ISET Journal of Earthquake Technology, Paper No. 478, Vol. 44, No. 1, March 2007, pp. 101–109

A NOTE ON SPATIAL VARIATIONS IN RESPONSE SPECTRA OF EARTHQUAKE GROUND MOTIONS

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ABSTRACT

Response spectra of earthquake ground motions are important in the earthquake-resistant design and reliability analysis of structures. The purpose of this paper is to examine the spatial variability of response spectra recorded at the same epicentral distance as a function of frequency and separation distance. To do this, we define response spectrum ratios as spatial intra-event variations of response spectra and examine their statistical characteristics. Then we analyze the probability distribution of the ratios and formulate equations for their probability density functions, mean values, standard deviations, and percentiles. These statistics are estimated using accelerometer arrays of the Chiba and SMART-1 databases, and their relationships with the station separation distance are analyzed. It has been found out that the means and standard deviations have almost linear relationship with the logarithms of the station separation distances ranging from several meters to several kilometers. Finally, based on these findings, the differences between response spectra at two different sites due to future earthquakes are discussed.

KEYWORDS: Spatial Variation, Response Spectrum, Dense Instrument Arrays, Statistical Study

INTRODUCTION

Past significant earthquakes have seriously damaged many engineering structures, and field studies have reported that the degree of damage to each structure varied significantly from one location to another, even if the two structures were similar and the distance between them was small. This variation in structural damage, according to the reliability theory, is due to the differences in structural strength and the ground motion amplitude at these two separate locations.

The variations in structural strength and ground motion amplitude are taken into account in the current building design codes, i.e., today's seismic design of structures is based on the reliability theory. Fragility curves showing the relationship between the probability of structural damage and the amplitude of ground motion are based on the damage statistics obtained from large earthquakes. It is important to note, however, that the amplitudes of ground motion, e.g., the peak ground acceleration and the response spectrum amplitude, are estimated from earthquake records detected using seismometers located nearest to the structure. This estimation of the ground motion amplitude is noticeably affected by the distance between the structure and the seismometer, i.e., the amplitude is expected to have been more precisely estimated if the distance between the structure and the sensor is short; otherwise it is expected to contain error. However, how much error is to be expected quantitatively? If the spatial variance in the ground motion amplitude is examined based on the observed seismic records, whose number has been increasing remarkably in recent years, a more precise ground motion amplitude and a more reliable fragility curve can be obtained.

In light of these considerations, we have conducted statistical analyses of the spatial variations in peak ground accelerations (Kawakami and Mogi, 1999, 2003; Mogi and Kawakami, 2000). These analyses, however, ignored the effect of period or frequency of the seismic motions, which is very important when considering damage to structures with specific natural periods. Therefore, in this paper, we examined the response spectrum amplitude, which is the central scaling tool in earthquake engineering (Biot, 1932, 1933, 1934; Gupta, 2004). Trifunac (1978) and Trifunac and Anderson (1977) proposed differences (residuals) between the estimated and observed response spectrum amplitudes and their probability distribution functions using regression analysis. As several important parameters necessary in building design were taken into account, their results are very significant and useful when designing earthquake-resistant structures. However, since the regression analysis was made by using data from many earthquakes simultaneously, residuals included intra-event as well as inter-event variability.

In this study, we focused on the intra-event variability in the ground motion amplitude of earthquakes. In other words, we considered the spatial variability in the damage and, more specifically, the spatial variability in the ground motion amplitude of each (one) earthquake. Moreover, it is not the purpose of this paper to discuss the variability in the damage or ground motion due to earthquakes having the same magnitude, focal depth, and epicentral distance at sites classified in the same category. The spatial variability in the ground motion amplitude of an earthquake, i.e., the intra-event variability, can be considered to be due to variations in: (1) the directions of the waves radiating from the epicenter, (2) the physical material encountered along the path, and (3) the surface soil conditions.

This study on the intra-event spatial variability in ground motion amplitude will be useful in generating input ground motions for the design of spatially extended structures, such as pipelines. When designing such structures, spatially distributed seismic motions must be applied to the structure. Hence, taking into account the effects of variation in seismic motion will help improve the design.

