calculated by measuring this movement recorded on a recorder. The vertical load is applied through another lever arm which is so fixed that it can rotate in both vertical as well as in horizontal planes freely obout a fixed point. To apply the dynamic loads, a small mechanical vibration generator was fixed between the top toothed plate of the shear box and the vertical load lever arm. The amplitude of the vibrations induced in the sample was measured with a vibration pick up (Philips-PR 9261) fitted between the top and botton surfaces of the sample as shown in the Fig. (3). With this arrangement frequencies over 100 cps could be attained. The details of the set up are shown in figs. 4, 5 and 6.

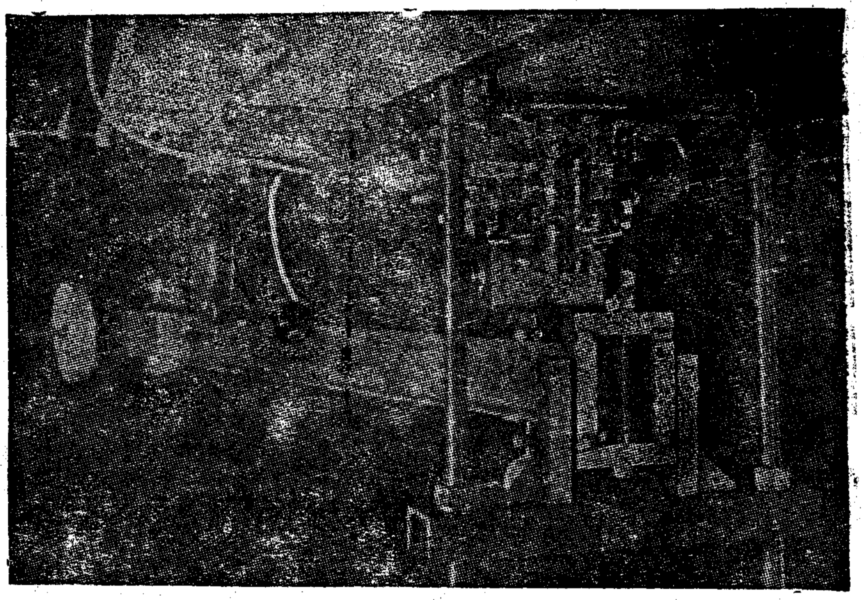

Fig. 6

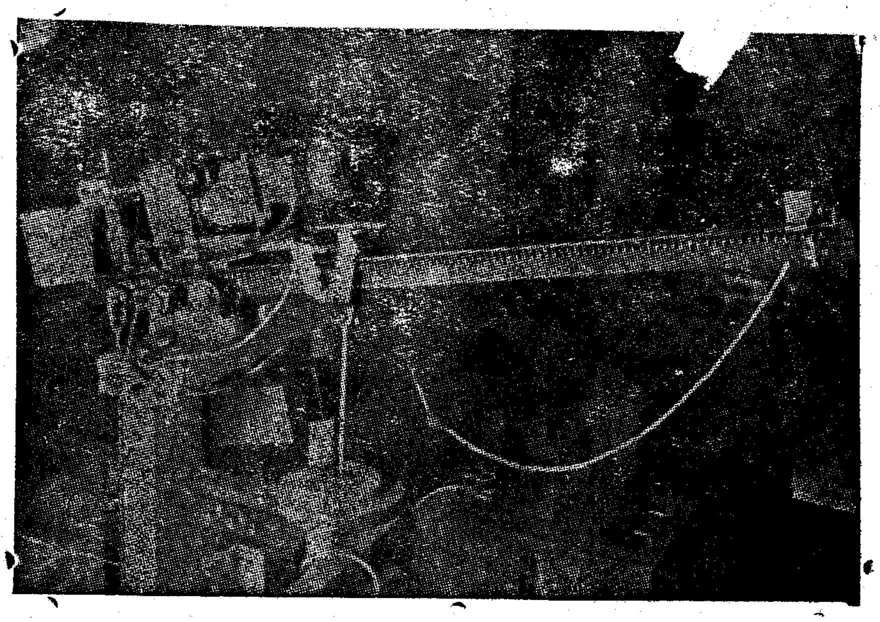

Fig. 7

To determine the void ratio of the sample at the start of the test, the shear box was before hand calibrated for volume against the height - X - as shown in Fig. 7, by casting wax samples in the box and determining its volume later. Thus, not only could the initial void ratio of the sample, but also its variation during the tests could be determined. All tests were conducted on Berlin Sand, whose sieve analysis is reproduced in Fig. 8. To avoid the pore pressure effects, only air dry sand was used for all tests.

