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RESPONSE SPECTRA AS A USEFUL DESIGN AND ANALYSIS TOOL FOR PRACTICING STRUCTURAL ENGINEERS

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ABSTRACT

Although response spectra have been in general use for decades by researchers, academics, and geotechnical professionals, their use by structural design professionals has generally been limited. However, as response spectra and dynamic analysis are being included within the newer building codes and as performance-based design (PBD) techniques are becoming acceptable, there is a need for the design professional to more clearly understand the meaning and usefulness of response spectra. The purpose of this paper is to review the concept of response spectra for design engineers not familiar with their significance and to summarize a variety of uses that can be applied for purposes such as rapid evaluation for a large inventory of buildings, performance verification of new construction, evaluation of existing structures for seismic vulnerability, and post earthquake estimates of potential damage of buildings.

KEYWORDS: Response Spectra, Building Codes, Performance-Based Design, Seismic Vulnerability, Earthquake Intensity

INTRODUCTION

The concept of response spectra was first incorporated into the United States building codes in the late 1950's by means of the coefficient C in the lateral force equation $V = KCW$ by the Structural Engineers Association of California (SEAOC, 1960), where V is the total lateral force, K is a structural systems coefficient of 1.33, 1.0 or 0.67, and W is the total dead load. Over the decades, response spectra have been playing an increasing role in the development of earthquake design criteria. Much of this is due to research and the vast data obtained from recording earthquake motion from earthquakes in California, such as 1971 San Fernando, 1989 Loma Prieta, and 1994 Northridge, as well as from earthquakes worldwide.

The paper traces the development of building code provisions and the relationship to response spectra. Response spectra used for design tend to be smooth curves, whereas response spectra obtained from ground motion recordings are generally very ragged with sharp spikes and valleys. The effects of these differences are discussed along with recommendations on how to graphically smooth out the curves. In general, response spectra are used to analyze structures that respond within elastic-linear limits. The paper presents methods of using response spectra to evaluate structural response in the inelastic-nonlinear range. This includes easy to use graphical methods that compare the seismic demand represented by a response spectrum to the capacity of the structure represented by pushover force-displacement curves. Such methods are the capacity spectrum method (CSM) developed by the author (Freeman et al., 1975) as well as modifications (ATC, 1996; FEMA, 2005; Freeman, 2006), and procedures presented by others (Fajfar, 1998; Priestley et al., 1996). Other uses of response spectra include the development of an earthquake engineering intensity scale (EEIS) that extends the TriNet instrumental intensity scale (Wald et al., 1999) to estimate damage levels for a variety of building types.

INTRODUCTION TO RESPONSE SPECTRA

Response spectra provide a very handy tool for engineers to quantify the demands of earthquake ground motion on the capacity of buildings to resist earthquakes. Data on past earthquake ground motion is generally in the form of time-history recordings obtained from instruments placed at various sites that are activated by sensing the initial ground motion of an earthquake. The amplitudes of motion can be expressed in terms of acceleration, velocity and displacement. The first data reported from an earthquake record is generally the peak ground acceleration (PGA) which expresses the tip of the maximum spike of the acceleration ground motion (Figure 1).

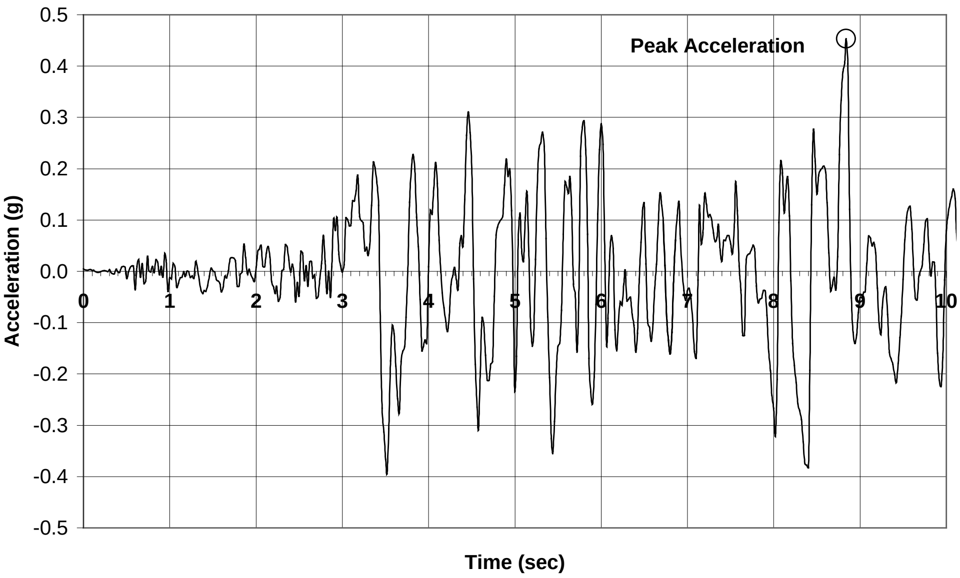


Fig. 1 Recorded ground motion (Holiday Inn, Van Nuys 1994, 270 degrees: 0–10 sec)

Although useful to express the relative intensity of the ground motion (i.e., small, moderate or large), the PGA does not give any information regarding the frequency (or period) content that influences the amplification of building motion due to the cyclic ground motion. In other words, tall buildings with long fundamental periods of vibration will respond differently than short buildings with short periods of vibration. Response spectra provide these characteristics. Picture a field of lollipop-like structures of various heights and sizes stuck in the ground. The stick represents the stiffness (K^*) of the structure and the lump at the top represents the mass (M^*). The period of this idealized single-degree-of-freedom (SDOF) system is calculated by the equation:

$$T = 2\pi (M^* / K^*)^{1/2} \tag{1}$$

If the peak acceleration (S_a) of each of these SDOF systems, when subjected to an earthquake ground motion, is calculated and plotted with the corresponding period of vibration (T), the locus of points will form a response spectrum for the subject ground motion. Thus, if the period of vibration is known, the maximum acceleration can be determined from the plotted curve. When calculating response spectra, a nominal percentage of critical damping is applied to represent viscous damping of a linear-elastic system, typically five-percent.