Several pioneering studies have investigated the use of the response spectrum in the design of extended structures. Trifunac and Todorovska (1997) and Trifunac and Gicev (2006) extended the common response spectrum method for synchronous ground motion to deal with extended structures experiencing differential ground motion and proposed the relative displacement response spectrum. To generate the spectrum, however, they assumed the propagation of waves and approximated strain as the particle velocity divided by the average shear wave velocity in the top 30 meters of soil.

Although the relative displacement response spectrum is very useful in designing extended structures, we used a different approach by focusing only on the intra-event variability in the ground motion amplitude, i.e., the relative displacement in each earthquake. We did not consider the inter-event variability (differences in the ground motion between two earthquakes having the same magnitude, focal depth, and epicentral distance) because it does not affect the strain or relative displacement in an extended structure.

It should be noted that the ground motion amplitudes in the current research were assumed to be lognormal random variables. Indeed, this assumption may not be accurate, as described elsewhere (Trifunac, 1978; Lee, 2002), and the lognormal distribution may only provide a rough approximation. However, as mentioned above, the safety probability in structural designs based on reliability theory is evaluated by comparing the structural strength and the ground motion amplitude and is ordinarily calculated by using their means and variances. Therefore, the variance in ground motion amplitude is as important in the engineering field as the mean value itself (Schuëller, 1981), and this assumption becomes very useful because the failure probability can be easily obtained analytically if the strength and the load (in this paper, the ground motion amplitude) are described by normal or lognormal distributions.

ANALYTICAL MODEL OF GROUND MOTION AMPLITUDES

1. Definition of Ratios

The ratio of ground motion amplitudes such as peak ground acceleration (PGA) and response spectrum represents a spatial intra-event difference between those amplitudes observed at two sites. These ratios are obtained by dividing the smaller value by the larger one for all possible station pairs for each earthquake (Kawakami and Sharma, 1999; Kawakami and Mogi, 2002; Mogi and Kawakami, 2002). Closer the values of the ratios are to one, higher is the correlation between the ground motion amplitudes in question. Statistical analyses of the ratios are valuable for the following two reasons: (1) they avoid estimating the mean value of the amplitudes at the sites, which depends on individual earthquakes; and (2) they can directly compare the statistical results for the different kinds of amplitudes because the ratios are non-dimensional.

2. Probability Density Function of Ratios

Ground motion amplitudes related to the peak value of a waveform such as PGA and response spectrum can be treated as a lognormal random variable (e.g., Katayama et al., 1978; Boore et al., 1980). In this study, we also assume that ground motion amplitudes, such as PGA and response spectrum amplitude, are the lognormal random variables and express their joint probability density function (PDF) as

$$f_{Z_1,Z_2}(z_1,z_2)=\frac{1}{2\pi\sigma_Z^2\sqrt{1-\rho^2}}\exp\left[-\frac{1}{2\sigma_Z^2(1-\rho^2)}\left\{(z_1-\mu_Z)^2-2\rho(z_1-\mu_Z)(z_2-\mu_Z)+(z_2-\mu_Z)^2\right\}\right] \quad (1)$$

where Z_1 and Z_2 are the logarithms of amplitudes X_1 and X_2 observed at the two sites, σ_Z is the standard deviation, and ρ is the correlation coefficient between Z_1 and Z_2.

From Equation (1), the PDF of $P=|Z_1-Z_2|$ can be derived as

$$f_P(p)=\frac{2}{\sqrt{2\pi}\sigma_{P'}}\exp\left(-\frac{p^2}{2\sigma_{P'}^2}\right),\quad p\ge 0 \quad (2)$$

where $\sigma_{P'}$ is the standard deviation of $P'=Z_1-Z_2$, given by

$$\sigma_{P'}=\sigma_Z\sqrt{2(1-\rho)} \quad (3)$$

Furthermore, by changing the variables from P to R such that $-P=\ln(R)$, the PDF of the ratios, R, becomes

$$f_R(r)=\frac{2}{\sqrt{2\pi}\sigma_{P'}r}\exp\left(-\frac{(\ln r)^2}{2\sigma_{P'}^2}\right),\ 0<r\le 1 \quad (4)$$

Because the difference $P'=Z_1-Z_2$ is a Gaussian random variable with zero mean and its PDF is not affected by the mean value, μ_Z, as shown in Equation (2), it is neither necessary to estimate earthquake-specific μ_Z nor to normalize by μ_Z. It is also evident that, since the standard deviation $\sigma_{P'}$ is the only parameter of the PDF of the ratios, it can be used to compare the scatter of various intra-event ground motion amplitudes.

