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A METHOD FOR ESTIMATING RESPONSE OF STRUCTURES DURING EARTHQUAKES

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Synopsis

A method has been developed to estimate seismic forces on structures at a site during future earthquakes. A set of new relationships has been obtained which relate the exciting potential of shock with its distance from the epicentre. The method takes into account the properties of ground motion and the dynamic behaviour of structures.

Introduction

Aseismic design of structures is a complex problem. For average structures the analysis and design is based on arbitrarily chosen static coefficients. However, for all important structures, it is essential that a dynamic analysis using the response spectrum technique(1) is carried out. This requires an accelerogram of expected earthquake shock at the site. If an accelerogram of past earthquake at the site is available it is the most reliable base to derive the response of structures precisely. If not, the data obtained elsewhere in similar conditions must be usefully utilised and taken for guidance in such computations. Realizing this, a detailed study has been made of the accelerograms that have been obtained so far during strong motion shocks at various places. Based on this study, a method has been developed for computing structural response at a site during future shocks. The same is described in following paragraphs.

Response of Structures During Earthquake

Considering an idealized structure (Fig. 1), with stiffness coefficient 'K', mass 'M' and damping 'C', subjected to ground motion represented by y, the equation of motion of the system could be written as follows :

$$M\ddot{x} + C(\dot{x} - \dot{y}) + K(x-y) = 0 \qquad (1)$$

where x is the absolute displacement of mass M as shown in Fig. 1, and dots represent differentiation with respect to time.

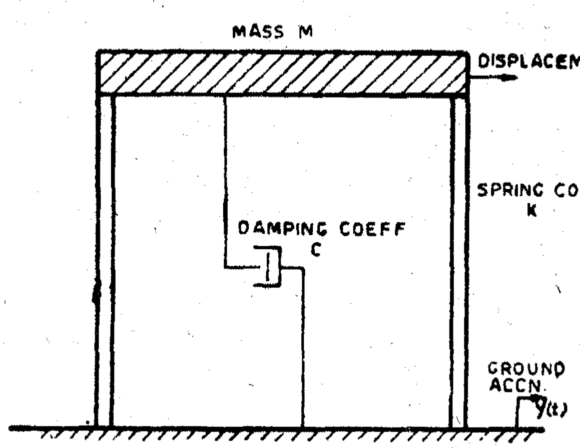

Fig. 1. The Idealised Structure

Subtracting $M\ddot{y}$ from both sides of eqn. (1) and calling $z = (x-y)$, we have,

$$M\ddot{z} + C\dot{z} + Kz = -M\ddot{y} \qquad (2)$$

Solution of eqn. (2) is well known(1) and is given by following :

$$z = \frac{1}{p\sqrt{1-\zeta^2}} \int_0^t \ddot{y}(\tau)\, e^{-p\zeta(t-\tau)} \sin p\sqrt{1-\zeta^2}\,(t-\tau)\, dt \qquad (3)$$

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where,

$p = \sqrt{K/M}$ = undamped circular frequency of system,

$\zeta = C/2Mp$ = critical damping ratio, and t, τ are time parameters.

For small values of ζ, as is present in most structures, the maximum value of z is obtained as

$$z_{max} = \frac{S_v}{p}$$

where,

$$S_v = \left[\int_0^t y(\tau)\, e^{-p\zeta(t-\tau)} \sin p\sqrt{1-\zeta^2}\,(t-\tau)\, d\tau \right]_{max} \quad (5)$$

This quantity S_v, popularly known as spectrum velocity, is the most important parameter is defining response of structures. With the value of z_{max} known, the maximum force caused on the structure can be worked out as