Response spectra can be plotted in a variety of formats. A format commonly used in the 1960s was the tripartite logarithmic plot, where the vertical scale is spectral velocity (S_v) and the horizontal scale is T in seconds or frequency (f) in Hertz. On diagonal lines are designated S_a and spectral displacement (S_d). An example is shown in Figure 2.

Mathematical relationships between the components of response spectra are given by the following equations:

$$S_v = (T / 2\pi) S_a \tag{2}$$

$$S_a = (2\pi / T) S_v \tag{3}$$

$$S_d = (T / 2\pi) S_v = S_a (T / 2\pi)^2 \tag{4}$$

$$f = 1/T \tag{5}$$

Figure 3 shows other graphical formats used to represent response spectra. Figure 3(a) is known as the ADRS format (Mahaney et al., 1993) that plots S_a versus S_d and shows the period, T, as radial lines. Curved lines representing S_v can also be added (not shown, see Figure 4(b)). ADRS is essentially the tripartite format in a rotated linear coordinate system. Figure 3(b) is the commonly used S_a versus T coordinate system. When S_d is the unit of interest, the S_d versus T format can be used (Figure 3(c)). The relationships among these curves are consistent with the equations listed above, which define S_v as a pseudo velocity.

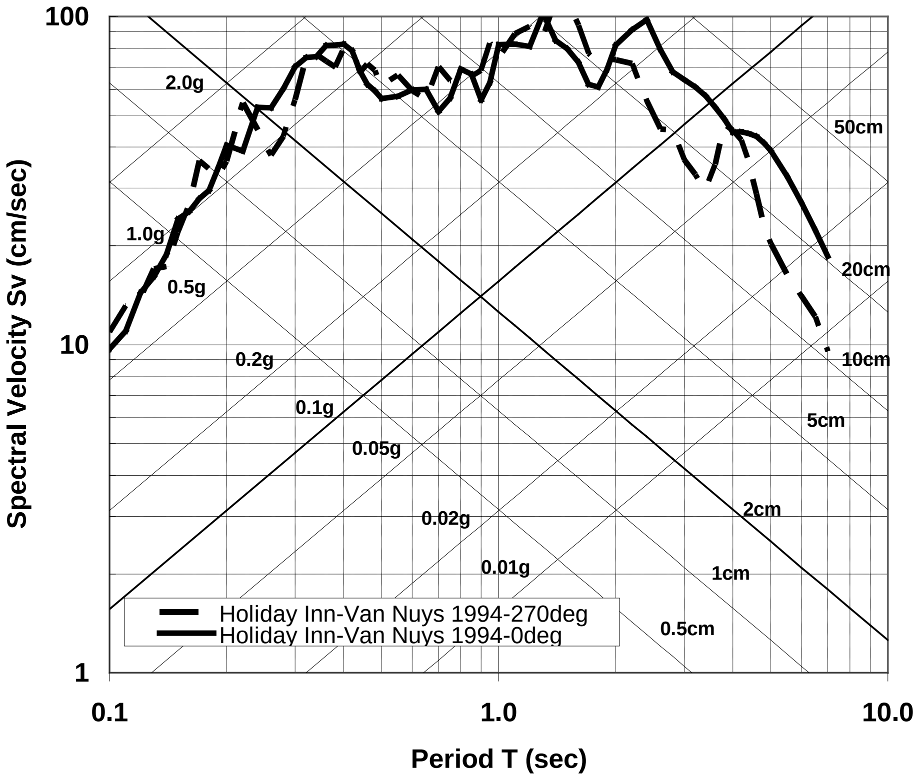


Fig. 2 Tripartite (logarithmic) response spectra

The response spectra shown in Figures 2 and 3 represent the ground motion recorded at the ground level of the Holiday Inn hotel structure during the Northridge earthquake of January 1994 in California, U.S.A. The continuous curves represent the horizontal motion in the 0-degree direction and the dashed curves represent the horizontal orthogonal motion in the 270-degree direction. Vertical motion was also recorded (not shown). Ground motion, as well as building motion, was recorded for many other locations during the Northridge event. Response spectra have also been obtained during the 1971 San Fernando earthquake as well as from other earthquakes in the Los Angeles, California area. This data bank, as well as data from earthquakes from all over the world, provides useful tools for studying the effects of earthquake ground motion on building structures and for the development of code provisions for the design of buildings.

It is observed that the response spectra shown in Figures 2 and 3 are rather jagged with sharp peaks and valleys; and there are significant variations in the two directions of motion. It can also be shown that there are large variations in ground motion characteristics at other sites for the same earthquake, as well as for the same site from other earthquakes. The peaks and valleys illustrate the sensitivity of the response of structures to a slight variation in the natural period of vibration. The large variations in ground motion characteristics illustrate the difficulties in accurately predicting demands of future earthquakes. This leads us to the challenge to develop standard response spectra that give a reasonable probability of having credible design provisions.

Methods of constructing smooth response spectra for design purposes have been developed to compensate for the peaks, valleys, and shape variations in actual response spectra; for example, the use of a constant S_a for short periods of response, constant S_v for the mid range, and constant S_d for long period response to develop probabilistic design spectra (Newmark and Hall, 1982; Newmark et al., 1973). An example of smooth spectra is shown in Figure 4 based on a building code design response spectrum for a site of high seismicity.

