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SEISMIC ANALYSIS OF A PRE-HEATER TOWER FOR A CEMENT PLANT

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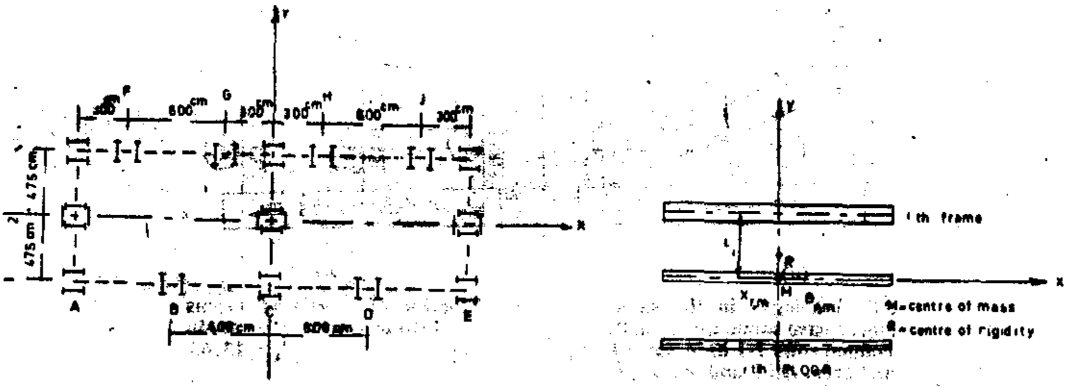
INTRODUCTION :

It is recommended that dynamic analysis be required for all important structures, structures that are dynamically unsymmetrical to seek out and reinforce weak places because adequate design criterion are not available. A major simplification in the dynamic analysis of multistorey structures is achieved if the analysis can be limited to a plane structure composed of braced and plane frames subjected to horizontal forces in their own plane. This is possible when a structure subjected to symmetrical loading is composed of a set of symmetrical parallel forces. A number of published papers[1-3] deal with this type of two dimensional problem. But, the analysis of a framed building consisting, normally, of a number of variable multi-bay frames pose a problem of determining the basic structural parameters like stiffness and mass matrices to compute frequencies and mode shapes due to their large kinematic degrees of freedom. Such a problem becomes more important when it is to be solved on a medium size computer with no magnetic tapes and discs available. In order to reduce the computer time and therefore, the cost of analysis, a simple procedure of doing dynamic analysis of multi-storey, multi-bay frame structures is described in this paper.

The plan layout of the Steel pre-heater tower is shown in fig. 1 (a). Frame elevation



Plan Layout (a)

Resisting Frames in X Direction (b)

Fig. 1. Plan layout of the steel pre-heater tower

on grid A and E are identical and consist of diagonally braced frames as shown in fig. 2(a). The function of these two frames in the shorter direction is similar to that of shear

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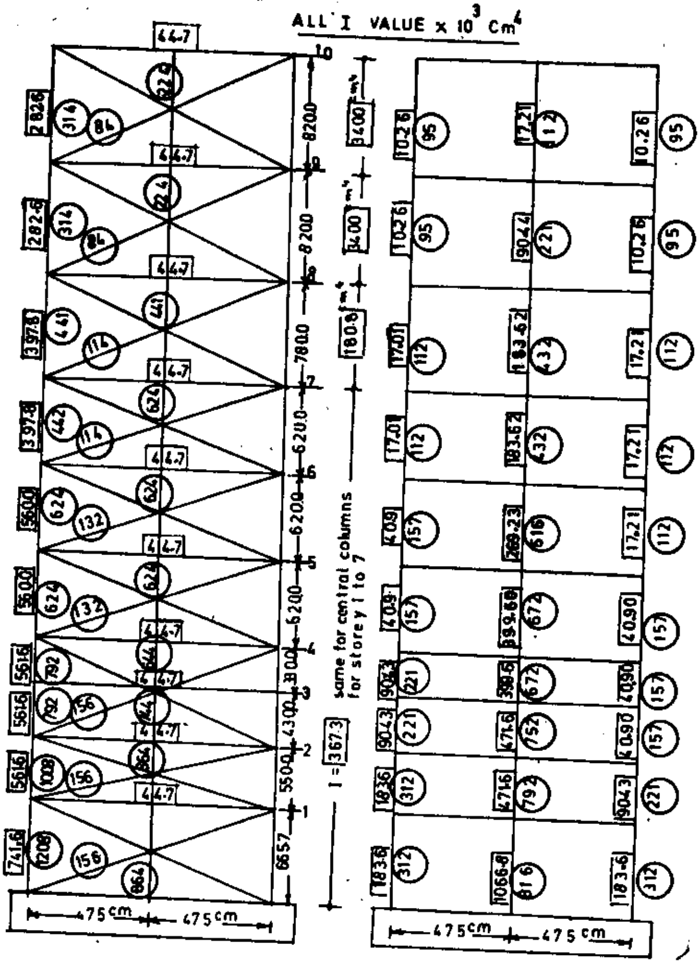
walls in tall buildings. Frame on Grid C is shown in Figure 2b. As mentioned later on in this paper, the total stiffness matrix $[K]_L$ of the structure defined as the reduced stiffness



(a)

All bracings in the same storey have same areas both column rows 1-1 have same I and area. All I and areas are doubled.

