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QUASISTATIC SEISMIC LOADING DISTRIBUTIONS FOR HYPERBOLIC COOLING TOWERS

BY

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INTRODUCTION

Hyperbolic cooling towers, Fig. 1, must frequently be designed for seismic effects. This computation should be performed using rational methods of dynamic analysis; however, for preliminary design, quasistatic methods are quite useful. A recent paper (1) has pointed out the considerable difference in the meridional stresses obtained from using a base shear distribution proportional to just the mass of the shell (M distribution) as against that obtained by using a distribution proportional to both the mass and the height of any differential ring element of the shell above the base (MH distribution). The MH distribution is, of course more severe.

In a discussion to the aforementioned paper (2), Professor Arya noted that the MH distribution of the base shear gave unconservative results when compared to accelerations computed from a dynamic analysis, while a distribution proportional to the mass and the square of the height of the differential ring element above the base (MH^2 distribution) seemed to exhibit closer correlation.

Accordingly, the purposes of this paper are : [1] to derive closed form expressions for the membrane stress resultants for a hyperboloid of revolution under a quasistatic MH^2 distribution of the base shear; and [2] to present a comparative study showing the differences in stress resultants obtained from the M, MH and MH^2 distributions.

SURFACE GEOMETRY

The geometry of a hyperboloid of revolution, shown in Fig. 1, is defined by

$$r_0^2-(k^2-1)y^2=1 \quad \ldots(1)$$

in which $r_0=\dfrac{R_0}{a},\ y=\dfrac{Y}{a},\ k^2=1+\dfrac{a^2}{b^2}$ ...(2)

R_0 = the horizontal radius, Y = vertical coordinate, a = throat radius and

$$b=\frac{aT}{\sqrt{t^2-a^2}}=\frac{aS}{\sqrt{s^2-a^2}} \quad \ldots(3)$$

In Eq. 3 s = the base radius, t = the top radius, and S and T = the vertical distances from the throat to the base and the top of the shell, respectively. Also, the top and base of the shell are denoted by φ_t and φ_b. In Fig. 1 the coordinate system is also defined and the positive directions of φ, θ, z, and the load components per unit area of middle surface, P_φ, P_θ, and P_z, are indicated. The principal radii of curvature, R_θ and R_φ, are given by

$$R_\theta=ar_\theta=\frac{a\sqrt{k^2-1}}{\sqrt{k^2\ \sin^2\varphi-1}} \quad \ldots(4)$$

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and

$$R_\varphi = ar_\varphi = \frac{-a\sqrt{k^2-1}}{(k^2 \sin^2 \varphi - 1)^{3/2}} \quad ...(5)$$

in which r_θ and r_φ=the nondimensional principal radii of curvature.

MH² QUASISTATIC SEISMIC LOADING

Components of Surface Loading

For a horizontal quasistatic seismic force per unit area of middle surface, the components of surface loading take the nondimensional form (3)

$$p_\varphi = p_{\varphi 1}(\varphi)\cos\theta,\ p_\theta = p_{\theta 1}(\varphi)\sin\theta,\ p_z = p_{z1}(\varphi)\cos\theta \cdot \quad ...(6a, b, c)$$

Mass Height² Proportional Loading Distribution

Considering a differential ring segment of the hyperboloid located at a height h_y above the base of the shell, the portion of the base shear assigned to this level is

$$F_y = \frac{V dW \xi^2}{\Sigma dW \xi^2} = \frac{V \frac{dW}{d\xi}\xi^2\, d\xi}{\int_0^{\frac{S+T}{a}} \frac{dW}{d\xi}\, \xi^2\, d\xi} \quad ...(7)$$

in which

dW=the weight of the differential segment

V=the base shear

$$\xi = \frac{h_y}{a} = \frac{S-Y}{a} \quad ...(8)$$

Following an identical argument to that presented in Reference 1, the components of the surface loading are

$$p_{\varphi 1} = -C\,\bar{\zeta}\xi^2 \cos\varphi\ p_{\theta 1}, = C\,\bar{\zeta}\xi^2,\ p_{z1} = -C\,\bar{\zeta}\xi^2 \sin\varphi \quad ...(9)$$

in which

C=the seismic coefficient taken as a percentage of the gravity loading

$$\bar{\zeta} = \frac{\zeta_1(\varphi_b) - \xi_1(\varphi_t)}{\zeta_5(\varphi_b) - \zeta_5(\varphi_t)} \quad ...(10)$$

$$\zeta_1(\varphi) = -\int r_\varphi r_\theta \sin\varphi\, d\varphi \quad ...(11)$$

and

$$\zeta_5(\varphi) = \int r_\varphi r_\theta \sin\varphi\, \xi^2 d\varphi \quad ...(12)$$

$\zeta_1(\varphi)$ and $\zeta_5(\varphi)$ are given in explicit form in Appendix 1.

