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EVALUATION OF BUCKLING STRENGTH OF CYLINDRICAL SHELL ROOF MODELS BY VIBRATION METHODS

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INTRODUCTION

The present trend in shell construction is to make them as thin as possible which makes it desirable to know more about buckling strength. At present very little is known about the stability of cylindrical shell roofs. It is the aim of this paper to propose a non-destructive testing method to predict the buckling load of thin cylindrical shell roof models made of plexiglas.

The mathematical similarity between simple problems in free vibration and elastic instability is well known. In a beam-column, the effect of axial compressive force P is to decrease the natural frequency of transverse vibration. In addition, the variation of the square of the frequency with P is linear and finally the frequency falls to zero when P approaches P_{cr} which is the buckling load of the beam-column. The question naturally arises about the generality of the variation and whether such a behaviour can be expected in structures other than simply supported beams. Physical intuition would lead one to believe that if axial compression does decrease the natural frequency, frequency of any structure would fall to zero when buckling is reached (5).

In order to investigate the validity of the above proposed method to predict the buckling load of cylindrical shell roofs for which many methods are not available, theoretical and experimental studies were made by the author on closed cylinders at first, then on open circular cylinderical shells made of plexiglas and finally on full scale models of microconcrete. This work deals with the experiments on plexiglas shell roof models of different spans.

REVIEW OF LITERATURE

Dulacska(2) (1965) has derived a general expression connecting vibration and buckling of an infinitely long cylindrical shell roof.

Steinert(6) (1967) has done some limited number of tests on cylindrical shell roof models of plexiglas and tried to establish experimentally the linear relationship between the square of the frequency and the lateral load.

EXPERIMENTAL SETUP AND PROCEDURE

The Plexiglas shell roof models (Fig. 1) were of radius 33.4 cm and of span 100 cm, 78 cm and 56 cm. The thickness of the shell was 3 mm and the shell was laterally loaded by means of dead weights. These shells were manufactured by M/s. Josef Weiss Plastic Company, München, West Germany.

Fig. 2 shows the diagramatic representation of the loading for the plexiglas model shell roof which consisted of circular mild steel discs of 90 mm diameter weighing 500g and 100g. 34 such discs were used a time for each step of loading. Each disc was supported by a four legged stirrup as such the load was applied on the surface of the shell over 136 point, thus realising uniformly distributed loads. The four legged stirrups were not directly resting on the shell but on small aluminium discs of 12 mm diameter. The aluminium discs were fixed on to the shell by a patended paste.

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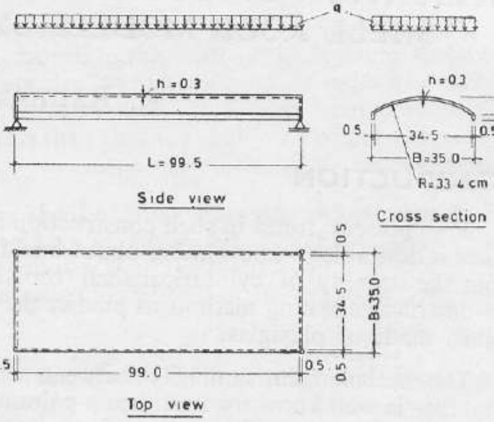

Fig. 1. Dimension of the Plexiglas shell roof Model. →

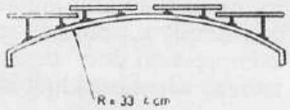

Fig. 2. Loading Device for the Model Shell →

The instrumentation for measuring the frequencies and for determining the mode of vibration in the longitudinal and transverse direction is shown in Figs. 3 and 4. A small Philips Electro-dynamic Exciter was used to excite the shell. The mode of vibration in the longitudinal direction was found out by measuring the amplitude in 19 points by using a 30 g B.K. electromagnetic transducer and the sign of each mode being determined by observing the Lissajou figures from an oscilloscope. The same was repeated for the transverse direction also at two sections, one at midspan and the other at 1/4th span.

