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DYNAMIC RESPONSE OF A PLATE RESTING ON A VISCOELASTIC SUBGRADE

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INTRODUCTION

The theory of beams and plates on elastic foundations is one of the most important fields of study in mechanics. Large number of papers have appeared on the subject[1] and most of the studies in this field were based on the Winkler-Zimmermann hypothesis. H. Hertz (1886) was the first to give a complete analysis of floating elastic plates. Wymen[2] and Meyerhof[3] analysed ice sheets as floating plates on liquid foundations. An excellent account of plates on elastic foundations is available in the classical book of Timoshenko and Woinsky-Kreiger[4]. E. Reissner[5] analysed thin plates on Winkler foundation having variety of boundary conditions including non-linear effects. Sinha[6] solved the problem for large deflections and Woinsky-Kreiger[7] and Kaczkowski[8] solved the problem for anisotropic plates. Y.Y. Yu[9] solved the problem for simultaneous axial and lateral loads.

Freudenthal and Lorsch[10] and Hoshkin and Lee[11] have presented investigations of infinite beams and plates on Maxwell and Kelvin type of foundations. Pister and Williams[12] and Pister[13] show the application of Reissner type model representing differential shear. The dynamic response of a beam on a viscoelastic subgrade has been discussed by Achenbach and Sun[14].

The present paper deals with the evaluation of dyanamic response of a plate resting on a viscoelastic subgrade and subjected to dynamic surface loads with spatial force distributions corresponding to that for rigid base, uniform and parabolic distributions.

FORMULATION OF THE PROBLEM

In the present case an infinite elastic plate is considered resting on a three-element standard linear viscoelastic solid represented by a spring k_1 in series with a Kelvin-Voigt element having spring k_2 and dashpot c. It appears as if an infinite number of viscoelastic elements are required to represent this system. But due to the linearity of the viscoelastic solid the analysis may be carried by considering spring and dashpot constants per unit of area.

The differential equation of motion for a homogeneous, isotropic and thin elastic plate resting on a viscoelastic subgrade is given, for axisymmetric case, by the following;

$$D \nabla^4{}_1 W(R, t) + m \frac{\partial^2 W(R, t)}{\partial t^2} = P(R, t) - Q(R, t) \qquad \ldots(1)$$

where,

$$\nabla^4{}_1 = \left(\frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} \right)^2$$

and the relation between the deflection of the viscoelastic element and the subgrade reaction due to its deformation is expressed by the following equation;

$$\{(k_1 + k_2)/k_1\} Q(R, t) + (c/k_1) \frac{\partial Q(R, t)}{\partial t} = k_2 W(R, t) + c \frac{\partial W(R, t)}{\partial t} \qquad \ldots(2)$$

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The initial conditions of the system are specified by the following relations;

$$\left.\begin{aligned} W(R\ t) &= f_1(R) \\ \frac{\partial W(R\ t)}{\partial t} &= f_2(R) \end{aligned}\right\} (t=0) \qquad \ldots(3)$$

The formulation for the problem is completed by specifying deflection and its normal derivative on the boundary and these boundary conditions are given by the following;

$$\left.\begin{aligned} W(R,t) &= g_1(R,t) \\ \frac{\partial W(R,t)}{\partial n} &= g_2(R,t) \end{aligned}\right\} \text{(On boundary, } t > 0) \qquad \ldots(4)$$

where n represents the directions of the outward normal at a point on the boundary.

Now, the following nondimensional parameters are defined;

$$r = R/\delta$$

$$\tau = \tfrac{1}{2}(h/\delta)^2 \left[\frac{E}{3(1-\nu^2)\,\rho_1\, h_1}\right]^{1/2} . t$$

$$\nabla^4 = \delta^4\, \nabla^4_1$$

$$\eta = (\delta/h)^2 \left[12(1-\nu^2)\frac{c^2}{\rho_1 . E}\right]^{1/2}$$

$$\mu_{1,2} = (\delta/h)^2 \left[12(1-\nu^2)\frac{hk_{1,2}}{E}\right]^{1/2}$$

$$w(r,\tau) = W(R,t)/\delta$$

$$p(r,\tau) = \{12(1-\nu^2)/E\}(\delta/h)^3\, P(R,t)$$

$$q(r,\tau) = \{12(1-\nu^2)/E\}(\delta/h)^3\, Q(R,t)$$

$$F_1(r) = \frac{1}{\delta} f_1(R)$$

$$F_2(r) = \frac{1}{\delta} f_2(R)$$

$$G_1(r,\tau) = \frac{1}{\delta} g_1(R,t)$$

$$G_2(r,\tau) = \frac{1}{\delta} g_2(R,t)$$

δ = plate characteristic dimension

In addition to the above parameters the following quantities are defined;