(b)

Moment of inertia of beams from storey 1 to 9 =20.46
top =13.63

Height in cms

Fig. 2. Geometrical dimension of frames in the shorter direction

Figures in rectangular blocks = I values
Figures in circles = Areas

matrix of the complete structure relating horizontal storey force with the corresponding storey level displacement, in the shorter direction can be obtained by adding the individual stiffness matrices $[\bar{K}]_L$ of each frame lying parallel to that direction. As the frames on grid A and E are identical and, are symmetrically placed with respect to Grid C, the total stiffness of these two frames in the shorter direction will be two times that of single frame.

As such the figures in rectangular blocks and circles of fig. (2a) represent twice the values of moment of Inertia, I, and area, A, of each member. Elevation of multi-bay frames on grid 1, 2 and 3 are shown in figure 3a, 3b and 3c.

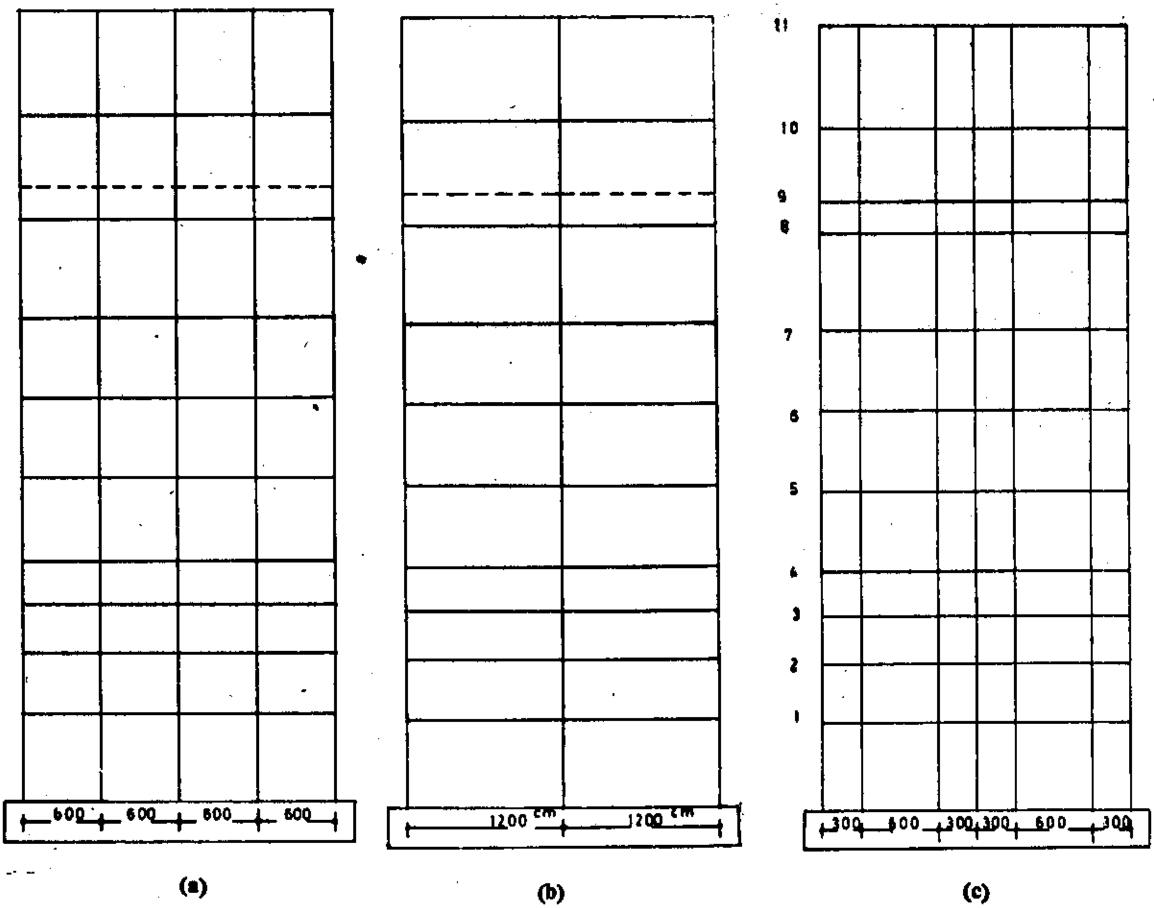


Fig. 3. Outlines of frames grid on 1, 2 and 3

SCOPE AND ASSUMPTIONS :

It was required to make the following studies of the tower building :

1. Periods of vibration in two perpendicular directions.
2. Mode shapes of vibration.
3. Shear force diagram due to earthquake loads acting at storey level.
4. Inter-storey as well as total deflection diagram, and,
5. Design forces and moments in the various members of the structure.

