Paper No. 36

DYNAMIC EARTH PRESSURE ON RETAINING WALLS DUE TO GROUND EXCITATIONS

A.S. Arya* and Y.P. Gupta**

SYNOPSIS

The magnitude of dynamic pressures due to cohesionless fills behind retaining walls and their points of application are computed on the basis of wedge theory assuming that the acceleration of the mass of the wedge relative to that of the ground varies linearly from zero at the base of the wall to maximum design value at its top level. The magnitudes of dynamic pressures are given as fraction of the static pressures. Tabular data given is suitable for design office use.

INTRODUCTION

Shaking Table Tests [1,2†] have previously been carried out on two types of model retaining walls. Fixed walls were those whose base and sides were held by the walls of the test box against any movement and movable walls were those which were free to tilt about the base. Some tests[2] have also been carried out in the field on retaining walls standing on ground by imparting ground vibrations by a D.C. motor at a certain distance from the wall. Obviously the movable wall or the one used in the field represents more closely the conditions of the normal retaining wall as compared with the fixed wall. The results of the above testing work for movable and fixed walls may be summarised briefly as follows:

1. The lateral earth pressure during vibration consisted of two parts—First the increase in the static earth pressure which would remain even after the vibration had stopped and second, a varying pressure with time whose intensity might be expressed by its amplitude. Unless the vibrations were too severe (the acceleration being more than 500 gals) the residual earth pressure would be considerably more than the vibrating amplitude. Therefore, for design purposes, the residual earth pressure only may be considered.

* Professor & Assistant Director, School of Research & Training in Earthquake Engineering, University of Roorkee, Roorkee.

** Lecturer in Civil Engineering, School of Research & Training in Earthquake Engineering, University of Roorkee, Roorkee.

† Refers to serial number of references listed at the end of the paper.

2. "Inspite of the great complexity of the lateral earth pressure corresponding to various characteristics of walls, backfill and given vibrations, the magnitude of the total lateral earth pressure coincides with, or is lower than the value given by Mononobe-Okabe formula. At the present state of research, it is recommended that Mononobe-Okbe formula can be used in the design of gravity type quay walls" [1].

3. In the case of the field test[2], the relative acceleration of the soil mass behind the wall was found to vary linearly from zero at base of wall to maximum at top of wall.

4. The point of application of the residual earth pressure was found to lie between 0.33 and 0.4 ofthe height of wall measured from the bottom the shifting up of the point of application being more for higher values of acceleration.

Based upon the above results, suitable assumptions have been made for deriving the numerical results herein.

1. The earthfill behind the wall is cohesionless.

2. The failure surface is a plane starting from the heel of the wall and the Mononobe-Okabe formula is applicable for calculating the pressure exerted on the wall by the failure wedge.

3. The horizontal acceleration of the mass at the wedge varies linearly from zero at base of wall to maximum value at the top level.

TOTAL RESIDUAL ACTIVE EARTH PRESSURE

For uniform acceleration a_h of the mass of the wedge the Mononobe-Okabe formula for total active earth pressure including static as well as dynamic components is as follows.

$$P_a = \tfrac{1}{2} w H^2 C_a, \tag{1}$$

$$C_a = \frac{\cos^2(\phi-\theta-\alpha)}{\cos\theta \cos^2\alpha \cos(\phi_1+\alpha+\theta)} \times \frac{1}{\left[1+\left\{\dfrac{\sin(\phi+\phi_1)\sin(\phi-\delta-\theta)}{\cos(\alpha-\delta)\cos(\phi_1+\alpha+\phi)}\right\}^{\frac{1}{2}}\right]^2} \tag{2}$$

Where,

P_a = active earth pressure on the wall including the increase due to ground acceleration,

w = unit weight of soil,

H = height of wall,

C_a = Coefficient of total active earth pressure,

δ = angle of surcharge of earthfill,

α = angle of back of wall with vertical,

ϕ = angle of internal friction of soil,

ϕ_1 = angle of friction between the wall and the earth fill,

θ = $\tan^{-1} a_h$

α_h = horizontal seismic coefficient, that is, ratio of design acceleration to acceleration due to gravity.

The point of application of P_a lies at H/3 from the bottom.