$E_R = (\mu^2_1\, \mu^2_2)/(\mu^2_1+\mu^2_2)$ = relaxed elastic modulus,

$\tau_\epsilon = \eta/(\mu^2_1+\mu^2_2)$ = time of relaxation of load under the condition of constant deflection,

$\tau_\sigma = \eta/\mu^2_2$ = time of relaxation of deflection under the condition of constant load.

Using the parameters defined above, the equations (1) to (4) are rewritten as follows;

$$\nabla^4 w(r,\tau) + \frac{\partial^2 w(r,\tau)}{\partial \tau^2} = p(r,\tau) - q(r,\tau) \qquad \ldots(6)$$

$$E_R\, w(r,\tau) + E_R\, \tau_\sigma \frac{\partial w(r,\tau)}{\partial \tau} = q(r,\tau) + \tau_\epsilon \frac{\partial q(r,\tau)}{\partial \tau} \qquad \ldots(7)$$

$$\left.\begin{aligned} w(r,\tau) &= F_1(r) \\ \frac{\partial w(r,\tau)}{\partial \tau} &= F_2(r) \end{aligned}\right\} (\tau=0) \qquad \ldots(8)$$

$$\left.\begin{aligned} w(r,\tau) &= G_1(r,\tau) \\ \frac{\partial w(r,\tau)}{\partial n} &= G_2(r,\tau) \end{aligned}\right\} (\text{On boundary, } \tau > 0) \qquad \ldots(9)$$

In the case of an infinite plate the problem is completely defined by equations (6) to (8) and the conditions that as $r \to \infty$, $w \to 0$ and as $r \to 0$, $w \neq \infty$.

METHOD OF SOLUTION

The Hankel transform $\bar{f}(\rho, \tau)$ of a function $f(r, \tau)$ is defined as follows;

$$\bar{f}(\rho, \tau) = \int_0^\infty r\, J_0(\rho r)\, f(r, \tau)\, dr \qquad \ldots(10)$$

Applying the Hankel transforms (10), by the multiplication of $rJ_0(\rho r)$ to both sides of the equations (6) to (8) and integrating between the limits 0 to ∞; the following sets of equations are obtained;

$$\rho^4 \bar{w}(\rho, \tau) + \frac{d^2\bar{w}(\rho, \tau)}{d\tau^2} = \bar{p}(\rho, \tau) - \bar{q}(\rho, \tau) \qquad \ldots(11)$$

$$E_R \bar{w}(\rho, \tau) + E_R \tau_\sigma \frac{d\bar{w}(\rho, \tau)}{d\tau} = \bar{q}(\rho, \tau) + \tau_\varepsilon \frac{d\bar{q}(\rho, \tau)}{d\tau} \qquad \ldots(12)$$

$$\left.\begin{aligned} \bar{w}(\rho, \tau) &= \bar{F}_1(\rho) \\ \frac{d\bar{w}(\rho, \tau)}{d\tau} &= \bar{F}_2(\rho) \end{aligned}\right\} (\tau=0) \qquad \ldots(13)$$

The Laplace transform to $\bar{\bar{f}}(\rho, s)$ of a function $\bar{f}(\rho, \tau)$ is defined by the following relation;

$$\bar{\bar{f}}(\rho, s) = \int_0^\infty e^{-s\tau} \bar{f}(\rho, \tau)\, d\tau \qquad \ldots(14)$$