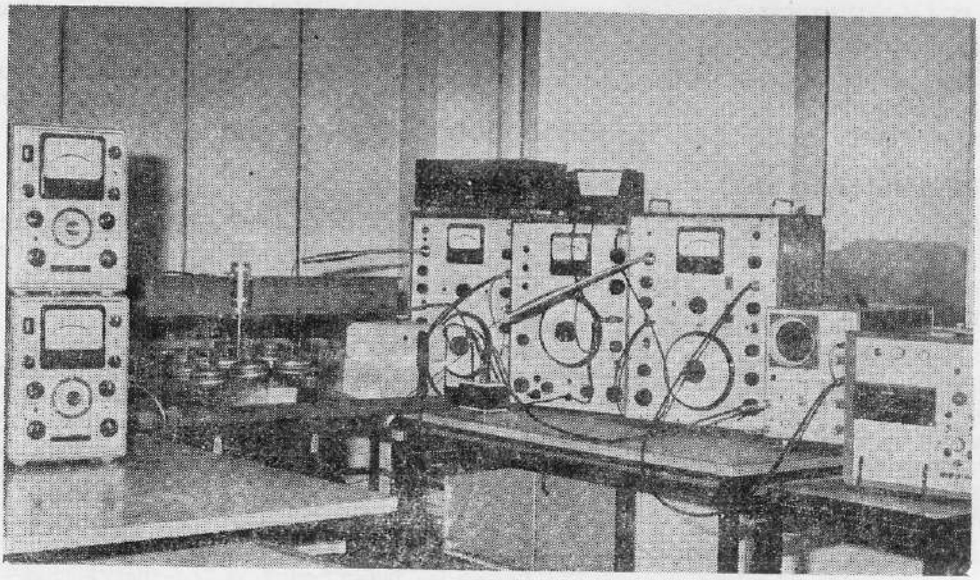

Fig. 3. Test Setup for Plexiglas Model Shell

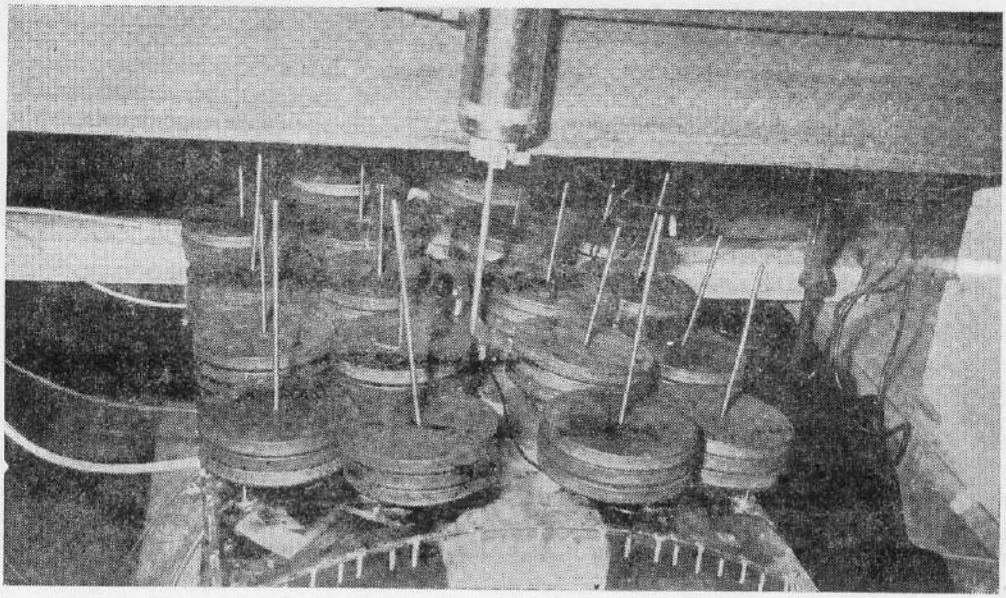

Fig. 4. View Showing the Pickup, Exciter, etc.

When the frequency went below 20 H_z which was the lowest range of the filter in the analyser, an automatic recording instrument known as Lumiscript (shown in Fig 3.) was used to measure the frequency. The exciter was removed and a very thin metallic piece was fixed on to the crown of the shell at midspan. A contactless electro-mechanical transducer was used to sense the vibrations which were directly recorded by the Lumiscript on a sensitive developing paper from which the frequency was calculated. The excitations were made by pressing and releasing the shell with a mild steel rod. The procedure was repeated twice, once on each side of the transducer and the average frequency was taken.