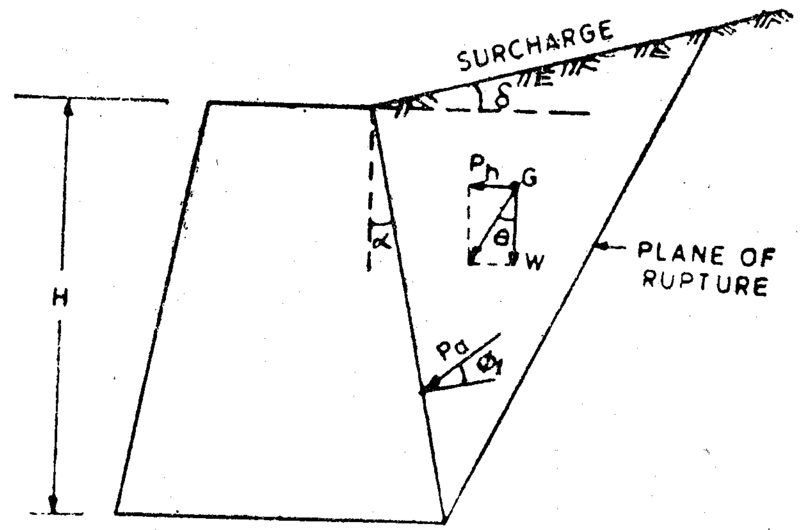

Figure 1

When the design acceleration is assumed to vary linearly, the value of θ will change and the point of application of the resultant is likely to shift up from the lower third point. The angle θ is actually equal to the angle that the resultant of the weight of the wedge and the seismic force acting on it makes with the vertical. This is shown in Fig. 1. To determine its general value for the present case, consider the failure wedge shown in Figure 2 in which the failure plane starts from any point Y on the back of the wall. Let the weight of the wedge be W and the horizontal force on it be P_h. Then,

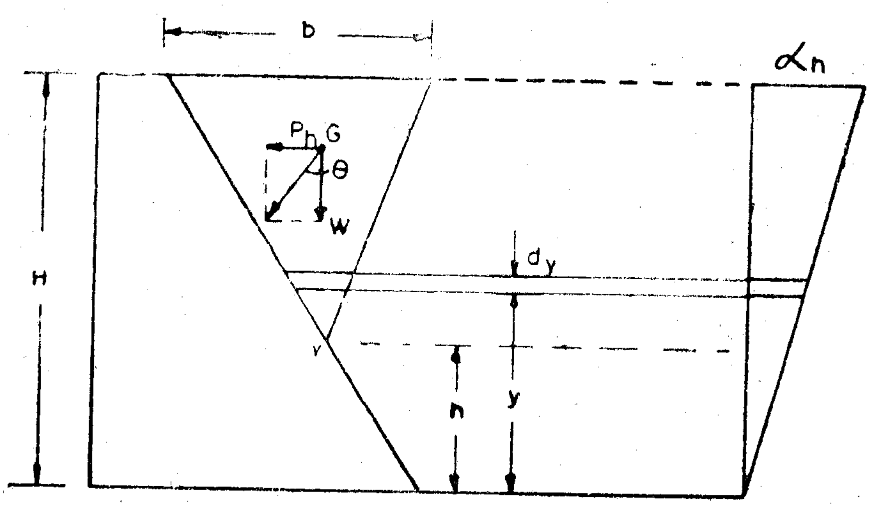

Figure 2

$$W = \frac{w}{2} bH\left(1 - \frac{h}{H}\right)$$

$$P_h = \int_h^H w\left(\frac{y-h}{H-h} \times b\right)\frac{\alpha_h}{H}\, y\, dy$$

$$= \frac{wb\,\alpha_h H}{6\left(1-\frac{h}{H}\right)} \times \left[2-3\frac{h}{H}+\frac{h^3}{H^3}\right]$$

$$\therefore \quad \tan\theta = \frac{P_h}{W}$$

$$= \frac{\alpha_h}{3}\,\frac{2-\frac{3h}{H}+\frac{h^3}{H^3}}{\left(1-\frac{h}{H}\right)^2}$$

$$= \frac{2}{3}\alpha_h\left(1+\frac{h}{2H}\right)$$

$$\therefore \theta = \tan^{-1}\left[\frac{2}{3}\alpha_h\left(1+\frac{h}{2H}\right)\right] \tag{3}$$

For the failure plane starting from the base of the wall

$$\theta = \tan^{-1}\left[\frac{2}{3}\alpha_h\right] \tag{4}$$

Substituting this value of θ in Equation 2, the total active earth pressure may be determined.

