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AN INVESTIGATION INTO VIBRATION OF STACKS

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INTRODUCTION

The vibration of chimneys has been studied by a number of investigators. Chimneys are treated as vertical cantilever beams. When the beam vibrates, there are bending, shear and rotatory inertia effects. For beams which are long as compared to the cross-sectional dimensions, bending effects are predominant and analysis of such cases has been carried out by Housner [3]. Chimneys have been analysed based on the above three effects by Chandrasekharan.[1] The mass of beam in addition to its movement in the lateral direction also exerts a downward force. This vertical load may have some influence on the vibration characteristics of the stacks. The purpose of this paper is to study the extent of the influence of the vertical load on the natural frequencies and mode shapes of the chimneys.

EQUATION OF MOTION

Fig. 2 represents the freebody diagram of an elementary length of chimney shown in Fig. 1. The following equations may be written:

$$y = y_s + y_b \quad \text{...(1)}$$

$$V = \mu AG \frac{\partial y_s}{\partial x} \quad \text{...(2)}$$

$$M = -EI \frac{\partial^2 y_b}{\partial x^2} \quad \text{...(3)}$$

$$\frac{\partial M}{\partial x} = V - \rho I \frac{\partial^3 y_b}{\partial x \partial t^2} + N \frac{\partial y}{\partial x} \quad \text{...(4)}$$

$$\frac{\partial V}{\partial x} = m \frac{\partial^2 y}{\partial t^2} \quad \text{...(5)}$$

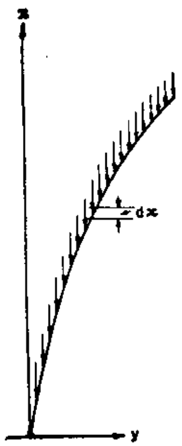

Fig. 1.
A Vertical Cantilever

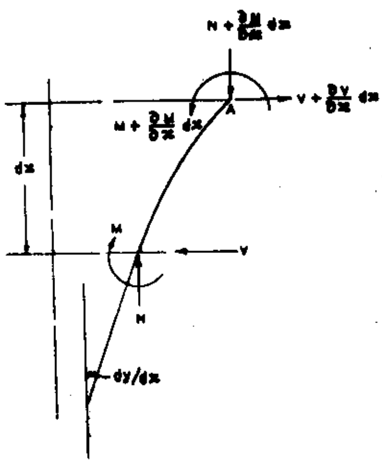

Fig. 2.
Free Body Diagram

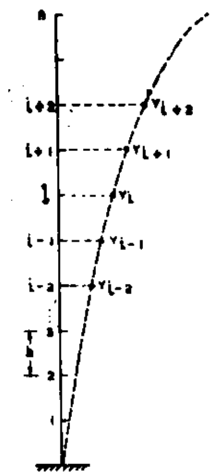

Fig. 3.
Chimney Lebelled at Discrete Points

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where

y = total deflection of the beam

y_b = contribution to the deflection of bending moment (including rotatory inertia)

y_s = contribution to the deflection of shear deformation

V = shear force at any cross-section

M = bending moment at any cross-section

E = modulus of elasticity of the beam

I = moment of inertia of the cross-section.

N = axial force at any section x of the beam

ρ = mass per unit volume

m = mass per unit length = ρA

μ = shape factor in shear

A = cross-sectional area of the chimney

Equations (4) and (5) have been obtained by considering dynamical equilibrium of the free-body of an element dx of the beam (Fig. 2). Equations (1) to (5) can be combined into a single equation as given below for uniform beams.

$$EI\frac{\partial^4 y}{\partial x^4} - \frac{EI\rho}{\mu G}\frac{\partial^4 y}{\partial x^2 \partial t^2} + \rho A\frac{\partial^2 y}{\partial t^2} - \rho I\frac{\partial^4 y}{\partial x^2 \partial t^2} + N\frac{\partial^2 y}{\partial x^2} + \frac{\partial N}{\partial x}\cdot\frac{\partial y}{\partial x} = 0 \quad ...(6)$$

It is known that

$$N = \rho g A\,(l - x) \quad ...(7)$$

and

$$\frac{dN}{dx} = -\rho g A \quad ...(8)$$

Solution of the Eq. (6) in the following form may be assumed

$$y = Y \sin(pt - \alpha) \quad ...(9)$$

where Y is a function of x only.