Equilibrium Equations

If the upper edge of the shell is assumed to be stress free, the solution for the membrane theory stress resultants is given by (3, 4)

$$n_{\varphi 1} = \frac{1}{r_\theta^2 \sin^3_\varphi}\Phi_1(\varphi) \quad ...(13a)$$

$$n_{\theta 1} = r_\varphi\left(p_{z1} - \frac{n_{\varphi 1}}{r_\varphi}\right) \quad ...(13b)$$

$$n_{\theta\varphi 1}=n_{\varphi 1}\cos\varphi-\frac{1}{r_\theta \sin\varphi}\Phi_2(\varphi) \quad ...(13c)$$

in which

$$\Phi_1(\varphi)=\int_{\varphi_t}^{\varphi}\Phi_2\, r_\varphi \sin\varphi\, d\varphi \quad ...(14)$$

and

$$\Phi_2(\varphi)=\int_{\varphi_t}^{\varphi}-(p_{z1}\sin\varphi+p_{\varphi 1}\cos\varphi-p_{\theta 1})\, r_\varphi\, r_\theta \sin\varphi\, d\varphi \quad ...(15)$$

In Eqs. 13.

$$n_\varphi=\frac{N_\varphi}{Pa}=n_{\varphi 1}(\varphi)\cos\theta,\; n_\theta=\frac{N_\theta}{Pa}=n_{\theta 1}(\varphi)\cos\theta,\; n_{\theta\varphi}=\frac{N_{\theta\varphi}}{Pa}=n_{\theta\varphi 1}(\varphi)\sin\theta \quad ...(16)$$

where N_φ, N_θ and $N_{\theta\varphi}$ are the stress resultants per unit length of middle surface, positive as shown in Fig. 1, and P is a constant reference load intensity taken as the weight of the shell per unit area of middle surface.

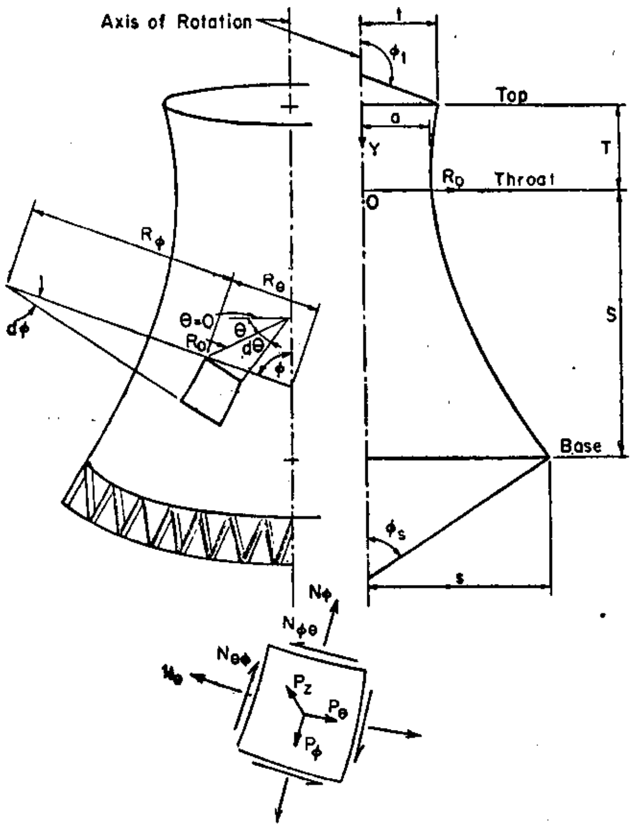

Fig. 1. Hyperboloid of Revolution

Substituting Eq. 9 into Eq. 14 and performing the necessary integrations,

$$\Phi_2(\varphi)=2C\zeta\Big[\zeta_8\Big]_{\varphi t}^{\varphi} \qquad ...(17)$$

Then, substituting Eq. 17 into Eq. 15 and again integrating yields

$$\Phi_1(\varphi)=2C\bar{\zeta}\Big[\zeta_1\zeta_8+\zeta_9+\xi\zeta_8(\varphi_t)\Big]_{\varphi t}^{\varphi} \qquad ...(18)$$

The functions $\zeta_1-\zeta_9$ are summarized in Appendix 1. The stress resultants then follow from Eq. 13.

COMPARISON OF SEISMIC LOADING DISTRIBUTIONS

In Reference 1, expressions are derived for the membrane stress resultants produced by the M and MH distributions as well as the gravity loading. For a hyperboloid of typical proportions, $a/s=0{\cdot}65$, $a/t=0\ 90$, $k^2=1{\cdot}12$, a comparison of the meridional stress resultant $n_{\varphi 1}$ for the M, MH and MH² distributions is shown in Fig. 2.