For the formulation of dynamic analysis of the pre-heater tower, the following assumptions were introduced :

1. The in-plane stiffness of the floors is large compared with the stiffness of diagonally braced frames or columns so it is common to assume that each floor diaphragm is

rigid in its own plane such that the lateral displacement components at each floor level are same for all members meeting at that level.

2. The dynamic analysis of the structure in the shorter direction involves no rotation of floors because the centre of mass and the centre of rigidity of each floor lie on one of the principal axis of rigidity as shown in fig. 1b. But the analysis of the tower structure in the longer direction requires taking into consideration rotation of the floors because the centre of mass does not coincide with the centre of rigidity. In the present analysis, the response in the longer direction is determined by neglecting the rotation of floors for the ease of analysis on a medium size computer with a core storage of 16 K. However, this assumption is otherwise not necessary. The complete procedure for carrying out the dynamic analysis when floor rotations are included is discussed in the appendix.
3. The mass of the building is lumped at the floor levels.
4. The structure behaviour is linearly elastic so that normal mode theory can be applied.

DETERMINATION OF EIGEN VALUES AND EIGEN VECTORS IN TWO DIRECTIONS

The equation of motion of a multi-degree freedom system undergoing free vibrations, when damping is neglected, is given by :

$$\left[{}^{\nwarrow}M_{\searrow}\right]\{\ddot{x}\}+[K]_L\{x\}=0 \qquad \ldots(1)$$

where $\left[{}^{\nwarrow}M_{\searrow}\right]$ is the diagonal mass matrix and $[K]_L$ is the reduced stiffness matrix of the structure-relating horizontal storey force with the storey level displacement, {D}.

If the structure is assumed to be vibrating in one of its normal mode of vibrations, say nth, then one can write :

$$\{x\}_n=\{q_n\}\sin p_nt \qquad \ldots(2)$$

or

$$\{\ddot{x}\}_n=-p^2_n\{q\}_n\sin p_nt=-p^2_n\{x\}_n \qquad \ldots(3)$$

Substitution of $\{\ddot{x}\}_n$ from equation (3) into equation (1) yields :

$$[K]_L\{x\}_n=p^2_n\left[{}^{\nwarrow}M_{\searrow}\right]\{x\}_n$$

or

$$[K]_L^{-1}\left[{}^{\nwarrow}M_{\searrow}\right]\{x\}_n=\frac{1}{p^2_n}\{x\}_n$$

or

$$[D]\{x\}_n=\frac{1}{p^2_n}\{x\}_n \qquad \ldots(4)$$

Where [D] is called the dynamical matrix and is equal to $[K]_L^{-1}\left[{}^{\nwarrow}M_{\searrow}\right]$. The size of $[K]_L$ matrix will be $N\times N$ where N is the number of storeys of the structure. The equation (4) was solved by using matrix iteration procedure together with the conditions of orthogonality to yield natural frequencies and mode shapes. As the matrix [D] contains a product of $[K]_L^{-1}$, the iteration converged to frequencies in the ascending order starting from first frequency. Computer programme was written to solve this eigenvalue problem.

FORMATION OF REDUCED STIFFNESS MATRIX $[K]_L$

The complete stiffness matrix $[K]_L$ of the tower structure can be obtained by adding the individual stiffness matrix $[\bar{K}]_L$ of each frame taken in the considered direction. The reduced matrix $[\bar{K}]_L$ for a frame can be obtained from its complete stiffness matrix [K] by separating the floor displacement from the nodes rotation and vertical displacement components as illustrated below :

$$\begin{Bmatrix} \{P_D\} \\ \{P_R\} \end{Bmatrix} = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} \begin{Bmatrix} \{D\} \\ \{R\} \end{Bmatrix} \quad ...(5)$$

where R defines the generalised displacement component of a frame excluding the lateral displacement components. When only lateral loads are acting at each storey level, equation (5) can be re-written as :

$$\{0\} = [K_{21}]\{D\} + [K_{22}]\{R\} \quad ...6(a)$$

and

$$\{P_D\} = [K_{11}]\{D\} + [K_{12}]\{R\} \quad ...6(b)$$

From equation 6(a), the value of {R} is given by :

$$\{R\} = -[K_{22}]^{-1}[K_{21}]\{D\} \quad ...6(c)$$

The substitution of {R} from equation 6(c) in equation 6(b) yields :