3. Mean Value and Percentile

The mean values of the ratios and their logarithms, μ_R and μ_P, can be obtained from Equations (4) and (2), respectively, as

$$\mu_R=\exp\left(\frac{\sigma_{P'}^2}{2}\right)\left\{1-\mathrm{erf}\left(\frac{\sigma_{P'}}{\sqrt{2}}\right)\right\} \quad (5)$$

$$\mu_P=\sqrt{\frac{2}{\pi}}\,\sigma_{P'} \quad (6)$$

where $\mathrm{erf}(\cdot)$ is the error function (Abramowitz and Stegun, 1972). We define the γth percentile of a ratio r_γ, and its logarithm p_γ, as the value for which *R* and *P* in the range of

$$r_\gamma\le R\le 1,\ 0\le P\le p_\gamma \quad (7)$$

have the probability of γ percent. In this study, we focused on the 50th and 95th percentiles; the former is used because it is a median value of the ratio, and the latter is used because it is a minimum (*R*) or maximum (*P*) expected value (i.e., 5% significance level is assumed). These percentiles were estimated by

$$p_{50}=0.68\sigma_{P'},\quad r_{50}=\exp(-p_{50})$$
$$p_{95}=1.96\sigma_{P'},\quad r_{95}=\exp(-p_{95}) \quad (8)$$

based on the properties of the Gaussian distribution.

ARRAY DATABASES

The dense-array databases of the Chiba array in Japan and SMART-1 array in Lotung, Taiwan, were used to statistically analyze the ratios. The instrument arrangements of the Chiba and SMART-1 arrays are shown in Figures 1 and 2, respectively.

The separation distances are unevenly distributed because of the configuration of seismometers. Taking this distribution into account, the PGA ratios were divided into several groups depending on the distance between two stations. Table 1 lists (a) Chiba array groups and (b) SMART-1 array groups, with the records at a rock site (E02 station in Figure 2) removed, for the PGA and response spectrum ratios. Statistical analyses of the ratios were carried out for each station separation group.

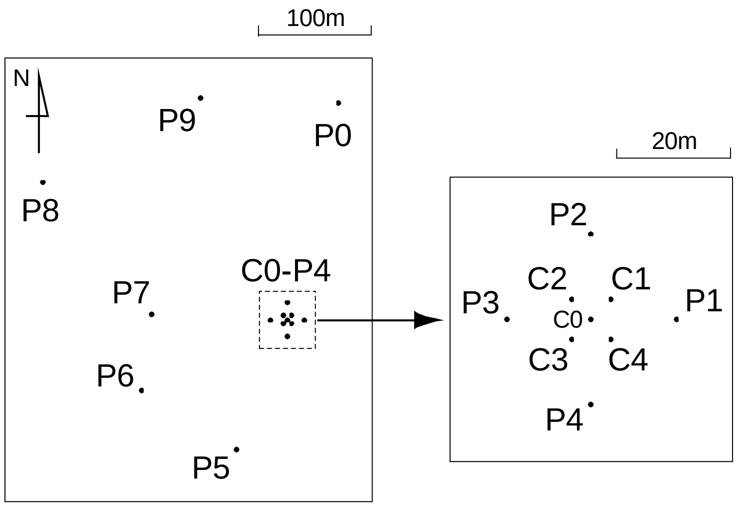

Fig. 1 Instruments in Chiba array (Katayama et al., 1990)