Applying the Laplace transform to (11) and (12) and using the initial conditions (13) the following equations are obtained;

$$(\rho^4 + s^2)\, \bar{\bar{w}}(\rho, s) + \bar{\bar{q}}(\rho, s) = \bar{\bar{p}}(\rho, s) + s\bar{F}_1(\rho) + \bar{F}_2(\rho) \qquad \ldots(15)$$

$$(E_R + sE_R\tau_\sigma)\, \bar{\bar{w}}(\rho, s) - (\tau_\varepsilon s + 1)\, \bar{\bar{q}}(\rho, s) = E_R \tau_\sigma \bar{F}_1(\rho) - \bar{q}(+0). \qquad \ldots(16)$$

Equations (15) and (16) are two simultaneous equations have $\bar{\bar{w}}(\rho, s)$ and $\bar{\bar{q}}(\rho, s)$ as the unknown. Solving these equations result in the following;

$$\bar{\bar{w}}(\rho, s) = (1/\Delta) \begin{vmatrix} \bar{\bar{p}}(\rho, s) + s\bar{F}_1(\rho) + \bar{F}_2(\rho) & 1 \\ \{E_R\tau_\sigma \bar{F}_1(\rho) - \tau_\varepsilon \bar{q}(+0)\} & -(\tau_\varepsilon s + 1) \end{vmatrix} \qquad \ldots(17)$$

$$\bar{\bar{q}}(\rho, s)=(1/\Delta)\begin{vmatrix} (\rho^4+s^2)\ \{\bar{\bar{p}}(\rho,s)+s\bar{F}_1(\rho)+\bar{F}_2(\rho) \\ E_R(1+\tau_\sigma s)\ \{E_R\tau_\sigma \bar{F}_1(\rho)-\tau_\varepsilon\bar{q}(+0)\} \end{vmatrix} \qquad \ldots(18)$$

where

$$\Delta=\begin{vmatrix} (\rho^4+s^2) & 1 \\ E_R(1+\tau_\sigma s) & -(1+\tau_\varepsilon s) \end{vmatrix}$$

Applying the inverse Laplace and Hankel transforms to (17) and (18) yields the following for $w(r, \tau)$ and $q(r, \tau)$;

$$w(r,\tau)=\frac{1}{2\pi i}\int_0^\infty\int_{\gamma-i\infty}^{\gamma+i\infty}\rho J_0(\rho r).$$

$$\cdot\frac{\{\bar{\bar{p}}(\rho,s)+s\bar{F}_1(\rho)+\bar{F}_2(\rho)\}(1+\tau_\varepsilon s)+\{E_R\tau_\sigma\bar{F}_1(\rho)-\tau_\varepsilon q(+0)\}}{(\rho^4+s^2)(1+\tau_\varepsilon s)+E_R(1+\tau_\sigma s)}e^{s\tau}\,ds\,d\rho \qquad \ldots(19)$$

$$q(r,\tau)=-\frac{1}{2\pi i}\int_0^\infty\int_{\gamma-i\infty}^{\gamma+i\infty}\rho J_0(\rho r).$$

$$\cdot\frac{(\rho^4+s^2)\{E_R\tau_\sigma\bar{F}_1(\rho)-\tau_\varepsilon\bar{q}(+0)\}-E_R(1+\tau_\sigma s)\{\bar{\bar{p}}(\rho,s)+s\bar{F}_1(\rho)+\bar{F}_2(\rho)}{(\rho^4+s^2)(1+\tau_\varepsilon s)+E_R(1+\tau_\sigma s)}.\ e^{s\tau}\,ds\,d\rho$$

$$\ldots(20)$$

EVALUATION OF DYNAMIC RESPONSE FOR IMPULSIVE SURFACE LOADING

Equations (19) and (20) are the general expressions for the dynamic response and the viscoelastic subgrade reaction for any type of given dynamic load and initial conditions. In this section the analysis is presented for the case of an impulsive surface loading with homogeneous initial conditions.

Let the dynamic surface load be given by the following;

$$p(r,\tau)=p(r)\,\delta(\tau) \qquad \ldots(21)$$

where, $\delta(\tau)$ is the Dirac-delta function with Laplace transform equal to unity and $p(r)$ is a function of spatial coordinate r only.