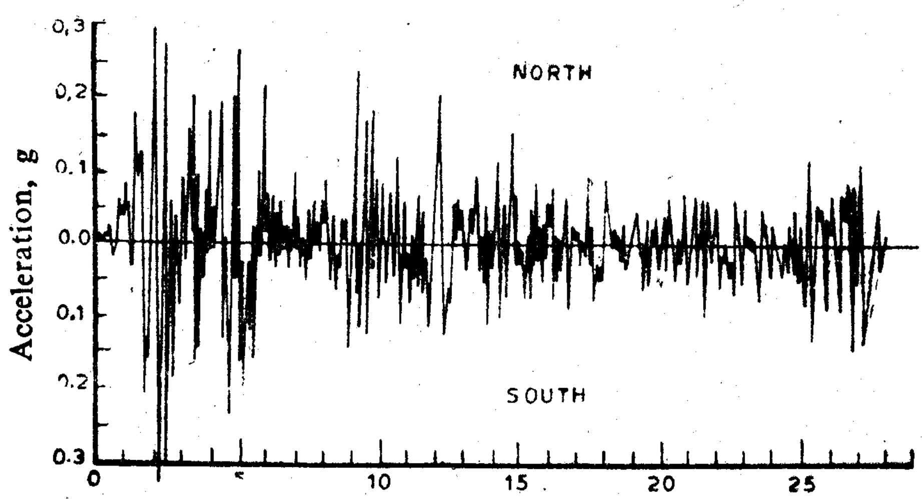
$$F_{max} = K \,.\, z_{max},$$

$$\text{or} \quad F_{max} = \frac{K}{p} S_v \quad (6)$$

An examination of eqn (5) would show that S_v depends on the ground accelerations as function of time, and the properties of the structure viz. p and ζ. For a particular earthquake accelerogram therefore, a set of values of S_v will be obtained for various p and ζ values. It is customary to plot S_v against the undamped natural period, T, of the system $(T=2\pi/p)$. This plot is termed as "velocity spectra". Fig 2 shows the accelerogram of one of the components of El Centro shock of May 18, 1940. The velocity spectra of this accelerogram is also shown in the same figure.

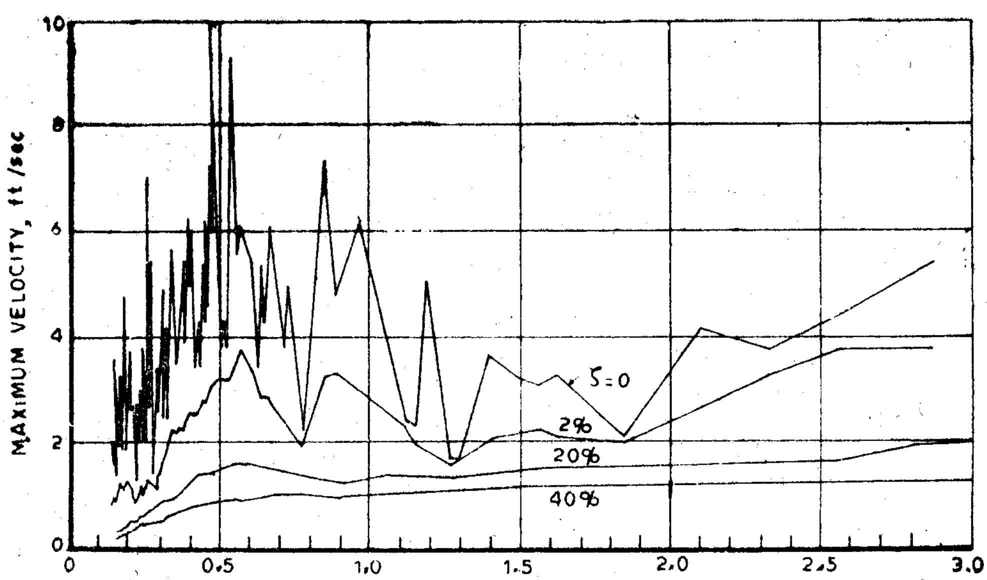
It is now recognized[2,3] that the general shape of spectra is same for all earthquakes although the absolute values of S_v are different in all cases. Based on this, Housner[3] has proposed what are called ' average spectrum curves". Multiplying factors are given for some important Californian earthquakes for which response could be obtained directly. This was a very important step towards simplifying response computations. However, the determination of such a multiplying factor remained to be decided for various sites. A method to work out this factor was developed by Jai Krishna[4] using the magnitude-distance-acceleration relationships given by Gutenderg and Richter[5]. Multiplying factor could then be calculated in proportion to the peak acceleration expected at the site. It is now realized that the peak acceleration alone does not represent the exciting potential of an earthquake shock. The frequencies associated with the acceleration pulses are as important as the peak accelerations. This fact carries special significance because accelerograms recorded at different distances from epicentre of a shock will contain different frequencies. The higher frequency components die out within a small distance from the epicentre and an accelogram at a greater distance will have the low frequency components—of course, the amplitudes of acceleration pulses will be much smaller in this case. Therefore, the question of a multiplying factor should be viewed with respect to the response spectrum rather than the peak accelerations alone. In fact, the excitation potential of an earthquake shock at a site would be better represented by the quantity spectral intensity (SI) which gives a quantitative idea regarding the spectral response of structures having periods varying from 0.1 sec to 2.5 sec. Mathematically,