To plot the load-deflection curve in order to observe the deflection at the buckling load of the shell, a dial gauge was fixed on the crown of the shell at midspan and the load was increased at intervals 50 g per disc which amounted to 1.7 kg of total load on the surface of the shell, till the dial gauge showed large deformation indicating buckling.

A wooden beam supported by a hydraulic jack was kept at the bottom side of the shell running parallel to the crown line so that the excessive deformation before buckling could not destroy the shell.

Predeformed Plexiglas shell roof model

A plexiglas shall roof of the dimensions shown in Fig. 1 was allowed to predeform before loading so that the crown of the shell at midspan with respect to the other two ends had sunk down. This pre-deformation was necessary in order to study the buckling process in general, especially the reduction of the buckling load of shallow shells caused by creep deformation. After the shell was made to deform by heating it in a closed glass case, it was taken out, cooled and the exact amount of sinking of the crown of the shell was found by measuring with a dial gauge. The shell was then loaded as explained previously in steps of 500 g per disc and the frequency spectrum was found out.

Plexiglas shell roof model without edge beams

The dimensions and material of the shell were exactly the same as shown in Fig. 1, except for the fact that the edge beams were removed in this case. The loading technique and the method of measuring of frequency spectra were the same as explained above. Because of the absence of the edge beams, the stiffness of the shell had been considerably reduced and the fundamental frequency was less than that the shell with edge beams.

Plexiglass shell roof model of 78cm span

In order to study the effect of l/r ratio on the buckling strength of cylindrical shell roofs, the one metre long shell was reduced to 78 cm length by cutting off two sets of load points and the nondestructive tests described above were conducted on the shell. The loading intensities on the shell remained the same as in the case of 100 cm long shell.

Plexiglas shell roof of 56 cm span

Further case of l/r ratio was studied by cutting the shell by another test load points and making the length as 56 cm and the experiment was repeated as explained above.

Static buckling test on plexiglas model shell roofs

To compare the actual buckling load with the calculated value obtained from the non-destructive vibration test, the plexiglas model shell roof of the dimension shown in Fig. 1 was loaded as explained above in steps of 500g per disc to destruction. The same procedure was repeated for the cases of pre-deformed shell roof model and the shell roof without edge beams also.

DISCUSSION OF TEST RESULTS

Long plexiglas shell roof with edge beams

Fig. 5 shows the load deflection diagram from the static tests as well as the relationship between the square of the frequency and the lateral load obtained from vibration tests for the plexiglas model shell roof shown in Fig. 1. Table 1 shows the values of the buckling load calculated by the proposed non-destructive vibration method and also its comparison with the experimental values obtained by Kordina[4] (1963) on the same type of shell and the semi-empirical value obtained by Ivanyi[3] (1968). This table also gives the results obtained by loading the shell up to collapse. It can be seen that all these results agree to a fair degree of accuracy.

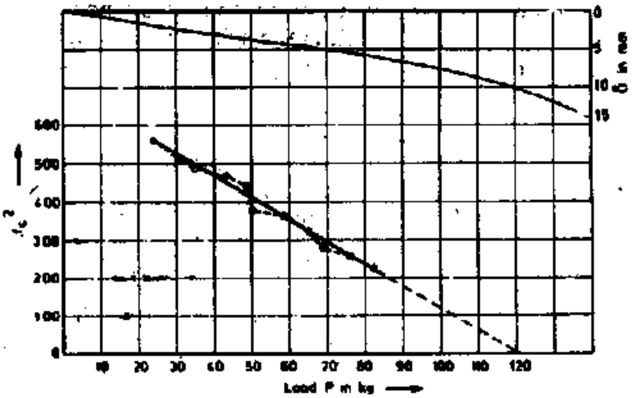

Fig. 5. Load Vs Square of Frequency (L = 100 cm)