DYNAMIC INCREMENT OF PRESSURE

If θ is put equal to zero in Equation 2, the ordinary Coulomb static pressure will be obtained. The difference of the total and the static pressure will give the net dynamic increment of pressure on the retaining wall.

POINT OF APPLICATION OF THE RESULTANT PRESSURE

On the straight back of a retaining wall, the distribution of Coulomb static active earth pressure is linear, varying from zero at the top to maximum at the base. The distribution of the dynamic increment is also similar if α_h was assumed uniform, thus giving the line of action of the resultant at H/3 above the base for both static and dynamic components. But when the horizontal acceleration is taken linear as assumed, the distribution of dynamic increment of pressure becomes different from linear, the line of action of the resultant pressure shifting above the lower third point. In order to determine the distribution of dynamic pressure the following procedure was followed.

In the Coulomb's Wedge theory, the assumption is made that every point on the back of the retaining wall is the foot of the plane of the incipient failure wedge. Let us consider two points A and B at heights h_A and $h_B = h_A + h_{AB}$ from the base of the retaining wall. The total static and the dynamic active earth pressures acting on the parts of the wall above the points A and B can now be calculated as explained above. For determining the total active pressure, θ_A and θ_B will be found from Eq. 3 by substituting the corresponding heights h_A and h_B. Let the dynamic increments determined be P_{dA} and P_{dB} respectively. Then the dynamic pressure acting on the height h_{AB} of the wall between points A and B will be $P_{dA} - P_{dB}$. If the height h_{AB} is sufficiently small, the pressure $P_{dA} - P_{dB}$ may be assumed to be distributed uniformly on this height. Thus the intensity of pressure on height h_{AB},

$$P_{AB} = \frac{P_{dA} - P_{dB}}{h_{AB}} \tag{5}$$

Now dividing the height of wall in convenient number of divisions, the intensity of pressure can be obtained at the centre of each division. Thus the distribution of dynamic pressure can be derived. Knowing the distribution of pressure, the centre of gravity of the resultant dynamic pressure can be found by taking moments of the forces acting on the divisions about the base.

THE COMPUTER PROGRAM

For working out the numerical results, a computer program has been written in the FORTRAN language. The following quantities have been computed:

The total earth pressure coefficient C_a,

The static earth pressure coefficient C_{as},

The dynamic earth pressure coefficient $C_{ad} = C_a - C_{as}$,

(These coefficients are to be multiplied with $\frac{1}{2}$ wH^2 to get the magnitudes of resultant pressures).

The ratio of dynamic to static pressures C_{ad}/C_{as},

The distance of point of application of resultant dynamic pressure above base of retaining wall as a fraction of height. This is denoted as CG.

The base pressure coefficient C_{pb} to be multiplied with 'wh' to give the base pressure.

The ratios of pressure intensities at various heights to base pressure.

RESULTS

The values of the pressures etc. stated above were computed for the following values of the variables:

ϕ — 30°, 36°

δ — 0°, 20°

α — 0°, 20°

ϕ_1 — 10°, 20°

α_h — .02, .04, .06, .08, .10, .12

The resulting values of C_{as}, C_{ad}, C_{ad}/C_{as}, CG, and C_{pb} are presented in Table 1. The ratios of pressure intensities at various heights to the base pressure were computed for all combinations of the variables but only the average values are presented here. It was found that the pressure distributions are not appreciably different from each other. Based on the calculated data, the following resutls could be arrived at.

(1) Dynamic Increment of Pressure

The influence of the variables ϕ, δ, α, ϕ_1 and a_h on dynamic part of pressure are found to be as follows when only the variable under consideration is changed, all the others taken fixed in magnitude.

(a) As the angle of internal friction ϕ increases, not only the static component decreases, the ratio of dynamic to static component also decreases. Thus the dynamic component decreases appreciably when ϕ is increased.

(b) When the angle of surcharge δ is increased. the static part increases. The dynamic increment is found to increase more rapidly and its ratio to static pressure also increases.

(c) When the angle of the back of the wall with the vertical α is increased, the static pressure increases, and the ratio of dynamic to static also increases. Thus the dynamic increment increases faster than the static part.

(d) As the angle of friction of soil with back of wall ϕ_1 increases, the static pressure decreases but the ratio of dynamic to static pressure increases. The effect is that the magnitude of dynamic increment does not appreciably change.