$$SI = \int_{0.1}^{2.5} S_v(T, \zeta) .\, dT \quad (7)$$



Duration sec

(a) Accelerogram



Undamped Natural Period, sec

(b) Velocity Spectra

Fig. 2. Acceleregram and Velocity Spectra of El Centro, May 18, 1940 Shock (Component N–S)

For the purpose of comparing the potential of various shocks, it is desirable that only undamped spectral intensities should be worked out[3,6]. This has been used in developing the new method which is described below.

Outline of the Proposed Method

The method presented here is an attempt to provide answer to very serious problem of estimating structural response at a site during expected earthquakes in future. The method is developed after a study of response spectra of a number of strong motion shock that have occured in the past. The following are the main assumptions in the method :

1. That acceleration—time record of earthquakes at any distance, D, from epicentre, is a function of the magnitude, M, and depth of focus, h, of the earthquake beside the quantity D itself.
2. That the frequencies present in a particular accelerogram at a site are functions of the distance, D, of the site from epicentre. Low frequencies are present if D is large.
3. That undamped spectral intensities of an accelerogram (SI_0) represent the exciting potential of an earthquake and is to be used to compare the various shocks.
4. That the general ground conditions are similar everywhere, and firm foundations would be available. For unusally loose foundation conditions, special study is called for, and is not included here.

Solution of the problem

The problem is divided into following parts :

1. Determination of undamped spectral intensity of an earthquake with the peak ground acceleration as unity. This will be referred to as normalized spectral intensity hereinafter and will be denoted by $(SI_0)_n$. A relationship between $(SI_0)_n$ and distance D from epicentre is sought.
2. Determination of peak ground acceleration 'a' expected at a site.
3. Exciting potential of earthquake, Q, could then be worked out at any place from the following equation,

$$Q = a. \quad (SI_0)_n \tag{8}$$

4. Q, thus calculated, may be interpreted as a multiplying factor for the standard spectra.

Regarding item (2) above, detailed work has been done earlier and the author was associated in development of relationship expressing maximum ground acceleration a' as a function of magnitude 'M' and depth of focus 'h' and distance D, from epicentre. The relationship is discussed in great detail in another publication[7]. In this 'a' as fraction of acceleration due to gravity is expressed as

$$\frac{a}{g} = \frac{2.925 \dfrac{10^{(M-5)}}{h}}{1+4.5 \dfrac{10^{(M-5)}}{h}} e^{-0.26 (D/h)^{3/2}} \tag{9}$$

This is shown graphically in Fig. 3 for a focal depth h = 15 miles. Eqn. (9) has been found to give good correlation with the actually recorded values of 'a' in strong motion earthquakes. It is therefore proposed to use this relationship in the present work.