Predeformed Plexiglas model shell roof

Having established experimentally that the proposed non-destructive vibration method could be satisfactorily applied to predict the buckling load, the study was extended for the case of predeformed shell also. Kordina[4] (1963) had performed a series of tests in order to study the buckling process in general, especially the reduction of the buckling load of shallow shells caused by creep deformation (Fig. 6). Plexiglas was used as a model material because it shows elastic properties at normal temperature, but quickly increasing deformation at high temperatures, part of which—similar to creep deformation in concrete—do not recede after unloading. Hence it was possible to produce any kind of creep deformation by rise of temperature within a few minutes, an affinity to those which would occur in real concrete shells could be expected.

$$\left[10^{-3}\right]\varphi_{kr} = \frac{q_{kr}}{E}\left(\frac{b}{t}\right)^2$$

q_{kr} =bucking load/sqare cm

f_0 =pre-deflection (without load)

E =modulus of elastisity

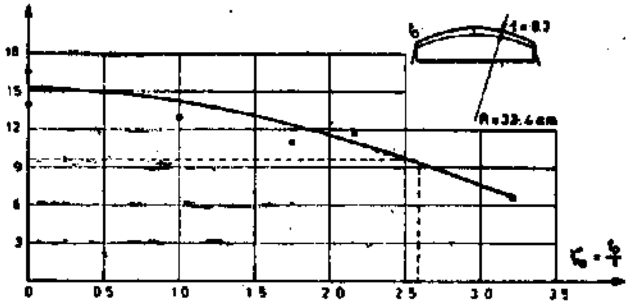

Fig. 6. Critical Load Versus Predeflection.

Fig. 7 shows the relationship between the square of the fundamental frequency and the lateral load obtained from the dynamic tests on the same shell tested by Kordina for his creep study, and also the value of the buckling load that can be extrapolated from the results of the dynamic tests. These values are calculated according to Kordina's pro-

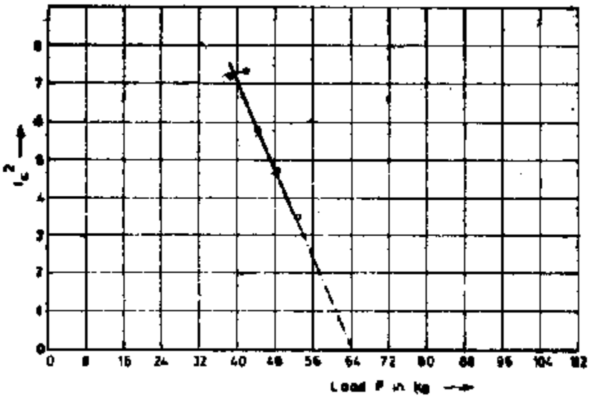

Fig. 7. Load Vs Square of Frequency (Predeformed shell) (L=100 cm)

posed method for the same shell as shown in Fig. 6. The actual buckling load on this predeformed shell obtained by static test is given in Table 1. It can be seen from Table 1 that all these results agree quite well.

TABLE I

COMPARISON OF THE BUCKLING LOADS ON CYLINDRICAL SHELL ROOFS

Material	Plexiglas
Dimensions	Radius 33.4 cm
	Thickness 0.3 cm

S. No.	Span in cm	Buckling Load		Theoretical values of Buckling Load		Remarks
		Expereimental Proposed method)	Actual	Ivanyi	Kordina	
1	100	120 kg	115 kg	136 kg	125 kg	—
2	100	64 kg	62 kg	—	65 kg	Predeformed shell
3	78	130 kg	—	—	—	—
4	56	120 kg	—	—	—	—

Plexiglas model shell without edge beams

Fig. 8 shows the graph between the lateral load versus the square of the frequency for a shell roof model having the same dimensions as shown in Fig. 1 and as tested and described above but without edge beams. Since the rigidity of the cross section without the edge beams is very much less when compared with a shell with edge beams, the spectral response of the shell was not clear enough. Further the carrying capacity of the shell is so low that it failed not by buckling but by the usual mode of collapse of a laterally loaded member. This has also been brought out by Steinert[6] in his thesis. The collapse load in bending was of the order of 64 kg and the predicted buckling load was 129 kg. Obviously the shell failed by bending failure as was observed in the experiment.