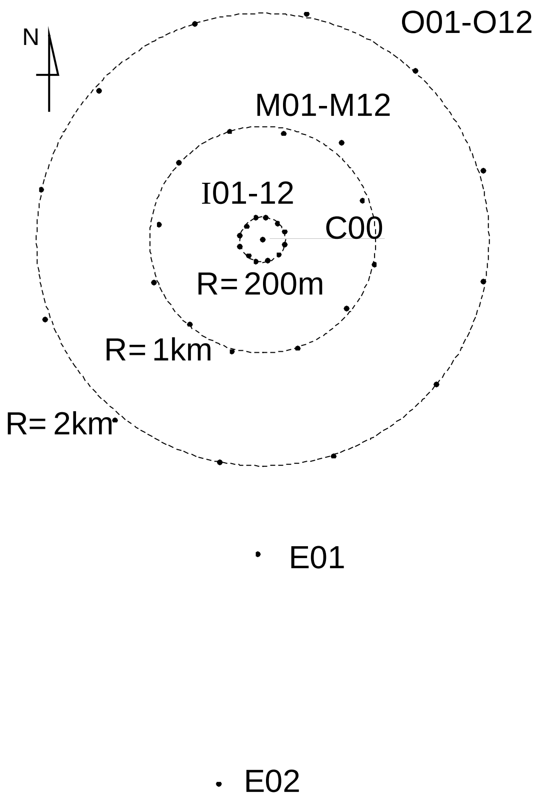

Fig. 2 Instruments in SMART-1 array (Bolt et al., 1982; Figueras et al., 1992)

Table 1: Station Separation Groups and Corresponding Number of Data Per Component

(a) Chiba Array

Group	Station Separation L (m)	Number of Data EW	NS	UD
A	$0 < L \leq 40$	1,368	1,368	1,368
B	$40 < L \leq 160$	819	819	819
C	$160 < L$	108	108	108
Total		2,295	2,295	2,295

(b) SMART-1 Array

Group	Station Separation L (m)	Number of Data EW	NS	UD
a	$0 < L \leq 650$	1,368	1,389	1,357
b	$650 < L \leq 1{,}600$	3,628	3,628	3,583
c	$1{,}600 < L \leq 2{,}400$	3,803	3,777	3,787
d	$2{,}400 < L \leq 3{,}200$	1,315	1,297	1,305
e	$3{,}200 < L$	571	563	571
Total		10,685	10,654	10,603

STATISTICAL RESULTS

1. Standard Deviations and Percentiles of Ground Motion Amplitudes

The standard deviations of the ratios of the ground motion amplitudes versus the station separations are plotted in Figure 3. The station separation distance is the average value of separation for each station separation group. The left-side ordinate is the standard deviation (sixty-eighth percentile) and its corresponding ratio, while the right-side ordinates are the fiftieth and ninety-fifth percentiles and their corresponding ratios. In this figure, the results for the PGA ratios (EW and NS components) and for the acceleration and velocity response spectrum ratios (EW and NS components) at 2 and 6 Hz are plotted. These frequencies were chosen because standard deviations $\sigma_{P'}$ of acceleration response spectrum ratios are at their minimum and maximum values at 2 and 6 Hz, respectively (see Figure 4).

It should be noted in Figure 3 that the scatters of ground motion amplitudes generally increase as distance between stations increases, though there are discontinuities in the plots between the Chiba and SMART-1 arrays. The response spectrum ratios at 6 Hz show the largest scatter among these results. The response spectrum ratios at 2 Hz show smaller scatter for station separation up to about 300 m, but their scatter increases abruptly for station separation of about 1 km. It can also be observed that the velocity response spectrum ratios have a slightly larger scatter than the acceleration ratios at either frequency. Furthermore, it can be observed that the scatter of the PGA ratios increases linearly with station separation and is generally between the scatter of the response spectrum ratios at 2 and 6 Hz for station separation less than about 1 km.

As shown in Figure 3, the 50th percentiles of the response spectrum ratios, r_{50}, are approximately 0.9–0.95 and 0.83–0.87 at 2 and 6 Hz, respectively, for Group A of the Chiba array and are 0.7–0.72 and 0.65–0.67 at 2 Hz and 6 Hz, respectively, for Group e of the SMART-1 array. The 95th percentiles, r_{95}, are approximately 0.8 and 0.6–0.66 at 2 and 6 Hz, respectively, for Group A of the Chiba array and are 0.33–0.38 and 0.26–0.3 for Group e of the SMART-1 array.