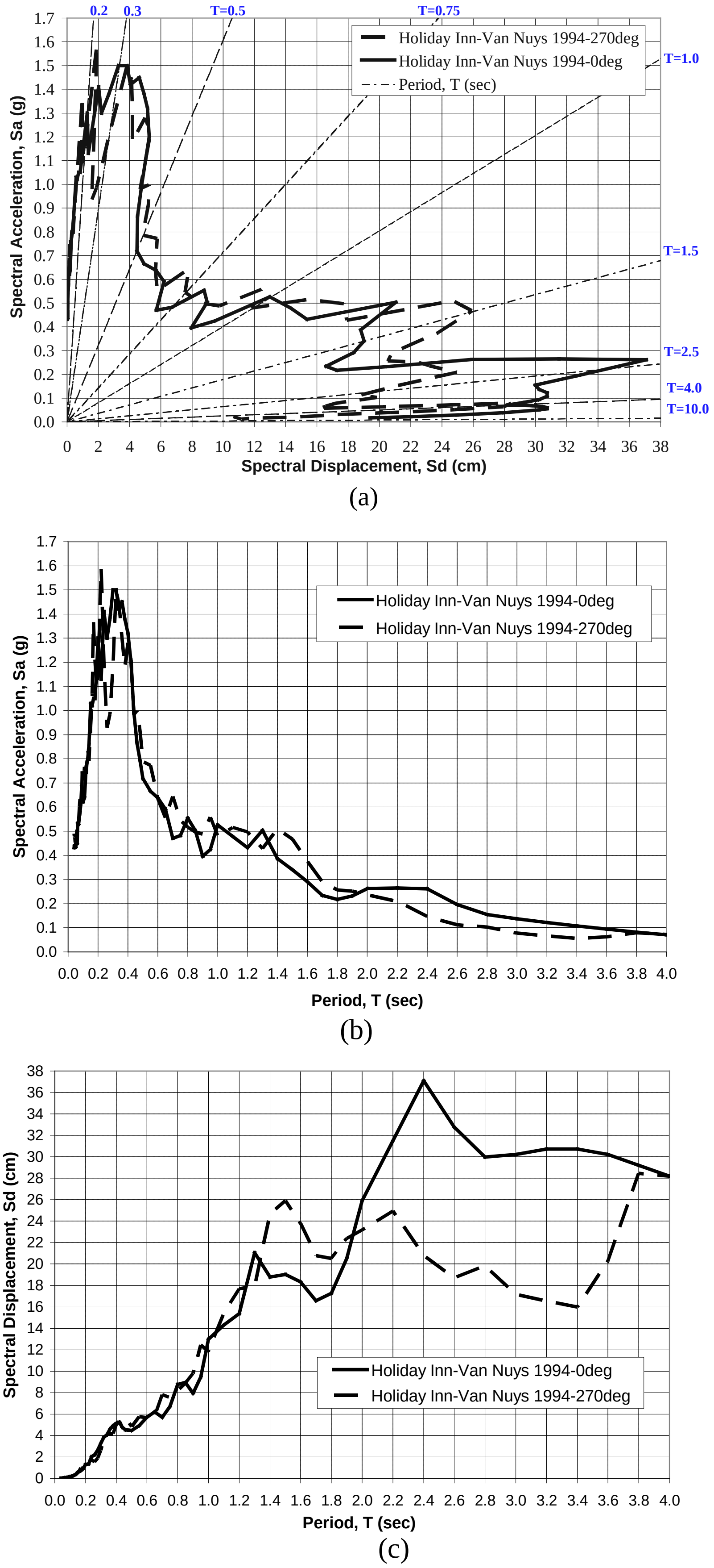
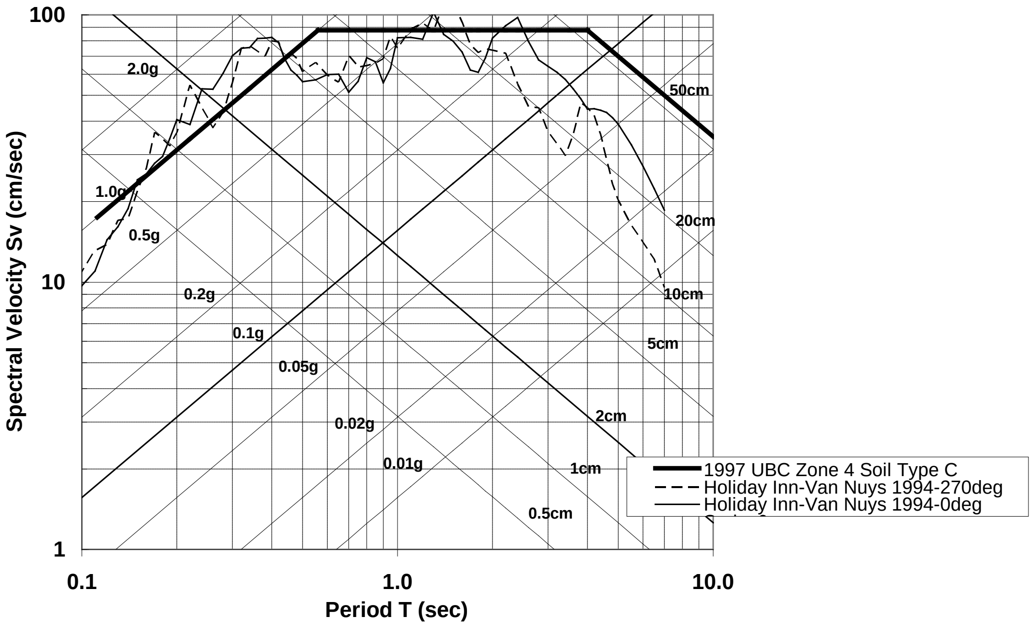
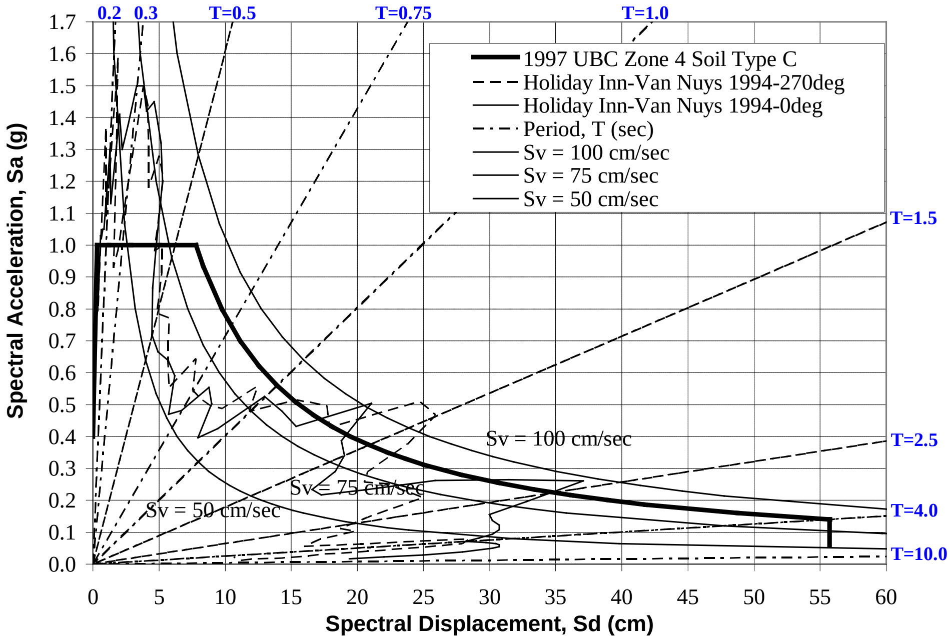