Relationship between $(SI_0)_n$ and distance 'D'

In order to develop such a relationship, a study has been made of the available data from the sixteen well recorded strong motion shocks. These shocks were recorded on firm

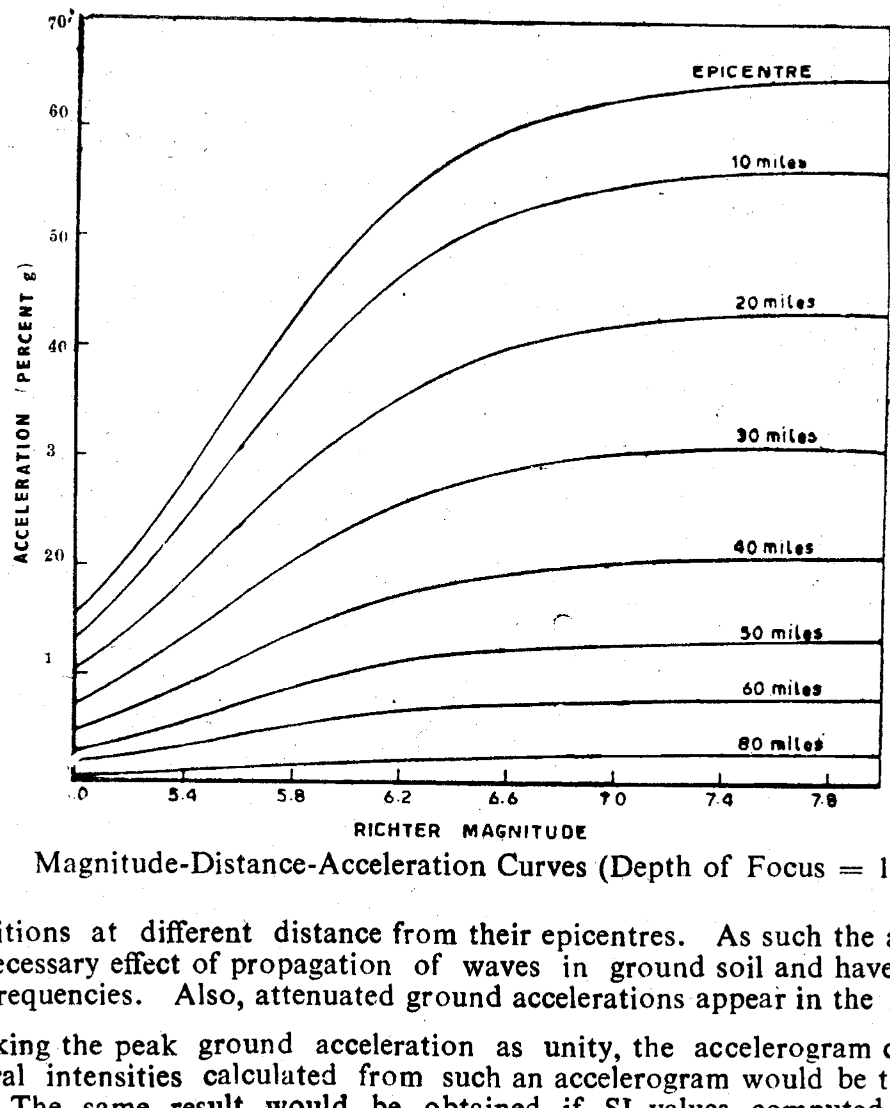


Fig. 3. Magnitude-Distance-Acceleration Curves (Depth of Focus = 15 miles)

ground conditions at different distance from their epicentres. As such the accelerograms contain the necessary effect of propagation of waves in ground soil and have components of different frequencies. Also, attenuated ground accelerations appear in the records.

By making the peak ground acceleration as unity, the accelerogram can be normalized. Spectral intensities calculated from such an accelerogram would be the normalized values $(SI)_n$. The same result would be obtained if SI values computed from original accelerogram, are divided by the peak recorded acceleration.