$$\{P_D\} = \Big[[K_{11}] - [K_{12}][K_{22}]^{-1}[K_{21}] \Big] \Big\{ D \Big\}$$

or

$$\{P_D\} = [\bar{K}]_L \{D\} \quad ...(7)$$

where

$$[\bar{K}]_L = \Big[[K_{11}] - [K_{12}][K_{22}]^{-1}[K_{21}] \Big]$$

It should be observed that the computer time required to find $[K_{22}]^{-1}$ will depend upon the size of this matrix or on other words on the total number of joint rotations and vertical displacement components of the frame. If the stiffness matrix $[K_{22}]$ was obtained by using equations (5) to (7) for the frames of the complete structure participating in the particular direction when placed side by side connected by axiality stiff pin connected members at each storey level, its size would have been very large. As the computer time is approximately proportional to the cube of the matrix size, it will be advantageous to reduce the time of computation by following a simple procedure, if possible. By following equation from (5) to (7), the reduced stiffness matrix $[\bar{K}]_L$ for each frame lying in the particular direction of analysis was computed. In such cases, the maximum size of $[K_{22}]$ for a frame depended upon the degree of freedom of that frame only, and this size of $[K_{22}]$ was much smaller than that of the complete tower structure when considered in the particular direction. The total reduced stiffness matrix $[K]_L$ was obtained by simple addition of individual matrices. This was possible due to the assumption that lateral displacement component at a storey level is same for all frames meeting at that level.

ASSEMBLAGE OF THE COMPLETE STIFFNESS MATRIX OF A FRAME

It was required that column extensions should be taken into account for the dynamic analysis. The frame consisted of three types of elements, that is, column, beam and bracing members. For a column element three degrees of freedom are permitted at each node as shown in figure 4(a). In case of a beam element only two degrees of freedom were allowed at each node as shown in figure 4(b) ; and for the bracing element, two degrees of translation u and v were specified at each node point as shown in figure 4(c). As the stiffness matrices of these elements are well documented, these are not given here and can be taken from any standard text book on matrix methods of structural analysis. For assembling the stiffness matrix of a frame, the node points were carefully numbered so as to separate storey horizontal displacement component from the remaining displacement components. This was achieved by first numbering the storey level displacement components.

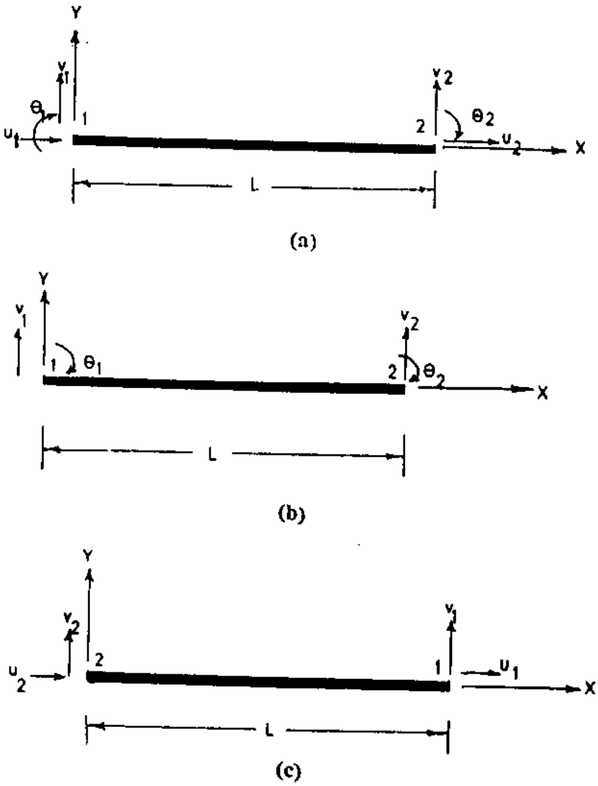


Fig. 4. Beam element with two and three degrees of freedom at each end

MASS MATRIX

The mass is considered as being lumped at each floor level and the mass matrix is formed by placing these masses along the leading diagonal of a square matrix with all other co-efficients equal to zero. The floor weights are given in the second column of table I and II.