Fig. 3 Response spectra formats (Holiday Inn, Van Nuys 1994): (a) ADRS format, (b) S_a versus T format, (c) S_d versus T format



(a)



(b)

Fig. 4 Building code type smooth response spectra: (a) Tripartite format, (b) ADRS format

The example shown is for a 1997 Uniform Building Code criterion for seismic zone 4 at a soil category C site. The PGA is 0.4*g* (i.e., 40% of gravity), the constant S_a is 2.5 times the PGA (= 1.0*g*). Constant velocity is based on S_a at one second that equals 1.4 times PGA (= 0.56*g*). This translates to a S_V equal to 87 cm/sec (using Equation (2)). Assuming a cut-off period of 4 sec, the constant displacement becomes 56 cm (using Equation (4)).

Once design response spectra are established, it is fairly simple to establish seismic design forces for a building. For low-rise buildings, where the fundamental mode of vibration (in each direction) is predominant, we estimate the period of vibration of the building and find the corresponding S_a. This may be used as a base shear coefficient for determining the lateral forces on the building or adjustments may be made for dynamic participation factors. For tall buildings, where the dynamic effects of higher modes of vibration are significant, spectral accelerations for each of the several modes may be quickly determined using the estimated periods. If the period estimates are revised, the lateral forces can be easily adjusted proportionally to the revised spectral accelerations.

INFLUENCE OF RESPONSE SPECTRA ON BUILDING CODE PROVISIONS

The basis for the development of current seismic building code provisions had their beginnings in the 1950s. A joint committee of the San Francisco section of ASCE and the Structural Engineers Association of Northern California prepared a "model lateral force provision" based on a dynamic analysis approach and response spectra (Anderson et al., 1952). The proposed design curve, $C = K/T$, was based on a compromise between a standard acceleration spectrum by M.A. Biot (Biot, 1941, 1942) and an El Centro analysis by E.C. Robison (Figure 5). It is interesting to note that the PGA of 0.2*g* in the Biot curve has a peak spectral acceleration of 1.0*g* at a period of 0.2 sec. The curve then descends in proportion to 1/*T* (i.e., constant velocity). If the peak spectral acceleration is limited to 2.5 times the PGA, the Biot spectrum is very close to the 1997 UBC design spectrum for a PGA of 0.2*g* (dashed line without symbols in Figure 5). The proposed design lateral force coefficient was $C = 0.015/T$, with a maximum of 0.06 and a minimum of 0.02 (line with dots in Figure 6). These values were considered consistent with the current practice, and the weight of the building included a percentage of live load.

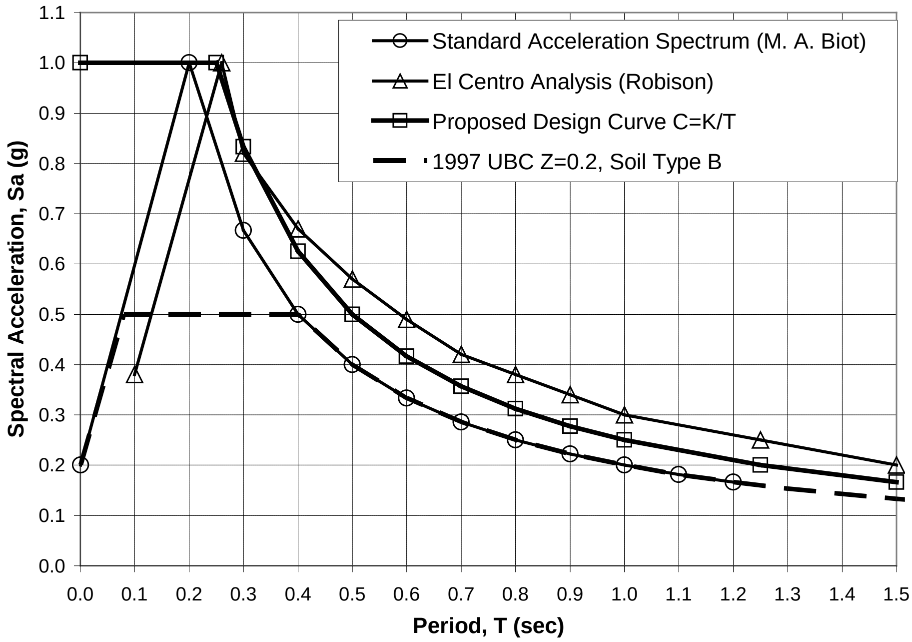


Fig. 5 1952 Joint Committee Response Spectra (Anderson et al., 1952)

In 1959, the Seismology Committee of the Structural Engineers Association of California published "Recommended Lateral Force Requirements" (generally referred to as the SEAOC bluebook) and included "Commentary" in 1960 (SEAOC, 1960). Influenced by the Joint Committee (many of the members were on both committees), recommendations were proposed that were adopted for the 1961 Uniform Building Code (UBC) (ICBO, 1961). The new recommended design lateral force coefficient was $C = 0.05/T^{1/3}$, and the live loads were not included in the weight (except for a percentage in storage facilities). By using *T* to the one-third power, the equation could account for higher modal participation and give a larger load factor for tall buildings. In addition it avoided the need for a minimum cut-off. The maximum was set at $C = 0.10$ (Figure 6). Also shown in Figure 6 is a comparably adjusted version of the 1997 UBC.

Over the years, the SEAOC bluebook and the UBC went through many revisions, generally influenced by some events such as the 1971 San Fernando, 1989 Loma Prieta, and 1994 Northridge earthquakes, and by data relating to soil effects. The comparable curves shown in Figure 7 have been adjusted to represent strength design response spectra and include factors representing soil classification type D. At this level of design, the structures would be expected to remain linear-elastic with some reserve capacity before reaching yield. In order to survive a major earthquake ground motion (e.g., PGA = 0.4*g*) the structure is expected to experience nonlinear post-yielding response.

