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LAGREST VALUE THEORY APPLIED TO PROBABILITY OF EARTHQUAKE OCCURRENCE

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INTRODUCTION

In the absence of a good understanding of the physical process governing the origin of earthquakes the problem of earthquake occurrence be studied only by applying the various statistical models. The usual objectives are either the estimation of the probability of occurrence of large shocks, or the properties of earthquakes sequences as a whole. In the present investigation, the theory of largest values is applied to estimate the probability of occurrence and return period (T) of the largest earthquakes in the North-West Kashmir region with the assumption that the future largest shocks would behave similarly to those in the past interval for which data is available.

THEORY AND PROCEDURE

The theory of largest values is known and was published by Gumbel (1). Gumbel applied this theory mainly to the prediction of floods but later on it was applied in the field of seismology by Nordquist (2). Some development was made by Epstein and Lomnitz (3) who assumed that the number of earthquake per year is a Poisson random variable. Later on this theory was applied by Karnik and Schenkova (4) for estimating the magnitude of the largest possible earthquake in the Balkan and European region.

The basic assumptions of this theory are the following: (a) the number of earthquakes in a year is a poisson random variable and (b) the earthquake magnitude X is considered as a random variable described by the cummulative distribution function:—

$$F(x)=1-e^{-\beta x}, x \geqslant 0 \quad (1)$$

The cummulative probability distribution function of the largest event in a given interval (say for one year) then has the form of the double exponental

$$G(y)=\exp[-\alpha \exp(-\beta y)], \ y=c(x-u) \quad (2)$$

where α, β, c, u are parameters. The probability $F(\hat{X}_m)$ of the mth largest annual magnitude is obtained by interpolation between the probability of the most probable smallest and largest observations $F(\bar{x}_1)$ and $F(\bar{x}_n)$ respectively i.e.,

$$F(X_m)=F(x_1)+\frac{m-1}{n-1}[F(x_n)-F(x_1)] \quad (3)$$

The return period (occurrence interval) $T(\hat{X}_m)$ of earthquakes having a magnitude equal or larger than a given threshold valve $\hat{X}_m$ are defined:

$$T(\hat{X}_m)=[1-F(\hat{X}_m)]^{-1} \quad (4)$$

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In practice, the largest annual magnitude $\hat{X}_1, \hat{X}_2 \ldots\ldots\ldots \hat{X}_n$ are arranged in order of increasing magnitude. The representative values of $\hat{X}_m$ are grouped into classes, and the values corresponding to upper limits of classes are associated with the probabilities $F(\hat{X}_m)$ calculated according to (3). The pairs $[\hat{X}_m, F(\hat{X}_m)]$ are plotted on extremal probability paper. According to theory the points should fit a straight line and the extrapolated line will yield the largest magnitude which will be exceeded with a given probability. The magnitudes which will be exceeded with probability of $P=1\%$ (i.e., $T=100$ years) can be taken as largest possible earthquake in a region.

Further useful information about the future activity can be obtained from the Time inequalities T (M max, obs)$\gtrless n$, where n is the length of the period of observations expressed in years, M (max. obs.) is the largest magnitude actually observed. According to the Gumbel (1) probability theory T(M max, obs)$<n$ means that one can expect largest shock in the future in the set of the comparable size and T(M max, obs) $>n$ that one cannot expect such a shock.

DATA AND ITS ANALYSIS

In this study shocks which have occurred in the period 1902 to 1974 have been taken. The data were taken from USCGS, ISC and other sources. The one year interval has been used as most appropriate unit. Shorter intervals are not suitable because then the number of empty intervals increases. Fig. 1, shows the spatial distribution of NW Kashmir earthquakes for the period 1902 to 1974.

First the annual largest magnitudes are arranged in order of increasing magnitude and the corresponding F $(\hat{X}_m)$ values are computed according to equation (3).

Fig. 2 shows the probability distribution of the largest value of earthquakes for North West Kashmir earthquakes in three periods 1902-1925, 1902-1954 and 1902-1974.