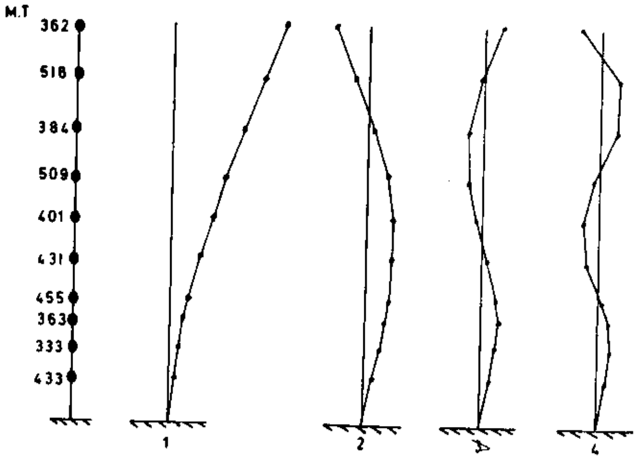
EIGEN VALUES AND EIGEN VECTORS

Having formulated the stiffness matrix $[K]_L$ and lumped mass matrix [M], the eigen values and the eigen vectors were obtained by using matrix iteration procedure on the computer. The computer results of eigen values and eigen vectors of the tower structure in the two perpendicular directions are tabulated in tables I and II. The mode shapes are plotted in figure 5(a) and (b). Lumped masses are also shown in the same figures.

MODAL ANALYSIS

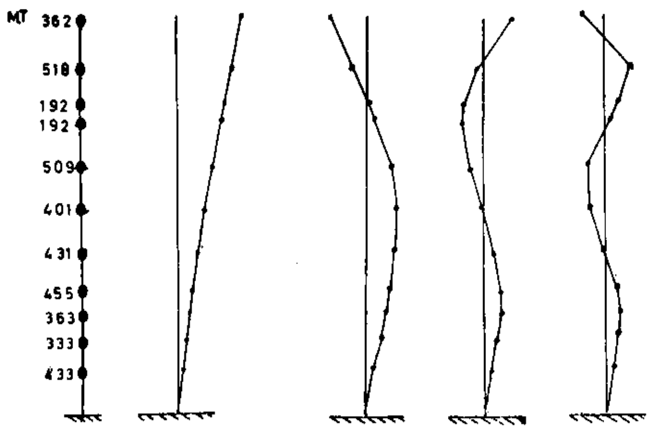
The modal analysis of the tower was carried out in accordance with IS Code (4) 1893 (1970). The load acting at any floor (i), (ii) due to vibrations taking place in r th mode is given by :

$$Q_i^{(r)} = W_i \phi_i^{(r)} C_r \frac{S_a^{(r)}}{g} \beta \cdot F \quad ...(8)$$



Shorter direction

(a)



Longer direction

(b)

Fig. 5. Theoretical modes shapes of vibration

Where Wi, $Qi^{(r)}$, $Sa^{(r)}$, β and F having the same meaning as defined in the IS Code; and

$$C_r = \frac{\sum_{i=1}^{n} W_i \phi_i^{(r)}}{\sum_{i=1}^{n} W_i {\phi_i^{(r)}}^2}$$

Factois β and F are assigned the values equal to 1·0 and 0·7 respectively. Modal response was computed uisng equation (8). The shear V_i, acting at the i th floor level was obtained by finding the root mean square value of first three modes of vibration.

TABLE

STOREY SHEAR CALCULATIONS FOR FRAMES IN

Floor No.	First Mode T=2.37 Sec.					Second Mode T=0.724 Sec.			
	W_i	ϕ_{i1}	$W_i\phi_{i1}$	$W_i\phi_{i1}^2$	Q_{i1}	ϕ_{i2}	$W_i\phi_{i2}$	$W_i\phi_{i2}^2$	Q_{i2}
1.	433	1.0	433	433	0.845	1.00	433	433	7.35
2.	333	2.39	795	1910	1.55	2.05	6 4	1400	11.60
3.	363	3.71	1345	5000	2.63	2.80	1015	2850	17.2
4.	455	4.856	2210	10750	4.31	3.14	1430	4480	24.2
5.	431	7.48	3220	27300	6.28	3.56	1535	5460	26.0
6.	401	10.497	4200	43800	8.20	3.47	1390	4820	23.6
7.	509	13.89	7060	98000	13.80	2.75	1400	3850	23.8
8.	384	18.43	7100	131000	13.85	0.96	369	354	6.25
9.	518	23.52	12200	186000	23.8	−1.75	−910	1590	15.45
10.	362	28.27	10250	290000	19.90	−4.27	−1545	6600	26.2
			48813	894193			5801	31837	