Substituting the value of y from Eq. (9) into Eq. (6), the following equation results.

$$EI\frac{d^4Y}{dx^4} + N.\frac{d^2Y}{dx^2} - \rho g A\frac{dY}{dx} = p^2\left[\rho AY - \left(\frac{EI\rho}{\mu G} + \rho I\right)\frac{d^2Y}{dx^2}\right] \quad ...(10)$$

The above equation contains fourth order differentials on the left hand side while the right hand side contains the eigenvalue parameter p^2 and consists of terms upto second order differential. Equation (10) has to be solved subject to the following boundary conditions.

BOUNDARY CONDITIONS

At $x = 0$

$$y = 0 \quad ...(11a)$$

and

$$\frac{dy}{dx} = 0 \quad ...(11b)$$

At $x = l$

$$M = EI\frac{d^2y}{dx^2} = 0 \quad ...(12a)$$

and

$$V = EI\frac{d^3y}{dx^3} = 0 \quad ...(12b)$$

On substituting Eq. (9) into these boundary conditions one gets

at $x = 0$

$$Y = 0 \quad ...(13a)$$

and $$\frac{dY}{dx} = 0 \qquad ..(13b)$$

and at $$x = l$$

$$\frac{d^2Y}{dx^2} = 0 \qquad ..(14a)$$

and $$\frac{d^3Y}{dx^3} = 0 \qquad ..(14b)$$

METHOD OF SOLUTION

Finite Difference Equation

Solution of Eq. (10) can be obtained by numerical approach to the problem based on the replacement of the differential equation by the corresponding finite difference equation.

The chimney under consideration is shown in Fig. 3. The chimney is assumed to be made of a finite number of regularly spaced grid points. Figure 3 shows a portion of a grid where the point i is a typical interior point i.e. neither the point itself nor any of the adjoining points fall on any kind of boundary. When the chimney deflects, a section through the points $i-2$, $i-1$, i, $i+1$, $i+2$ is shown in Fig. 3; h is the finite distance between the two consecutive points. Y_{i-1}, Y_i, ... are the corresponding horizontal deflection of the points $i-1$, i..... In the above formulation standard central differences with constant order of truncation error has been used.

Transformed Equation

The deflection at any particular point can be expressed by a finite difference equation in terms of the deflection of the adjoining nodes. This can be presented conveniently in the form of a computational molecule given in Eq. (15).

$$\left[\ \boxed{C_{i-2}} - \boxed{C_{i-1}} - \boxed{C_i} - \boxed{C_{i+1}} - \boxed{C_{i+2}}\ \right] Y = \lambda \left[\ \boxed{B_{i-1}} - \boxed{B_i} - \boxed{B_{i+1}}\ \right] \qquad ...(15)$$

where

$$C_{i-2} = EI$$
$$C_{i-1} = -4EI + Nh^2 - \tfrac{1}{2} wAh^3$$
$$C_i = 6EI - 2Nh^2$$
$$C_{i+1} = -4EI + Nh^2 - \tfrac{1}{2} wAh^3$$
$$C_{i+2} = EI$$
$$B_{i-1} = -\left(\frac{EI\rho}{\mu G} + \rho I\right)$$
$$B_i = 2\left(\frac{EI\rho}{\mu G} + \rho I\right)$$
$$B_{i+1} = -\left(\frac{EI\rho}{\mu G} + \rho I\right)$$
$$\lambda = p^2h^2$$
$$p = \text{natural frequency} \qquad ...(16)$$

Finite Difference Form of the Boundary Conditions

At $$x = 0$$

$$Y = 0 \qquad ..(17a)$$

$$\left[(-1) \text{---} (0) \text{---} (1) \right] Y = 0 \qquad \ldots(17b)$$

At $x = l$

$$\left[(1) \text{---} (-2) \text{---} (1) \right] Y = 0 \qquad \ldots(18a)$$

$$\left[(-1) \text{---} (2) \text{---} (0) \text{---} (-2) \text{---} (1) \right] Y = 0 \qquad \ldots(18b)$$

Outline of the Solution

By using Eq. (15) one can write the finite difference equation at each discrete point including the boundry points. Lebelling the discrete point as shows in Fig. 3 and subscripting the unknown displacements and the known quantities with the label of the associated discrete points, the force-equilibrium Eq. (15) which is valid for the whole domain is reduced to discrete eigenvalue problem formulated as a set of linear algebraic simultaneous equations.