Applying Laplace and Hankel transforms to equation (21) results in the following;

$$\bar{\bar{p}}(\rho,s)=\bar{p}(\rho) \qquad \ldots(22)$$

Substituting for $\bar{\bar{p}}(\rho, s)$ in (19) from (22) and applying the given initial conditions results in the following expression for the dynamic deflection;

$$w(r,\tau)=\frac{1}{2\pi i}\int_0^\infty\int_{\gamma-i\infty}^{\gamma+i\infty}\rho J_0(\rho r)\frac{\bar{p}(\rho)(1+\tau_\varepsilon s)e^{s\tau}\,ds\,d\rho}{(\rho^4+s^2)(1+\tau_\varepsilon s)+E_R(1+\tau_\sigma s)} \qquad \ldots(23)$$

Using the tabulated inverse Laplace transforms [15] for the evaluation of the inner integral of (23) and substituting gives the following;

$$w(r,\tau)=\int_0^\infty\rho J_0(\rho r)\bar{p}(\rho)\left[\frac{1+\tau_\varepsilon a_1}{(a_1-a_2)(a_1-a_3)}e^{a_1\tau}+\frac{1+\tau_\varepsilon a_2}{(a_2-a_1)(a_2-a_3)}e^{a_2\tau}+\frac{1+\tau_\varepsilon a_3}{(a_3-a_1)(a_3-a_2)}e^{a_3\tau}\right]d\rho \qquad \ldots(24)$$

where,
$$a_1 = A_0 + B_0 - b/3a \;;$$
$$a_2 = -\frac{a_1}{2} + i\,(\sqrt{3}/2)\,(A_0 - B_0) - b/3a \;;$$
$$a_3 = -\frac{a_1}{2} - i\,(\sqrt{3}/2)\,(A_0 - B_0) - b/3a \;;$$

are the roots of the equation,
$$\tau_\varepsilon s^3 + s^2 + (\tau_\varepsilon \rho^4 + E_R \tau_\sigma)\, s + \rho^4 + E_R = 0 \qquad \ldots(25)$$

and
$$A_0 = [-b/2 + (b^2/4 + a^3/27)^{\frac{1}{2}}]^{\frac{1}{3}}$$
$$B_0 = -[b/2 + (b^2/4 + a^3/27)^{\frac{1}{2}}]^{\frac{1}{3}}$$
$$a = \rho^4 + E_R \frac{\tau_\sigma}{\tau_\varepsilon}$$
$$b = \frac{1}{\tau_\varepsilon}(\rho^4 + E_R) - \frac{1}{3\tau_\varepsilon^2}(\tau_\varepsilon \rho^4 + E_R \tau_\sigma) + \frac{2}{27\,\tau_\varepsilon^3}.$$

APPROXIMATE DYNAMIC RESPONSE OF THE SYSTEM

It is quite evident that the evaluation of the integral in (24) is exceedingly difficult. But an approximate expression for the dynamic deflection may be obtained when the relaxation time (τ_ε) is very small and can be neglected for all practical purposes. For this case, w (r,τ) is given by the following;

$$w\,(r,\tau) = \int_0^\infty \rho\, J_0(\rho r)\, \bar{p}(\rho)\, \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{s\tau}\, ds\, d\rho}{[(1 - \tau_\sigma \tau_\varepsilon E_R)\, s^2 + E_R(\tau_\sigma - \tau_\varepsilon)\, s + (\rho^4 + s^2)]} \qquad \ldots(26)$$

if,
$$A = 1 - \tau_\sigma \sigma_\varepsilon E_R$$
$$B = E_R(\tau_\sigma - \tau_\varepsilon)$$
$$C = \rho^4 + E_R$$

then (26) may be written as follows :
$$w\,(r,\tau) = \int_0^\infty \rho\, J_0(\rho r)\, \bar{p}(\rho)\, \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{s\tau}\, ds}{As^2 + Bs + C}\, d\beta. \qquad \ldots(27)$$