DISCUSSION

The distribution of points $[\hat{X}_m, F(\hat{X}_m)]$ plotted on the extremal probability paper fits fairly well a straight line for all the three periods. This proves that the actual physical process fits well this statistical model of distribution of the extreme value of magnitude. The general feature of most distributions of points $(\hat{X}_m, F(\hat{X}_m)]$ is that with increasing duration of period the slope of the straight line decreases or the line as a whole is shifted to lower magnitude. Consequently, for large magnitude the probability of occurrence of magnitudes larger than a certain value decreases.

Table 1 shows the return periods for the individual shocks having a magnitude equal to or larger than a selected value. The values T (M) can be used as informative values for estimating earthquake risk within the seismic region as a whole. Table 2 gives a comparison of magnitudes which will be exceeded with the probability of 1%($T=100$ years) in future with the largest actually observed magnitude during 1902-1974 and the corresponding return periods T (M). This probability level has been chosen assuming the values $M_{Max,\ T=100}$ are close to the 'maximum possible' magnitudes. The values of $M_{1\%}$ (values of M which will be exceeded in the future with a probability of 1%) are read from the straight line approximating the probability paper. The difference dM between $M_{1\%}$ and the largest magnitude actually observed M (max, obs) do not much exceed the standard error of magnitude determination ($\pm\cdot 3$) except in