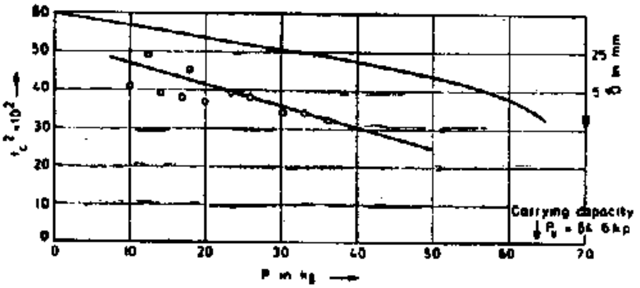

Fig. 8. Load Vs Square of Frequency shell Without Edge Beam (L=100 cm)

Plexiglas model shell roof L=78cm (with edge beams)

Fig. 9 shows the load deflection curve and the load versus square of the fundamental frequency for the plexiglas model shell roof of span 78 cm. This being an intermediate shell and the proposed method is applicable only for long shells ($l/r \geqslant 3$), the buckling load obtained from the graph seems to be fictitious. For this case, the buckling is not induced by the axial compression at the crown section at midspan as in the case of long shells, since the effect of longitudinal moment M_x and stress resultant N_ϕ in the transverse direction have great influence and cannot be ignored as in the previous case. It can be seen that the actual curve got from the experiment extends indefinitely parallel to the load axis.

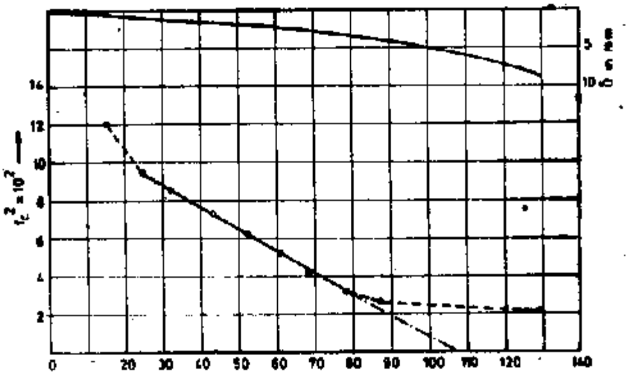

Fig. 9. Load Vs Square of Frequency (L=78 cm)

Plexiglas model shell roof l=56cm (with edge beams)

Similarly for the shell with length of 56 cm the l/r ratio becomes still smaller. The behaviour of the shell is exactly the same as explained above. When the span of the shell becomes smaller and smaller, as one would anticipate, the buckling load should be higher. But from Figs. 9 and 10, this does not seem to be the case, thus ascertaining the limitation of the proposed method only for long shells (length to radius ratio is equal to or greater than three).

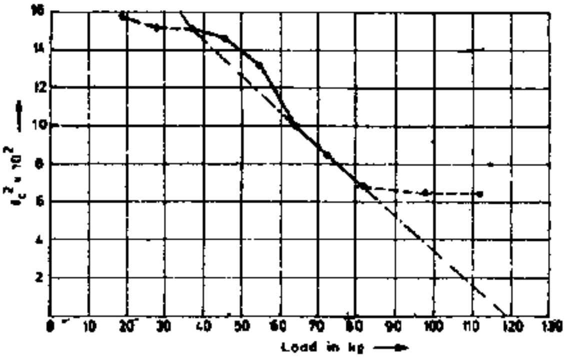

Fig. 10. Load Vs Square of Frequency (L=56 cm)

In order to investigate this more fully, the shell was loaded upto 120 kg and the natural frequencies obtained are shown in Fig. 10. It can be seen that the actual curve got from experiment does not follow the extrapolated line but extends indefinitely parallel to the load axis indicating no probability of buckling.

CONCLUSION

The following are the major findings of this investigation.

1. The tests on plaxiglas models indicate that this method is useful in cases where the buckling loads precede the bending failure.
2. These studies also indicate that the application of the method shall be limited to long shells i.e. $1/r > 3$, where axial compression predominates. For short and intermediate shells this method is not applicable.

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