The evaluation of the inner integral in (27) leads to the following three cases, depending on whether

$$B^2 - 4AC > 0,\ B^2 - 4AC = 0 \text{ or } B^2 - 4AC < 0\;;$$

$$w\,(r,\tau) = \int_0^\infty \rho\, J_0(\rho r)\, \bar{p}(\rho)\, \frac{1}{\mu}\,(e^{-\beta\tau} - e^{-\alpha\tau})\, d\rho, \text{ if } B^2 - 4AC > 0, \qquad \ldots(28)$$

$$w\,(r,\tau) = \int_0^\infty \rho\, J_0(\rho r)\, \bar{p}(\rho)\, 1/A\; \tau e^{-\alpha_1 \tau}\, d\rho, \text{ if } B^2 - 4AC = 0, \qquad \ldots(29)$$

$$w\,(r,\tau) = \int_0^\infty \rho\, J_0(\rho r)\, \bar{p}(\rho)\, \frac{2}{\sigma}\, e^{-\alpha_1 \tau} \sin\frac{\sigma\tau}{2A}\, d\rho, \text{ if } B^2 - 4\,AC < 0, \qquad \ldots(30)$$

where
$$\alpha = \{1/(2A)\}\,(B + \mu)\;;$$
$$\beta = \{1/(2A)\}\,(B - \mu)\;;$$
$$\mu = (B^2 - 4AC)^{\frac{1}{2}}\;;$$
$$\alpha_1 = B/(2A)\;;$$
$$\sigma = (4AC - B^2)^{\frac{1}{2}}\,.$$

Since for dynamic response $B^2 - 4AC < 0$, therefore, (30) represents the solution of the present problem.

EVALUATION OF p (r)

It is important to note that the settlement of soil and similar materials under uniform surface loads is not uniform. This requires the foundation to be flexible so as to conform to the deflection. But if the flexible foundation is replaced by a rigid one carrying the same load then the contact pressure distribution would change. In soil-foundation systems three types of pressure distributions are commonly used, viz. the rigid base approximation, uniform and parabolic load distributions. Hence following Sung[16] the pressure distributions may be expressed by means of the following Fourier-Bessel integral;

$$p(r)=\{p/(\pi R_0)\}\int_0^\infty M(\xi R_0)\, J_0(\xi r)\, d\xi \qquad \ldots(31)$$

where,

$$\begin{aligned} M(\xi R_0) &= (1/2)\sin(\xi R_0), \text{ for rigid base;} \\ &= J_1(\xi R_0), \text{ for uniform loading;} \\ &= 4 J_0(\xi R_0)/(\xi R_0), \text{ for parabolic loading;} \end{aligned}$$

and R_0 = radius of the circular base.

The integral in (31), when evaluated for the three types of spatial pressure distributions, gives the following:

(i) For rigid base :

$$p(r)=\begin{cases} \dfrac{P}{2\pi R_0^2 (R_0^2-r^2)^{1/2}} & \text{for } r<R_0 \\ 0 & \text{for } r>R_0 \end{cases} \qquad \ldots(32)$$

(ii) For uniform load :

$$p(r)=\begin{cases} P/(\pi R_0^2) & \text{for } r<R_0 \\ 0 & \text{for } r>R_0 \end{cases} \qquad \ldots(33)$$

(iii) For parabolic load :

$$p(r)=\begin{cases} \{2P/(\pi R_0^2)\}\{1-(r/R_0)^2\} & \text{for } r<R_0 \\ 0 & \text{for } r>R_0 \end{cases} \qquad \ldots(34)$$

RESPONSE EVALUATION FOR RIGID BASE APPROXIMATION

The spatial force distribution for this case is given by (32), therefore, applying Hankel transform to it gives the following for $\bar{p}(\rho)$;

$$\begin{aligned} \bar{p}(\rho) &= \int_0^\infty r J_0(\rho r)\, \frac{P}{2\pi R_0 (R_0^2-r^2)^{1/2}}\, dr \\ &= \{P/(2\pi R_0)\}\, \frac{1}{\rho} \sin(\rho R_0) \end{aligned} \qquad \ldots(35)$$

Substitution of $\bar{p}(\rho)$ from (35) into (30) results in the following :

$$w(r,\tau)=\frac{P}{2\pi R_0}\, e^{-\alpha_1 \tau}\int_0^\infty J_0(\rho r)\, \frac{1}{\sigma}\, 2\sin(\rho R_0)\sin\frac{\sigma\tau}{2A}\, d\rho \qquad \ldots(36)$$

RESPONSE EVALUATION FOR UNIFORM LOAD DISTRIBUTION

The spatial force distribution is given by (33) and applying Hankel transform to it gives the following for $\bar{p}(\rho)$;

$$\bar{p}(\rho)=\int_0^\infty rJ_0(\rho r)\frac{P}{\pi R_0^2}dr$$
$$=\{P/(\pi R_0^2)\}(R_0/\rho)J_1(\rho R_0) \qquad \ldots(37)$$

