A NOTE ON FLEXURAL VIBRATIONS OF AN INTERCONNECTED BEAM SYSTEM

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SYNOPSIS

A method is developed to find the lowest natural frequency of vibration of an interconnected beam system (Fig. 1) with different edge conditions. The analysis is based on the elastic equivalence of a grid frame work to an orthotropic plate. The method used for the analysis is Galerkin's method which belongs to the same class as that of Rayleigh and Ritz. But this method makes the formulation of the physical problem simple and suitable for numerical calculation.

INTRODUCTION

The elastic equivalence of a rectangular gridwork to an orthotropic plate was first suggested by Timoshenko (1959). Yonezawa and Naruoka (1958) used this principle for studying the free vibrations of bridge decks made of interconnected beam systems by the frequency analysis of the equivalent orthotropic plate. Recently, Rajappa (1964) has used this method for finding out the flexural vibration of an interconnected beam system point supported at the corners. The principle applied in this paper is the same and the Galerkin's method which has been used by Stanisic and McKinley (1961) for isotropic plate, is used here to determine the frequency of the orthotropic plate. Frequencies are calculated for two different types of edge conditions.

ANALYSIS

The equivalence of a beam system to the orthotropic plate has been described in detail in a paper by Rajappa (1964). The expression for 'p' is given by:

$$p^2 = \frac{g\,\lambda^2}{\gamma h} - \left(\frac{gk}{2\gamma h}\right)^2 \qquad (1)$$

where,

$$\lambda^2 = \frac{\iint \left(D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2\, \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4}\right) f(x, y)\, dx\, dy}{\iint f^2(x, y)\, dx\, dy.} \qquad (2)$$

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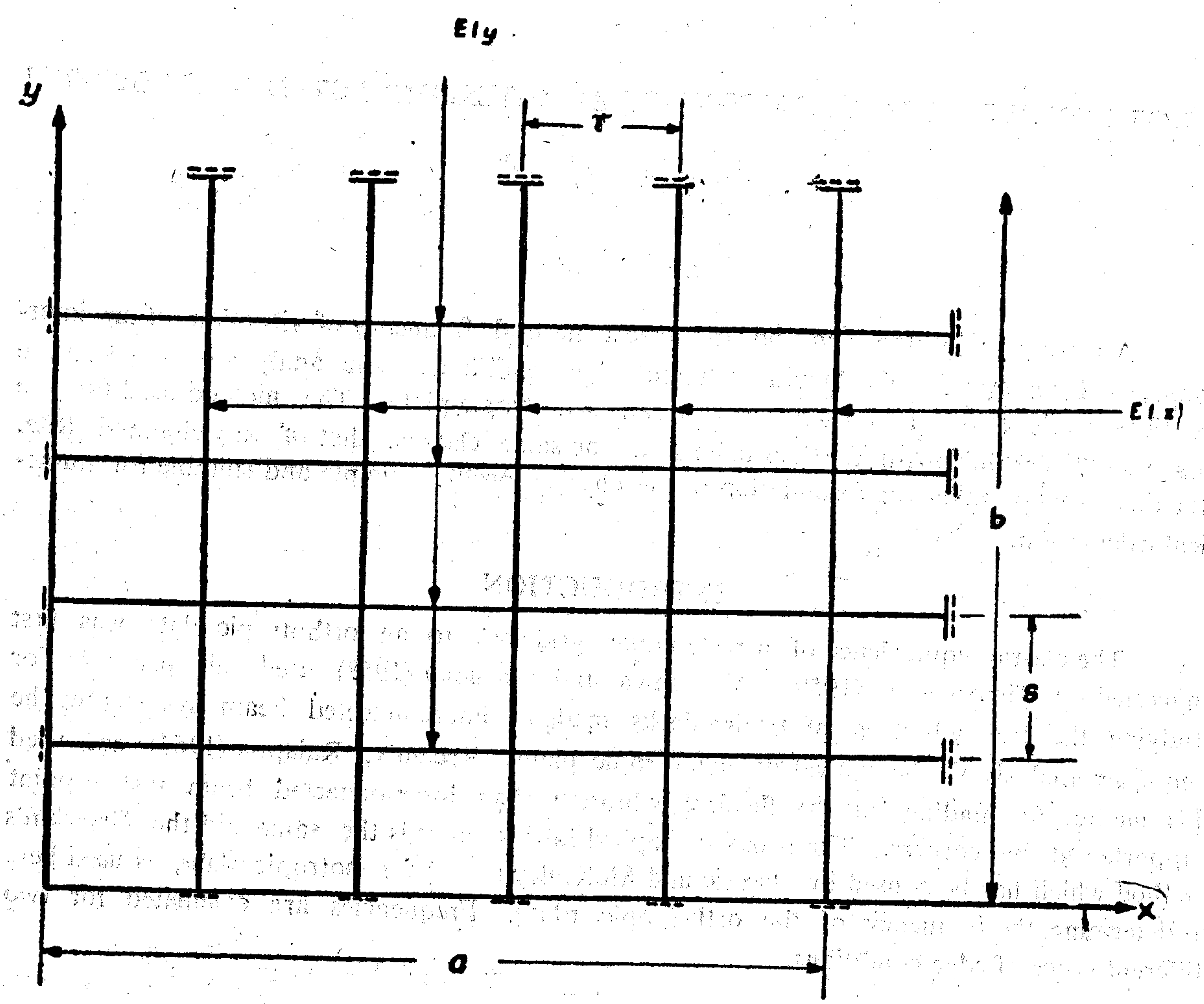