(e) As the seismic coefficient a_h increases, the dynamic pressure increases slightly more than proportionately. A typical relationship is shown in Fig. 4.

(2) Line of Action of Increment of Pressure

Effects of the variables on the position of line of action are as follows :

(a) Value of angle of internal friction ϕ has little influence on the line of action of the dynamic increment.

(b) When the surcharge angle δ is increasea from 0° to 20°, the line of action shifts up. In the worst case, the shift is 2.9%.

(c) When the angle of back of wall α is changed from 0° to 20°, the line of action is not appreciably affected. The maximum shift is 0.25% upward.

(d) When the angle of friction between soil and wall ϕ_1 is varied from 10° to 20°, the line of action remains almost unchanged.

(e) The line of action is affected maximum by the increase in the seismic coefficient a_h. In the worst case when a_h is varied from .02 to 0.12, the line of action shifts upwards by about 4%.

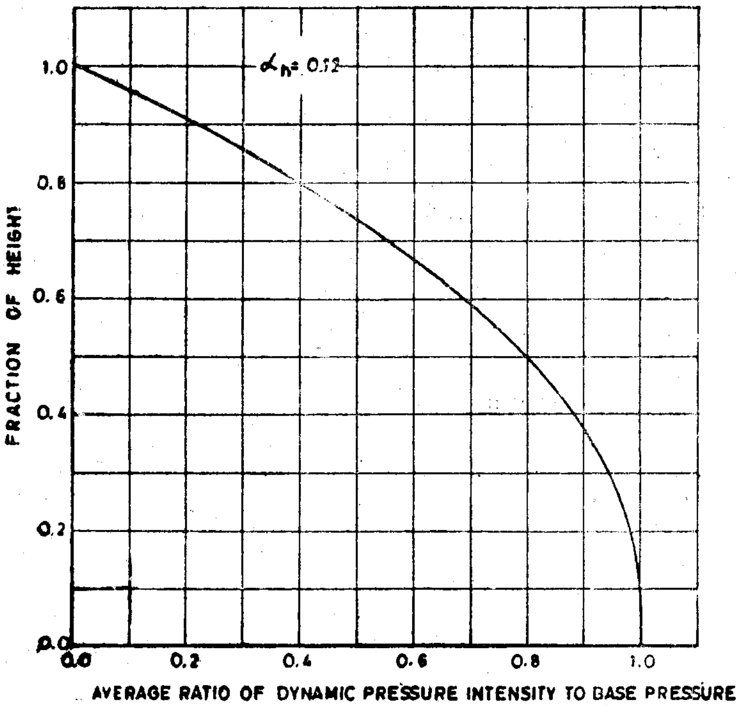

Fig. 3 Typical Dynamic pressure Distribution

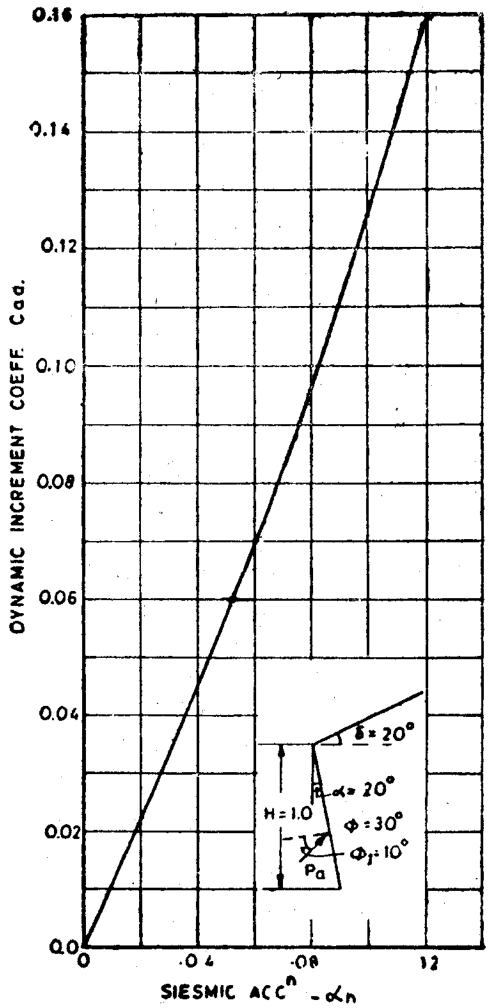

Fig. 4 Typical Variation of Dynamic Increment with seismic Coefficient

(3) Distribution of Dynamic Increment of Pressure

Since the line of action represents the centre of gravity of the pressure distribution, the latter is affected by the change of variables in a similar way as the former. Thus the angle of internal friction ϕ, the angle of back of wall α, and the angle of friction between soil and wall ϕ_1 do not change the pressure distribution appreciably. The other two variables δ and α_h are also found to change the intensities of pressure only slightly.