Substitution of $\bar{p}(\rho)$ from (37) into (30) and using the following limit,

$$\lim_{R_0\to 0} J_1(\rho R_0)/(\rho R_0)=1/2$$

results in the following:

$$w(r,\tau)=(P/\pi)e^{-\alpha_1\tau}\int_0^\infty \rho J_0(\rho r)\frac{1}{\rho}\sin\frac{\sigma\tau}{2A}d\rho \qquad \ldots(38)$$

RESPONSE EVALUATION FOR PARABOLIC LOAD DISTRIBUTION

The spatial force distribution for this case is given by (34) and applying Hankel transform to it gives:

$$\bar{p}(\rho)=\int_0^\infty rJ_0(\rho r)\frac{2P}{\pi R_0^2}\{1-r^2/R_0^2\}dr$$
$$=\frac{2P}{\pi R_0^4}\left[\frac{4R_0}{\rho^3}J_1(\rho R_0)-\frac{2R_0^2}{\rho^2}J_0(\rho R_0)\right] \qquad \ldots(39)$$

Substitution of $\bar{p}(\rho)$ from (39) into (30) and on simplification the following is obtained:

$$w(r,\tau)=\frac{4P}{\pi R_0^4}e^{-\alpha_1\tau}\left[\int_0^\infty \rho J_0(\rho r)\left\{\frac{4R_0}{\rho^3}J_1(\rho R_0)-\frac{2R_0^2}{\rho^2}J_0(\rho R_0)\right\}\frac{1}{\sigma}\sin\frac{\sigma\tau}{2A}d\rho\right. \qquad \ldots(40)$$

DYNAMIC RESPONSE EVALUATION FOR PULSE LOADING

Let the dynamic surface load be given by

$$p(r,\tau)=p(r)F(\tau) \qquad \ldots(41)$$

where $F(\tau)$ is a general type of pulse loading given by

$$F(\tau)=\begin{cases}\dfrac{1-e^{2\theta\tau/T}}{1-e^{\theta}}, & \text{for } 0\leqslant\tau\leqslant T/2\\[2ex] \dfrac{1-e^{2\theta(1-\tau/T)}}{1-e^{\theta}} & \text{for } \dfrac{T}{2}\leqslant\tau<T\end{cases} \qquad \ldots(42)$$

It can be observed from equation (42) that for $\theta\to-\infty$, the pulse shape approaches a rectangle, for $0\to+\infty$, a spike and for $\theta=0$. It represents a triangular pulse. The other pulse shapes of common interest can be obtained by giving suitable values to 0.

The response of the system with rigid base approximation under a pulse loading given by equation (41) and can be obtained as below by applying convolution integral to equation (36):

$$w(r,\tau)=\frac{P}{2\pi R_0}\int_0^\tau\int_0^\infty\frac{2J_0(\rho r)\sin(\rho R_0)}{\rho(1-e^\theta)}e^{-\alpha_1\tau_1}\sin\frac{\sigma\tau_1}{2A}\cdot$$
$$\cdot\left\{1-e^{2\theta\left(1-\frac{\tau-\tau_1}{T}\right)}\right\}d\rho\, d\tau_1 \qquad \ldots(43)$$

Similarly, the responses for uniform and parabolic load distributions can be obtained by applying convolutions to (38) & (40) as

$$w(r,\tau)=\frac{P}{\pi}\int_0^{\tau}\int_0^{\infty}\rho\, J_o(\rho r)\frac{e^{-\alpha_1\tau_1}}{\sigma(1-e^{\theta})}\sin\frac{\sigma\tau_1}{2A}\left\{1-e^{2\theta\left(\frac{\tau-\tau_1}{T}\right)}\right\}d\rho\, d\tau_1 \qquad \ldots(44)$$

$$w(r,\tau)=\frac{4P}{\pi R_0^4}\int_0^{\tau}\int_0^{\infty}J_0(\rho r)\frac{4R_0}{\rho^2}J_1(\rho R_0)\frac{1}{\sigma(1-e^{\theta})}\cdot$$

$$\cdot\, e^{-\alpha_1\tau_1}\sin\frac{\sigma\tau_1}{2A}\left\{1-e^{2\theta\left(1-\frac{\tau-\tau_1}{T}\right)}\right\}d\rho\, d\tau_1 \qquad \ldots(45)$$