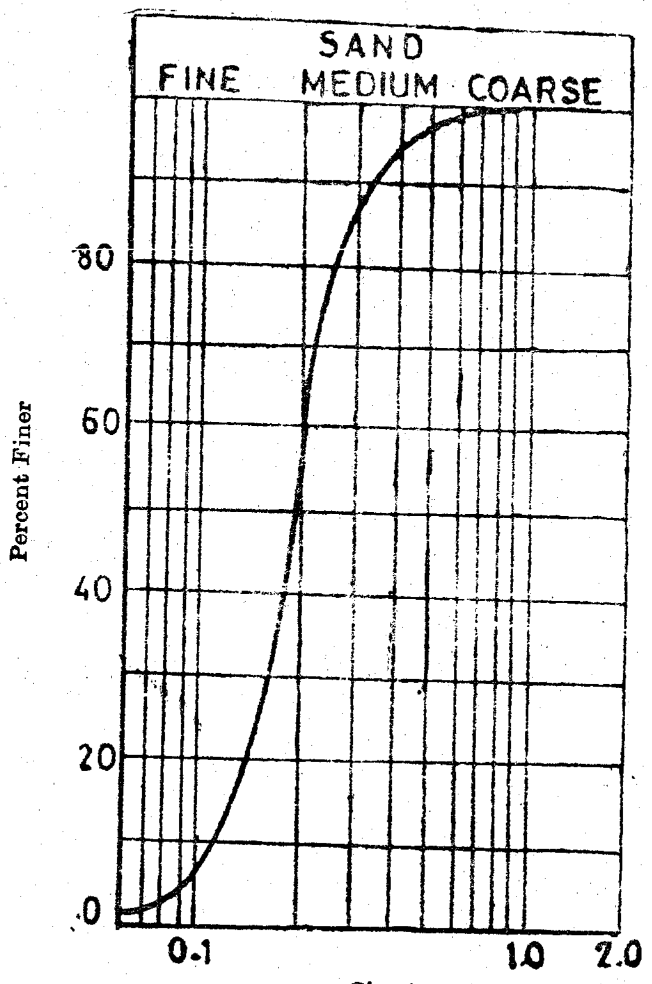

Fig. 8 Grain size distribution of Berlin sand

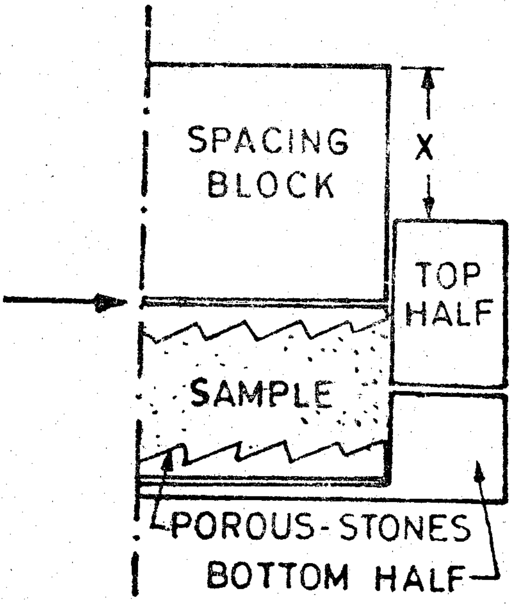

Fig. 7

In the analysis of the results, the τ values corrdsponding to the R-points have been considered. This R-point appeared in all cases and was very conspicuous. A typical curve is shown in fig. (2). From the analysis, it was found that the shear strength equation for this sand is

$$\tau_{st} = 0.1 + \sigma_{st} \tan (10^\circ + \alpha) \quad (3)$$

On the basis of the readings of vertical deformations, trials were made to correlate the change in void ratio with the initial void ratio and the normal loads. But no systematic results could be obtained. Every sample got itself compacted to an arbitrary void ratio. This might have been due to the variation in void ratio within the sample itself.

In order to further verify the validity of the equation (2), the published test results on sand and other cohesionless materials conducted by various investigators have been used. In fig. 9. the angle of interal friction (ϕ) measured in the experiments have been plotted against the void ratio and a smooth curve passing through the points have been drawn. On the same figure, the theoretical variation of α with void ratio (i.e. when $\varphi = 0$ in equation 2) has also been plotted. In can be seen that all the curves run parallel to each other. The distance between any particular curve obtained from the experimental results and the α - curve represents the value of φ in equation 2. This value is independent of void ratio. According to this method of analysis in the experiments of Idell, the angle of friction between glass spheres works out to be 18.5° and that between steel ones 8°. Wittke [15] has experimentally determined the Coefficients of friction of the steel and glass spheres used in the tests and their values are 0.15 and 0.40 respectively, corresponding to the φ values of 8.5° and 21.8°. These values agree well with those obtained by present method of analysis.