Using the finite difference representation, Eq. (15) with boundary condition is placed into the matrix form :

$$[C]\{Y\} = \lambda [B]\{Y\} \qquad \ldots(19)$$

where [C] and [B] are square matrix of order n, where n is the degree of freedom of discrete representation and {Y} is the eigenvector or the horizontal deflection of the deflected shape. Materices [C] and [B] are unsymmetric in nature-because the governing Eq. (15) contains the coefficient involving the independent variable w. Thus the solution of the system of equations as given by Eq. (19) contains complex eigenvalues.

On premultiplying by $[B]^{-1}$, Eq. (19) reduces to

$$([B]^{-1}[C] - \lambda [I])\{Y\} = 0 \qquad \ldots(20)$$

The values of λ are obtained by using QR transformation method.[2]
Corresponding to the values of λ, {Y} is computed.

DISCUSSION OF RESULTS

The cantilever has been analysed for three different slenderness ratios keeping the material and cross-sectional properties same in all three cases. The chimney has been considered as a beam having uniform cross-section. The following data were assumed

$$E = 1.5 \times 10^{5}\ \text{kg/cm}^2$$
$$I = 6.0 \times 10^{8}\ \text{cm}^4$$
$$w = 0.0024\ \text{kg/cm}^3$$
$$s = 0.0000244$$
$$A = 30000\ \text{cm}^2$$
$$\mu = 7.5$$

Three different spans considered were 705cm, 2820cm and 7050cm so that slenderness ratios were 5, 20 and 50.

The following four cases were studied for each of the above slenderness ratio :

Case I Effect of bending, shear, rotatory inertia and axial load

Case II Effect of bending, shear and rotatory inertia
Case III Effect of bending, rotatory inertia and axial load
Case IV Effect of bending and rotatory inertia

The analysis for each category was based on five discrete points. Entire analysis was carried out by IBM 1620 at the Computer Centre at I.I.T., Kharagpur, by a suitably developed program as mentioned(2).

The first three natural frequencies and mode shapes for the above four cases for each slenderness ratio are given in Tables 1, 2 and 3. The mode shape coefficients given in the Tables 2 and 3 corresponded to the values of one-fifth point starting from the fixed end.

A look into Table 1 reveals that on comparison between case I and Case II for chimneys having slenderness ratios of 5 and 20, axial force has absolutely no effect on the natural frequencies and for more slender chimney (l/r ratio = 50) the axial force has only marginal effect, the difference in fundamental frequency being 2.5 percent. Same remarks are valid when Case III and Case IV where shear effects were not considered, have been compared. As is obvious elimination of shear did not have appreciable effect on slenderest chimney but considerably increased the natural frequency for the shortest chimney.

TABLE 1.

NATURAL FREQUENCIES OF CHIMNEYS

	Length cm	1st Mode	Percent exact	2nd Mode	Percent exact	3rd Mode	Percent exact
Case I	705.0	59.990	100.0	326.95	100.0	765.99	100.0
	2820.0	10.750	100.0	59.28	100.0	140.02	100.0
	7050.0	2.190	100.0	12.33	100.0	29.65	100.0
Case II	705.0	59.990	100.0	326.95	100.0	765.97	100.0
	2820.0	10.78	100.0	59.28	100.0	140.02	100.0
	7050.0	2.24	102.5	12.41	100.5	29.65	100.0
Case III	705.0	244.5	407.7	1094.50	336.0	2082.50	272.4
	2820.0	15.00	139.5	82.10	138.4	191.90	136.6
	7050.0	2.36	107.9	13.23	107.2	31.58	106.2
Case IV	705.0	244.52	407.7	1094.53	336.0	2082.62	272.4
	2820.0	15.00	139.5	82.20	138.5	192.00	136.6
	7050.0	2.40	109.5	13.30	107.9	31.80	107.2

For longest chimney, mode shapes for beams with and without axial force differs only in fourth decimal places for first three modes. For other two chimney similarity is more pronounced.