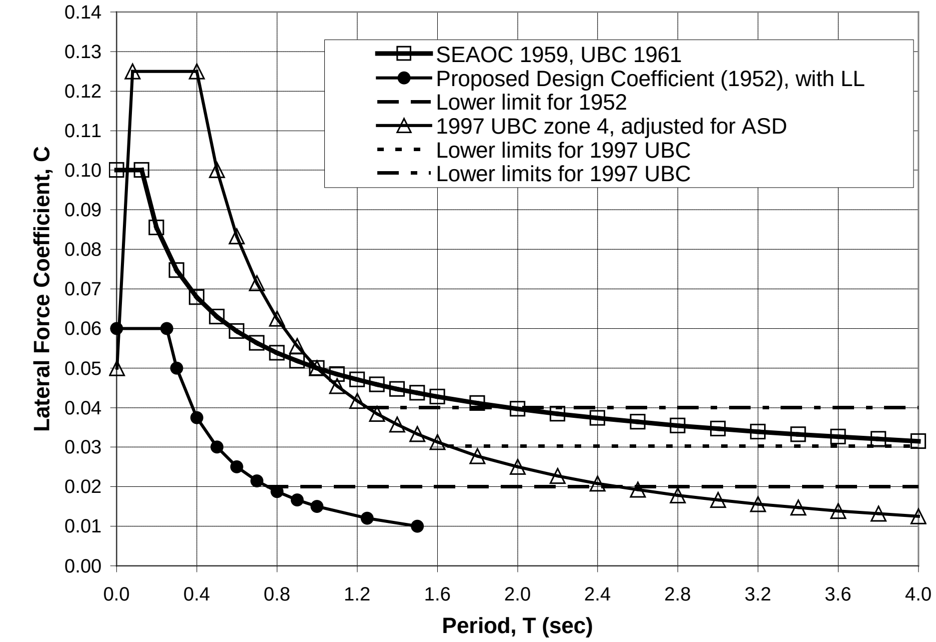


Fig. 6 1959 design lateral force coefficients (SEAOC, 1960)

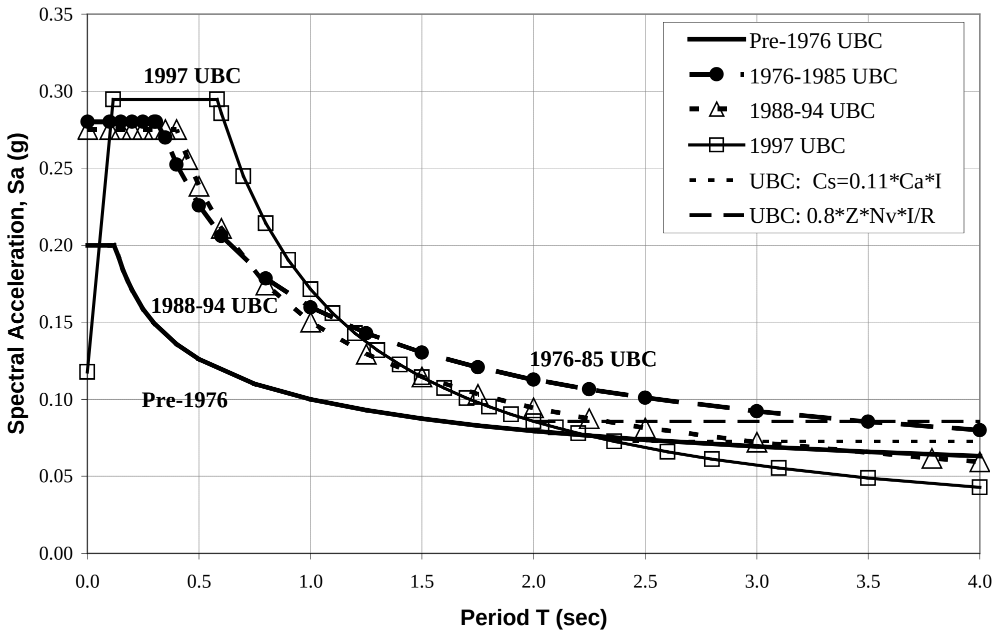


Fig. 7 UBC strength design response spectra (Zone 4, Soil D equivalents)—1961 to 1997

RESPONSE SPECTRA FROM GROUND MOTION RECORDINGS

It is convenient for design to have smooth response spectra; however, in the real world response spectra come in a large variety of sizes and shapes. Therefore, data on PGA and intensity do not give the full picture of an earthquake event. Examples of response spectra from three locations in the Los Angeles, California area from the 1994 Northridge earthquake are shown in Figure 8. The locations are Santa Monica, Newhall and Sylmar, which experienced PGAs greater than 0.6*g* (code's maximum probable PGAs are generally considered to be 0.4*g*). The ADRS format is used and, for scale, constant spectral velocity is shown for 150 and 75 cm/sec by double-dot-dash curves. For Santa Monica the demand is great for very short period buildings ($T < 0.3$ sec) and moderate for tall buildings ($T > 1.5$ sec). In the mid-period range the demands are relatively small. On the other hand, Newhall has a huge demand in the mid-period range with a broad double hump (T from 0.6 to 1.5 sec). The Sylmar spectrum has moderate demands in the mid-period range, but has a very large displacement demand for long periods (T from 2 to 4 sec). It is tempting to envelope these and a whole family of response spectra to illustrate that the ground motion was about twice the expected average 475-year event, but that would be misleading. For each of the locations, buildings would respond differently, and because of energy absorption (in soil and in the building), nonlinearity and changing periods, many buildings avoided catastrophic results.