FIG.1. SIMPLY SUPPORTED RECTANGULAR GRID

and $\frac{\gamma h}{g}$ is the mass per unit area of the plate and 'k' is the coefficient of viscous damping. The value of 'p' can always be found whenever the value of 'λ' is calculated for any plate whose characteristic function f (x, y) is given :

CASE 1.

For a plate simply supported on all sides, f (x,y) for the fundamental mode of vibration is given by :

$$f(x, y) = A \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \qquad (3)$$

where, x=0, x=a, y=0 and y=b are the edges of the plate. Substituting the value of f (x,y) in equation (2), we get :

$$\lambda^2 = \pi^4\left[\frac{D_x}{a^4} + \frac{2H}{a^2b^2} + \frac{D_y}{b^4}\right] \tag{4}$$

and,

$$p^2 = \frac{g\,\pi^4}{\gamma h}\left[\frac{D_x}{a^4} + \frac{2H}{a^2\,b^2} + \frac{D_y}{b^4}\right] - \left(\frac{gk}{2rh}\right)^2 \tag{5}$$

For the isotropic plate $D_x = D_y = H = D$ and hence:

$$p^2 = \frac{gD\pi^4}{\gamma h}\left[\frac{1}{a^4} + \frac{2}{a^2b^2} + \frac{1}{b^4}\right] - \left(\frac{gk}{2rh}\right)^2 \tag{6}$$

and for a square isotropic plate where damping is neglected :

$$p^2 = \frac{4gD\pi^4}{\gamma h\ a^4} \tag{7}$$

which is a well-known result (Timoshenko, 1959).

CASE 2.

For a plate clamped along its sides :

$$f\,(x,\,y) = B\ \left(1 - \cos\ \frac{2\pi x}{a}\right)\left(1 - \cos\ \frac{2\pi y}{b}\right) \tag{8}$$

and substituting in equation (2) :

$$\lambda^2 = \frac{16\pi^4}{9}\left[\frac{3b\ D_x}{a^4} + \frac{2\ H}{a^2b^2} + \frac{3b\ D_y}{b^4}\right] \tag{9}$$

On simplifying :

$$\lambda^2 = 523.5\left(\frac{D_x}{a^4}\right) + 349\ \left(\frac{H}{a^2\ b^2}\right) + 523.5\ \left(\frac{D_y}{b^4}\right) \tag{10}$$

By employing an algebraic function for f (x, y) as :

$$f\,(x,\,y) = \ x^2\,(x - a)^2\,y^2\,(y - b)^2 \tag{11}$$

the frequency will be obtained (Rajappa, 1963) as :

$$\lambda^2 = 504\ \left(\frac{D_x}{a^4}\right) + 288\ \left(\frac{H}{a^2\ b^2}\right) + 504\ \left(\frac{D_y}{b^4}\right) \tag{12}$$

which is smaller than the one obtained in equation (10). Hence, the value of f (x,y) given by equation (11) is preferable for practical purposes.

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NOTATIONS

a,b	—	Dimensions of grid along x and y axis
D	—	Flexural rigidity of an isotropic plate
D_x, D_y, H	—	Flexural and Torsional rigidities of an orthotropic plate
g	—	Acceleration due to gravity
k	—	Co-efficient of viscous damping
γ	—	Weight density of the plate
h	—	Thickness of the plate material
r,s	—	Spacing of the beam along x and y axis