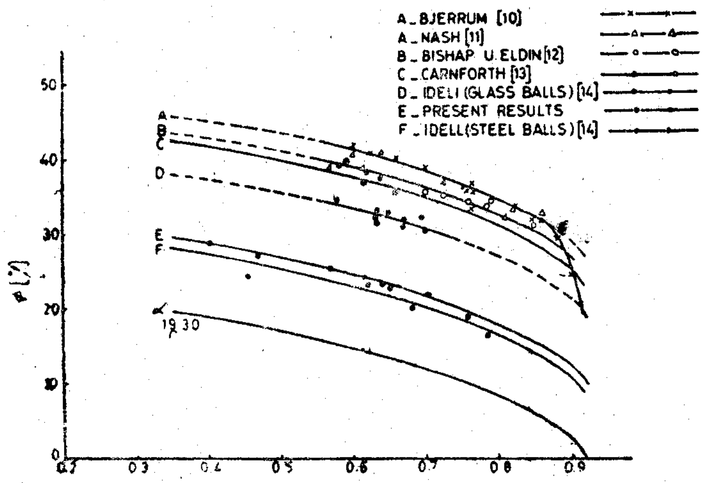

Fig. 9

Thus, the angle in Coulomb's equation has been replaced by a sum of two angles, a frictional angle and an angle depicting the void ratio of the sample. In an ideal case of a packing of uniform spheres, the frictional angle between the spheres can be assumed to a good accuracy. In case of sands however it is very difficult. As such, it is suggested that, at least two well controlled experiments on disturbed samples be conducted in the laboratory so that from known values of σ_{st} and α as well as from the measured value of τ, the two unknow quantities, c and φ can be evaluated to form the shear strength equation for the sand under consideration.

In the case of dynamic shear tests also the shape of the V-Diagrammes had the R-points as in the case of static tests. Only for samples with high void ratios subjected to dynamic loads almost equal to static loads did the sample consolidate continuously upto failure. In such cases the τ value by failure was considered for the analysis.

As in the static case, here also a linear relation between τ/σ vs e, void ratio, was recognized (Fig. 10). The influence of amplitude and acceleration of vibrations on the shear strength was not very regular. This was due to the fact that for every Eccentric mass combination, there existed a particular natural frequency, but this natural frequency had no influence on the shear strength. The influence of the frequency was however more regular. The decrease in shear strength at any particular frequency depends upon the eccentric mass in the vibrator. Fig 11 shows this effect for a particular σ_{st} value.

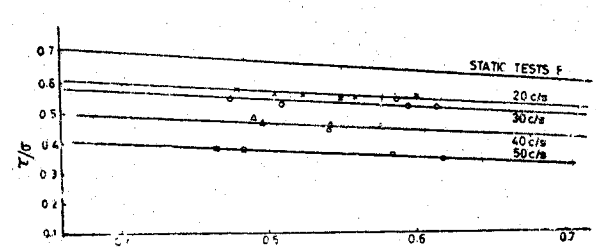

Fig. 10

In Fig. 12 the relation between ($\sigma_{st} - \sigma_{dyn}$) and τ is plotted from all cases of σ_{st} for a particular value of void ratio, e = 0.55. The angle φ for this case is 25.50° same as in static case. Similar curves were also plotted for other values of e and it was found that the values so obtained were the same both in dynamic and static tests. Thus, the shear strength equation under dynamic loads came as

$$\tau_{dyn} = 0.1 + (\sigma_{st} - \sigma_{dyn}) \tan (10^\circ + \alpha) \quad (4)$$

thus confirming the equation (1)

From equations (3) and (4), it follows that

$$\frac{\sigma_{dyn}}{\sigma_{st}} = \frac{\tau_{st} - \tau_{dyn}}{\tau_{st} - 0.1} \quad (5)$$

This results has been plotted in Fig. (13). Hence, in conclusion, it can be said that these theoretical as well as the experimental results have confirmed the views of Lorenz (5) that the value of dynamic loads is the controlling factor in the reduction of shear strength. The shear strength of a sand is related with the density and the dynamic loads as given by equation 1.