No attempts has been made to improve the results, by Richardson's extrapolation technique; because from this limited study conclusion is obvious i.e. the axial load has no effect on the dynamic characteristics of the chimneys.

TABLE 2
MODE SHAPES FOR CASE I AND CASE II

Mode	SPAN					
	705cm		2820cm		7050cm	
	Case I	Case II	Case I	Case II	Case I	Case II
1st Mode	0.0503174	0.0503176	0.0515883	0.0515998	0.0514223	0.0516014
	0.1740123	0.1740125	0.1783874	0.1784008	0.1781597	0.1783705
	0.3446759	0.3446760	0.3533101	0.3533164	0.3531137	0.3532158
	0.5390739	0.5390739	0.5525405	0.5525384	0.5523718	0.5523424
	0.7403411	0.7403410	0.7588030	0.7587940	0.7586328	0.7584943
2nd Mode	0.2685493	0.2685493	−0.2625324	−0.2625250	−0.2629041	−0.2627883
	0.5501675	0.5501673	−0.5378899	−0.5378713	−0.5387173	−0.5384253
	0.4691382	0.4691383	−0.4609838	−0.4609793	−0.4633181	−0.4632473
	−0.0329213	−0.0329213	0.0248060	0.0248077	0.0191032	0.0191217
	−0.7399263	−0.7399268	0.7098580	0.7098882	0.6997047	0·7001530
3rd Mode	0.5400053	0.5400049	0.5393583	0.5393367	0.5394370	0.5391175
	0.4193896	0.4193899	0.4157968	0.4158158	0.4129917	0.4132943
	−0.3434947	−0.3434943	−0.3473200	−0.3472954	−0.3506029	−0.3502271
	−0.3397567	−0.3397572	−0.3529925	−0.3530230	−0.3628675	−0.3633460
	0.6550774	0·6550775	0.6255425	0.6255397	0.6029325	0.6028784

TABLE 3
MODE SHAPES FOR CASE III AND CASE IV

Mode	SPAN					
	705 cm		2820 cm		7050 cm	
	Case III	Case IV	Case III	Case IV	Case III	Case IV
1st Mode	0.0739921	0.007399232	0.05030605	0.050031729	0.05142220	0.0516013
	0.2580877	0.258088000	0.17399360	0.174012400	0.17816160	0.1783722
	0.5137006	0.513700900	0.34466880	0.344676500	0.35311950	0.3532216
	0.8058129	0.805813300	0.53907500	0.539075400	0.55238300	0.5523534
	1.1085515	1.108552000	0.74034910	0.740344000	0.75864970	0.7585110
2nd Mode	−0.2388641	−0.2388639	−0.2685577	−0.2685485	−0.2628865	−0.2627706
	−0.4746203	−0.4746197	−0.5501874	−0.5501651	−0.5386807	−0.5383887
	−0.3189569	−0.3189565	−0.4691159	−0.4691085	−0.4631872	−0.4631162
	0.2844111	0.2844119	0.0330070	0.0330095	0.1941867	0.1943800
	1.0976135	1.0976157	0.7400565	0.7400853	0.7002376	0.7006880
3rd Mode	0.5289980	0.5289969	0.5400315	0.5400107	0.5394407	0.5391206
	0.5393463	0.5393467	0.4194100	0.4194303	0.4131247	0.4134272
	−0.0820578	−0.0820568	−0.3434708	−0.3434467	−0.3504393	−0.3500630
	0.1881148	0.1881148	−0.3395680	−0.3395994	−0.3622909	−0.3627693
	1.5096308	1.5096324	0.6554186	0.6554195	0.6041849	0.6041315

CONCLUSION

The weight of the vertical stacks acting downwards has only marginal effect on the lateral vibration of chimneys and as such can be neglected for all practical purposes.

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