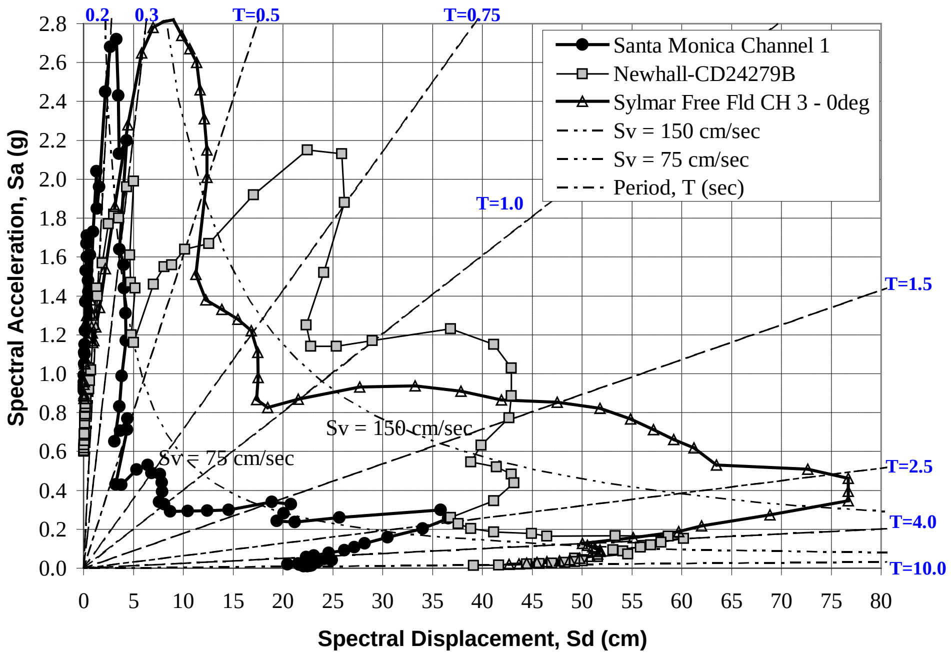


Fig. 8 Three Northridge, 1994 response spectra (5% damped)

In Figure 9, response spectra are shown for the Holiday Inn hotel structure, which experienced damage from both the 1971 San Fernando and 1994 Northridge earthquakes. The spectra with circles show two directions for 1994 and the curves with squares show 1971. The building experienced damage and was softened up by the 1971 earthquake (Murphy, 1973). The initial period was about 0.5 sec; after the earthquake it was about 1 sec. The 7-story pushover curve represents the capacity of the structure (e.g., lateral force versus roof displacement, transformed to S_a versus S_d). The curve shown in Figure 9 was obtained by an evaluation of the recorded building motion (Gilmartin et al., 1998) and is consistent with calculations. Figure 9(a) shows 5% damped spectra and Figure 9(b) shows 20% damped spectra. The structure is overwhelmed by the 5% damped spectra; however, the use of 20% damped spectra to represent inelastic-nonlinear response spectra (Freeman, 2004), illustrates how the building survived without total collapse (i.e., the capacity curve breaks through the response spectra envelopes). In this example 20% damping represents roughly a displacement ductility of 2.5 (Freeman, 2006).

SMOOTHING RESPONSE SPECTRA

If there is a desire to construct a smooth spectrum from a jagged response spectrum Figure 10(a) illustrates a very simple method. Using the ADRS format, we identify the peak spectral acceleration and draw a horizontal line (constant acceleration). We do the same for the peak spectral displacement, drawing a vertical line for the maximum constant displacement. Then, moving out along radial lines from the origin, we locate the maximum spectral velocity (this may be more visually clear on the tripartite graph in Figure 10(b)). Connecting the lines forms a maximum smooth spectrum. A similar procedure is used to form the minimum smooth spectrum (for the minimum acceleration we use the spectral acceleration at $T = 0.1$ sec to avoid selecting the peak ground acceleration). Taking an average of the maximum and minimum curves will result in a reasonable estimation of a smooth spectrum. Also shown on the Figure 10 graphs are peak ground motion (PGM) spectra, which are formed using the measured peak acceleration, velocity and displacement. An interesting use of these graphs is to estimate dynamic amplification factors (DAFs) by dividing spectral values by ground motion values. For example, if average constant acceleration (1.05*g*) is divided by the peak ground acceleration (0.4*g*) the DAF is about 2.5. For velocity the DAF is about 1.7, and for displacement the DAF is about 2.3.