2. Mean and Probability Density Functions of Ground Motion Amplitude Ratios

As pointed out above, standard deviation $\sigma_{P'}$ is a useful index for examining the scatter of the ground motion amplitudes. However, to recognize the scatter of the ground motion amplitudes intuitively, the mean value of the ratios, μ_R, calculated from Equation (6), is more useful than the standard deviation $\sigma_{P'}$.

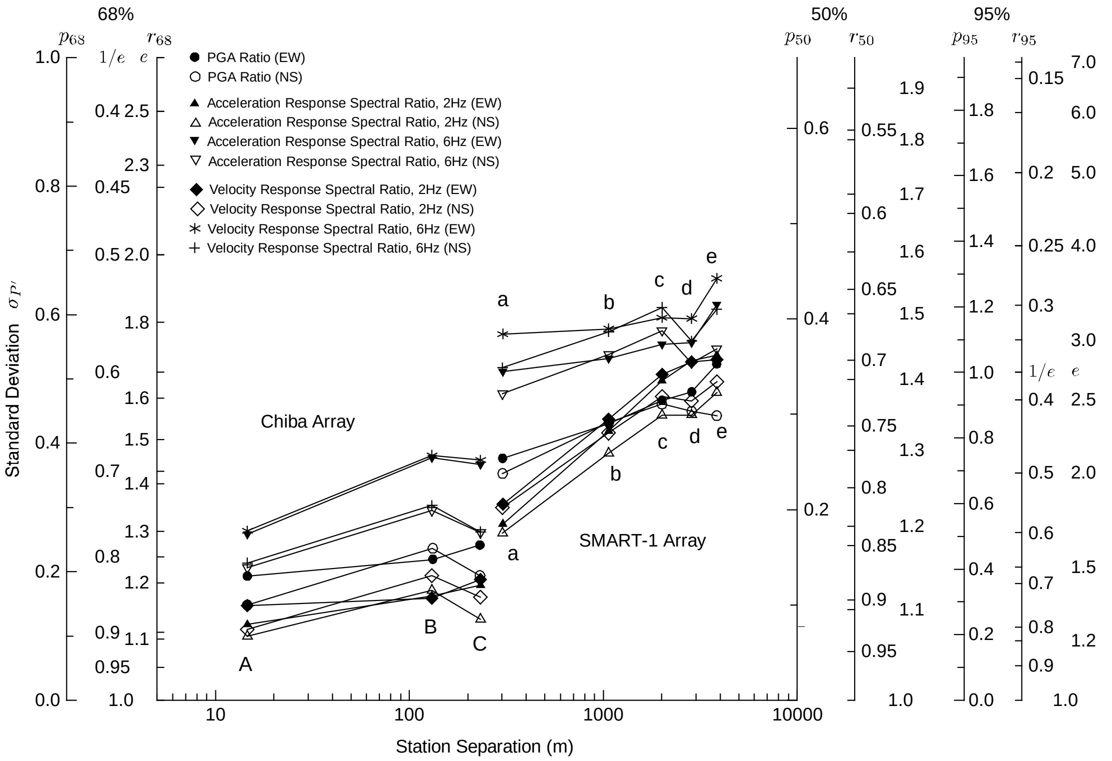

Fig. 3 Standard deviations and 50th and 95th percentiles of ratios *R* and differences *P*