$Sa = 50\ cm/Sec^2$

$$Cr = \frac{48813}{894193} = 0.0546$$

$Sa = 130\ cm/Sec^2$

$$Cr = \frac{5801}{31837} = 0.1825$$

SHORTER DIRECTION—DAMPING—5% OF CRITICAL

Third Mode $T_3=0.37$ Sec.				V_{i1}	V_{i2}	V_{i3}	$\sum_{j=1}^{3} V_{ij}^2$	$\sqrt{\sum_{j=1}^{3} V_{ij}^2}$
ϕ_{i3}	$W_i\phi_{i3}$	$W_i\phi_{i3}^2$	Q_{i3}					
1.00	433	433	8.10	0.845	7.35	8.10	120.465	10.90
1.845	615	1132	11.92	2.395	18.95	20.02	772.736	27.80
2.294	810	1850	19.2	5.025	36.15	39.22	2870.250	53.55
1.89	860	1625	16.1	9.335	60.35	55.32	6787.000	82.30
0.696	269	1905	5.41	15.615	86.35	60.73	11583.820	107.50
−0.766	−308	236	−5.78	23.815	109.95	54.95	15634.000	125.10
−1.992	−1012	2020	−19.0	37.615	133.75	35.95	20611.000	143.40
−2.00	−768.0	1536	−14.4	51.465	140.00	21.55	26898.606	164.00
−0.17	−88.1	15.0	−.65	75.265	150.00	20.90	34540.000	186.00
2.167	785.0	1700	14.72	95.165	181.20	35.60	43230.000	208.00
	1615.9	10737						

$Sa = 175$ cm/Sec^2

$$Cr = \frac{1615.9}{10737} = 0.15$$

TABLE

STOREY SHEAR CALCULATIONS FOR FRAMES IN

Floor No.	First Mode T=2.42 Sec.					Second Mode T=0.965 Sec.			
	W_1	ϕ_{11}	$W_1\phi_{11}$	$W_1\phi_{12}$	Q_{11}	ϕ_{12}	$W_1\phi_{12}$	$W_1\phi_{12}^2$	Q_{12}
1.	433	1.00	433	433	1.405	1.00	433	433	4.93
2.	333	2.176	724	1574	2.35	2.063	685	1415	7.79
3.	363	3.04	1102	3350	3.58	2.707	1005	2720	11.41
4.	455	3.67	1670	6130	5.43	3.046	1385	4220	15.72
5.	431	5.35	2304	12340	7.48	3.61	1560	5610	17.72
6.	401	7.15	2870	20500	9.31	3.691	1490	5450	16.80
7.	509	9.023	4600	41400	14.95	3.10	1580	4880	17.95
8.	192	11.74	2250	26400	7.30	1.00	192	192	2.18
9.	192	12.53	2402	30100	7.80	0.214	41	8.78	0.465
10.	518	14.59	7560	11000	24.62	−2.17	−1125	2440	−12.8
11.	362	16.59	5850	97000	19.02	−4.92	−1780	8750	−22.02
			31765	349327			54560	36118.78	

Sa=50 Cm/Sec²

$$Cr=\frac{31765}{349327}=0.091$$

Sa=105 Cm/Sec²

$$Cr=\frac{54560}{361188}=0.1515$$

II

LONGER DIRECTION—DAMPING—5% OF CRITICAL

Third Mode T=0.556				V_{i1}	V_{i2}	V_{i3}	$\sum_{j=1}^{3} V_{ij}^2$	$\sqrt{\sum_{j=1}^{3} V_{ij}^2}$
ϕ_{i3}	$W_i\phi_{i3}$	$W_i\phi_{i3}^2$	Q_{i3}					
1.00	433	433	5.45	1.405	4.93	5.45	56.125	7.50
1.91	635	1215	7.97	3.755	12.72	13.42	353.60	18.80
2.258	820	1850	10.30	7.335	24.13	23.72	1197.40	34.60
2.22	1000	2220	12.6	12.765	39.85	36.32	3073.00	55.40
1.275	550	700	6.91	20.245	57.57	43.23	5606.00	74.90
−0.30	−120	36	−1.51	29.555	74.37	41.72	8129.00	90.10
−1.92	−976	1875	−12.25	44.505	92.32	29.47	11380.00	106.50
−2.92	−560	1638	−7.05	51.805	94.50	22.42	12132.50	110.10
−2.74	−527	1012	−6.13	59.605	94.965	16.29	12815.00	113.10
−1.198	−620	740	−7.80	84.225	82.165	8.49	13922.10	118.00
3.49	1265	4410	15.9	103.245	60.145	24.39	14369.40	119.80
	1900	16129						

$Sa = 150\ cm/Sec^2$

$$Cr = \frac{1900}{16129} = 0.1175$$

The storey shear force diagram calculated for the tower along longitudinal and transverse directions are plotted in figure 6.