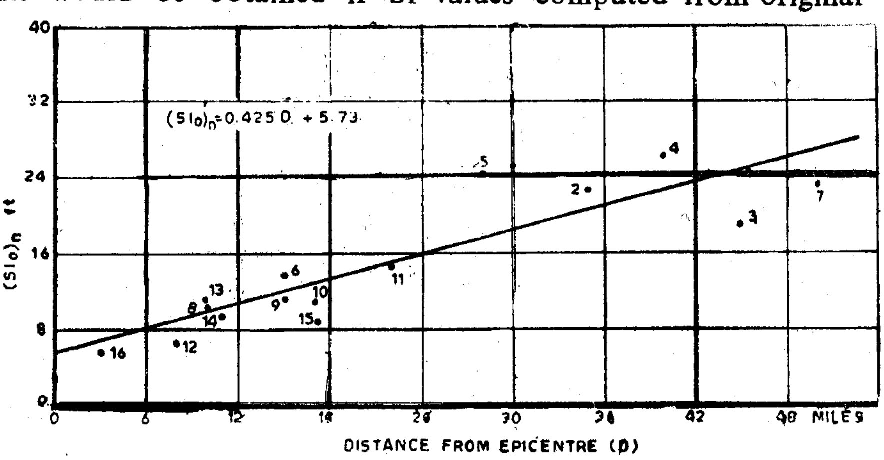


Fig. 4. Variation of normalized undamped intensity with distance from epicenter. Numbers on the dots refer to the serial no. of the shock. (Table 1)

Table 1 lists the sixteen shocks alongwith the pertinent data regarding the shock viz. magnitude, depth of focus, distance from epicentre and the maximum recorded acceleration. Average SI_0 values for these shocks are available[3,8] and are tabulated in the same table. From this, $(SI_0)_n$ values are computed. A plot of $(SI_0)_n$ values against the distance from epicentre, shown in Fig. 4, reveals

Table 1. Comparative Study of Well Recorded Shocks

Sl. No.	Name of the Station and data of shock	M	D	h	a Max. accn. recorded x g	Average SI_o	SI_o/a $=(SI_o)_n$
1	El Centro May 18, 1940	7.1	30	15	0.33	8.35	25.3
2	El Centro Dec. 30, 1934	6.5	35	15	.26	5.88	22.6
3	Olympia April 13, 1949	7.1	45	45	.31	5.82	18.75
4	Taft July 21, 1952	7.7	40	15	.18	4.69	26.1
5	Vernon May 10, 1933	6.3	28	15	.19	4.62	24.3
6	Santa Barbara June 30, 1941	5.9	15	19	.24	3.29	13.7
7	Ferendale Oct. 3, 1941	6.4	50	15	.13	2.99	23.0
8	Hollister March 9, 1949	5.3	10	15	.23	2.36	10.5
9	Helena Oct. 31, 1935	6.0	15	25	.16	1.82	11.38
10	Vernon Oct. 2, 1933	5.3	17	15	.12	1.32	11.0
11	L.A. Subway Term. Oct. 2, 1933	5.3	22	15	.065	0.96	14.75
12	S.F. Golden Gate March 22, 1957	5.3	7.8	7	.13	0.84	6.46
13	S.F. State Bldg. March 22, 1957	5.3	9.8	7	.10	1.12	11.2
14	S.F. Alexander Bldg. March 22, 1957	5.3	10.8	7	.05	0.48	9.6
15	S.F. Oakland March 22, 1957	5.3	17.2	7	.05	0.38	7.6
16	Koyna Dec. 11, 1967	6.5	3	5	.63	3.72	5.9

very significant trends. A straight line fitted by the method of least squares yields the following relationship

$$(SI_o)_n = 0.425\ D + 5.73 \tag{10}$$

Eqn. (10) brings out that structural response would be higher at larger distances provided that the peak accelerations are equal. In other words, accelerograms with lower frequency components would yield a higher structural response compared to its counterpart with higher frequencies. This could be explained with reference to the resonance curves (Fig. 5). Ruling out possibility of resonance, it may be seen that for a particular natural frequency p, the response of system is less at higher values of forcing frequency (w) than that at a lower frequency. It may be mentioned that we are referring to the w/p values greater than one since structural periods of interest range from 0.1 sec. to about 2.5 secs., and frequencies associated with earthquakes are generally higher than these. The trends indicated by eqn. (10) are therefore justified.