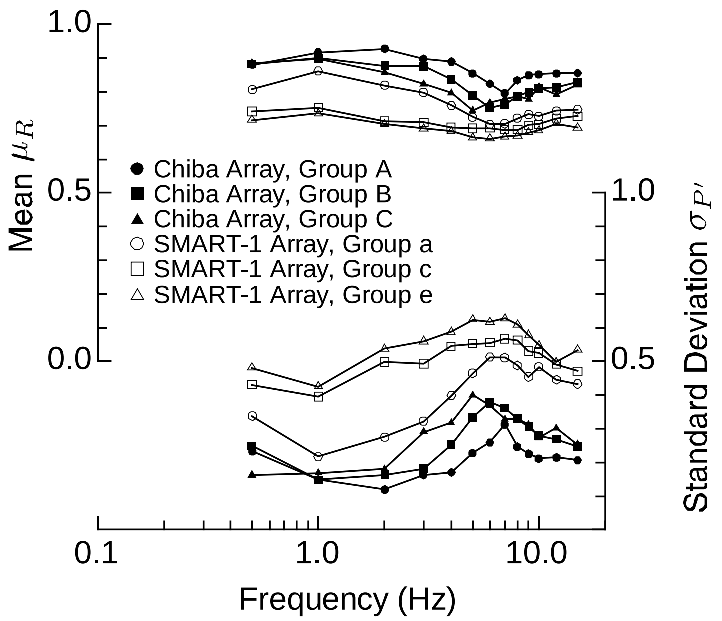

Fig. 4 Mean and standard deviation of acceleration spectrum ratios for EW components with damping ratio $h = 0.05$

The mean value of the ratios is plotted against station separation in Figure 5 for the same sets of ground motion amplitudes as considered in Figure 3. An inverse relation between mean value of ratios and station separation is shown in this figure; e.g., for the station pair with the station separation less than a few tens of meters (Group A in the Chiba array) the mean value of the ratio is 0.8 to 0.95, and for the pair with the station separation of three kilometers (Group d in the SMART-1 array) the mean value is

0.65 to 0.8. This inverse relation between mean value of ratios and station separation implies that ground motion amplitudes with larger standard deviations generally have smaller mean values.

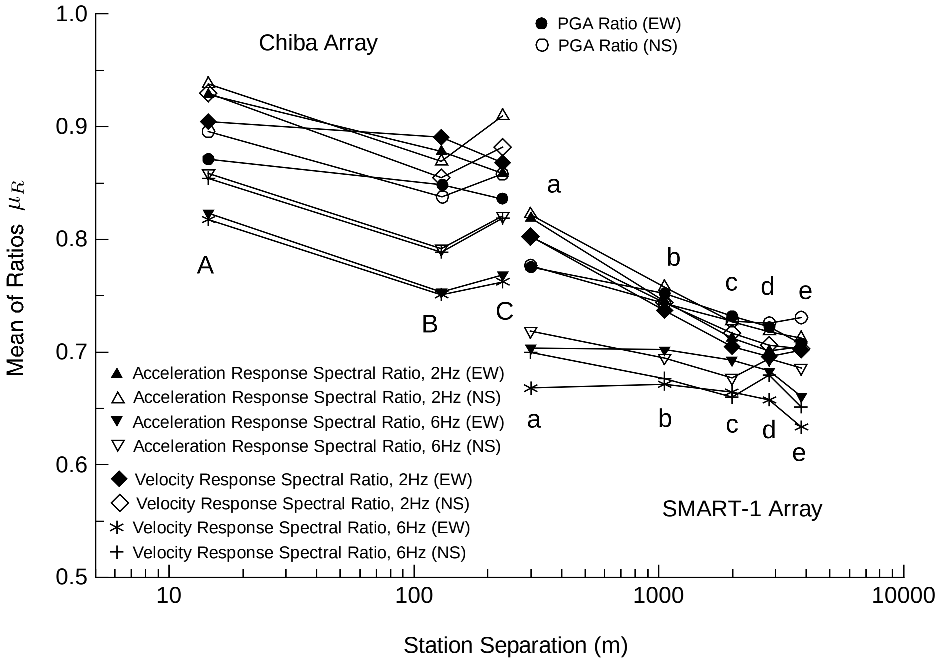

Fig. 5 Mean of ratios R versus station separation

The probability density functions (PDFs) of Group A in the Chiba array and Group d in the SMART-1 array are shown in Figure 6. For both groups in this figure, the PDFs estimated from the frequencies of occurrence are shown by the lines with symbols, and the analytical functions calculated from the observational standard deviations using Equation (4) are shown by the smooth solid lines. Figure 6 shows that the analytical expression of the PDF in Equation (4) is a good approximation of the probability distribution of the ground motion amplitude ratios.