From the above study it therefore follows that the dynamic pressure distribution and its line of action could be averaged within a maximum variation of $\pm$ 2%. The average ordinates are given in Table 2. A typical curve for the value of α_h = 0.12 is shown in Figure 3.

CONCLUSIONS

The magnitude of total active earth pressure behind retaining walls due to ground excitations may be computed by Mononobe-Okabe formula as given in IS: 1893-1962, taking $\theta = \tan^{-1} (\frac{3}{2}\alpha_h)$ for any combination of other variables. The same expression may be used for calculating the static component by taking $\theta=0$. The dynamic increment is found by taking the difference of the above two values. It may be taken to act at heights given in Table 1.

Numerical values of relevant data required for design of retaining walls are tabulated for several combinations of variables involved for ready reference.

ACKNOWLEDGEMENTS

The degital computor work was carried out at the SERC, Roorkee.

REFERENCES

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TABLE 1

$\alpha_h = 0.02$

Ø	30°								36°							
Ø'	10°				20°				10°				20°			
α	0°		20°		0°		20°		0°		20°		0°		20°	
δ	0°	20°	0°	20°	0°	20°	0°	20°	0°	20°	0°	20°	0°	20°	0°	20°
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
C_{as}	.3085	.4195	.4782	.6867	.2973	.4142	.4794	.7143	.2425	.3105	.4080	.5528	.2354	.3060	.4111	.5714
C_{ad}	.0079	.0158	.0089	.0215	.0082	.0171	.0102	.0258	.0069	.0113	:0082	.0158	.0071	.0120	.0092	.0184
C_{ad}/C_{as}	.0256	.0378	.0186	.0314	.0278	.0414	.0213	.0362	.0284	.0365	.0201	.0286	.0303	.0392	.0224	.0321
C.G. from Base.	.3769	.3778	.3770	.3780	.3770	.3779	.3771	.3782	.3769	.3774	.3769	.3775	:3769	.3775	.3770	.3776
C_{pb}= Coeff. x10^{-2}	.0589	.1173	.0660	.1590	.0614	.1269	.0761	.1910	.0513	.0841	.0610	.1172	.0531	.0890	.0685	.1363

$\alpha_h = 0.04$

C_{as}	.3085	.4195	.4782	.6867	.2973	.4142	.4794	.7143	.2425	.3105	.4080	.5528	.2354	.3060	.4111	.5714
C_{ad}	.0160	.0327	.0181	.0447	.0167	.0355	.0207	.0537	.0139	.0232	.0166	.0324	.0145	.0246	.0187	.0378
C_{ad}/C_{as}	.0519	.0780	.0378	.0651	.0564	.0857	.0433	.0752	.0575	.0746	.0407	.0586	.0615	:0804	.0455	.8661
C.G. from Base	.3776	.3796	.3777	.3800	.3778	.3799	.3780	.3803	.3775	.3786	.3776	.3788	.3776	.3788	.3778	.3791
C_{pb}= Coeff x10^{-2}	.1188	.2400	.1340	.3270	.1242	.2599	.1535	:3920	.1036	.1711	.1230	.2386	.1070	.1812	.1386	·2779

Contd.

TABLE–1 (Contd.)

$\alpha_h = 0.06$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
C_{as}	.3084	.4195	.4782	.6867	.2973	.4142	.4793	.7143	.2426	.3105	.4080	.5528	.2354	.3060	.4111	.5714
C_{ad}	.0243	.0508	.0275	.0696	.0255	.0552	.0317	.0841	.0212	.0356	.0252	.0498	.0220	.0378	.0285	.0588
C_{ad}/C_{as}	.0790	.1211	.0576	.1014	.0859	.1334	.0661	.1177	.0874	.1147	.0618	.0902	.0935	.1237	.0693	.1021
C.G. from base.	.3783	.3816	.3786	.3823	.3786	.3821	.3789	.3829	.3782	.3798	.3783	.3802	.3784	.3802	.3786	.3807
C_{pb}= Coeff.x10^{-2}	.1800	.3680	.2031	.5030	.1880	.3990	.2332	.6050	.1568	.2607	.1865	.3643	.1625	.2766	.2100	.4250