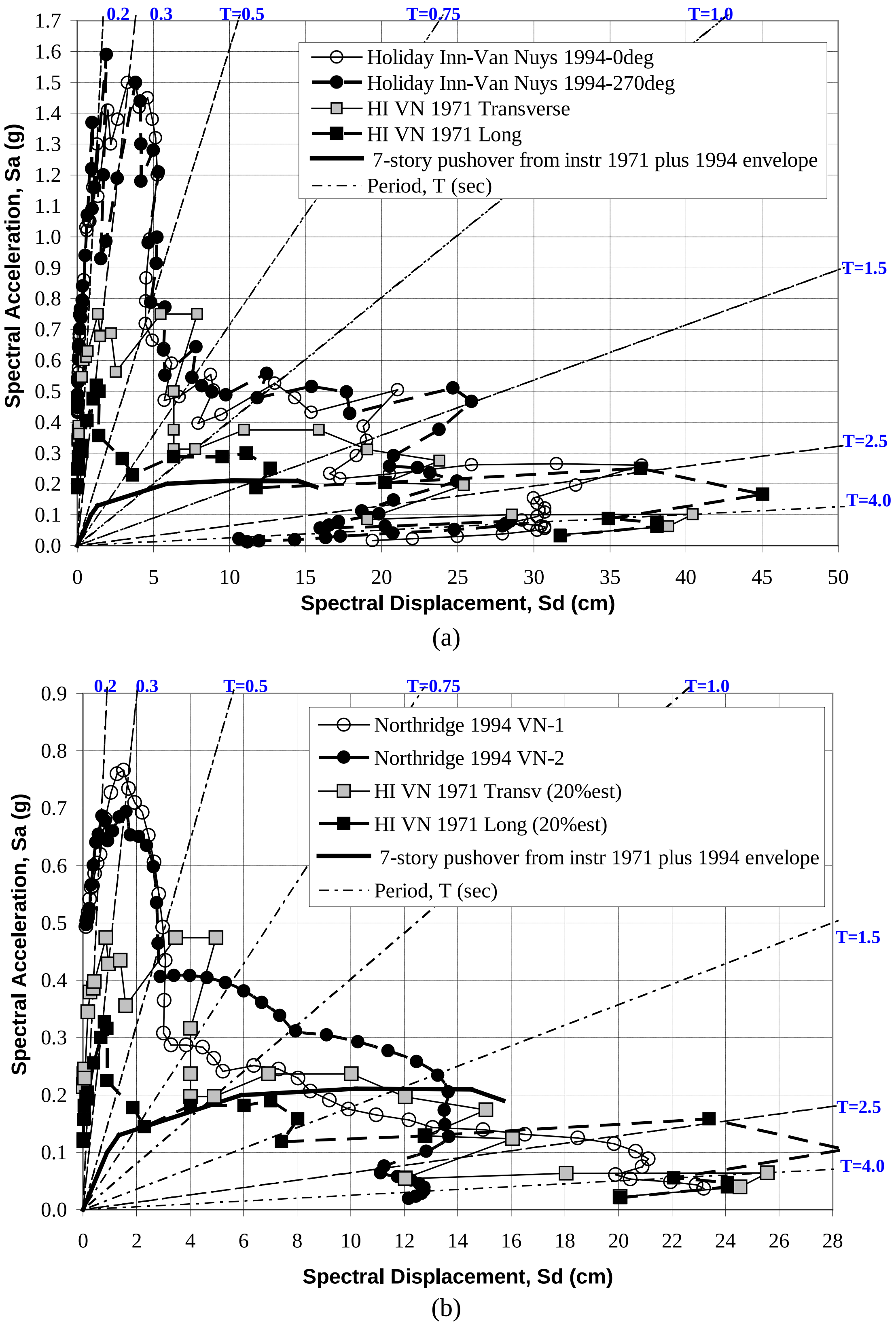
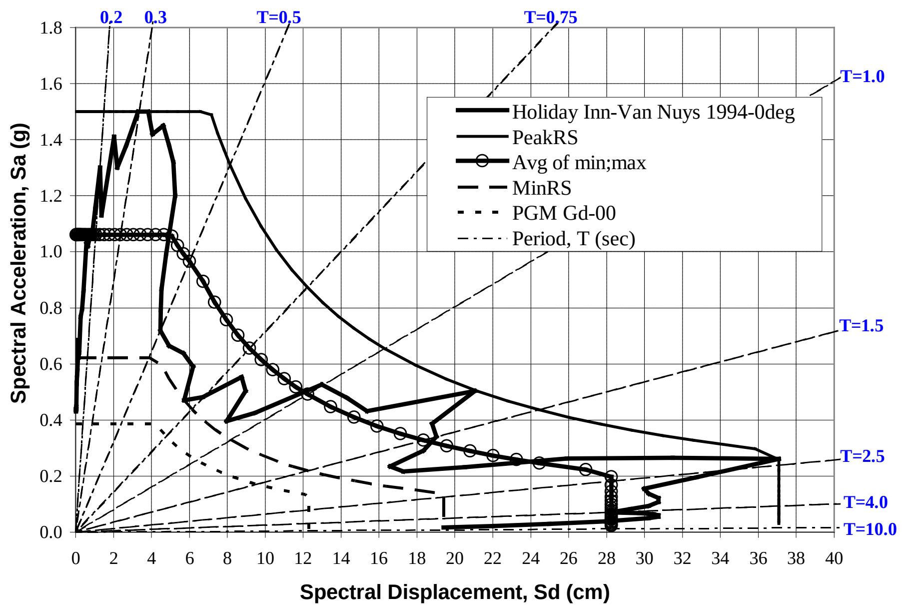
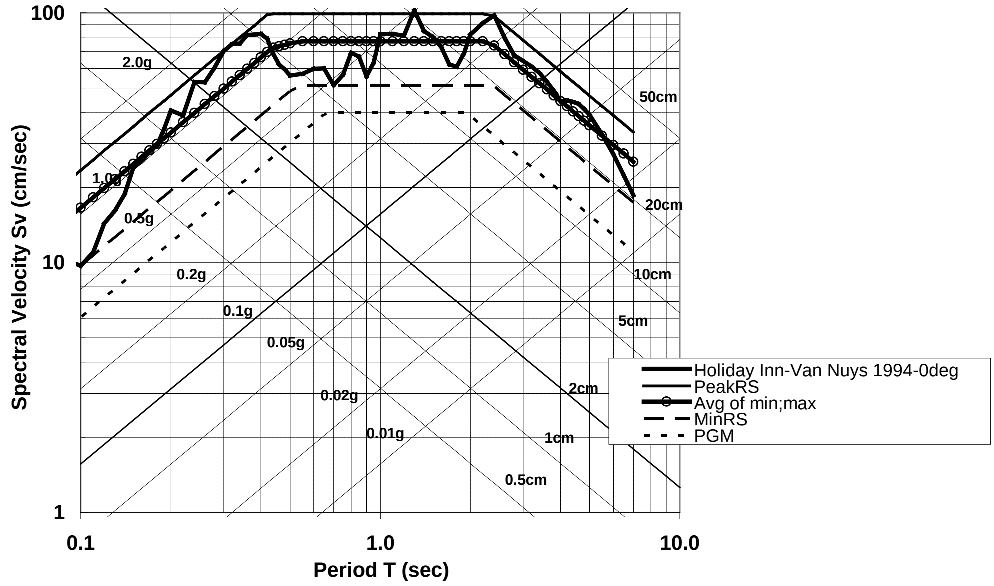


Fig. 9 Holiday Inn, Van Nuys response spectra for 1971 and 1994 earthquakes: (a) 5% damped, and (b) 20% damped



(a)



(b)

Fig. 10 Smoothing response spectra and ground motion spectra (Holiday Inn, Van Nuys 1994): (a) ADRS format, (b) Tripartite format

AN EARTHQUAKE ENGINEERING INTENSITY SCALE

Emergency response after an urban area earthquake requires incorporation of data from various sources. Main sources of data for engineering use are the so-called free-field instruments, as used in TriNet ShakeMap, and strong-motion instruments installed in buildings. The TriNet system is capable of providing a rapid instrumental intensity map for strong motion earthquakes on the basis of an array of recording instruments. The instrumental intensity scale (I_{mm}) is based on recorded peak ground accelerations (PGAs) and peak ground velocities (PGVs). Both are calibrated against historical Modified Mercalli Intensity (MMI) data, and are related to two parallel scales describing potential damage and perceived shaking (Wald et al., 1999). To improve emergency response, an Earthquake Engineering Intensity Scale (EEIS), built on a scale initially developed by the late John A. Blume in 1970s (Blume, 1970), is presented (Freeman et al., 2004). EEIS allows translation of ground shaking information in the form of response spectra at a site into response/shaking intensity for different kinds of buildings. When this translation is presented in Acceleration-Displacement Response Spectrum (ADRS) format, spectrum levels for different period ranges can be graded into various EEIS levels by relating them to the Instrumental Intensity (I_{mm}) scale developed for TriNet ShakeMap.