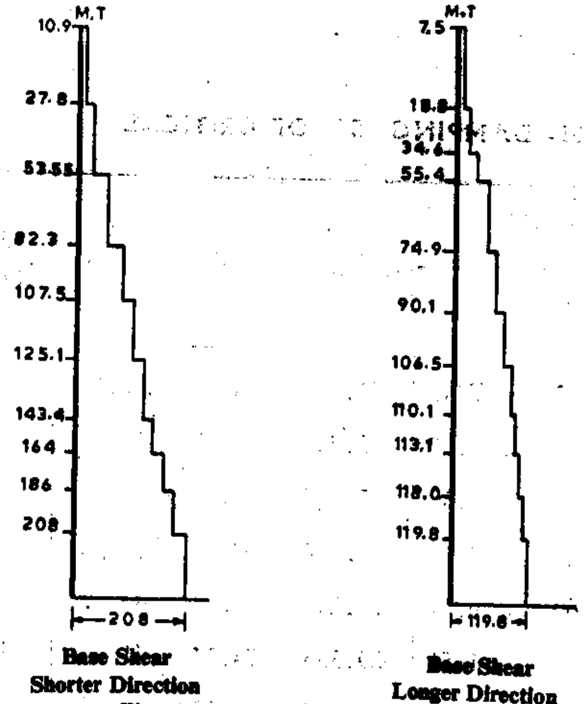


Fig. 6. Shear distribution diagram

It has been observed that the contribution of second and third mode to the total response is of great significance and, thus, proved the importance of dynamic analysis of the structure.

DETERMINATION OF LATERAL DEFLECTIONS OF THE TOWER STRUCTURE

Knowing the lateral loads due to an earthquake input at each storey level, the lateral displacement vector of each storey level were computed by performing the simple matrix operation:

$$\{P_D\}=[K]_L\{D\}$$

or

$$\{D\}=\left[K\right]_L^{-1}\{P_D\} \quad \ldots(9)$$

DETERMINATION OF JOINT MOMENT AND SHEAR FORCE FOR EACH MEMBER

For the design of the structure, it is necessary that moments and shear forces acting at the ends of each member should be known. Knowing the storey level displacement component vector {D}, which will be same for all members participating in the particular direction of analysis, the vector {R}, for each frame can be computed from equation 10 given below:

$$\{R\}=-[K_{22}]^{-1}[K_{21}]\{D\} \quad \ldots(10)$$

Where all the above matrices pertain to a frame under consideration. Now, the joint forces for each member can be determined in terms of global coordinates by using the following relationship:

$$\{P\}_m = [K]_m \{D\}_m \qquad \ldots(11)$$

Where $\{P\}_m$, $[K]_m$, and $\{D\}_m$ define the load vector, stiffness matrix and the displacement vector for the member. It will be seen from the results that moment equilibrium condition will be satisfied at each joint independently. The output from the computer was so arranged that the final results were nicely tabulated for the convenience of design engineer.

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APPENDIX

DYNAMIC ANALYSIS WHEN FLOOR ROTATIONS ARE INCLUDED

In order to make the dynamic analysis of the tower self sufficient, a brief procedure for carrying out the dynamic analysis of a building which is not symmetric in plan is discussed below. In such structures the centre of mass does not coincide with centre of rigidity or twist at various floor levels.

When the motion of the structure is considered along y direction, (Fig. 1a) the earthquake forces $[M]\ddot{y}s$ pass through the centre of rigidity and act along one of the principle axis of rigidity. Hence, the motion in the direction of Y axis is not accompanied by rotation of the floor.

But while considering the motion of structure in the longer direction, it can be seen that the force $[M]\ddot{x}s$ due to ground acceleration will cause not only translation of each floor along x axis but will also produce floor rotations. Each floor will have two degrees of freedom, that is, the displacement xr, m and rotation θr, m; where xr, m and θr, m define the horizontal displacement along x axis and rotation of the centre of mass of rth floor of the structure. If the structure has n storeys then it will have 2n degrees of freedom

Let $[\bar{K}i]_L$ define the reduced stiffness matrix of an ith frame relating horizontal storey force with the storey level displacement, which is obtained as described in the paper.

{xm} and {θm} are the column vectors of translation and rotation of the centre of mass of various floor levels. Let {Li} define the column vector of the distance of ith frame from the centre of mass of each floor level. In the present structure as the centre of mass of all floors lie along same vertical line, Li is same for all floors.

The equation of motion of the tower structure for free vibrations in the x direction can be written in matrix from using D'Alembert's principle as

$$\left[{}^{\nwarrow}M_{\searrow}\right]\left\{\ddot{x}_m\right\} + \sum_{i=1}^{s}\left[\bar{K}_i\right]_L\left\{x_i\right\} \qquad (A_1)$$

where $\{x_i\}=\{x_m\}+L_i\{\theta_m\}$ **(A_2)**

and S=Number of frames parallel to x direction

$$\left[{}^{\nwarrow}M_{\searrow}\right]\left\{\ddot{x}_m\right\}+\sum_{i=1}^{s}\left[\bar{K}_i\right]_L\left[\left\{x_m\right\}+L_i\left\{\theta_m\right\}\right]=0$$

or

$$\left[{}^{\nwarrow}M_{\searrow}\right]\left\{\ddot{x}_m\right\}+\sum_{i=1}^{s}\left[\bar{K}_i\right]_L\left\{x_m\right\}+\sum_{i=1}^{s}L_i\left[\bar{K}_i\right]_L\left\{\theta_m\right\}=0 \qquad (A_3)$$

and

$$\left[{}^{\nwarrow}I_{m\searrow}\right]\left\{\ddot{\theta}_m\right\}+\sum_{i=1}^{s}\left[\bar{K}_i\right]_L\left\{x_i\right\}L_i+\sum_{i=1}^{s}\left[T_i\right]\left\{\theta_m\right\}=0 \qquad (A_4)$$

where [Ti] is the matrix of torsional regidity of an i th frame. For a beam column frame,

$$\sum_{i=1}^{s}[T_i]\{\theta_m\}$$

is normally small and can be ignored.