$\alpha_h = 0.08$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
C_{as}	.3084	.4195	.4782	.6867	.2973	.4142	.4793	.7143	.2426	.3105	.4080	.5528	.2354	.3060	.4111	.5714
C_{ad}	.0329	.0702	.0373	.0967	.0346	.0766	.0430	.1172	.0286	.0486	.0341	.0683	.0298	.0518	.0386	.0801
C_{ad}/C_{as}	.1068	.1674	.0779	.1408	.1163	.1850	.0897	.1641	.1181	.1567	.0836	.1235	.1265	.1694	.0940	.1402
C.G. from base.	.3790	.3840	.3794	.3850	.3794	.3847	.3799	.3859	.3788	.3813	.3790	.3818	.3792	.3817	.3795	.3824
C_{pb}= Coeff.x10^{-2}	.2423	5020	.2737	.6880	.2538	.5461	·3148	.8290	.2110	.3520	.2511	.4945	.2189	.3750	.2835	.5780

Contd.

TABLE 1 (Contd.)

$\alpha_h = 0.10$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
C_{as}	.3084	.4195	.4782	.6867	.2973	.4142	.4793	.7143	.2426	.3105	.4080	.5528	.2354	.3060	.4111	.5714
C_{ad}	.0417	.0913	.0473	.1263	.0439	.0999	.0547	.1537	.0363	.0623	;0433	.0877	.0378	.0666	.0491	.1033
C_{ad}/C_{as}	.1354	.2176	.0990	.1839	.1477	.2413	.1142	.2152	.1496	.2008	.1060	.1587	.1605	.2176	.1194	.1807
C.G.from. Base.	.3798	.3869	.3803	.3803	.3803	.3878	.3809	.3895	.3795	.3828	.3798	.3835	;3799	.3854	.3804	.3843
C_{pb}= oeff. x10^{-2}	.3058	.6426	.3459	.8820	.3208	.6999	.3984	1.0666	.2660	.4490	.3169	.6290	.2765	.4780	.3583	.7372

$\alpha_h = 0.12$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
C_{as}	.3084	.4195	.4782	.6867	.2973	.4142	.4793	.7143	.2426	.3105	.4080	.5528	.2354	.3060	.4111	.5714
C_{ad}	.0508	.1142	.0577	.1588	.0535	.1255	.0669	.1943	.0441	.0768	.0527	.1083	.0460	.0822	.0599	.1280
C_{ad}/C_{as}	.1648	.2723	.1207	.2313	.1801	.3031	.1396	.2720	.1819	.2473	.1291	.1960	.1955	:2687	.1457	.2240
C.G.from. Base	.3806	.3905	.3812	.3923	.3811	.3917	.3820	.3940	.3802	.3845	.3806	.3854	.3807	.3852	.3813	.3865
C_{pb}= Coeff. x10^{-2}	.3706	.7890	.1436	1.0850	.3890	.8607	.4840	:3156	.3223	.5480	.3840	.7689	.3350	.5840	.4350	·9025

TABLE 2

Average values of Dynamic Pressure Intensities and Line of Action of Dynamic Increment.

		Value of α_h						General Average
		0.02	0.04	0.06	0.08	0.10	0.12	
Line of action		0.3774	0.3786	0.3799	0.3813	0.3830	0.3849	0.3808
Ratio of pressure at given height to base pressure	Top.	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.9H	0.099	0.101	0.103	0.106	0.108	0.112	0.105
	0.8H	0.282	0.287	0.292	0.999	0.306	0.312	0.297
	0.7H	0.444	0.451	0.459	0.468	0.474	0.476	0.562
	0.6H	0.586	0.593	0.602	0.612	0.623	0.632	0.608
	0.5H	0.707	0.714	0.723	0.733	0.744	0.747	0.728
	0.4H	0.806	0.813	0.821	0.831	0.841	0.848	0.827
	0.3H	0.885	0.891	0.898	0.906	0.914	0.920	0.903
	0.2H	0.943	0.948	0.953	0.959	0.964	0.968	0.956
	0.1H	0.982	0.984	0.987	0.990	0.993	0.995	0.989
	Bottom	1.000	1.000	1.000	1.000	1.000	1.000	1.000