To construct the link, response spectra corresponding to the I_{mm} scale can be approximated by applying dynamic amplification factors to the TriNet PGA and PGV values. Studies dating back from the 1970s to the present have provided recommendations for these amplification factors (Newmark et al., 1973; Newmark and Hall, 1982). By multiplying the PGA values by the acceleration amplification factor for the short periods (i.e., constant acceleration range) and by multiplying the PGV values by the velocity amplification factor for the medium-to-long periods (i.e., constant velocity range), smooth response spectra can be formed into a structural response intensity scale. Amplification factors of 2.0 for the PGA and 1.7 for the PGV were selected as illustrated in Figure 11. The response spectra shown in Figure 8 are shown superimposed on transformed I_{mm} scales VII through X. Note that they have small bumps into X, but generally lie in intensity IX. Santa Monica lies in intensity VIII except at very short periods. The EEIS is also shown on the tripartite format (Figure 11(b)). Note the period bands that designate zones of short-, medium- and long-period buildings.

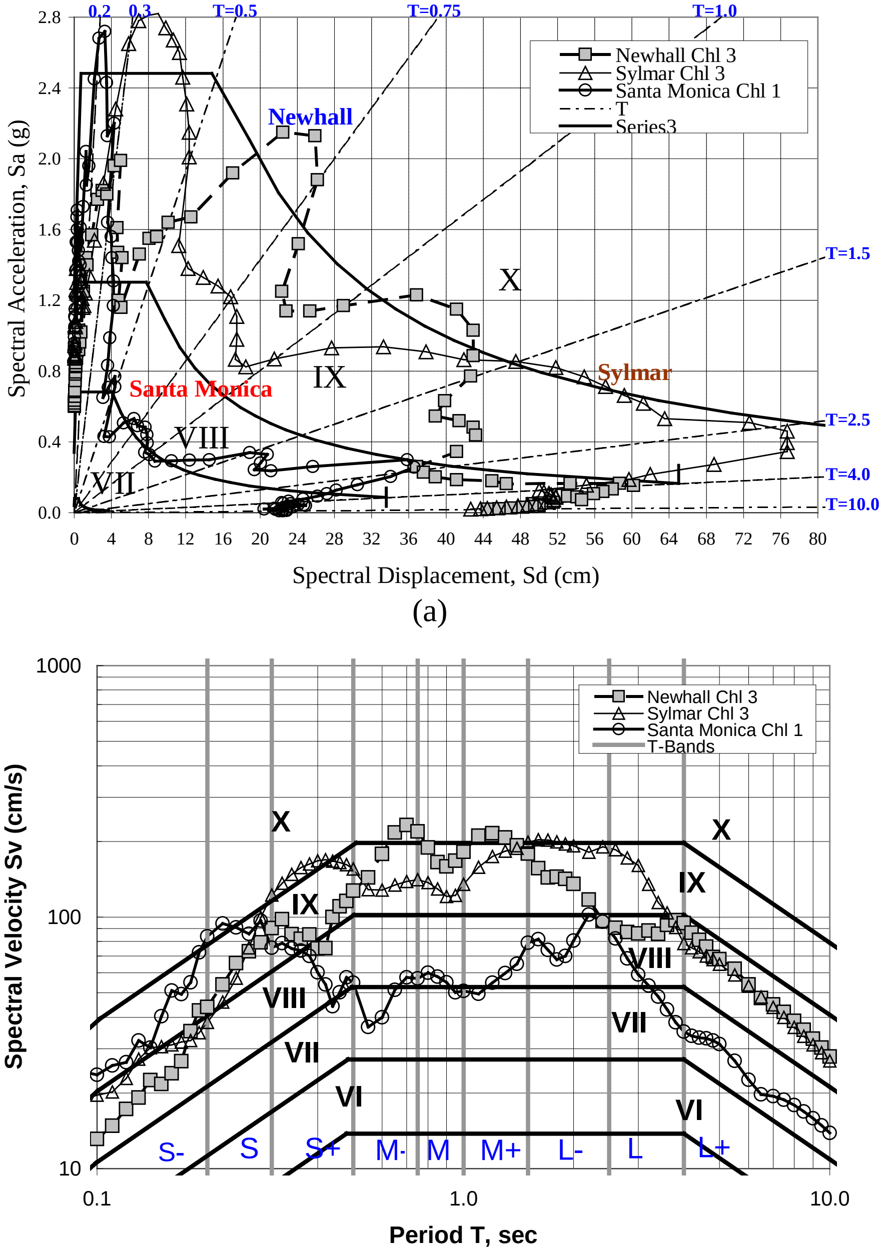


Fig. 11 Earthquake Engineering Intensity Scale (EEIS): (a) ADRS format, (b) Tripartite format

CLOSING

An introduction to response spectra has been presented, illustrating procedures that may be useful to professional engineers as an aid to design and evaluation of buildings and other structures. When earthquake ground motion data is available, the use of response spectra can be very useful in understanding how buildings perform and to identify deficiencies and damage potential.

However, response spectra, as in any other technique, must be used with caution and a good understanding of the process. For single-degree-of-freedom systems responding in a linearly elastic manner, response spectra give good credible results, assuming that the data is credible. For a measured earthquake response spectrum with sharp peaks and valleys, the variations due to uncertainty in actual structural period of vibration is visually apparent. For multi-modal systems, the combination of modes is generally done by SRSS (square root of the sum of the squares) or CQC (complete quadratic combination) rule. Although these rules are based on probability approximations, the results are generally reasonable. The more technical time-history method is generally considered more exact; however, due to sensitivity to small variations in accuracy of structural periods of vibration, there are also uncertainties in this procedure. When analysis is extended into the inelastic nonlinear realm of structural response, complexities of analysis multiply. Response spectrum techniques allow engineers to visually imagine how buildings will perform during major damaging earthquakes.

It is recommended that researchers and design professionals put more effort into detailed examinations of individual building response records. By deconstructing individual recorded floor motions into individual modes of vibration, there is the potential of better understanding how buildings perform during earthquake ground motions. This could lead to developing better methods of using response spectra.

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