With the values of $(SI_o)_n$ as obtained from eqn. (10) and the value of 'a' obtained from eqn. (9), exciting potential of earthquake, Q could be worked out using eqn. (8).

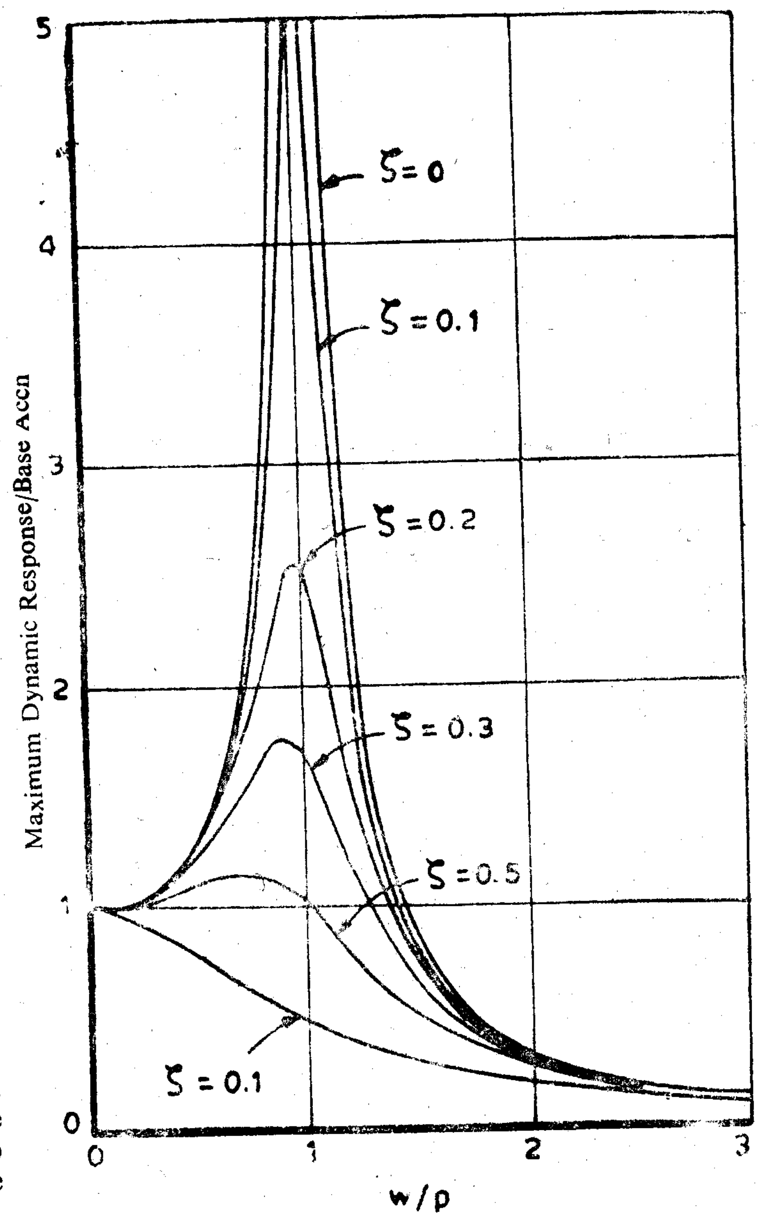


Fig. 5. Maximum dynamic response factor for damped Systems for sinusoidal loading

The Standard Spectra

For the purpose of obtaining response parameter S_v, a standard spectra must be defined in such a way that the exciting potential 'Q' of an earthquake may be used to obtain the multiplying factor. Also, the standard spectra must take care of the fact that in different shocks the peaks will have random distribution with respect to 'period' parameter. Housner's average spectra satisfies these requirements. These are obtained by averaging the spectrum values of the eight components of the four strongest ground motions recorded (El Centro 1934, El Centro 1940, Olympia 1949 and Taft 1952) and turn out in a neat smooth shape as shown in Fig. 6. Multiplying factors for these shocks have been assigned as 1.9, 2.7, 1.9 and 1.6 respectively.