$\left[{}^{\nwarrow}I_{m\searrow}\right]$ is the diagonal matrix of mass moment of inertias of various floors about their centre of mass.

Substituting eqn. A2 in A4, we get

$$\left[{}^{\nwarrow}I_{m\searrow}\right]\left\{\ddot{\theta}_m\right\}+\sum_{i=1}^{s}\left[\bar{K}_i\right]_L\left[\left\{x_m\right\}+L_i\left\{\theta_m\right\}\right].L_i=0$$

or

$$\left[{}^{\nwarrow}I_{m\searrow}\right]\left\{\ddot{\theta}_m\right\}+\sum_{i=1}^{s}L_i\left[\bar{K}_i\right]_L\left\{x_m\right\}+\sum_{i=1}^{s}L_i^2\left[\bar{K}_i\right]_L\left\{\theta_m\right\}=0 \qquad (A_5)$$

Equations A3 and A5 can be combined into a single matrix equation as given below

$$\begin{bmatrix}\left[{}^{\nwarrow}M_{\searrow}\right] & \\ & \left[{}^{\nwarrow}I_{m\searrow}\right]\end{bmatrix}_{2n\times 2n}\begin{Bmatrix}\left\{\ddot{x}_m\right\}\\ \left\{\ddot{\theta}_m\right\}\end{Bmatrix}_{2n\times 1}+\begin{bmatrix}\sum_{i=1}^{s}[\bar{K}_i]_L & \sum_{i=1}^{s}L_i[\bar{K}_i]_L\\ \sum_{i=1}^{s}L_i[\bar{K}_i]_L & \sum_{i=1}^{s}L_i^2[\bar{K}_i]_L\end{bmatrix}_{2n\times 2n}\begin{Bmatrix}\left\{x_m\right\}\\ \left\{\theta_m\right\}\end{Bmatrix}_{2n\times 1}=\begin{Bmatrix}\{0\}\\ \{0\}\end{Bmatrix}_{2n\times 1} \quad \ldots (A6)$$

When this structure is subjected to ground accelerations $\ddot{X}_S$ in the x direction the equation of motion A_6 takes the form

$$\left[{}^{\nwarrow}\bar{M}_{\searrow}\right]_{2n\times 2n}\left\{\ddot{Z}\right\}_{2n\times 1}+\left[K\right]_{2n\times 2n}\left\{Z\right\}_{2n\times 1}=-\ddot{X}_S\left[{}^{\nwarrow}\bar{M}_{\searrow}\right]_{2n\times 2n}\left\{I\right\}_{2n\times 1}$$

Where {I} is a vector which contains unity for first n terms corresponding to floor translations and zeros for remaining terms corresponding to floor rotations and,

$$Z_r = x_{r,m} - X_S \quad \text{for } r = 1, \ldots\ldots n$$

$$Z_r = \theta_{r,m} \quad \text{for } r = n+1, \ldots\ldots 2n$$

where X_S = ground displacement

Again, using normal mode theory, the dynamic analysis of the tower structure can be made for a given response spectra. The response obtained will yield the maximum values of displacement of the centre of mass and rotation of each floor level X_r^n, m and θ_r^n, m in any n th mode. The maximum displacements for a number of different modes would be likely to occur at different times and from probability studies it is generally accepted that a root mean square superposition of modes will yield realistic values, that is

$$x_{r,m}\,(\max{}^m) \simeq \sqrt{\sum \left(x^n_{r,m}\,(\max{}^m) \right)^2}$$

and

$$\theta_{r,}\,(\max{}^m) \approx \sqrt{\sum \left(\theta^n_{r,m}\,(\max{}^m) \right)^2}$$

The maximum probable deflection of an i th frame at a distance of Li from the centre of mass would be

$$\{x_i\}_{\max}\, m = \{x_m\}_{\max}\, m + L_i\, \{\theta_m\}_{\max}\, m \qquad \ldots (A7)$$

When the lateral displacements at each storey level of a frame are known, the joint moment and shear force for each member in a frame can be determined using equations 10 and 11 as discussed in the paper. The main of effect including floor rotations into analysis is to increase storey force in some frames and decrease in others depending on the location of the centre of mass.