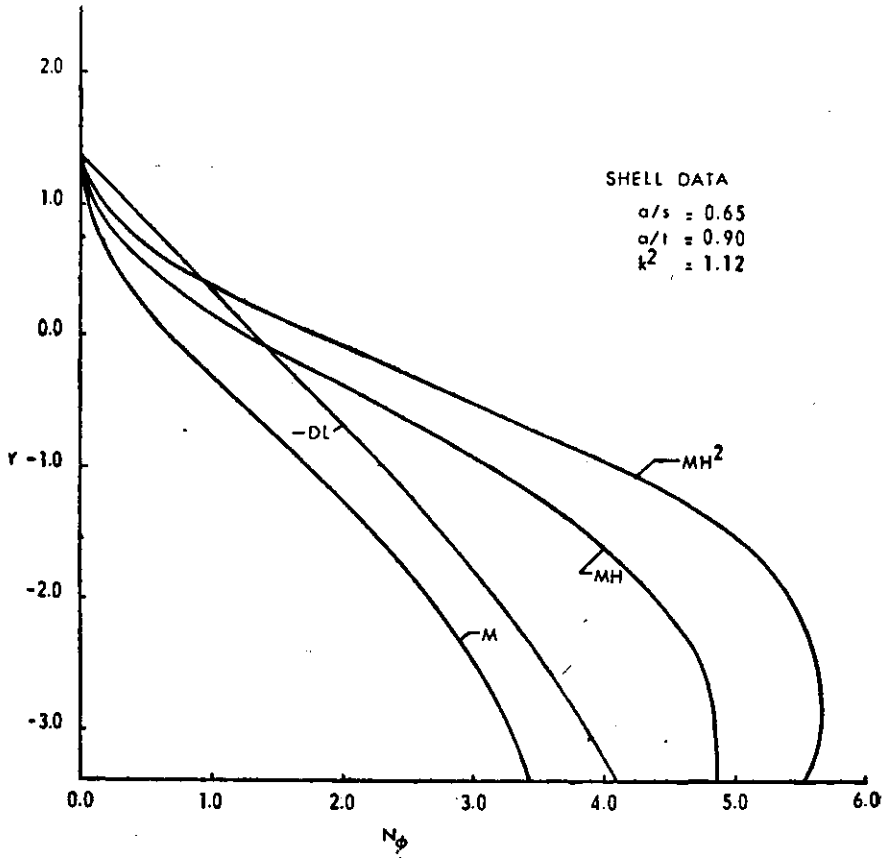

Fig. 2. Effect of Base Shear Distribution on Meridional Stress Resultant

These stress resultants are multiplied by a load factor of 1·6 (5) and are computed for C=0·2 (1). Also shown is the negative of the meridional stress produced by the dead load. The difference between the dead load and seismic load stresses represents the net tension at any section. It is seen that the M loading results in no net tension while the MH loading results in net tensile stresses over most of the shell; however, the MH² loading yields a net tension which is practically double that computed from the MH loading while the increase in the absolute value of $n_{\varphi 1}$ is less than 20%. Thus a modest increase in the stresses due to the lateral forces produce a much greater increase in the net stress which is important in design.

SUMMARY AND CONCLUSIONS

The membrane stress resultants for a hyperboloid of revolution were derived for a quasistatic seismic loading in which the base shear is distributed to any section of the shell in proportion to the mass of the section and the square of the height of the section above the base. A comparison of the net meridional stress resultants obtained from various distributions of the base shear revealed that the MH² distribution is by far the most severe.

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APPENDIX I. SUMMARY OF FUNCTIONS

$$\zeta_1(\varphi)=\frac{1}{4(k^2-1)}\left[\frac{\sqrt{k^2-1}}{k}\ \ln K-2r_\theta^2\cos\varphi\right] \qquad \ldots(A1)$$

in which

$$K=\frac{\sqrt{k^2-1}-k\cos\varphi}{\sqrt{k^2-1}+k\cos\varphi}$$

$$\zeta_2(\varphi)=\frac{r_\varphi}{3k^2}-\zeta_1\frac{S}{a} \qquad \ldots(A2)$$

$$\zeta_3(\varphi)=\frac{\cos\varphi\ r_\theta}{k^2-1} \qquad \ldots(A3)$$

$$\zeta_4(\varphi)=\frac{1}{2k^2}\zeta_3\left[\frac{1}{\sqrt{k^2-1}}\ \ln K+\frac{2}{k\cos\varphi}\right]+\frac{r_\varphi}{3k^2} \qquad \ldots(A4)$$

$$\zeta_5(\varphi)=\zeta_1\ \zeta_6+\xi\ \zeta_2+\zeta_7 \qquad \ldots(A5)$$

$$\zeta_6(\varphi)=\frac{1}{4k^2\ (k^2-1)}-\zeta_3\frac{S}{a} \qquad \ldots(A6)$$

$$\zeta_7(\varphi)=\frac{r_\varphi}{12k^2}\left[\zeta_3+4\ \frac{S}{a}\right] \qquad \ldots(A7)$$

$$\zeta_8(\varphi)=\frac{\xi+\frac{S}{a}}{4k^2\ (k^2-1)}+\zeta_3\left[\frac{1}{2k^2\ (k^2-1)}-\left(\frac{S}{a}\right)^2\right] \qquad \ldots(A8)$$

$$\zeta_9(\varphi)=\frac{r_\varphi}{3k^2}\left[\frac{\zeta_3\left(\xi+\frac{S}{a}\right)}{4}+\left(\frac{S}{a}\right)^2-\frac{1}{2k^2\ (k^2-1)}+\frac{r_\theta^2}{10k^2\ (k^1-1)}\right] \qquad \ldots(A9)$$

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