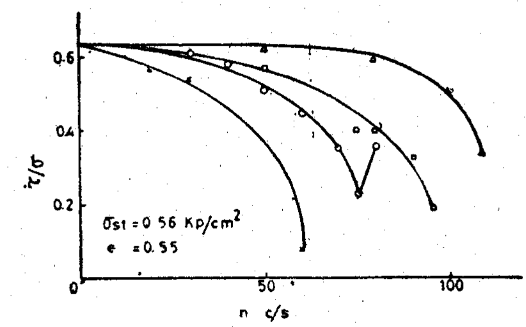

Fig. 11

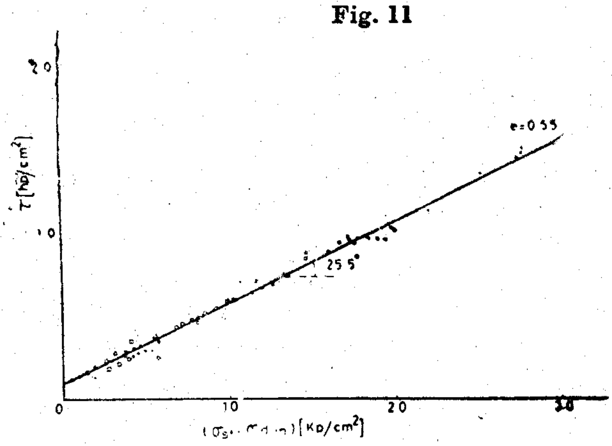

Fig. 12

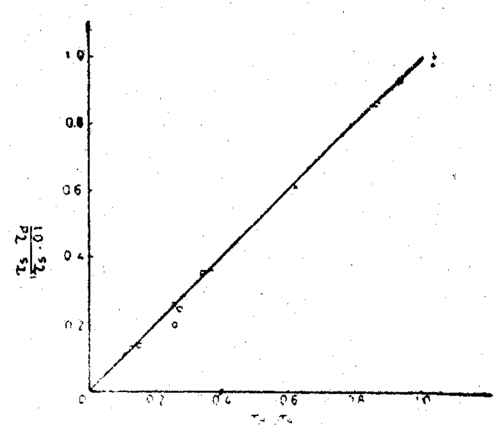

Fig. 13

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Referencs

1. Lorenz, H. (1962), Verdichtungh und Verängung als Mapstab der Tragfähigkeit rolliger Boden.—Sonderdruck d. Deutschen Gasellschaft fur Erd-und Grundbau eV.

2. Hertwig, A., Fruh, G. und Lorenz, H., (1933), Die Ermittlung der für das Bauwesen wichtigeten Eigen-schaften des Bodens durch erzwungene Schwingunger.—Veröffentl. Degebo, T.H. Berlin, H. 1. Berlin, Springer.

3. Hertwig, A., und Lorenz, H., (1935), Das dynamische Bodenuntersuchungsverfahren.—Bauingenieur, (16) Heft 25/26.

4. Lorenz, H., (1960), Grundbau-Dynamick, Springer.

5. Lorenz, H., Üder die Vorgänge im rolligen Baugrund bei Schwingungs-verdichtung.—Forschung sarbeiten aus dem Strassenbauwesen Neue Folge, H. 17, Bielefeld, Kirschbaum Verlag.

6. Mogami, T, and Kubo, K., (1953), The behaviour of sands during vibration.-Pros. 3rd Int. Conf. SMFE., Vol. 1. S. 152.

7. Mogami, T., (1964), (Personal Communication).

8. Kutzner, Christian., (1962), Über die Vogänge in Kornigen Schüttungen bei der Rüttel—verdichtung. Veröffentl. Inst. f. Erd.-u-Grundbau, TH, Karlsruhe Heft 9.

9. Sawtschenko, I.A., (1958), Der EinfluB der Vibration auf die innere Reibung von Sanden. (Überstzung) Dtsch. Gessel. f. Erd-u. Graundcau eV.

10. Bjerrum, L., Krinstage, S. and Kummeneje., The shear strength of a fine sand.—Proc. 5th Int. Conf. SMFE, Vol. 1, p. 29.

11. Nash, K.L., The Shearing resistance of a fine closely granded sand-Proc. 3. Int. Conf. SMFE, Vol. 1, pp. 160–164.

12. Bishop, A.W. and Eldin, AKG, The Effect of stress history on the relatien between Ø and porosity in sand. Proc. 3 Int. Conf. SMFE, Vol. 1 pp. 100–105.

13. Cornforth, D.H., (1964), Some experiments on the influence of strength of sand-Geotechnique 14, pp. 144—168.

14. Idell, K.H., (1960), Die Scherfestigkeit rolliger Erdstoffe.Veoff d. Inst. fur Bodenmechanik und Grundbau, Tech. Hochule, Karlsruhe, H. 2.

15. Wittke, W., (1962), Über die Scherfesting Keit rolliger Erdstoffe—T.H. Karlsruhe, H. 11.