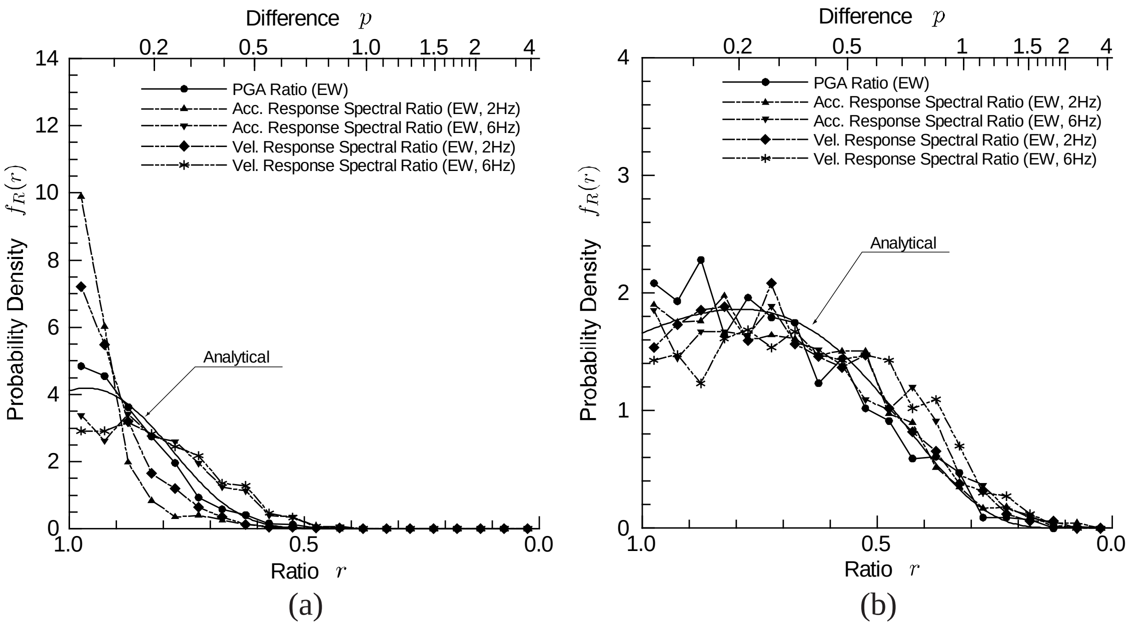

Fig. 6 Probability density functions of ratio R for (a) Group A of Chiba array, and (b) Group d of SMART-1 array

In addition, we can observe in Figure 6 the differences in the scatters of various kinds of amplitudes even for the same station-separation group. For example, in Figure 6(a) the PDFs of the acceleration and velocity response spectrum ratios at 6 Hz, which have larger standard deviations than the others, are flatter. Similar tendencies can also be observed in Figure 6(b). Thus, when the ratio is small, the probability of ratios with larger scatter is higher.

CONCLUSIONS

In this study, the scatter of ground motion amplitudes (peak ground acceleration (PGA) and response spectrum) was examined using accelerometer arrays of the Chiba and SMART-1 databases, and the scatters of the PGA ratios and response spectrum ratios were compared with each other based on the properties of the Gaussian distribution. Results can be summarized as follows:

1. The standard deviation $\sigma_{P'}$ increases monotonically as distance between stations increases. The standard deviation $\sigma_{P'}$ of the PGA ratios has an almost linear relationship with the logarithm of the station separation distances ranging from several meters to several kilometers.
2. The standard deviation $\sigma_{P'}$ of the response spectrum ratios is strongly influenced by frequency. In the statistical analysis, the response spectrum ratios at 6 Hz showed the largest scatter. Conversely, response spectrum ratios at 2 Hz showed the smallest scatter, but those increased abruptly for station separation greater than 1 km, and were almost equal to those at 6 Hz for distances between stations of about 3 km.

ACKNOWLEDGEMENTS

The authors wish to thank the Association for Earthquake Disaster Prevention of Japan and National Geophysical Data Center in U.S.A. for publishing the strong motion array record database and to the University of Tokyo for providing the strong motion records.

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