CONCLUSIONS

Expressions, for the dynamic response of a system of plate and viscoelastic subgrade, have been obtained for impulsive surface loading with three most commonly assumed pressure distributions in soil-foundation systems. The standard linear viscoelastic solid used for the subgrade may be assumed to represent soil as it exhibits an initial elastic response and a delayed elastic response. These two elastic responses are the limit cases corresponding to the case of an elastic plate on an elastic subgrade and to the case of an elastic plate on a Kelvin-Voigt type subgrade. These limit cases are discussed below:

(i) Elastic Plate on an Elastic Subgrade :

The dynamic response for this case is obtained from that of a standard linear viscoelastic subgrade by letting $\eta \to 0$ inequation (23) as

$$w(r,\tau)=\frac{1}{2\pi i}\int_0^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\rho\, J_0(\rho r)\frac{\bar{p}(\rho)\, e^{s\tau}\, ds}{\rho^4+s^2+(\mu_1^2\,\mu_2^2/(\mu_1^2+\mu_2^2)}\, d\rho \qquad \ldots(46)$$

On evaluation of the inner integral of (46) and then substituting the result in it, the response is given by the following :

$$w(r,\tau)=\int_0^{\infty}\rho\, J_0(\rho r)\,\bar{p}(\rho)\,\frac{\sin(\rho^4+\mu_1^2)^{1/2}}{(\rho^4+\mu_1^2)^{1/2}}\, d\rho \qquad \ldots(47)$$

(ii) Elastic Plate on Kelvin Voigt Type Subgrade :

The dynamic response of an elastic plate resting on a Kelvin Voigt solid corresponds to another limit case of the standard linear viscoelastic solid by allowing the constant of initial elasticity to increase beyond limits. The response for this case, as obtained from (23) by letting $\mu_1^2 \to \infty$, is given by the following;

$$w(r,\tau)=\frac{1}{2\pi i}\int_0^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\rho\, J_0(\rho r)\frac{\bar{p}(\rho)\, e^{s\tau}\, ds}{(\rho^4+s^2)+E'_R(1+\tau_\sigma s)}\, d\rho \qquad \ldots(48)$$

Evaluation of the inner integral of (48) and subsequent substitution of the result in it gives the following:

$$w(r,\tau)=e^{-\frac{1}{2}\eta\tau}\int_0^{\infty}\rho\, J_0(\rho r)\,\bar{p}(\rho)\,\frac{\sin(4\beta_1^2-\eta^2)^{1/2}}{(4\beta_1^2-\eta^2)^{1/2}}\, d\rho \qquad \ldots(49)$$

where, $E'_R=\mu_2^2$ and $\beta_1^2=(\rho^4+\mu_2^2)$.

The dynamic response given by (49) contains $e^{-\frac{1}{2}\eta\tau}$ showing a finite amplitude of the system under dynamic loads which is quite expected of a viscoelastic subgrade.

SUGGESTIONS

The method presented in the paper may be used to calculate dynamic response of a plate foundation system under any type of dynamic load by using the equation (19) and equation (20) for the evaluation of the subgrade reaction. This reaction force, when divided by the dynamic deflection, shall give, what may be called, the coefficient of dyanamic vis coelastic subgrades reaction.

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NOTATIONS

$D = Eh^3/12(1-\nu^2)$ = plate flexural rigidity:
$m = \rho_1 h$ = mass per unit area of the plate;
h = plate thickness;
ν = Poisson' ratio;
ρ_1 = mass density of the plate material;
$P(R, t)$ = applied transverse dynamic load per unit area of the plate;
$Q(R, t)$ = viscoelastic subgrade reaction, force per unit area;
k_1, k_2 = spring constants of viscoelastic model, force per unit area per unit deflection;
c = viscosity constant of the viscoelastic model expressed in force per unit area per unit velocity;
$W(R, t)$ = transverse deflection of the mid-plane of the plate.