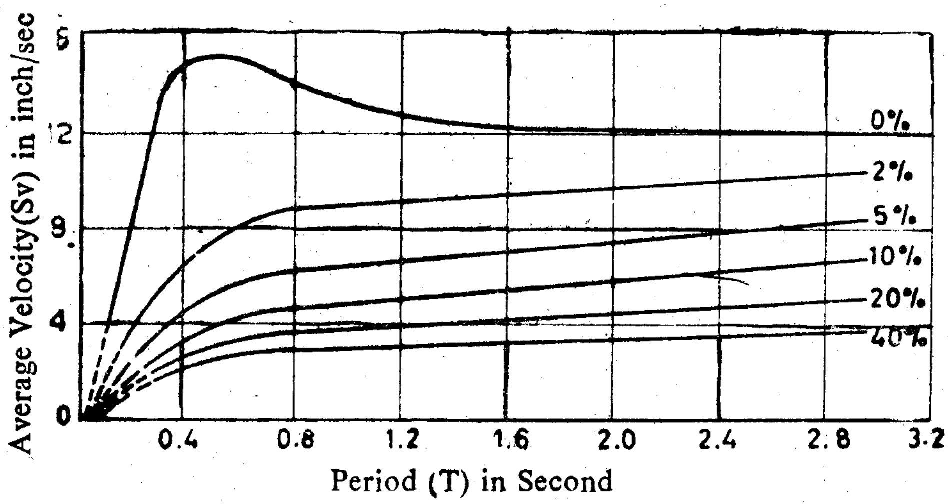


Fig. 6. The Standard Spectra

Observing that the average SI_0 values of these four shocks are 5.88, 8.35, 5.82 and 4.69 respectively, which are in about the same ratio as the multiplying factors proposed by Housner, it is conclusively established that the SI_0 values or Q values are directly proportional to the multiplying factor. Therefore, the average spectra (Fig. 6) can be very usefully and conveniently adopted as the standard spectra.

For determining the multiplying factor N for any earthquake, the following relationship may be used :

$$N = \frac{Q}{8.35} \times 2.7 \qquad (11)$$

Illustrative Example

It is desired to determine N for a site which is situated 50 miles from the epicentre of a possible earthquake with magnitude 6.2 and depth of focus 15 miles.

Step 1. Using eq. (9) for M 6.2, $D = 50$, $h = 15$, a/g works out as 0.10.

Step 2. Using eq. (10) for $D = 50$, $(SI_0)_n$ work out as 26.98.

Step 3. Using eq. (8), Q works out to be 2.698

Step 4. Using eq. (11) N work out as 0.871.

This factor N should be used as the multiplying factor for the spectral values obtained from Fig. 6, for the appropriate period and damping.

Conclusion

Eqns. 8–11 presented in this paper should be used to determine the multiplying factor for the standard spectra shown in Fig. 6. Response of any structure could then be found easily from this, by picking up the ordinate for the appropriate period and damping.

Acknowledgement

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The Ineternational Association for Earthquake Engineering are compiling an International Directory of Universities and Institutions engaged in earthquake engineering research. The fields of Research have been divided into the following categories for purposes of listing :

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Dear Member,

A notice for amending the constitution of Indian Society of Earthquake Technology was issued vide this office letter No. ISET/EC/ dated 20th July, 1969. The last date for return of the Ballot paper in this office was 15th August, 1969.

In all 55 ballot papers were returned and all have supported the amendment proposed in the letter.

It is therefore, my pleasant duty to inform all the members of the Society that the amendment of the constitution has been carried over and new copies of the constitution are available in this office which can be requested.

Yours sincerely,

Shamsher Prakash
Secretary

NOTICE

Sri Rama Rao Stanam had become a member of the Indian Society of Earthquake Technology last year, but unfortunately his address has been misplaced and is not available in our file. If any member is aware of his address, he may kindly convey the same to the Secretary, I.S.E.T.