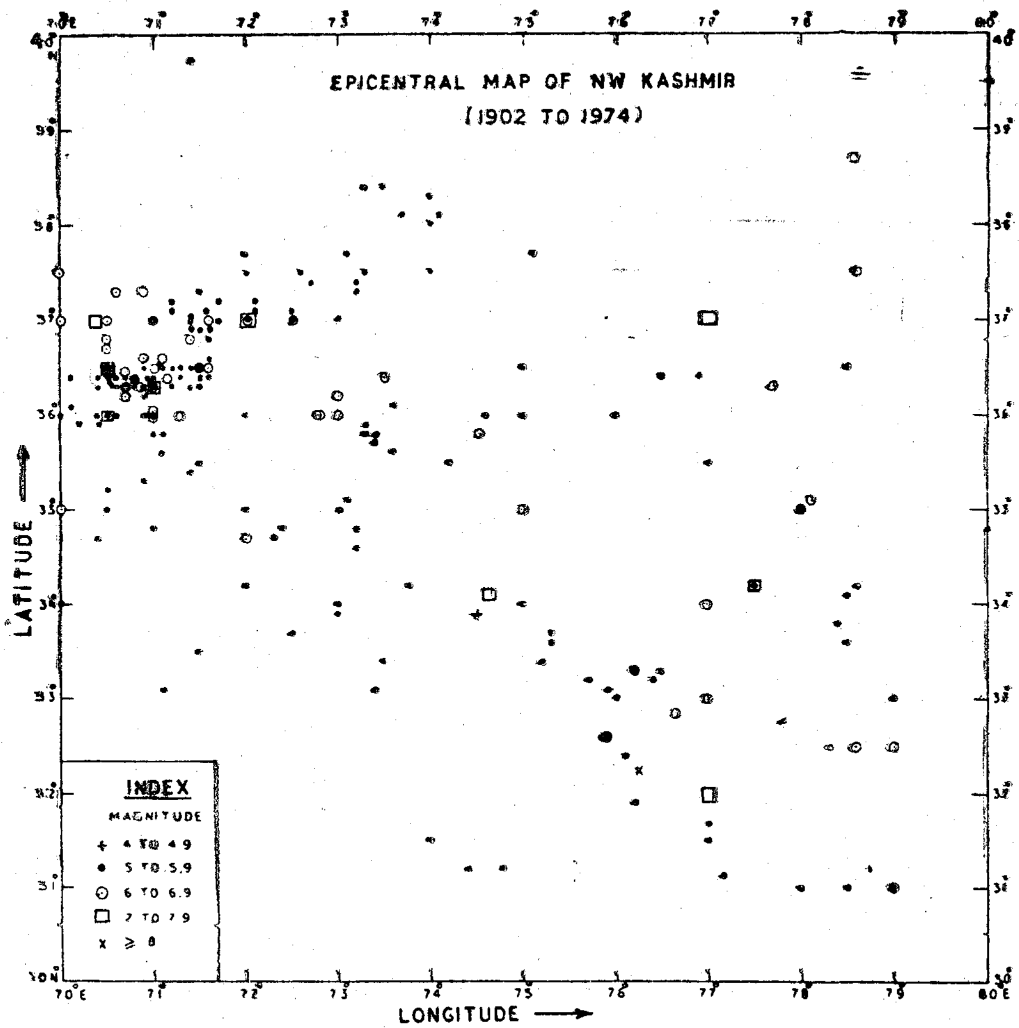

Fig. 1 Spatial distribution of NW Kashmir earthquakes for the period 1902–1974.

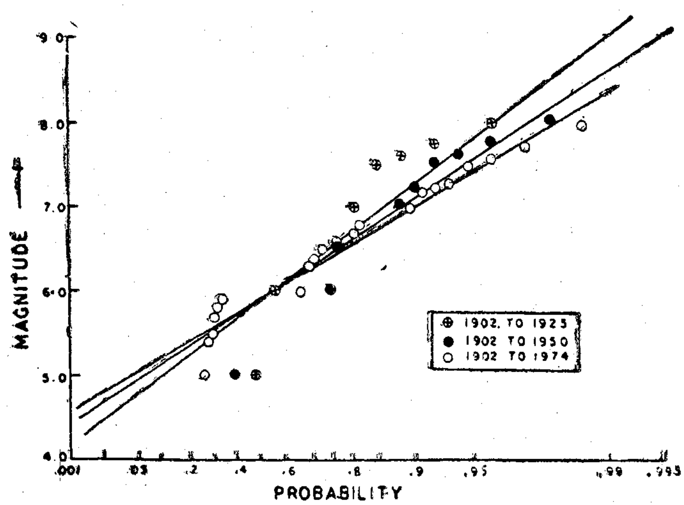

Fig. 2 Magnitude-Probability of maximum largest possible magnitude distribution for three different periods 1902–1925, 1902–1950, and 1902–1974.

the first period 1902-1925 which is evidently too short for statistical treatment. On the other hand the periods 1902-1950 and 1902-1974 yield very small differences. Moreover increase of length of the observed period does not have much effect on the return period of maximum possible earthquake. So our sampled period is representative period

TABLE 1

Region	M	$T(M)$ years		
		1902-1925	1902-1950	1902-1974
N-W Kashmir Region	4.5	1.01	1.002	—
	5.0	1.112	1.06	1.03
	5.5	1.428	1.351	1.29
	6.0	2.17	2.17	2.13
	6.5	3.703	4.00	4.16
	7.0	6.66	8.33	9.09
	7.5	12.50	17.24	20.83
	8.0	25.00	38.46	50.00

TABLE 2

Region	Period	M max $T=100$	M max, obs	T max, obs	Condition	$dM=M\%-$ M max, obs
	1902-1925	9.1	8.0	17.241	$T<n$	1.1
	1902-1950	8.65	8.0	35.714	$T<n$	0.65
	1902-1974	8.45	8.0	45.454	$T<n$	0.45

of observation. The time-inequality study shows that the return period of the maximum observed magnitude is less than the length of the period of observation in all three periods. This shows we can expect larger shock in the future in the set of same size.

CONCLUSIONS

The present analysis shows that the Gumbel probability theory of the largest values applied to the earthquake magnitudes can possibly be used for estimating the probability of occurrence of large earthquakes. The corresponding return periods may serve as a quantitative measure of seismicity. The above conclusions are based on the assumption that the earthquakes are random events which follow the same statistical regularities as in past.

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